ISyE 6739 Video Assignment 7

1. Suppose $X_1, X_2, \ldots, X_n \sim NID(0,1)$. What is the Cramer-Rao bound for this sample?
   Answer:
   $C - R = \frac{1}{n}$.

2. Write the definition of the mean square error.

   Answer:
   The mean square error of an estimator $\hat{\theta}$ of the parameter θ is defined as

   $$\text{MSE}(\hat{\theta}) = \text{E}[(\hat{\theta} - \theta)^2]$$

3. Suppose $\hat{\theta}_1$ and $\hat{\theta}_2$ are two estimators for some parameter θ. It is given that $\text{E}[\hat{\theta}_1] = \theta$, $\text{E}[\hat{\theta}_2] = \theta - 2$, $\text{Var}(\hat{\theta}_1) = 6$, $\text{Var}(\hat{\theta}_2) = 1$. Which estimator would you prefer?

   Answer:
   $\text{MSE}(\hat{\theta}_1) = (\text{Bias}[(\hat{\theta}_1])^2 + \text{Var}(\hat{\theta}_1) = 0 + 6 = 6$,
   $\text{MSE}(\hat{\theta}_2) = (\text{Bias}[(\hat{\theta}_2])^2 + \text{Var}(\hat{\theta}_2) = 4 + 1 = 5$.
   $\text{MSE}(\hat{\theta}_1) > \text{MSE}(\hat{\theta}_2)$
   $\Rightarrow$ estimator $\hat{\theta}_2$ is more efficient.