QUESTION 1.

This week, we talked about Parseval's theorem and how it transforms a norm in Hilbert space equivalent to a norm in l2. We also discussed the implications of introducing a change to the coefficients and saw how it would affect the error. DCT and JPEG were discussed with reference to their bases vectors and we saw how images are compressed using the JPEG standard. This led us to understand video compression as well. There could be non-orthogonal bases too. They provide better results in some case. Any non-orthogonal basis can be transformed into an orthogonal set by using the Gram-Schmidt orthogonalization procedure. A special type of non-orthogonal bases, called the Riesz bases, is one with infinte dimensions. We talked about how dual bases can be used to get the coefficients. One of the main applications of Riesz bases is to get numerical solutions to biharmonic equations. I am in the process of understanding what they are.

An important and useful method of solving a system of equations is using the method of least squares. This sets up non-linear regression as a linear inverse problem. We discussed about how to go about solving a system of symmetric equations. This involves computing the Eigen decomposition of the positive definite matrix and employing that to find extreme Eigen values, which help us to get the upper and lower bounds of the Riesz bases (as done in Q5.). We got a brief introduction to the Singular Value Decomposition. This takes apart an arbitrary matrix and expresses it as a product of diagonal matrix (which has the singular values) with two matrices with orthogonal vectors. It is the generalization of the Eigen decomposition of a positive definite normal matrix (for example, a symmetric matrix with positive eigenvalues) to any matrix. With all these concepts in mind, I look forward to the next lecture on least-squares solutions and the pseudoinverse.