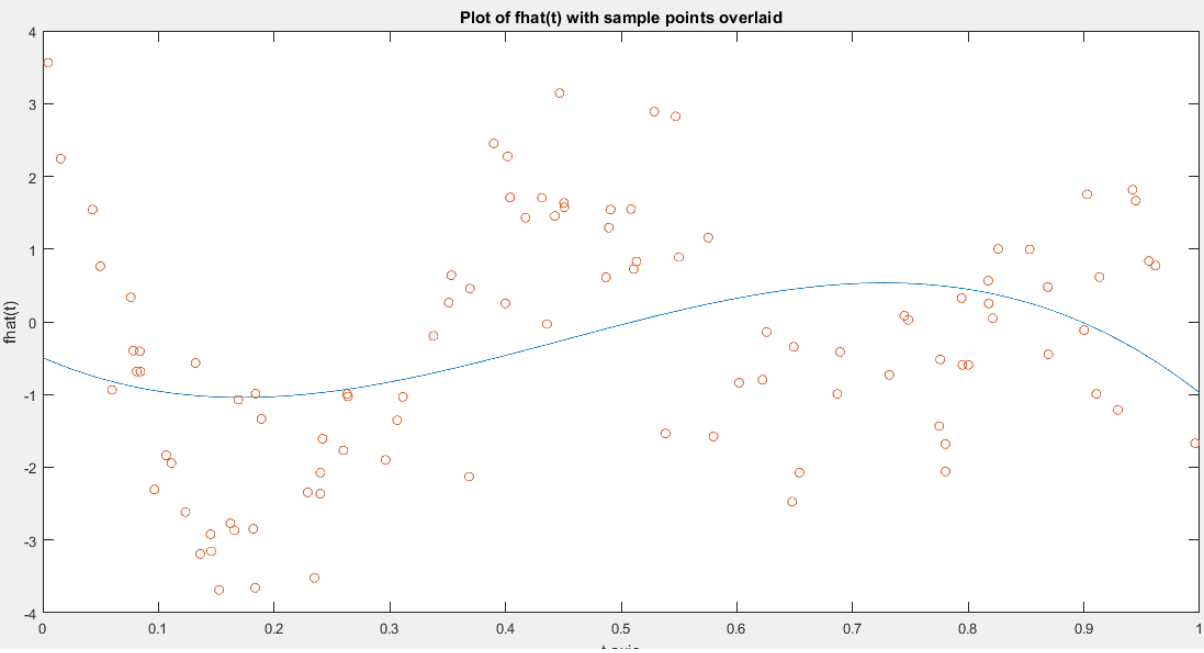
Question 4.

Codes are provided at the end.

a) Sample error: 15.520228104202692

The best cubic fit to this data:
W3 = -18.3959749927282
W2 = 24.8215751635097
W1 = -6.90972839870063
W0 = -0.492123793899227



b) Generalization error: 1.293859993771946

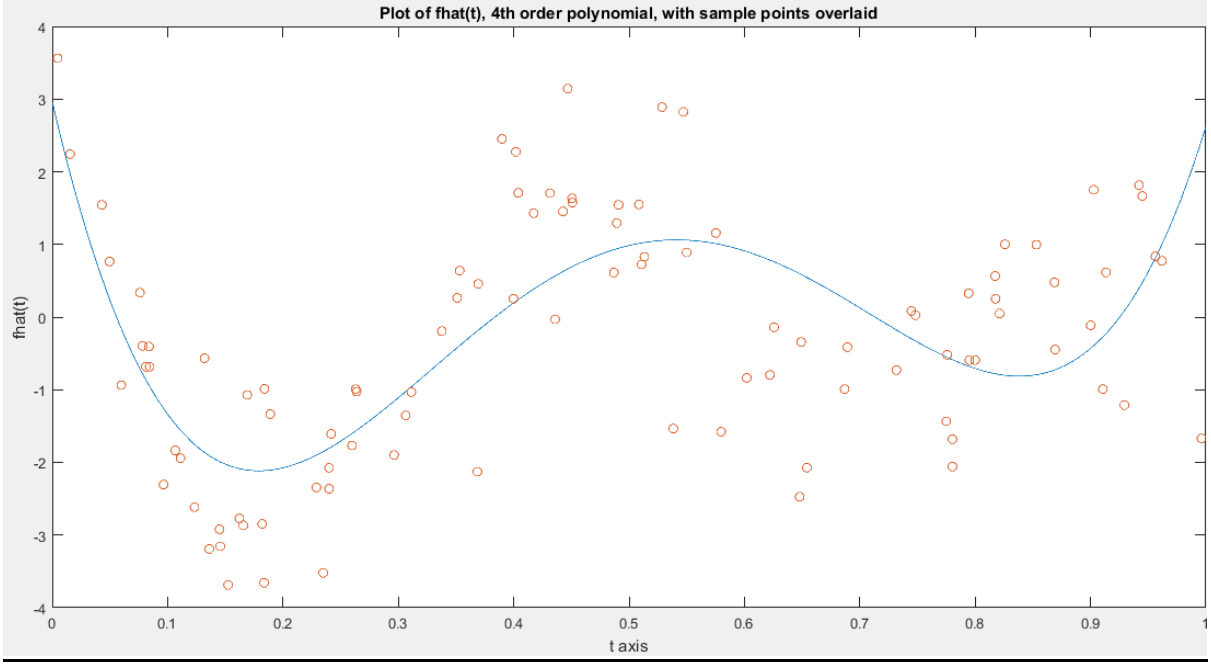
c)

