

# Advanced Engineering Mathematics

## Mathematical Techniques for Engineering

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# Definition: Heaviside Unit Step Function

The unit step function or Heaviside function  $u(t - a)$  is defined as:

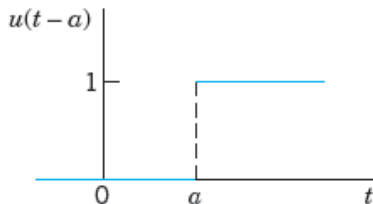
$$u(t - a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t > a \end{cases} \quad ; \quad a \geq 0$$

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Its graph is given as

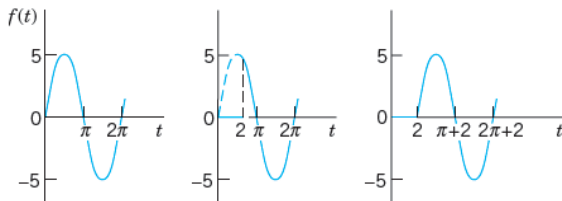


# Heaviside Unit Step Function: Explained

Let  $f(t) = 0$  for all negative  $t$ . Then  $f(t - a) u(t - a)$  with  $a > 0$  is  $f(t)$  shifted (translated) to the right by the amount  $a$ .

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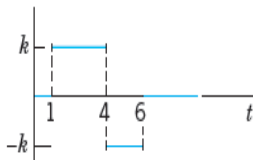
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(A)  $f(t) = 5 \sin t$ (B)  $f(t)u(t-2)$ (C)  $f(t-2)u(t-2)$ 

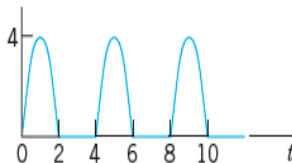
Effects of the unit step function: (A) Given function.

(B) Switching off and on. (C) Shift.

# Heaviside Unit Step Function: Explained



(A)  $k[u(t-1) - 2u(t-4) + u(t-6)]$



(B)  $4 \sin\left(\frac{1}{2}\pi t\right)[u(t) - u(t-2) + u(t-4) - \dots]$

Use of many unit step functions.

# Laplace Transform of Unit Step Function

LT of Heaviside unit step function may be found as

$$\begin{aligned}
 L(u(t-a)) &= \int_0^{\infty} e^{-st} u(t-a) dt \\
 &= \int_0^{\infty} e^{-st} (1) dt \quad \text{since } a \geq 0 \\
 &= \left[ \frac{e^{-st}}{s} \right]_{t=a}^{\infty} \\
 &= \frac{e^{-as}}{s} \quad \blacksquare
 \end{aligned}$$

## Second Shifting Theorem; t-Shifting

**Theorem:** (Second Shifting Theorem; t-Shifting) If  $f(t)$  has the transform  $F(s)$ , then the 'shifted function'

$$\tilde{f}(t) = f(t - a) u(t - a)$$



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$$L(f(t-a) u(t-a)) = e^{-as} F(s)$$

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**Remark:** Practically the t-shifting theorem amounts to this: if we know  $F(s)$ , we can obtain the transform of  $f(t-a) u(t-a)$  by just multiplying  $F(s)$  by  $e^{-as}$ .

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If the conversion of  $f(t)$  to  $f(t-a)$  is difficult, we may use following form as well:

$$L(f(t) u(t-a)) = e^{-as} L(f(t+a))$$

**Question:** Find LT of the piecewise function

$$f(t) = \begin{cases} 2 & \text{if } 0 < t < 1 \\ \frac{t^2}{2} & \text{if } 1 < t < \frac{\pi}{2} \\ \cos t & \text{if } t > \frac{\pi}{2} \end{cases}$$

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Applying LT on both sides

$$L(f(t)) = 2L(u(t)) - 2L(u(t-1)) + \frac{1}{2}L(t^2 u(t-1)) - \frac{1}{2}L(t^2 u\left(t - \frac{\pi}{2}\right)) + L(\cos(t) u\left(t - \frac{\pi}{2}\right))$$

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Writing each term in  $f(t)$  in the form  $f(t-a)$ , so that the LT of the form  $f(t-a)u(t-a)$ , of t-shifting theorem, may be applied.

