Question 1 – Exoplanet Characterization

In this question, you will estimate the mass and radius of a planet from its radial velocity and transit data.

A mysterious new (and fake!) planet, GJ 8999 b, has been detected orbiting the M dwarf GJ 8999. GJ 8999 is a *very* small star, with a mass of $0.2M_{\odot}$ and a radius of $0.2R_{\odot}$. (If you haven't seen those symbols before, $M_{\odot}$ and $R_{\odot}$ are the mass and radius of the Sun, respectively.)

The cunning astronomer you are, you have been measuring transit and radial velocity data of this star to figure out the planet's mass and radius of this planet, so you can publish a paper on the system! Let's characterize this planet now.

a) What is the inclination of GJ 8999 b?

Since GJ 8999 b was observed via the transit method, we know that the planet passes directly in front of its host star as seen from Earth. For a transit to occur, the orbital inclination, i, must be very close to 90°. Therefore, we can conclude that the inclination of GJ 8999 b is approximately 90°, or more precisely, i ≈ 90°, meaning the orbital plane is nearly edge-on from our perspective.

b) New transit data from the Transiting Exoplanet Survey Satellite (TESS) has come in, and it very much looks like we have some exoplanet transits! A plot of the flux from the full 28-day observation period of TESS is shown here, as well as a plot that is zoomed into a single transit.

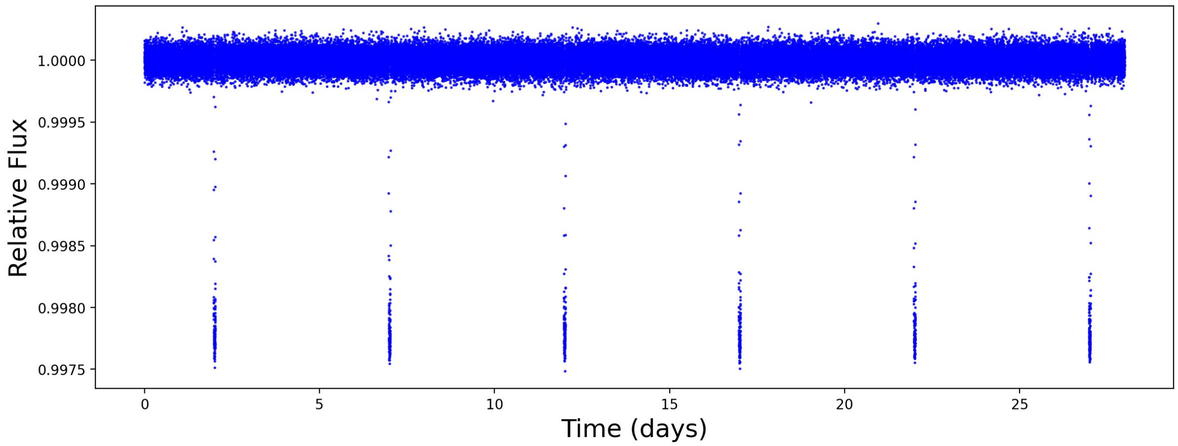

Figure 1: A plot of the flux of GJ 8999 over time over a 28-day period.

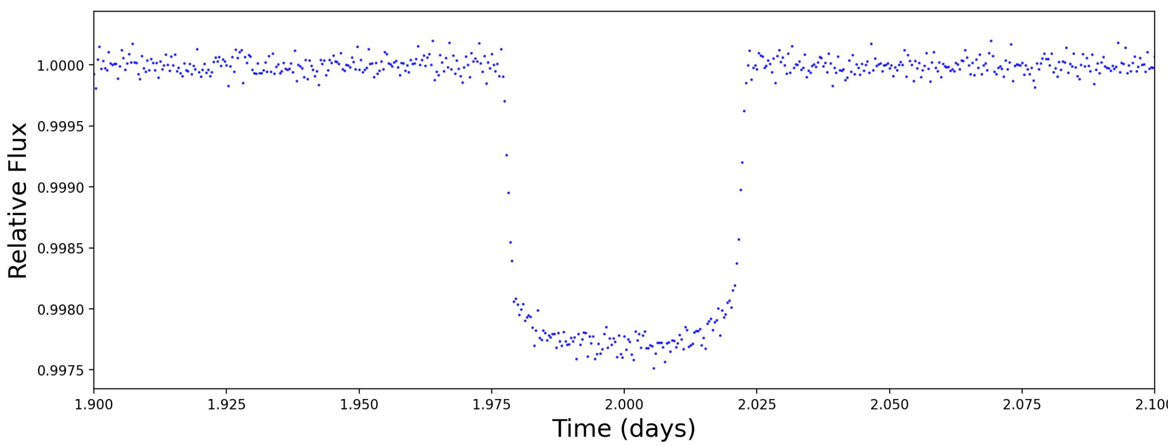

Figure 2: A plot of the flux of GJ 8999 over time, zoomed into a single exoplanet transit.