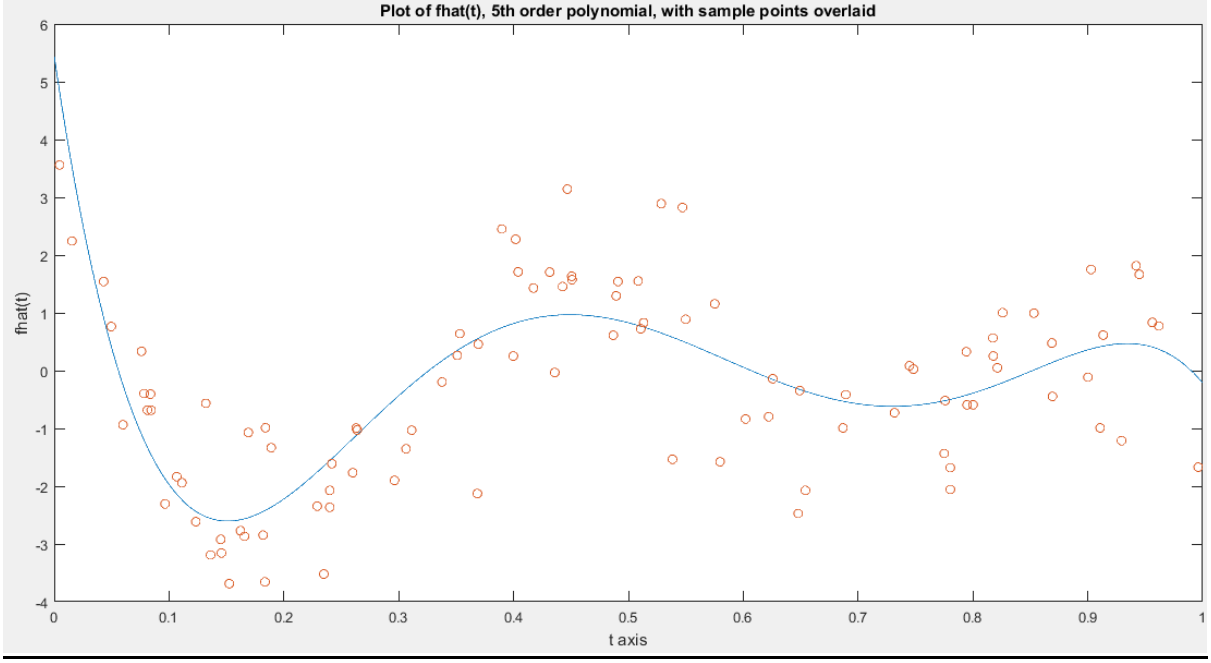
Polynomial of order p.

Order p	Sample error	Generalization error	Largest sing. value of A	Smallest sing. value of A
4	12.1340	0.7725	12.1560	0.00176
5	10.2337	0.4208	12.3196	0.0031
6	9.4812	0.4018	12.4474	5.8977x10^-4
7	9.3540	0.3261	12.5503	1.1005x10^-4
8	9.3167	0.3327	12.6351	1.8578x10^-5
9	9.2539	0.3309	12.7063	3.22x10^-6

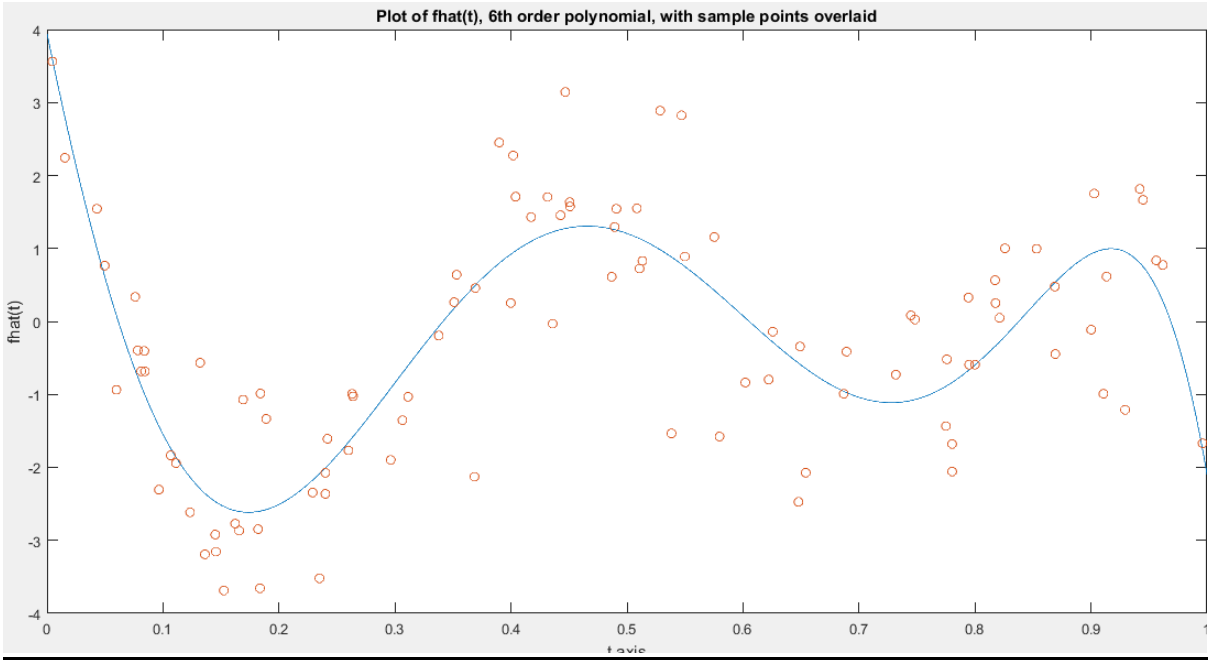
Question 4.

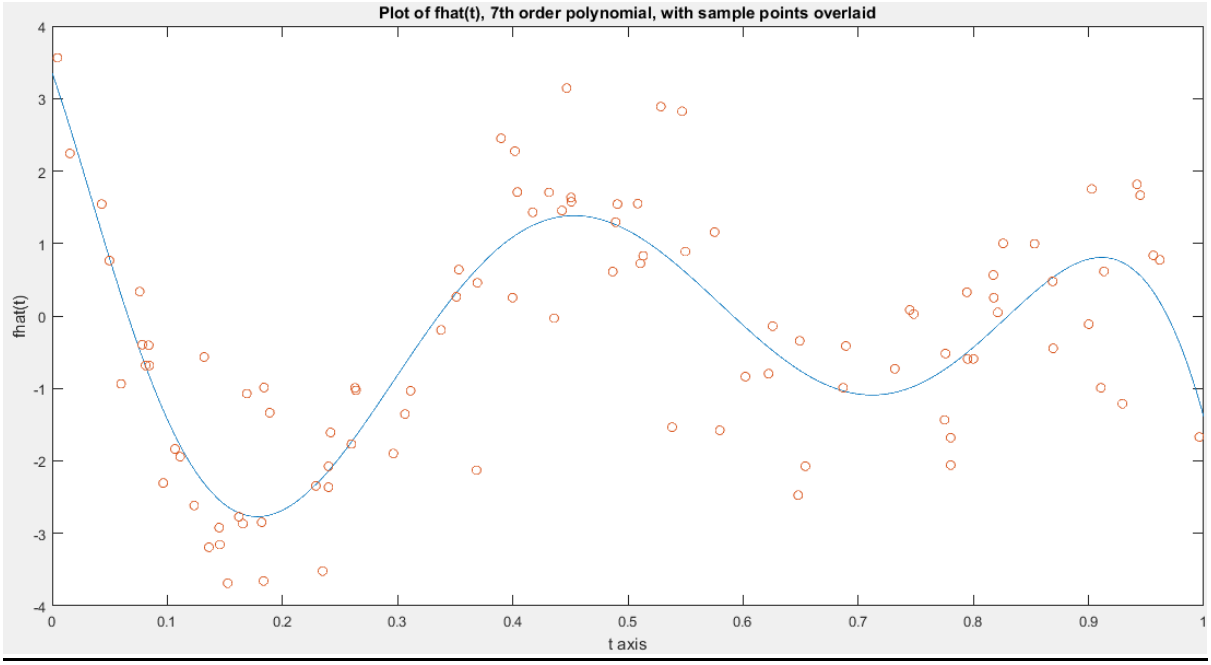
Plots:



