Assignment4 Exercise1

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Report on Distributions:

The distributions $p(\sigma^2|\alpha, x_{1:T})$ and $p(\beta^2|x_{1:T}, y_{1:T})$ represent the posterior distributions of the parameters σ^2 and β^2 given the observed data $y_{1:T}$ and other model parameters. These distributions are obtained using the Particle Gibbs (PG) sampler after discarding burn-in samples.

Trace Plots and Histograms:

Trace plots provide a visual representation of how the samples of the parameters evolve over iterations, indicating convergence. The trace plots for σ^2 and β^2 should stabilize after burn-in, showing convergence.

Histograms visualize the posterior distributions of σ^2 and β^2 by aggregating samples. The shape of the histograms provides insights into the uncertainty and variability in the estimates.

Approximate Marginal Likelihood:

The marginal likelihood, $p(y_{1:T}|\theta)$, represents the probability of observing the data $y_{1:T}$ given the model parameters θ. In the context of the PG sampler, the marginal likelihood is approximated by averaging over MCMC iterations:

$$p(y_{1:T}) \approx \prod_{i=1}^{T} \sum_{k=1}^{K} p(y_i|x_{k_i}, \theta)$$

This is an approximation because it involves integrating over the latent variables $x_{1:T}$, which are estimated using the PG sampler. The sum over k represents the Monte Carlo approximation.

Difference between $p(y_{1:T})$ and $p(y_{1:T}|\theta)$:

$p(y_{1:T})$ is the marginal likelihood, representing the overall probability of observing the data without conditioning on specific values of model parameters. It integrates over all possible parameter values.

$p(y_{1:T}|\theta)$ is the conditional likelihood, representing the probability of observing the data given specific values of the model parameters θ. It considers a fixed set of parameter values.

In summary, the marginal likelihood is a measure of how well the model explains the observed data, integrating over all possible parameter values. The PG sampler provides an efficient way to approximate this integral by sampling from the posterior distribution.