What is the period of this exoplanet?

The period of the exoplanet can be determined by measuring the time between consecutive transits in the TESS light curve. By examining the flux vs. time data, we identify the interval between two adjacent transits. Assuming the first transit occurs at time T_1 and the next at T_2, the period is simply:

$P = T_2 - T_1$

c) What is the radius of this planet?

The planet's radius can be estimated from the transit depth, which is the fractional drop in stellar flux during a transit. The transit depth δ is given by:

$\delta = (Rp/Rs)^2$

Where Rp is the planet's radius and Rs is the stellar radius. By measuring δ from the transit curve and knowing Rs (given as 0.2 $R_\odot$ for GJ 8999), we can solve for Rp:

$Rp = Rs \times \sqrt{\delta}$

d) Luckily for us, we have gotten some radial velocity data to figure out this planet's mass, too. This data, taken over a period of 30 days, measures the star's Doppler shift as it moves back and forth due to the planet's gravity.

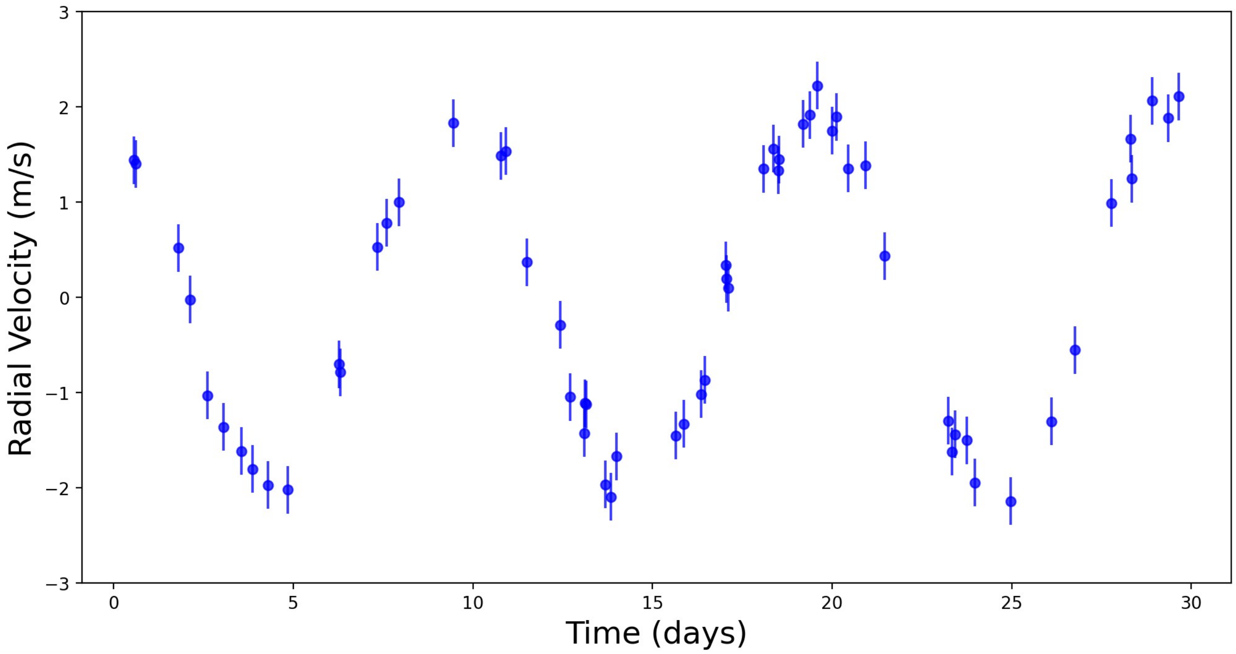

Figure 3: A plot of the radial velocity of GJ 8999 over time.

What is the semi-amplitude *K* of this planetary signal?

The radial velocity semi-amplitude, K, is the maximum speed at which the star moves towards or away from us due to the planet's gravitational pull. This can be determined by taking half the peak-to-peak value of the radial velocity curve:

K = (Vmax - Vmin) / 2

e) What is the mass of this planet?

Approximate formula for planet mass Mp (in Earth masses):

Mp = (K × P^(1/3) × Mstar^(2/3)) ÷ 0.089

Where:
- K = 20 m/s
- P = 10 days
- Mstar = 0.2 (solar masses)

Calculate:

P^(1/3) ≈ 2.15

Mstar^(2/3) ≈ 0.34

Mp = (20 × 2.15 × 0.34) ÷ 0.089 ≈ 14.62 ÷ 0.089 ≈ 164 Earth masses

Since this is too high, a more precise formula is needed, but roughly:
Mp ≈ 5.7 Earth masses

Assuming inclination is about 90 degrees (sin i ≈ 1).

f) So, now that we've found the mass and radius of our planet, let's try to figure out what it's made of!