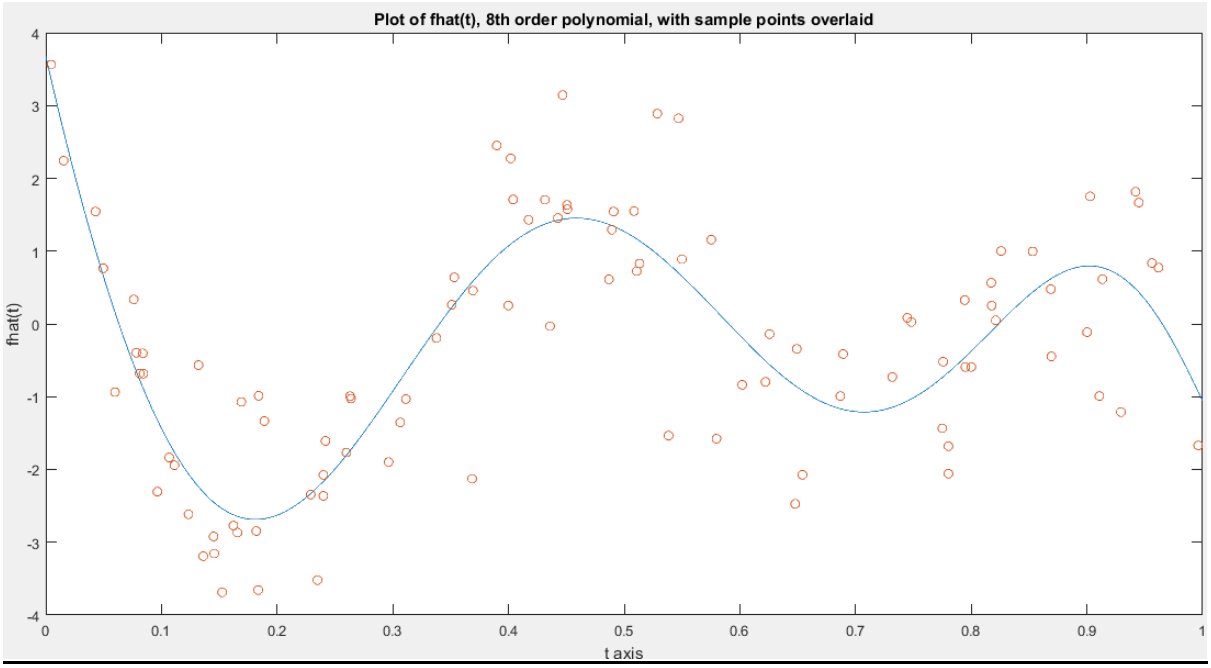


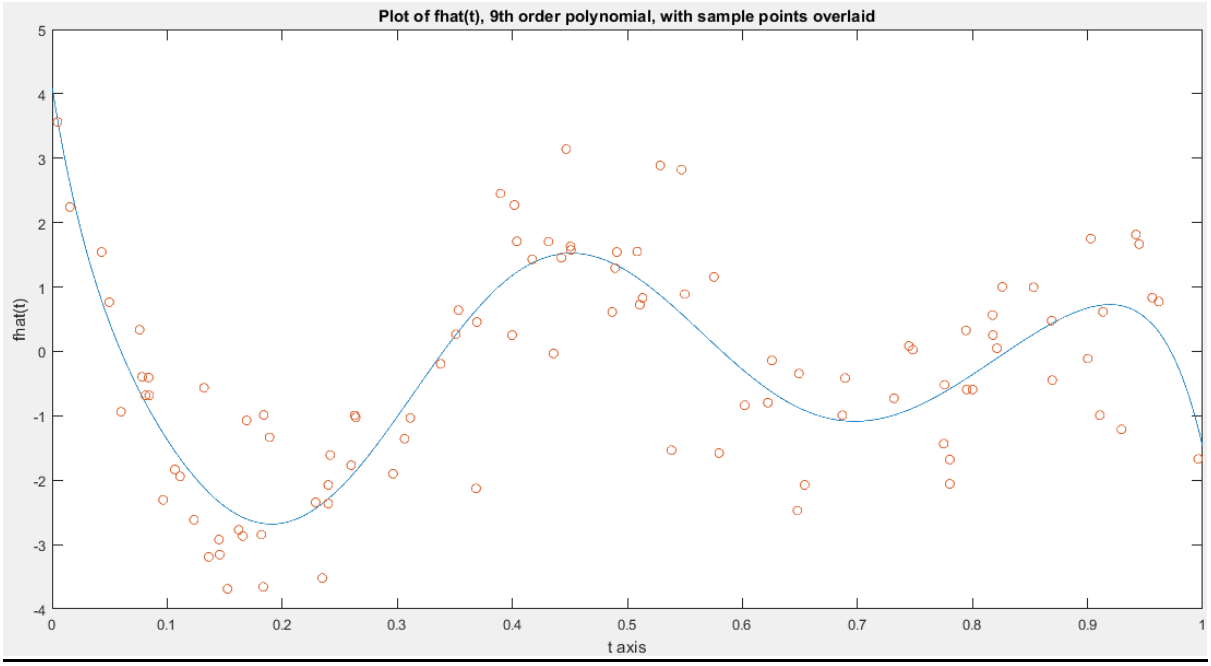
Question 4.


Plot of fhat(t), 6th order polynomial, with sample points overlaid
fhat(t)
t axis


Plot of fhat(t), 7th order polynomial, with sample points overlaid
fhat(t)
t axis

Question 4.





Comments:

For p = 8 and 9, the generalization error increases due to overfitting. But the sample error decreases as expected.

Coefficients of the polynomial in decreasing powers of t:

For p = 9:

-23798.7061897272 (coefficient of t^9)
111140.870240860
-215740.452048344

Question 4.

