Advanced Engineering Mathematics

Mathematical Techniques for Engineering

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Important Properties of Laplace Transform

- Linearity: $L\left\{af\left(t\right)+bg\left(t\right)\right\}=a\ L\left\{f\left(t\right)\right\}+b\ L\left\{g\left(t\right)\right\}$

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    - equivalently: $e^{at}f\left(t\right)=L^{-1}\left\{F\left(s-a\right)\right\}$

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Q (9ed-6.1-29)

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&= 4\left(\cos \pi t\right) - 3\left(\sin \pi t\right) \quad \blacksquare
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= \; & \sum_{k=0}^{n}a_k\left[L\left(t^k\right)\right]_{s-(-1)}=\sum_{k=0}^{n}a_k\left[\frac{k!}{s^{k+1}}\right]_{s+1} \quad \text{using}
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= \; & \sum_{k=0}^{n} a_k \left[L\left(t^k\right)\right]_{s-(-1)} = \sum_{k=0}^{n} a_k \left[\frac{k!}{s^{k+1}}\right]_{s+1} \quad \text{using } L\left(t^n\right) = \frac{n!}{s^{n+1}} \\
= \; & \sum_{k=0}^{n} a_k \left(\frac{k!}{(s+1)^{k+1}}\right) \quad \blacksquare
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= & \frac{3\,s+3-3-137}{(s+1)^2+(20)^2} = \frac{3\,(s+1)-140}{(s+1)^2+(20)^2} \\
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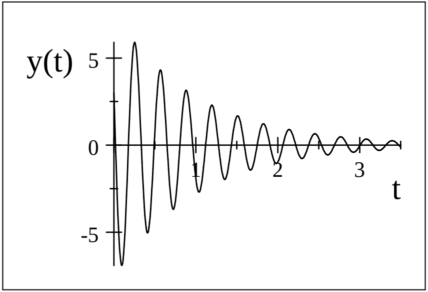
= & 3\frac{s-(-1)}{(s-(-1))^2+(20)^2} - 7\frac{20}{(s-(-1))^2+(20)^2} \\
= & 3e^{-t}\cos(20t) - 7e^{-t}\sin(20t) \\
= & e^{-t}\left(3\cos(20t) - 7\sin(20t)\right)
\end{aligned}$$

The damped osciliator found in this example is plotted as



$$L^{-1}\left(\frac{3\,s-137}{s^2+2\,s+401}\right) = y\,(t)$$
$$= e^{-t}\,(3\cos\,(20t)-7\sin\,(20t)) \quad \blacksquare$$

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Suppose $a = -\sqrt{2}$ in e^{at}, so tentatively we take $F(s) = \frac{1}{s^3}$ and $a = -\sqrt{2}$. Thus in first shifting theorem $\left(e^{at}f(t) = L^{-1}\left\{F(s-a)\right\}\right)$ we set
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$f\left(t\right) = L^{-1}\left(\frac{\sqrt{8}}{s^3}\right) = \sqrt{2}L^{-1}\left(\frac{2}{s^3}\right) = \sqrt{2}t^2$

$$\Rightarrow e^{at}f\left(t\right) = e^{-\sqrt{2}t}\left(\sqrt{2}t^2\right) = \sqrt{2}t^2e^{-\sqrt{2}t} \qquad \blacksquare$$

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$$\begin{aligned}
& L^{-1}\left(\frac{a(s+k)+b\pi}{(s+k)^2+\pi^2}\right) \\
= \ & ae^{-kt}L^{-1}\left(\frac{s}{s^2+\pi^2}\right) + be^{-kt}L^{-1}\left(\frac{\pi}{s^2+\pi^2}\right) \\
= \ & ae^{-kt}\cos(\pi t) + be^{-kt}\sin(\pi t) \\
= \ & e^{-kt}\left(a\cos(\pi t) + b\sin(\pi t)\right) \quad \blacksquare
\end{aligned}$$