Data Science - Lecture 6

Introduction To Data Science

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What is today's agenda?

Today we are going to learn following things :

- Data Understanding & Preprocessing

Similarity and Dissimilarity

- Similarity
  - Numerical measure of how alike two data objects are.
  - Is higher when objects are more alike.
  - Often falls in the range [0,1]
- Dissimilarity
  - Numerical measure of how different are two data objects
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- Proximity refers to a similarity or dissimilarity

Similarity / Dissimilarity for Simple Attributes

p and *q* are the attribute values for two data objects.

Attribute Type	Dissimilarity	Similarity
Nominal	$d = \begin{cases} 0 & \text{if } p = q \\ 1 & \text{if } p \neq q \end{cases}$	$s = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$
Ordinal	$d = \frac{\lvert p-q \rvert}{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{\lvert p-q \rvert}{n-1}$
Interval or Ratio	$d = \lvert p - q \rvert$	$s = -d,$

Table 5.1. Similarity and dissimilarity for simple attributes

Euclidean Distance

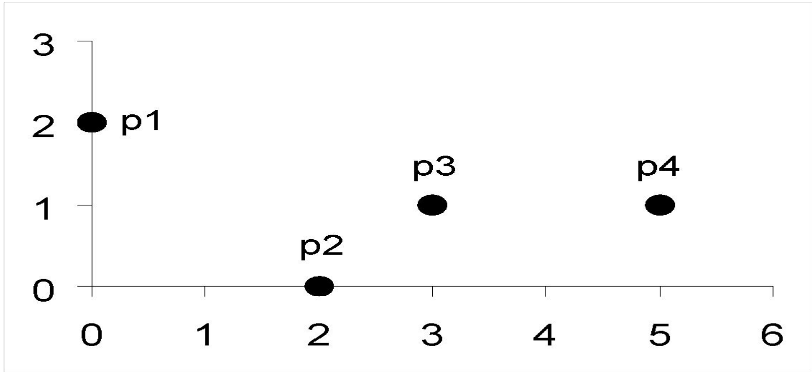
- Euclidean Distance

$$dist = \sqrt{\sum_{k=1}^{n} (p_k - q_k)^2}$$

Where n is the number of dimensions (attributes) and p_k and q_k are, respectively, the k^{th} attributes (components) or data objects p and q.

- Standardization is necessary, if scales differ.

Euclidean Distance

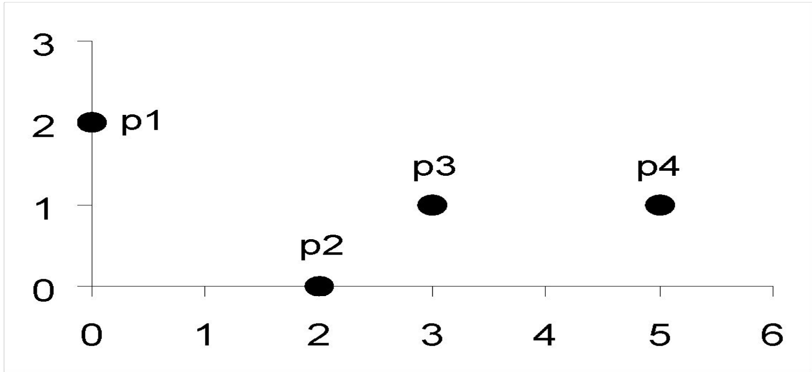


point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1				
p2				
p3				
p4				

Distance Matrix

Euclidean Distance



point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

Minkowski Distance

- Minkowski Distance is a generalization of Euclidean Distance

$$dist = \left(\sum_{k=1}^{n} | p_k - q_k |^r \right)^{\frac{1}{r}}$$

Where *r* is a parameter, *n* is the number of dimensions (attributes) and p_k and q_k are, respectively, the kth attributes (components) or data objects *p* and *q*.

Minkowski Distance : Example

- $r = 1$. City block (Manhattan, taxicab, L_1 norm) distance.
  - A common example of this is the Hamming distance, which is just the number of bits that are different between two binary vectors

- $r = 2$. Euclidean distance

- $r \to \infty$. "supremum" (L_{max} norm, L_∞ norm) distance.
  - This is the maximum difference between any component of the vectors

$$d_S(P, Q) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$$

- Do not confuse r with n, i.e., all these distances are defined for all numbers of dimensions.