223833.272265825
-133081.773965033
45550.6656529901
-8918.34362142543
1120.81742236563
-111.928796857435
4.10252141114323

For p = 8:

4486.79069046607
-15941.5379390372
20982.4060728154
-12101.4684617239
2617.74601894856
-167.870806474144
189.170029331071
-69.9739802710004
3.67715612192356

For p = 7:

1987.16478260814
-8234.99009811957
12844.8387557214
-9304.47448018932
2984.68950937058
-236.000470185899
-45.9737692145864
3.35566453150577

For p = 6:

-1308.64088521456
3300.24943619355
-2713.78661916290
603.624430019129
190.089586673773
-77.5780184278458
3.94573775725840

For p = 5:

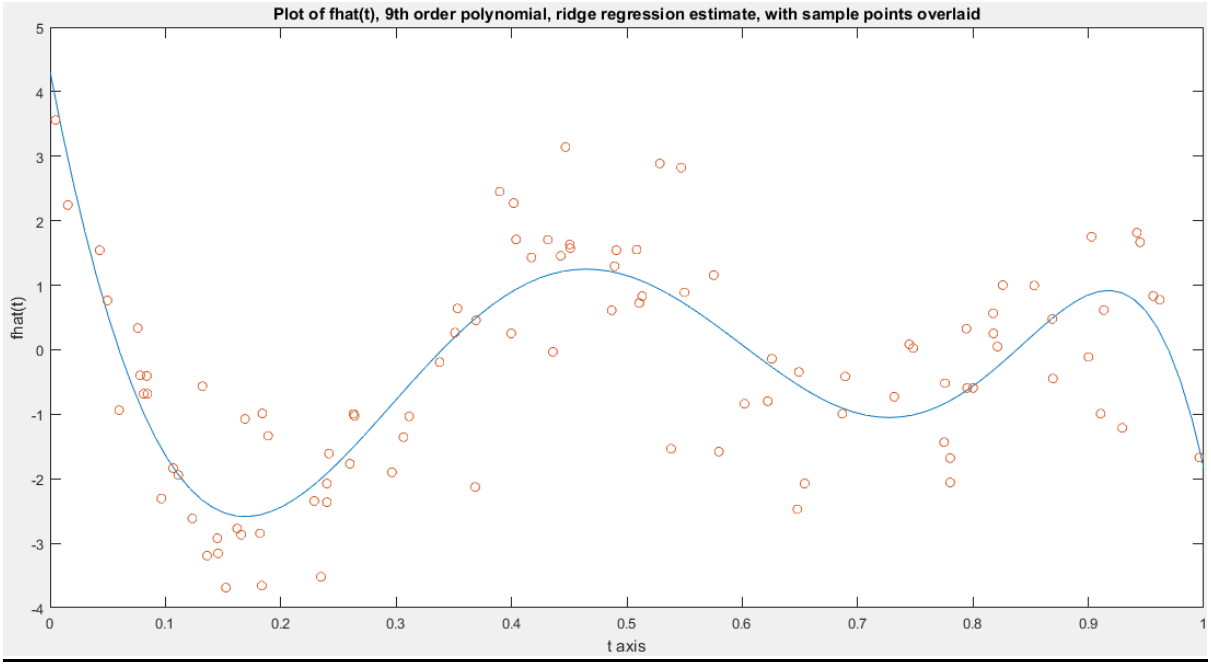
-582.906903286835
1650.04046269520
-1699.03484794543
761.058736321338
-134.810464992548
5.43777459866408

Question 4.

For p = 4:

210.193595834972
-436.753900569838
294.523108705144
-68.3034223143701
2.96351442284634

d)