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$$L(f(t)) = \frac{2}{s} - \frac{2}{s}e^{-s} + \frac{1}{2} \frac{(s^2 + 2s + 2)}{s^3} e^{-s} -$$

$$\frac{1}{8} \left( \frac{\pi^2}{s} + \frac{4\pi}{s^2} + \frac{8}{s^3} \right) e^{-\frac{\pi}{2}s} - \frac{1}{s^2 + 1} e^{-\frac{\pi}{2}s} \quad \blacksquare$$

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$$= \frac{1}{s^2-1} - \cosh(2) \left( \frac{1}{s^2-1} \right) e^{-2s} + \sinh(2) \left( \frac{s}{s^2-1} \right) e^{-2s}$$

$$= \frac{1}{s^2-1} - \left( \frac{\sinh(2)s + \cosh(2)}{s^2-1} \right) e^{-2s} \quad \blacksquare$$

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$$\begin{aligned} L^{-1} \left( \frac{4(e^{-2s} - 2e^{-5s})}{s} \right) &= L^{-1} \left( \frac{1}{s} (4e^{-2s} - 8e^{-5s}) \right) \\ &= 4L^{-1} \left( \frac{1}{s} e^{-2s} \right) - 8L^{-1} \left( \frac{1}{s} e^{-5s} \right) \end{aligned}$$



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Though not required, but this would be beneficial if student transforms above  $f(t)$  into a piecewise representation. For this  $f(t)$  it is given as

$$f(t) = 4u(t-2) - 8u(t-5) = \begin{cases} 4 & \text{if } 2 < t < 5 \\ -4 & \text{if } t > 5 \end{cases}$$

**Question:** (DIY, hints are given) Writing the Heaviside form of

$$f(t) = \begin{cases} \frac{1}{2}t & \text{if } 0 < t < \frac{1}{2} \\ 3t - 2 & \text{if } \frac{1}{2} < t < \frac{\pi}{2} \\ e^t & \text{if } t > \frac{\pi}{2} \end{cases}$$

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$$\underbrace{3t u\left(t - \frac{\pi}{2}\right) + 2u\left(t - \frac{\pi}{2}\right) - 2u\left(t - \frac{1}{2}\right)}_{\text{Second piece}} + \underbrace{e^t u\left(t - \frac{\pi}{2}\right)}_{\text{Third piece}}$$

**Question:** (DIY, hints are given) Writing the Heaviside form of

$$f(t) = \begin{cases} \frac{1}{2}t & \text{if } 0 < t < \frac{1}{2} \\ 3t - 2 & \text{if } \frac{1}{2} < t < \frac{\pi}{2} \\ e^t & \text{if } t > \frac{\pi}{2} \end{cases}$$

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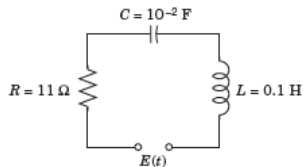
$$\underbrace{3t u\left(t - \frac{\pi}{2}\right) + 2 u\left(t - \frac{\pi}{2}\right) - 2 u\left(t - \frac{1}{2}\right)}_{\text{Second piece}} + \underbrace{e^t u\left(t - \frac{\pi}{2}\right)}_{\text{Third piece}}$$

Applying t-shifting theorem to obtain the LT as

$$L(f(t)) = \frac{1}{2} \frac{1}{s^2} + \frac{5}{4} \frac{(s+2)}{s^2} e^{-\frac{1}{2}s} - \frac{3}{2} \left( \frac{2}{s^2} + \frac{\pi}{s} \right) e^{-\frac{\pi}{2}s} + \frac{2}{s} e^{-\frac{\pi}{2}s} - \frac{2}{s} e^{-\frac{1}{2}s} + \frac{e^{-\frac{\pi}{2}(s-1)}}{s-1}$$



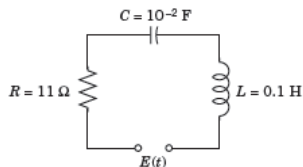
**Question:** Find the response (the current) of the RLC-circuit given in figure, where  $E(t)$  is sinusoidal,



acting for a short time interval only, say

$E(t) = 100 \sin(400t)$  if  $0 < t < 2\pi$  and  $E(t) = 0$  if  $t > 2\pi$ .