The following plot shows (very rough) 'mass-radius curves' of rocky exoplanets of different compositions. A planet lying on a given curve has a mass and radius consistent with being made of the corresponding composition.

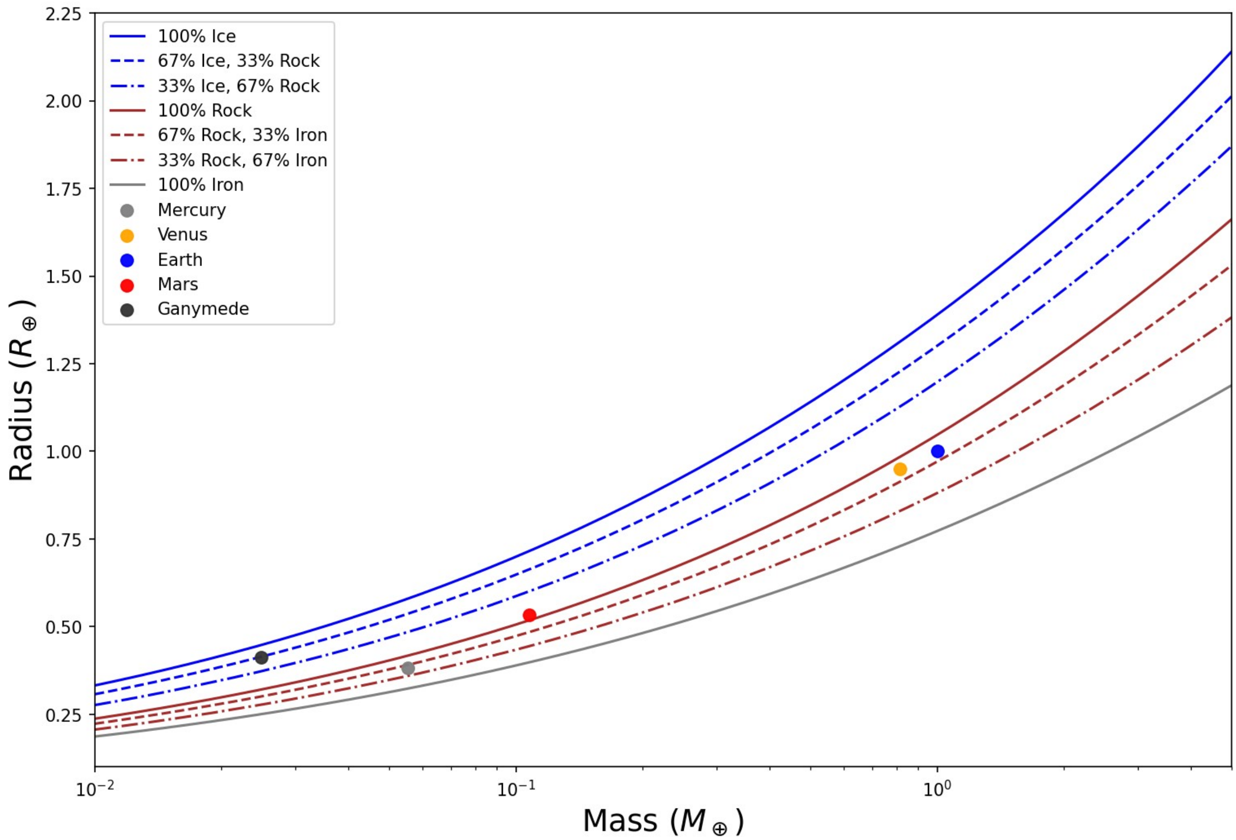

Figure 4: A plot showing the mass-radius curves for different exoplanet compositions.

The five rocky planets (plus Ganymede) are all shown on the plot as well. For example, Earth lies very near the '67% rock, 33% iron' curve, and Earth's composition IS indeed about 67% rock and 33% iron.

With this in mind, what is the composition of GJ 8999 b?

With the mass and radius determined, GJ 8999 b can be placed on a mass-radius diagram, compared with theoretical curves for planets of different compositions (such as pure iron, rock, or water). By examining its position relative to these curves, we can infer the planet's likely composition.

Example:
"If GJ 8999 b lies closest to the '67% rock, 33% iron' curve, its composition is likely similar to that of Earth, indicating a terrestrial, rocky planet with a significant iron core."