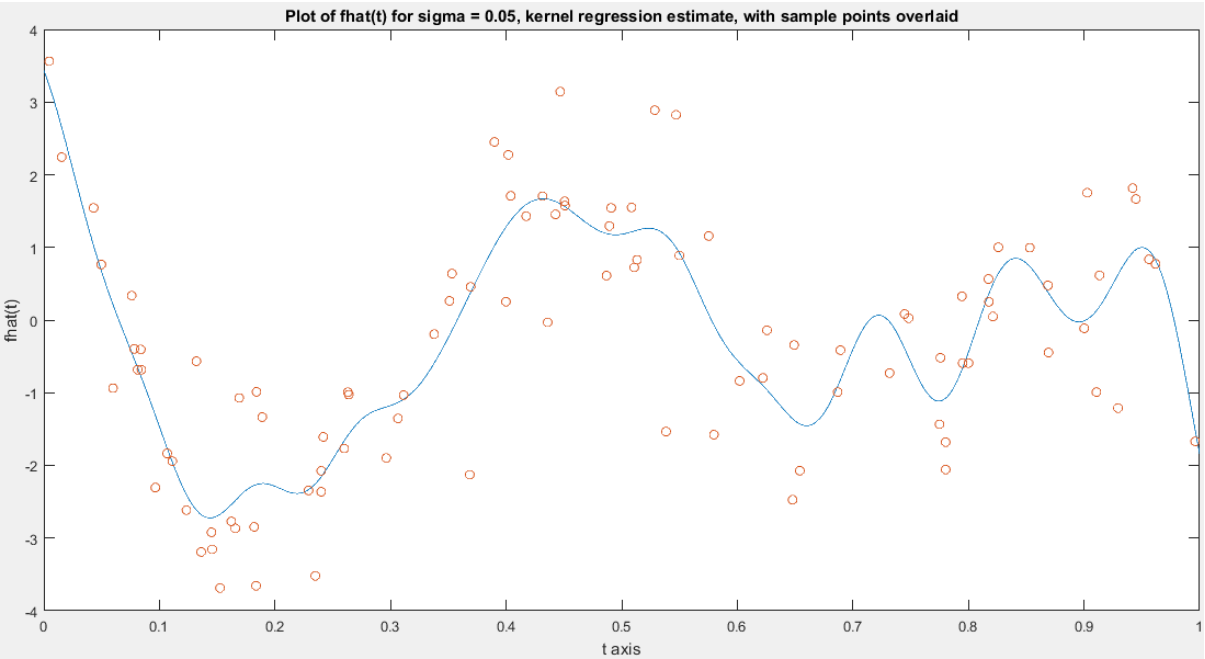


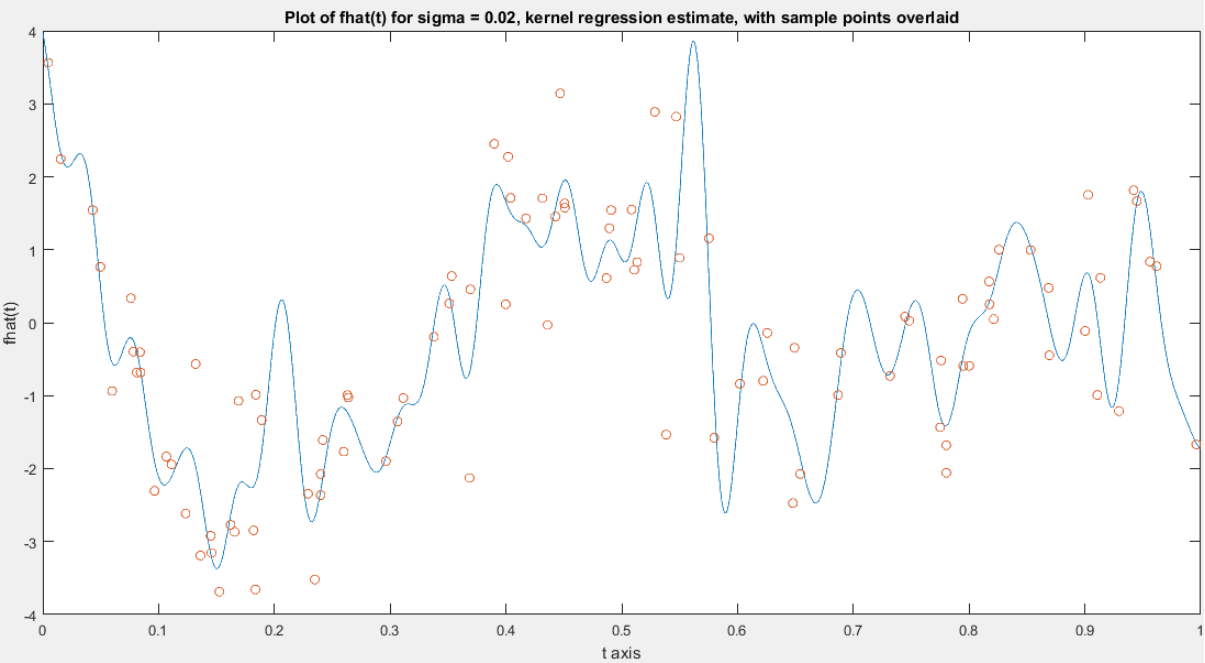
Largest singular value of A: 12.7063
Smallest singular value of A: 3.22x10^-6
Generalization error: 0.3854
Sample error: 9.5167

e)

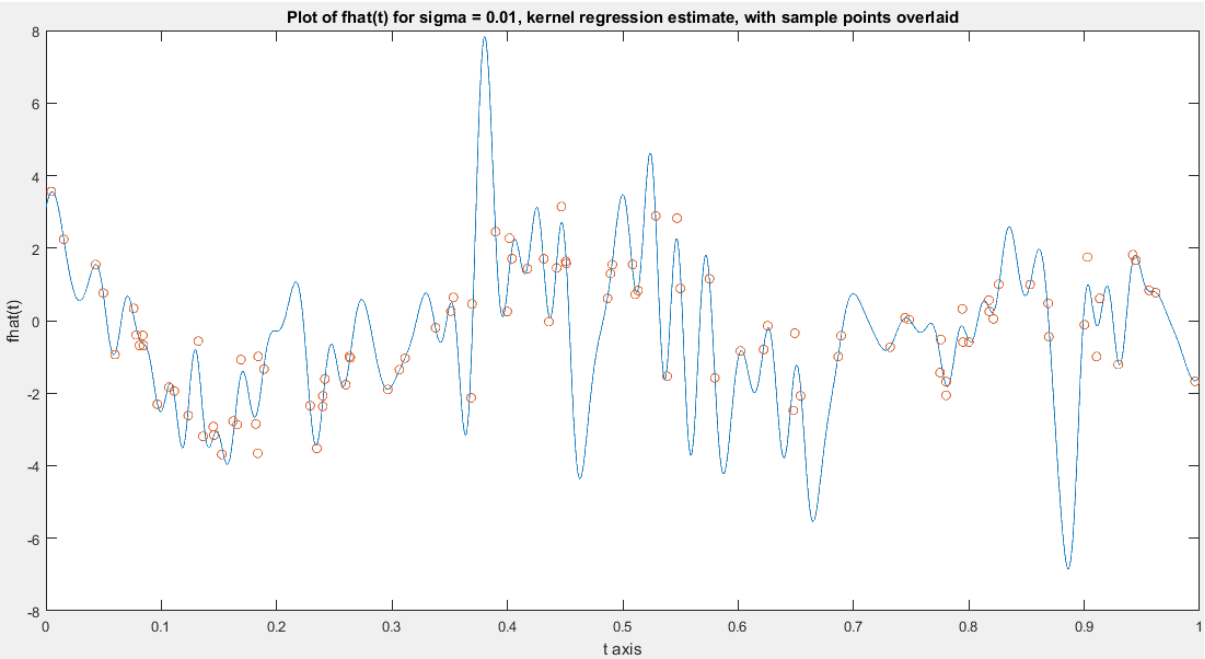
Sigma value	1/20	1/50	1/100	1/300	1/1000
Sample error	8.4528	6.6019	3.8359	1.4968	0.2951
Generalization error	0.5069	0.9975	1.8938	1.3260	1.1644

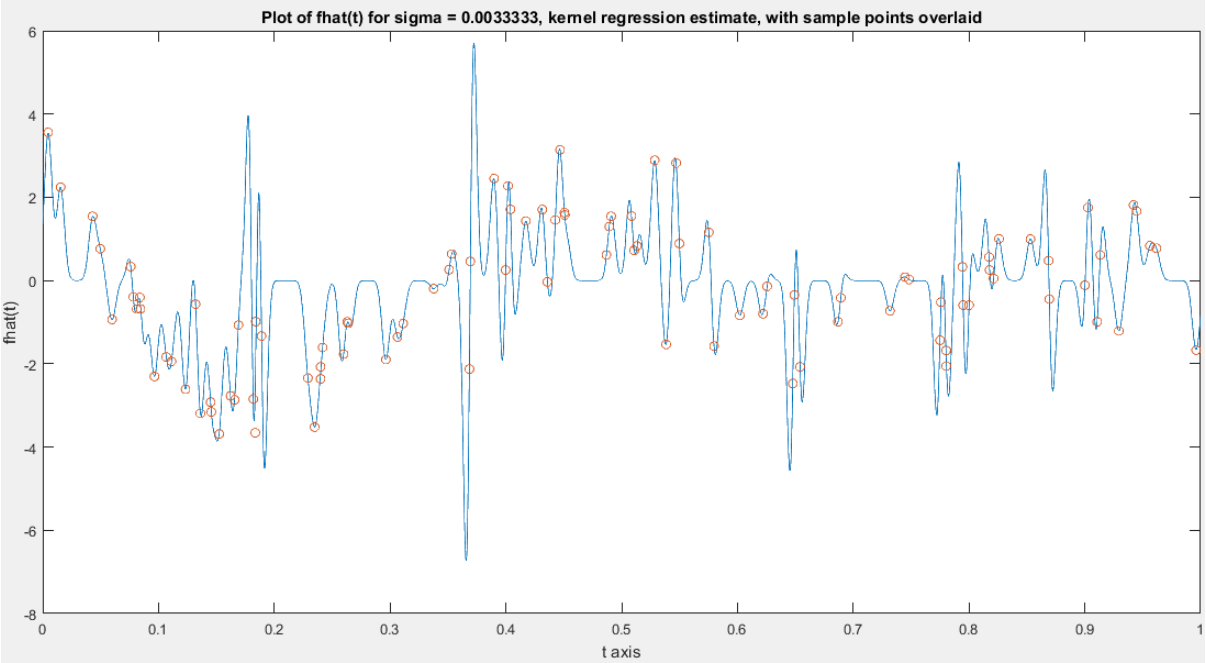
Question 4.