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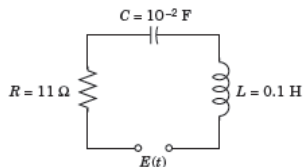
**Solution:** The forcing function

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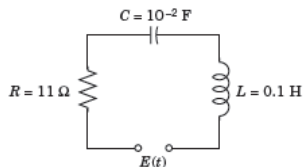
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Model for current  $i(t)$  is the integro-differential equation

$$0.1 \frac{d}{dt} i(t) + 11 i(t) + 100 \int_0^t i(\tau) d\tau = -100 \sin(400t) u(t - 2\pi) + 100 \sin(400t) u(t) \quad ; \quad i(0) = i'(0) = 0$$

$$0.1sY - 0.1i(0) + 11Y + 100\frac{1}{s}Y = \frac{40000}{(s^2 + 160000)} (1 - e^{-2\pi s})$$

$$0.1sY - 0.1i(0) + 11Y + 100\frac{1}{s}Y = \frac{40000}{(s^2+160000)} (1 - e^{-2\pi s})$$

$$Y = -\frac{(-i(0)s^2+400000 e^{-2\pi s}-160000 i(0)-400000)s}{(s^2+160000)(s^2+110 s+1000)}$$

$$Y = -\frac{(-400000+400000 e^{-2\pi s})s}{(s^2+160000)(s^2+110 s+1000)} = \frac{400000 s-400000 s e^{-2\pi s}}{(s^2+160000)(s^2+110 s+1000)}$$

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By partial fractions of first term,  $Y =$

$$\left( \frac{400}{153(s+100)} - \frac{4000}{14409(s+10)} - \frac{\frac{63600}{27217}s - \frac{7040000}{27217}}{s^2+160000} \right) - \left( \frac{400000 s e^{-2\pi s}}{(s^2+160000)(s^2+110s+1000)} \right)$$



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taking Inverse LT

$$i(t) = \left( \frac{400}{153} e^{-100t} - \frac{4000}{14409} e^{-10t} - \left( \frac{63600}{27217} \cos 400t - \frac{17600}{27217} \sin 400t \right) \right) \\ - u(t - 2\pi) \\ \left( \frac{400}{153} e^{200\pi-100t} - \frac{4000}{14409} e^{20\pi-10t} - \frac{63600}{27217} \cos 400t + \frac{17600}{27217} \sin 400t \right) \quad \blacksquare$$

**Question:** (10ed-6.3-23)

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = f(t) = \begin{cases} 3\sin(t) - \cos(t) & \text{if } 0 < t < 2\pi \\ 3\sin(2t) - \cos(2t) & \text{if } t > 2\pi \end{cases} \quad \text{where}$$

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Taking LT on both sides of the given DE

$$s^2 Y - y'(0) - sy(0) + sY - y(0) - 2Y =$$

$$\frac{1}{s^2+1} \left( 3 - s + 3 \frac{e^{-2s\pi}(s+2)(s-1)}{s^2+4} \right)$$

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$$Y = \frac{s^4 - s^3 + 3e^{-2s\pi}s + 6s^2 - 3e^{-2s\pi} - 4s + 8}{(s-1)(s^2+1)(s^2+4)}$$



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$$y(t) = \left\{ \right.$$

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**Question:** (10e-6.3-27) Put  $\tilde{t} = t - 1$  in

$$E(t) = \begin{cases} 8t^2 & 0 < t < 5 \\ 0 & t > 5 \end{cases} \quad \text{and writing it in Heaviside terms:}$$

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$$\frac{d^2}{d\tilde{t}^2}y(\tilde{t}) + 4y(\tilde{t}) = E(\tilde{t}) \quad ; \quad y(0) = 1 + \cos(2), \quad y'(0) = 4 - 2\sin(2)$$

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$$s^2\tilde{Y} - y'(0) - sy(0) + 4\tilde{Y} = 16\frac{1}{s^2} + 8\frac{1}{s} + 8\frac{-e^{-4s}(25s^2+10s+2)+2}{s^3}$$

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$$\tilde{Y} = \frac{1}{s^2 + 4} \left( \frac{16}{s^2} + \frac{8}{s} + 8 \frac{-e^{-4s}(25s^2 + 10s + 2) + 2}{s^3} + 4 - 2 \sin(2) + s(1 + \cos(2)) \right)$$

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$$\tilde{Y} = \frac{-s^4 \cos(2) + 2 \sin(2)s^3 - s^4 + 200e^{-4s}s^2 - 4s^3 + 80e^{-4s}s - 8s^2 + 16e^{-4s} - 16s - 16}{s^3(s^2 + 4)}$$

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$$y(\tilde{t}) = 1 + \cos(2) \cos(2\tilde{t}) - \sin(2) \sin(2\tilde{t}) + 2\tilde{t}^2 + 4\tilde{t} - u(\tilde{t} - 4) \\ (100 (\sin(\tilde{t} - 4))^2 + 2\tilde{t}^2 + \cos(2\tilde{t} - 8) - 10 \sin(2\tilde{t} - 8) + 4\tilde{t} - 49)$$

$$y(\tilde{t}) = \begin{cases} 1 + \cos(2) \cos(2\tilde{t}) - \sin(2) \sin(2\tilde{t}) + 2\tilde{t}^2 + 4\tilde{t} & \tilde{t} < 4 \\ -50 + \cos(2) \cos(2\tilde{t}) - \sin(2) \sin(2\tilde{t}) + 100 (\cos(\tilde{t} - 4))^2 - \cos(2\tilde{t} - 8) + 10 \sin(2\tilde{t} - 8) & \tilde{t} > 4 \end{cases}$$