Minkowski Distance

point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

L1	p1	p2	p3	p4
p1	0	4	4	6
p2				
p3				
p4				

L2	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2				
p3				
p4				

L_∞	p1	p2	p3	p4
p1	0	2	3	5
p2				
p3				
p4				

Distance Matrix

Minkowski Distance

point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

L1	p1	p2	p3	p4
p1	0	4	4	6
p2	4	0	2	4
p3	4	2	0	2
p4	6	4	2	0

L2	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

L_∞	p1	p2	p3	p4
p1	0	2	3	5
p2	2	0	1	3
p3	3	1	0	2
p4	5	3	2	0

Distance Matrix

Common Properties of a Distance

- Distances, such as the Euclidean distance, have some well known properties.

1. $d(p, q) \geq 0$ for all p and q and $d(p, q) = 0$ only if $p = q$. (Positive definiteness)
2. $d(p, q) = d(q, p)$ for all p and q. (Symmetry)
3. $d(p, r) \leq d(p, q) + d(q, r)$ for all points p, q, and r. (Triangle Inequality)

where $d(p, q)$ is the distance (dissimilarity) between points (data objects), p and q.

Common Properties of a Similarity

- Similarities, also have some well known properties.

  - $s(p, q) = 1$ (or maximum similarity) only if $p = q$.

  - $s(p, q) = s(q, p)$ for all p and q. (Symmetry)

  where $s(p, q)$ is the similarity between points (data objects), p and q.

Similarity between Binary Vectors

- Common situation is that objects, *p* and *q*, have only binary attributes
- Compute similarities using the following quantities

M_{01} = the number of attributes where p was 0 and q was 1

M_{10} = the number of attributes where p was 1 and q was 0

M_{00} = the number of attributes where p was 0 and q was 0

M_{11} = the number of attributes where p was 1 and q was 1

- Simple Matching and Jaccard Coefficients

SMC = number of matches / number of attributes

$= (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00})$

J = number of 11 matches / number of not-both-zero attributes values

$= (M_{11}) / (M_{01} + M_{10} + M_{11})$

SMC versus Jaccard : Example

$p = 1\,0\,0\,0\,0\,0\,0\,0\,0\,0$

$q = 0\,0\,0\,0\,0\,0\,1\,0\,0\,1$

$M_{01} = 2$ (the number of attributes where p was 0 and q was 1)

$M_{10} = 1$ (the number of attributes where p was 1 and q was 0)

$M_{00} = 7$ (the number of attributes where p was 0 and q was 0)

$M_{11} = 0$ (the number of attributes where p was 1 and q was 1)

$SMC = (M_{11} + M_{00})/(M_{01} + M_{10} + M_{11} + M_{00}) = ?$

$J = (M_{11}) / (M_{01} + M_{10} + M_{11}) = ?$

SMC versus Jaccard : Example

$p =$ 1 0 0 0 0 0 0 0 0 0

$q =$ 0 0 0 0 0 0 1 0 0 1

$M_{01} = 2$ (the number of attributes where p was 0 and q was 1)

$M_{10} = 1$ (the number of attributes where p was 1 and q was 0)

$M_{00} = 7$ (the number of attributes where p was 0 and q was 0)

$M_{11} = 0$ (the number of attributes where p was 1 and q was 1)

$$SMC = (M_{11} + M_{00})/(M_{01} + M_{10} + M_{11} + M_{00}) = (0+7) / (2+1+0+7) = 0.7$$

$$J = (M_{11}) / (M_{01} + M_{10} + M_{11}) = 0 / (2 + 1 + 0) = 0$$

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

$$\cos(d_1, d_2) = (d_1 \cdot d_2) / ||d_1||\ ||d_2||,$$

where · indicates vector dot product and || d || is the length of vector d.

- Example:

d_1 **= 3 2 0 5 0 0 0 2 0 0**

d_2 **= 1 0 0 0 0 0 0 1 0 2**

$d_1 \cdot d_2 = ?$

$||d_1|| = ?$

$||d_2|| = ?$

cos(d_1, d_2 **) = ?**

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

$$\cos(d_1, d_2) = (d_1 \cdot d_2) / ||d_1|| \, ||d_2|| ,$$

where $\cdot$ indicates vector dot product and $|| d ||$ is the length of vector d.