Plot of fhat(t) for sigma = 0.05, kernel regression estimate, with sample points overlaid
fhat(t)
t axis

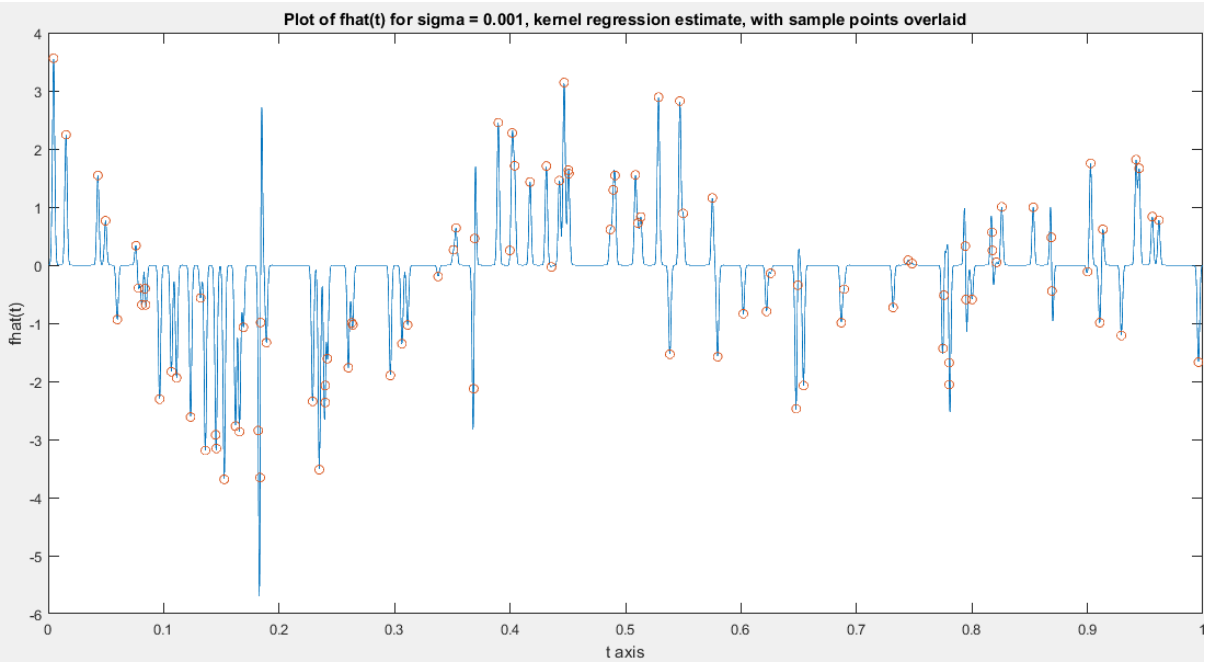

Plot of fhat(t) for sigma = 0.02, kernel regression estimate, with sample points overlaid
fhat(t)
t axis

Question 4.


Plot of fhat(t) for sigma = 0.01, kernel regression estimate, with sample points overlaid
fhat(t)
t axis
8
6
4
2
0
-2
-4
-6
-8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Plot of fhat(t) for sigma = 0.0033333, kernel regression estimate, with sample points overlaid
fhat(t)
t axis
6
4
2
0
-2
-4
-6
-8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Question 4.



Comments:

As the plots clearly indicate, for sigma = 1/1000, almost all the points are fit by the curve. This is a case of overfitting. If we observe the generalization errors, we can see that they increase up to sigma = 1/100 and then decrease. So, we have a maximum overfitting at this sigma = 1/100. For very low sigma, the generalization error reduces because the functions become similar to spikes and they approximate a point well. The case of sample error shows that it decreases monotonically, as expected.

Question 4.

CODES:

4.A.

```
clear all
close all
load('hw5p4_data.mat');
for i=1:100
    A(i,:) = [T(i)^3 T(i)^2 T(i) 1]; % ---- A is a 100x4 matrix.
end
B = pinv(A);
x = B*y;
se = 0;
%sample error:
for i=1:100
    se = se + ((y(i) - (x(1)*(T(i)^3) + x(2)*(T(i)^2) + x(3)*(T(i)) +
x(4)))^2);
end
sampleerror = sqrt(se);
t = linspace(0,1,5000);
plot(t,(x(1)*(t.^3) + x(2)*(t.^2) + x(3)*t + x(4)));
ylabel('fhat(t)');
xlabel('t axis');
title('Plot of fhat(t) with sample points overlaid');
hold on;
plot(T,y,'o');
```

4.B.

```
%For 3rd order polynomial
%Sampling 5000 points between 0 and 1 and at intervals of 1/5000.
t = linspace(0,1,5000);
for i = 1:5000
a1(i) = (x(1)*(t(i).^3) + x(2)*(t(i).^2) + x(3)*t(i) + x(4)) -
((sin(12*(t(i) + 0.2)))/(t(i) + 0.2));
b1(i) = a1(i)^2;
end
c1 = mean(b1);
generror3 = c1^(0.5);
```

Question 4.

