# Advanced Engineering Mathematics

## Mathematical Techniques for Engineering

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  $L\left\{e^{at}f\left(t\right)\right\}=F\left(s-a\right)$
  - equivalently: $L\left\{e^{at}f\left(t\right)\right\}=F\left(s\right)\left.\right|_{\left(s-a\right)}$ i.e. $F\left(s\right)$ at $\left(s-a\right)$

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  - equivalently: $e^{at}f(t) = L^{-1}\{F(s-a)\}$

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$$L\left(\cosh at\right) = \frac{1}{2}\left(L\left(e^{at}\right) + L\left(e^{-at}\right)\right)$$

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&= \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right) = \frac{s}{s^2 - a^2} \\
L\left(\sinh at\right) &= \frac{1}{2}\left(L\left(e^{at}\right) - L\left(e^{-at}\right)\right) \\
&= \frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right) = \frac{a}{s^2 - a^2} \quad \blacksquare
\end{aligned}$$

# Q (9ed-6.1-29)

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&= 4L^{-1}\left(\frac{s}{s^2+\pi^2}\right) - 3L^{-1}\left(\frac{\pi}{s^2+\pi^2}\right) \\
&= 4\left(\cos \pi t\right) - 3\left(\sin \pi t\right) \quad \blacksquare
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&= 5\left[\frac{\omega}{s^2+\omega^2}\right]_{s+a} \\
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Hence

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$$e^{-t}\left(a_0+a_1t+\ldots+a_nt^n\right)=e^{-t}\left(\sum_{k=0}^{n}a_kt^k\right)=\sum_{k=0}^{n}a_kt^ke^{-t}.$$

Hence

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& L\left(e^{-t}\left(a_0+a_1t+\ldots+a_nt^n\right)\right) \\
= & L\left(\sum_{k=0}^{n}a_kt^ke^{-t}\right)=\sum_{k=0}^{n}a_kL\left(t^ke^{-t}\right) \quad \text{using linearity} \\
= & \sum_{k=0}^{n}a_k\left[L\left(t^k\right)\right]_{s-(-1)}=\sum_{k=0}^{n}a_k\left[\frac{k!}{s^{k+1}}\right]_{s+1} \quad \text{using}
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= \; & \sum_{k=0}^{n} a_k \left[L\left(t^k\right)\right]_{s-(-1)} = \sum_{k=0}^{n} a_k \left[\frac{k!}{s^{k+1}}\right]_{s+1} \quad \text{using } L\left(t^n\right) = \frac{n!}{s^{n+1}} \\
= \; & \sum_{k=0}^{n} a_k \left(\frac{k!}{(s+1)^{k+1}}\right) \quad \blacksquare
\end{aligned}$$

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&= \frac{3s-137}{s^2+2s+1+400} \\
&= \frac{3s-137}{(s+1)^2+(20)^2} \\
&= \frac{3s+3-3-137}{(s+1)^2+(20)^2} = \frac{3(s+1)-140}{(s+1)^2+(20)^2} \\
&= 3\frac{s-(-1)}{(s-(-1))^2+(20)^2} - 7\frac{20}{(s-(-1))^2+(20)^2}
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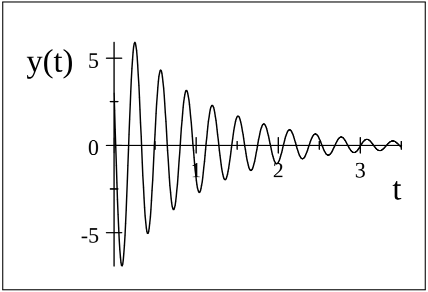
= \quad & 3\frac{s-(-1)}{(s-(-1))^2+(20)^2} - 7\frac{20}{(s-(-1))^2+(20)^2} \\
= \quad & 3e^{-t}\cos(20t) - 7e^{-t}\sin(20t) \\
= \quad & e^{-t}(3\cos(20t) - 7\sin(20t))
\end{aligned}
$$

The damped osciliator found in this example is plotted as




$$
\begin{aligned}
L^{-1}\left(\frac{3\,s-137}{s^2+2\,s+401}\right) &= y\left(t\right) \\
&= e^{-t}\left(3\cos\left(20t\right)-7\sin\left(20t\right)\right) \quad \blacksquare
\end{aligned}
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Suppose $a = -\sqrt{2}$ in $e^{at}$, so tentatively we take $F\left(s\right) = \frac{1}{s^3}$ and $a = -\sqrt{2}$.

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Suppose $a=-\sqrt{2}$ in $e^{at}$, so tentatively we take $F\left(s\right)=\frac{1}{s^3}$ and $a=-\sqrt{2}$. Thus in first shifting theorem ($e^{at}f\left(t\right)=L^{-1}\left\{F\left(s-a\right)\right\}$) we set

$f\left(t\right)=L^{-1}\left(\frac{\sqrt{8}}{s^3}\right)=\sqrt{2}L^{-1}\left(\frac{2}{s^3}\right)=\sqrt{2}t^2$

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In the Inverse Laplace Table, we look for the nearest similar entry to $\frac{\sqrt{8}}{\left(s+\sqrt{2}\right)^3}$. This is $\frac{2}{s^3}$. Next we guess an application of s-shifting i.e. $L\left\{e^{at}f(t)\right\} = F(s-a)$.

Suppose $a = -\sqrt{2}$ in $e^{at}$, so tentatively we take $F(s) = \frac{1}{s^3}$ and $a = -\sqrt{2}$. Thus in first shifting theorem $\left(e^{at}f(t) = L^{-1}\left\{F(s-a)\right\}\right)$ we set

$f(t) = L^{-1}\left(\frac{\sqrt{8}}{s^3}\right) = \sqrt{2}L^{-1}\left(\frac{2}{s^3}\right) = \sqrt{2}t^2$

$$\Rightarrow e^{at}f(t) = e^{-\sqrt{2}t}\left(\sqrt{2}t^2\right) = \sqrt{2}t^2e^{-\sqrt{2}t} \qquad \blacksquare$$

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$$
\begin{aligned}
& L^{-1}\left(\frac{a(s+k)+b\pi}{(s+k)^2+\pi^2}\right) \\
= \;& ae^{-kt}L^{-1}\left(\frac{s}{s^2+\pi^2}\right) + be^{-kt}L^{-1}\left(\frac{\pi}{s^2+\pi^2}\right) \\
= \;& ae^{-kt}\cos(\pi t) + be^{-kt}\sin(\pi t) \\
= \;& e^{-kt}\left(a\cos(\pi t) + b\sin(\pi t)\right) \quad \blacksquare
\end{aligned}
$$