1st Computer Assignment

Consider the linear system of the form $A_{nn}x = b$ where the matrix A_{nn} has the form

$$\begin{pmatrix} 6I_n + B_n & -4I_n & I_n & \cdots & 0 \\ -4I_n & 6I_n + B_n & -4I_n & \ddots & \vdots \\ I_n & \ddots & \ddots & \ddots & I_n \\ \vdots & \ddots & -4I_n & 6I_n + B_n & -4I_n \\ 0 & \cdots & I_n & -4I_n & 6I_n + B_n \end{pmatrix}$$

where B_n the $n \times n$ symmetric 5-diagonal matrix

$$\begin{pmatrix} 6 & -4 & 1 & \cdots & 0 \\ -4 & 6 & -4 & \ddots & \vdots \\ 1 & \ddots & \ddots & \ddots & 1 \\ \vdots & \ddots & -4 & 6 & -4 \\ 0 & \cdots & 1 & -4 & 6 \end{pmatrix}$$

I_n the $n \times n$ identity matrix, b the n^2 vector with all of its components equal to 1.

1. Solve fast and efficiently from the point of view of flops, memory and accuracy the above system for $n = 32, 64, 128$ using any direct method (and a variance of its) you prefer. How the necessary execution time increases in relation to the dimension n ? How the accuracy behaves in relation to the dimension ?

2. Solve the above system using SOR efficiently developed for the structure of the system. Use as tolerance $\tau = 10^{-8}$ and run your program for $\omega = 1 : 0.05 : 1.95$. Estimate numerically the ω_{opt}. From the numbers of iterations, conclude which is better: To over estimate or to under estimate ω_{opt}.

Remarks:

a) Observe that the matrix A can be written as a sum of Kronecker products. Take advantage of this analysis for task 2.

b) The above system appears in real applications in the numerical solution of Poisson and Laplace equations. For example they model the steady-state distribution of heat on a plane region whose boundary is being held at specific temperatures. In addition many instances of potential energy (electrostatic, gravitational) are modeled by the Poisson equation. Finally, the aerodynamics of airfoils at low speeds, known as incompressible irrotational flow, are a solution of the Laplace equation.

c) More generally, block tridiagonal, block upper Hessemberg or matrices arise in real application in many problems. For example modeling the buffer of a telecommunication system using Markov chain lead to such kind of matrices.

d) You can use any function you want and it is provided in library SciPy, NumPy or MatLab. Take into account that contrary to Matlab the monitor of flops, and memory is used by the program is not so accurate. The file that you will upload it must contain the source code with -many- comments, and a document file containing comments and remarks and figures on the results.