4.C.

```
%For 4th order polynomial
for i=1:100
    A1(i,:) = [T(i)^4 T(i)^3 T(i)^2 T(i) 1]; % ---- A is a 100x5 matrix.
end
B = pinv(A1);
x = B*y;
se = 0;
%sample error:
for i=1:100
    se(i) = (y(i) - ((x(1)*(T(i)^4) + x(2)*(T(i)^3) + x(3)*(T(i)^2) +
x(4)*(T(i)) + x(5))));
    de(i) = se(i)^2;
end
sampleerrorA1 = sqrt(sum((de)));
figure;
t = linspace(0,1,5000);
plot(t,(x(1)*(t.^4) + x(2)*(t.^3) + x(3)*(t.^2) + x(4)*t + x(5)));
ylabel('fhat(t)');
xlabel('t axis');
title('Plot of fhat(t), 4th order polynomial, with sample points
overlaid');
hold on;
plot(T,y,'o');
s = svd(A1);
smallestsingA1 = min(s);
largestsingA1 = max(s);

t = linspace(0,1,5000);
for i = 1:5000
a2(i) = (x(1)*(t(i).^4) + x(2)*(t(i).^3) + x(3)*(t(i).^2) + x(4)*t(i) +
x(5)) - ((sin(12*(t(i) + 0.2)))/(t(i) + 0.2));
b2(i) = a2(i)^2;
end
c2 = mean(b2);
generror4 = sqrt(c2);

%For 5th order polynomial
for i=1:100
    A2(i,:) = [T(i)^5 T(i)^4 T(i)^3 T(i)^2 T(i) 1]; % ---- A is a 100x6
matrix.
end
B = pinv(A2);
x = B*y;
se = 0;
%sample error:
for i=1:100
    se(i) = (y(i) - ((x(1)*(T(i)^5) + x(2)*(T(i)^4) + x(3)*(T(i)^3) +
x(4)*(T(i)^2) + x(5)*(T(i)) + x(6))));
    de(i) = se(i)^2;
end
sampleerrorA2 = sqrt(sum((de)));
figure;
t = linspace(0,1,5000);
plot(t,((x(1)*(t.^5) + x(2)*(t.^4) + x(3)*(t.^3) + x(4)*(t.^2) + x(5)*(t) +
x(6))));
ylabel('fhat(t)');
xlabel('t axis');
title('Plot of fhat(t), 5th order polynomial, with sample points
overlaid');
```

Question 4.

```
hold on;
plot(T,y,'o');
s = svd(A2);
smallestsingA2 = min(s);
largestsingA2 = max(s);

t = linspace(0,1,5000);
for i = 1:5000
a3(i) = (x(1)*(t(i).^5) + x(2)*(t(i).^4) + x(3)*(t(i).^3) + x(4)*(t(i).^2)
+ x(5)*(t(i)) + x(6)) - ((sin(12*(t(i) + 0.2)))/(t(i) + 0.2));
b3(i) = a3(i)^2;
end
c3 = mean(b3);
generror5 = sqrt(c3);


%For 6th order polynomial
for i=1:100
    A3(i,:) = [T(i)^6 T(i)^5 T(i)^4 T(i)^3 T(i)^2 T(i) 1]; % ---- A is a
100x7 matrix.
end
B = pinv(A3);
x = B*y;
se = 0;
%sample error:
for i=1:100
    se(i) = (y(i) - ((x(1)*(T(i)^6) + x(2)*(T(i)^5) + x(3)*(T(i)^4) +
x(4)*(T(i)^3) + x(5)*(T(i)^2) + x(6)*(T(i)) + x(7))));
    de(i) = se(i)^2;
end
sampleerrorA3 = sqrt(sum((de)));
figure;
t = linspace(0,1,5000);
plot(t,((x(1)*(t.^6) + x(2)*(t.^5) + x(3)*(t.^4) + x(4)*(t.^3) +
x(5)*(t.^2) + x(6)*t + x(7))));
ylabel('fhat(t)');
xlabel('t axis');
title('Plot of fhat(t), 6th order polynomial, with sample points
overlaid');
hold on;
plot(T,y,'o');
s = svd(A3);
smallestsingA3 = min(s);
largestsingA3 = max(s);

t = linspace(0,1,5000);
for i = 1:5000
a4(i) = (x(1)*(t(i).^6) + x(2)*(t(i).^5) + x(3)*(t(i).^4) + x(4)*(t(i).^3)
+ x(5)*(t(i).^2) + x(6)*t(i) + x(7)) - ((sin(12*(t(i) + 0.2)))/(t(i) +
0.2));
b4(i) = a4(i)^2;
end
c4 = mean(b4);
generror6 = sqrt(c4);

%For 7th order polynomial
for i=1:100
    A4(i,:) = [T(i)^7 T(i)^6 T(i)^5 T(i)^4 T(i)^3 T(i)^2 T(i) 1]; % ---- A
is a 100x8 matrix.
end
```

Question 4.

```
B = pinv(A4);
x = B*y;
se = 0;
%sample error:
for i=1:100
    se(i) = (y(i) - ((x(1)*(T(i)^7) + x(2)*(T(i)^6) + x(3)*(T(i)^5) +
x(4)*(T(i)^4) + x(5)*(T(i)^3) + x(6)*(T(i)^2) + x(7)*(T(i)) + x(8))));
    de(i) = se(i)^2;
end
sampleerrorA4 = sqrt(sum((de)));
figure;
t = linspace(0,1,5000);
plot(t,((x(1)*(t.^7) + x(2)*(t.^6) + x(3)*(t.^5) + x(4)*(t.^4) +
x(5)*(t.^3) + x(6)*(t.^2) + x(7)*t + x(8))));
ylabel('fhat(t)');
xlabel('t axis');
title('Plot of fhat(t), 7th order polynomial, with sample points
overlaid');
hold on;
plot(T,y,'o');
s = svd(A4);
smallestsingA4 = min(s);
largestsingA4 = max(s);

t = linspace(0,1,5000);
for i = 1:5000
a5(i) = (x(1)*(t(i).^7) + x(2)*(t(i).^6) + x(3)*(t(i).^5) + x(4)*(t(i).^4)
+ x(5)*(t(i).^3) + x(6)*(t(i).^2) + x(7)*t(i) + x(8)) - ((sin(12*(t(i) +
0.2)))/(t(i) + 0.2));
b5(i) = a5(i)^2;
end
c5 = mean(b5);
generror7 = sqrt(c5);


%For 8th order polynomial
for i=1:100
   A5(i,:) = [T(i)^8 T(i)^7 T(i)^6 T(i)^5 T(i)^4 T(i)^3 T(i)^2 T(i) 1]; % -
--- A is a 100x9 matrix.
end
B = pinv(A5);
x = B*y;
se = 0;
%sample error:
for i=1:100
    se(i) = (y(i) - ((x(1)*(T(i)^8) + x(2)*(T(i)^7) + x(3)*(T(i)^6) +
x(4)*(T(i)^5) + x(5)*(T(i)^4) + x(6)*(T(i)^3) + x(7)*(T(i)^2) + x(8)*(T(i))
+ x(9))));
    de(i) = se(i)^2;
end
sampleerrorA5 = sqrt(sum((de)));
figure;
t = linspace(0,1,5000);
plot(t,((x(1)*(t.^8) + x(2)*(t.^7) + x(3)*(t.^6) + x(4)*(t.^5) +
x(5)*(t.^4) + x(6)*(t.^3) + x(7)*(t.^2) + x(8)*t + x(9))));
ylabel('fhat(t)');
xlabel('t axis');
title('Plot of fhat(t), 8th order polynomial, with sample points
overlaid');
hold on;
plot(T,y,'o');
```