- Example:

d_1 = **3 2 0 5 0 0 0 2 0 0**

d_2 = **1 0 0 0 0 0 0 1 0 2**

$d_1 \cdot d_2 = 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5$

$||d_1|| = (3*3+2*2+0*0+5*5+0*0+0*0+0*0+2*2+0*0+0*0)^{0.5} = (42)^{0.5} = 6.481$

$||d_2|| = (1*1+0*0+0*0+0*0+0*0+0*0+0*0+1*1+0*0+2*2)^{0.5} = (6)^{0.5} = 2.245$

$$\mathbf{\cos(d_1, d_2) = .3150}$$

Summary Statistics

- Summary statistics are numbers that summarize properties of the data

  - Summarized properties include frequency, location and spread

    - Examples: location - mean
spread - standard deviation

  - Most summary statistics can be calculated in a single pass through the data

Frequency and Mode

- The frequency of an attribute value is the percentage of time the value occurs in the data set
  - For example, given the attribute ‘gender’ and a representative population of people, the gender ‘female’ occurs about 50% of the time.
- The mode of an attribute is the most frequent attribute value
- The notions of frequency and mode are typically used with categorical data

Percentiles

- For continuous data, the notion of a percentile is more useful.

Given an ordinal or continuous attribute *x* and a number *p* between 0 and 100, the *p*th percentile is a value of x such that *p*% of the observed values of x are less than .

- For instance, the 50th percentile is the value such that 50% of all values of x are less than .

Measures of Location : Mean and Median

- The mean is the most common measure of the location of a set of points.
- However, the mean is very sensitive to outliers.
- Thus, the median or a trimmed mean is also commonly used.

$$\text{mean}(x) = \overline{x} = \frac{1}{m}\sum_{i=1}^{m} x_i$$

$$\text{median}(x) = \begin{cases} x_{(r+1)} & \text{if } m \text{ is odd, i.e., } m = 2r+1 \\ \frac{1}{2}(x_{(r)} + x_{(r+1)}) & \text{if } m \text{ is even, i.e., } m = 2r \end{cases}$$

Measures of Spread : Range and Variance

- Range is the difference between the max and min
- The variance or standard deviation is the most common measure of the spread of a set of points.

$$\text{variance}(x) = s_x^2 = \frac{1}{m-1}\sum_{i=1}^{m}(x_i - \overline{x})^2$$

- However, this is also sensitive to outliers, so that other measures are often used.

$$\text{AAD}(x) = \frac{1}{m}\sum_{i=1}^{m}|x_i - \overline{x}|$$

$$\text{MAD}(x) = median\Big(\{|x_1 - \overline{x}|, \ldots, |x_m - \overline{x}|\}\Big)$$

$$\text{interquartile range}(x) = x_{75\%} - x_{25\%}$$

Measures of Spread : Range and Variance

- Average Absolute Deviation

The average absolute deviation of a set $\{x_1, x_2, ..., x_n\}$ is

$$\frac{1}{n}\sum_{i=1}^{n}|x_i - m(X)|.$$

For example, for the data set {2, 2, 3, 4, 14}:

Measure of central tendency $m(X)$	Average absolute deviation
Mean = 5	$\frac{\lvert 2-5\rvert + \lvert 2-5\rvert + \lvert 3-5\rvert + \lvert 4-5\rvert + \lvert 14-5\rvert}{5} = 3.6$
Median = 3	$\frac{\lvert 2-3\rvert + \lvert 2-3\rvert + \lvert 3-3\rvert + \lvert 4-3\rvert + \lvert 14-3\rvert}{5} = 2.8$
Mode = 2	$\frac{\lvert 2-2\rvert + \lvert 2-2\rvert + \lvert 3-2\rvert + \lvert 4-2\rvert + \lvert 14-2\rvert}{5} = 3.0$

Measures of Spread : Range and Variance

- Median Absolute Distance
  - D: 1, 1, 2, **2**, 4, 6, 9
  - Median: 2
  - The absolute deviations about 2 are (1, 1, 0, 0, 2, 4, 7) which in turn have a median value of 1 (because the sorted absolute deviations are (0, 0, 1, **1**, 2, 4, 7)).
  - So the median absolute deviation for this data is 1.

Questions ?