ISyE 6739 Video Assignment 8

1. What is the method of moments estimators for the parameters of the normal distribution? Are they both unbiased estimators?

   Answer:

   $$\hat{\mu} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

   $$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

   No, $\hat{\sigma}^2$ is biased:

   $$\mathrm{E}[\hat{\sigma}^2] = \frac{n-1}{n}\sigma^2.$$

2. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with a pdf $f(x; \theta)$.

   (a) What is the likelihood function of this sample?

   (b) What is the log likelihood function of this sample?

   (c) What is the maximum likelihood estimator of the parameter θ?

   Answer:

   (a) $L(\theta; x) = \prod_{i=1}^{n} f(x_i; \theta)$.

   (b) $l(\theta; x) = \sum_{i=1}^{n} log(f(x_i; \theta))$.

   (c) $\hat{theta} = \arg\max_{\theta} L(\theta; x) = \arg\max_{\theta} l(\theta; x)$.