Question 4.

```
s = svd(A5);
smallestsingA5 = min(s);
largestsingA5 = max(s);

t = linspace(0,1,5000);
for i = 1:5000
a6(i) = (x(1)*(t(i).^8) + x(2)*(t(i).^7) + x(3)*(t(i).^6) + x(4)*(t(i).^5)
+ x(5)*(t(i).^4) + x(6)*(t(i).^3) + x(7)*(t(i).^2) + x(8)*t(i) + x(9)) -
((sin(12*(t(i) + 0.2)))/(t(i) + 0.2));
b6(i) = a6(i)^2;
end
c6 = mean(b6);
generror8 = sqrt(c6);

%For 9th order polynomial
for i=1:100
    A6(i,:) = [T(i)^9 T(i)^8 T(i)^7 T(i)^6 T(i)^5 T(i)^4 T(i)^3 T(i)^2 T(i)
1]; % ---- A is a 100x10 matrix.
end
B = pinv(A6);
x = B*y;
se = 0;
%sample error:
for i=1:100
    se(i) = (y(i) - ((x(1)*(T(i)^9) + x(2)*(T(i)^8) + x(3)*(T(i)^7) +
x(4)*(T(i)^6) + x(5)*(T(i)^5) + x(6)*(T(i)^4) + x(7)*(T(i)^3) +
x(8)*(T(i)^2) + x(9)*(T(i)) + x(10))));
    de(i) = se(i)^2;
end
sampleerrorA6 = sqrt(sum((de)));
figure;
t = linspace(0,1,5000);
plot(t,((x(1)*(t.^9) + x(2)*(t.^8) + x(3)*(t.^7) + x(4)*(t.^6) +
x(5)*(t.^5) + x(6)*(t.^4) + x(7)*(t.^3) + x(8)*(t.^2) + x(9)*t + x(10))));
ylabel('fhat(t)');
xlabel('t axis');
title('Plot of fhat(t), 9th order polynomial, with sample points
overlaid');
hold on;
plot(T,y,'o');
s = svd(A6);
smallestsingA6 = min(s);
largestsingA6 = max(s);

t = linspace(0,1,5000);
for i = 1:5000
a7(i) = (x(1)*(t(i).^9) + x(2)*(t(i).^8) + x(3)*(t(i).^7) + x(4)*(t(i).^6)
+ x(5)*(t(i).^5) + x(6)*(t(i).^4) + x(7)*(t(i).^3) + x(8)*(t(i).^2) +
x(9)*t(i) + x(10)) - ((sin(12*(t(i) + 0.2)))/(t(i) + 0.2));
b7(i) = a7(i)^2;
end
c7 = mean(b7);
generror9 = sqrt(c7);
```

Question 4.

4.D.

```
del = 10^(-6)*eye(100);
%For 9th order polynomial
for i=1:100
    Anew(i,:) = [T(i)^9 T(i)^8 T(i)^7 T(i)^6 T(i)^5 T(i)^4 T(i)^3 T(i)^2
T(i) 1]; % ---- Anew is a 100x10 matrix.
end
B = Anew'*inv((Anew*Anew') + del);
x = B*y;
se = 0;
%sample error:
for i=1:100
    se(i) = (y(i) - ((x(1)*(T(i)^9) + x(2)*(T(i)^8) + x(3)*(T(i)^7) +
x(4)*(T(i)^6) + x(5)*(T(i)^5) + x(6)*(T(i)^4) + x(7)*(T(i)^3) +
x(8)*(T(i)^2) + x(9)*(T(i)) + x(10))));
    de(i) = se(i)^2;
end
sampleerrorAnew = sqrt(sum((de)));
figure;
t = linspace(0,1);
plot(t,((x(1)*(t.^9) + x(2)*(t.^8) + x(3)*(t.^7) + x(4)*(t.^6) +
x(5)*(t.^5) + x(6)*(t.^4) + x(7)*(t.^3) + x(8)*(t.^2) + x(9)*t + x(10))));
ylabel('fhat(t)');
xlabel('t axis');
title('Plot of fhat(t), 9th order polynomial, ridge regression estimate,
with sample points overlaid');
hold on;
plot(T,y,'o');
s = svd(Anew);
smallestsingAnew = min(s);
largestsingAnew = max(s);

t = linspace(0,1,5000);
for i = 1:5000
a8(i) = (x(1)*(t(i).^9) + x(2)*(t(i).^8) + x(3)*(t(i).^7) + x(4)*(t(i).^6)
+ x(5)*(t(i).^5) + x(6)*(t(i).^4) + x(7)*(t(i).^3) + x(8)*(t(i).^2) +
x(9)*t(i) + x(10)) - ((sin(12*(t(i) + 0.2)))/(t(i) + 0.2));
b8(i) = a8(i)^2;
end
c8 = mean(b8);
generrord = sqrt(c8);
```

Question 4.

4.E.

```
sigma = [1/20 1/50 1/100 1/300 1/1000];
for k = 1:5

    for i = 1:100
    for j = 1:100
        K(i,j) = exp(-((T(i)-T(j))^2)/(2*sigma(k)*sigma(k)));
    end
    end

e = eig(K);
del(k) = max(e)/1000;
delta = del(k)*eye(100);
B = inv(K + delta);
alpha = B*y;

 for i = 1:100
     ft(i) = 0;
    for j = 1:100
        ft(i) = ft(i) + (alpha(j) * exp(-((T(i)-
T(j))^2)/(2*sigma(k)*sigma(k))));
    end
 end
se = 0;
for i=1:100
    se = se + ((y(i) - ft(i))^2);
end
sampleerror(k) = sqrt(se);

tx = linspace(0,1,5000);
for i = 1:5000
     ft(i) = 0;
    for j = 1:100
        ft(i) = ft(i) + (alpha(j) * exp(-((T(j)-
tx(i))^2)/(2*sigma(k)*sigma(k))));
    end
end

%Sampling 5000 points between 0 and 1 and at intervals of 1/5000.
t = linspace(0,1,5000);
for i = 1:5000
ax = (ft(i) - ((sin(12*(t(i) + 0.2)))/(t(i) + 0.2)));
b(i) = ax^2;
end
c = mean(b);
generror(k) = c^(0.5);

figure;
t = linspace(0,1,5000);
plot(t,ft);
ylabel('fhat(t)');
xlabel('t axis');
title(['Plot of fhat(t) for sigma = ' num2str(sigma(k)) ', kernel
regression estimate, with sample points overlaid']);
hold on;
plot(T,y,'o');
end
```