K-means Clustering

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2015

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- Given a bunch of spherical fruits, optimally describe types of fruits

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- **Applications:** Group movies by ratings, Segment shoppers

Solutions

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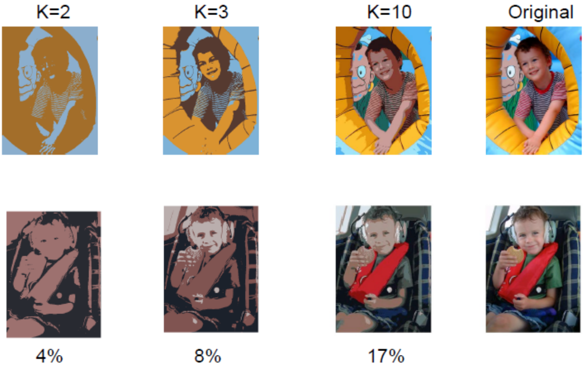
- Two popular methods:
  - Hierarchical clustering (computationally expensive)
  - k-means clustering (pre-specify k)

Source: Pattern Recognition and Machine Learning



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- Euclidean distance between observations, sum it over all observations

$$\min._{C_1,\ldots,C_K} \sum_{k=1}^{k} \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^{p} (x_{ij} - x_{i'j})^2 \tag{1}$$

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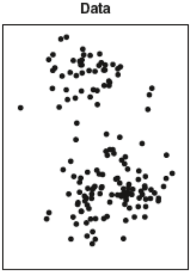

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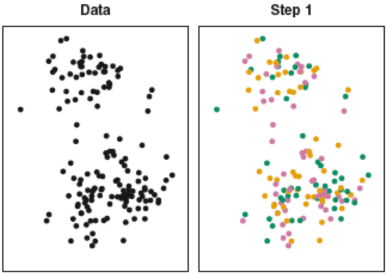

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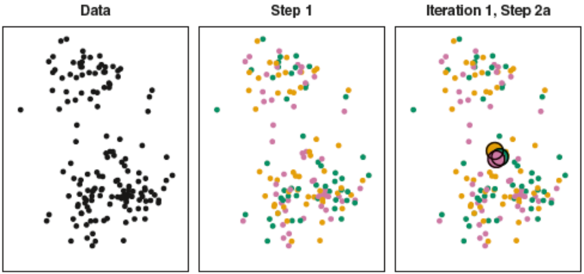

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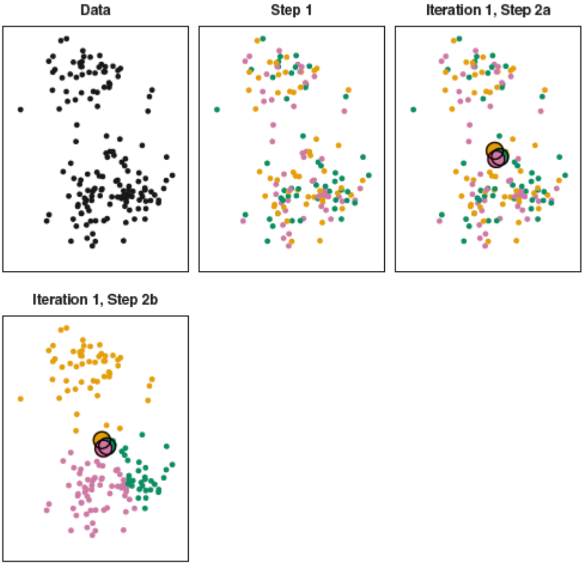

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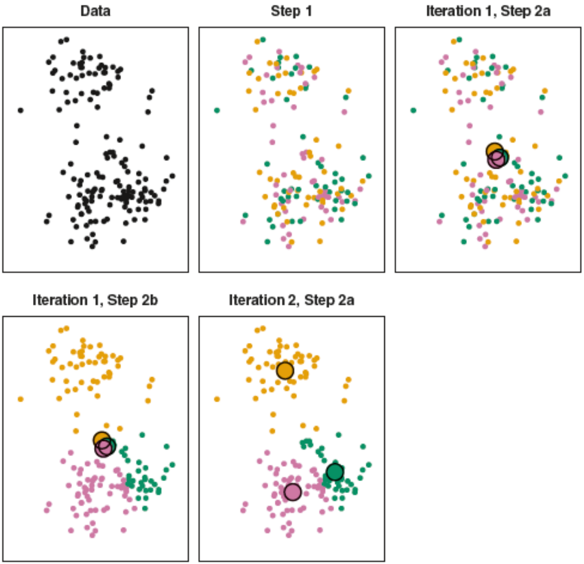

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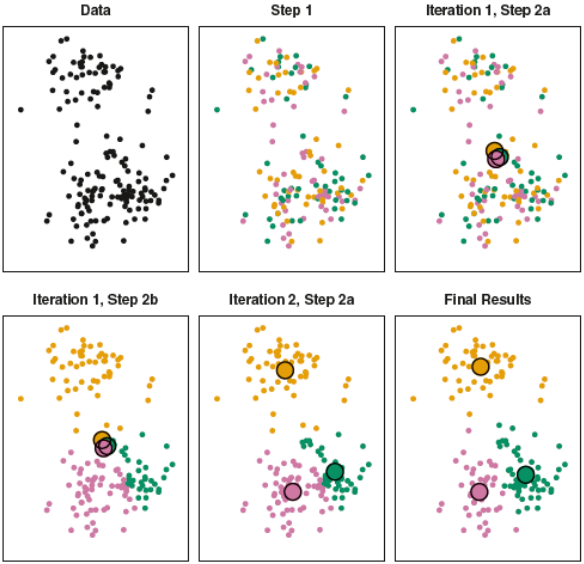

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  - Pick dispersed points as centroids. For e.g. k-means++ and variations of it.

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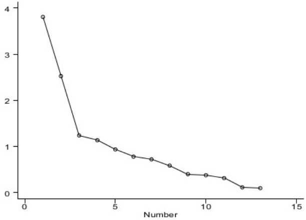
  - Plot them, look for the *knee*



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