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putting back  $\tilde{t} = t - 1$ ,  $y(t) =$

$$\begin{cases} -3 + \cos(2) \cos(2t-2) - \sin(2) \sin(2t-2) + 2(t-1)^2 + 4t & t < 5 \\ -50 + \cos(2) \cos(2t-2) - \sin(2) \sin(2t-2) + 100(\cos(t-5))^2 - \cos(2t-10) + 10 \sin(2t-10) & t > 5 \end{cases}$$

$$y(t) = \begin{cases} \cos(2t) + 2t^2 - 1 & \text{if } t < 5 \\ \cos(2t) + 49 \cos(2t-10) + 10 \sin(2t-10) & \text{if } t > 5 \end{cases}$$



**Question:** (10ed-6.3-30)  $0.5 \frac{d}{dt} i(t) + 10i(t) = E(t) =$

$$\begin{cases} 200t & , \quad 0 < t < 2 \\ 0 & , \quad t > 2 \end{cases} ; \quad i(0) = 0$$

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$$Y = -400 \frac{2e^{-2s}s + e^{-2s} - 1}{s^2(s+20)}$$

$$i(t) = -1 + 20t u(2-t) + e^{-20t} + 2 u(t-2) \left( e^{(20-10t)} \sinh(10t-20) + 20 e^{(-20t+40)} \right)$$

$$i(t) = \begin{cases} -1 + e^{-20t} + 20t & , \quad t < 2 \\ -1 + e^{-20t} + 2 e^{(20-10t)} \sinh(10t-20) + 40 e^{(-20t+40)} & , \quad t > 2 \end{cases}$$

**Question:** (10ed-6.3-40)

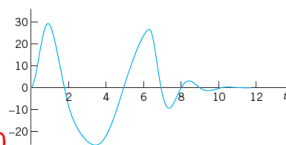
$$\frac{d}{dt}i(t) + 2i(t) + 10 \int_0^t i(\tau) d\tau = E(t) =$$

$$\begin{cases} 255 \sin(t) & 0 < t < 2\pi \\ 0 & t > 2\pi \end{cases} ; \quad i(0) = 0$$

**Question:** (10ed-6.3-40)

$$\frac{d}{dt}i(t) + 2i(t) + 10 \int_0^t i(\tau) d\tau = E(t) =$$

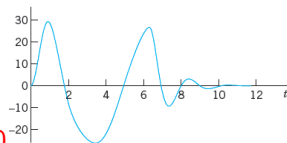
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$$E(t) = 255 \sin(t) u(t) - 255 \sin(t) u(-2\pi + t)$$

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$$255 \sin(t) u(t) - 255 \sin(t) u(-2\pi + t)$$

$$sY - i(0) + 2Y + 10 \frac{Y}{s} = 255 \frac{1 - e^{-2s\pi}}{s^2 + 1}$$

$$Y = 1 \left( 255 \frac{1 - e^{-2s\pi}}{s^2 + 1} + i(0) \right) (s + 2 + 10s^{-1})^{-1}$$

$$Y = 255 \frac{1 - e^{-2s\pi}}{(s^2 + 1) \left( s + 2 + \frac{10}{s} \right)}$$

$$Y = -255 \frac{(-1 + e^{-2s\pi})s}{(s^2 + 1)(s^2 + 2s + 10)}$$

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$$i(t) = 3 u(2\pi - t) (9 \cos(t) + 2 \sin(t)) +$$

$$(e^{2\pi - t} u(-2\pi + t) - e^{-t}) (27 \cos(3t) + 11 \sin(3t))$$

$$i(t) =$$

$$\begin{cases} 27 \cos(t) + 6 \sin(t) - (27 \cos(3t) + 11 \sin(3t)) e^{-t} & t < 2\pi \\ (27 \cos(3t) + 11 \sin(3t))(-e^{-t} + e^{(2\pi - t)}) & t > 2\pi \end{cases}$$

$$Y = 255 \frac{1 - e^{-2s\pi}}{(s^2 + 1) \left(s + 2 + \frac{10}{s}\right)}$$

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On plotting the current  $i(t)$  we have

