

ISyE 6739 – Group Activity 11

solutions

1. Assuming equal variances find the sample variance for the populations:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2}{n_1 + n_2 + n_3 - 3}$$

Then the corresponding CI is:

$$\begin{aligned} \frac{1}{2} (\bar{X}_1 + \bar{X}_2) - \bar{X}_3 - t_{\alpha/2, n_1+n_2+n_3-3} S_p \sqrt{\frac{1}{4n_1} + \frac{1}{4n_2} + \frac{1}{4n_3}} &\leq \\ &\leq \frac{1}{2} (\mu_1 + \mu_2) - \mu_3 \leq \\ &\leq \frac{1}{2} (\bar{X}_1 + \bar{X}_2) - \bar{X}_3 + t_{\alpha/2, n_1+n_2+n_3-3} S_p \sqrt{\frac{1}{4n_1} + \frac{1}{4n_2} + \frac{1}{4n_3}} \end{aligned}$$

\Rightarrow 95%-CI for $\frac{1}{2} (\mu_1 + \mu_2) - \mu_3$ is:

$$-1.354 \leq \frac{1}{2} (\mu_1 + \mu_2) - \mu_3 \leq -1.046$$

- 2.

$$n = 9, \quad \bar{X} = 10.5, \quad \sigma = 1, \quad \alpha = 0.05$$

- (a)

$$H_0 : \mu = 10, \quad H_1 : \mu > 10$$

- (b) The CI for hypothesis testing is following:

$$\left[\bar{X} - Z_{\alpha} \frac{\sigma}{\sqrt{n}}, +\infty \right) = \left[10.5 - 1.64 \frac{1}{3}, +\infty \right) = [9.95, +\infty)$$

$10 \in [9.95, +\infty) \Rightarrow$ fail to reject the null hypothesis.

- (c) Compute the test statistic:

$$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{10.5 - 10}{1/3} = 1.5 < 1.64 = Z_{0.05}$$

\Rightarrow fail to reject H_0 .

- (d) Find p-value:

$$\text{p-value} = 1 - \Phi \left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \right) = 1 - \Phi(1.5) = 1 - 0.933 = 0.067 > 0.05 = \alpha$$

\Rightarrow fail to reject H_0 .

- (e) The significance level of this test is $\alpha = 0.05$.

- 3.

$$n = 28, \quad \bar{X} = 1.1786, \quad \sigma = 16, \quad \alpha = 0.05$$

- (a)

$$H_0 : \mu = 0, \quad H_1 : \mu \neq 0$$

(b) The CI for hypothesis testing is following:

$$\left[\bar{X} - Z_{\alpha} \frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\alpha} \frac{\sigma}{\sqrt{n}} \right] = [-4.75, 7.11]$$

$0 \in [-4.75, 7.11] \Rightarrow$ fail to reject the null hypothesis.

(c) Compute the test statistic:

$$|Z_0| = \left| \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \right| = \frac{1.1786 - 0}{16/\sqrt{28}} = 0.39 < 1.96 = Z_{0.025}$$

\Rightarrow fail to reject H_0 .

(d) Find p-value:

$$\text{p-value} = 2 \left[1 - \Phi \left(\left| \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \right| \right) \right] = 2 [1 - \Phi(0.39)] = 0.697 > 0.025 = \alpha$$

\Rightarrow fail to reject H_0 .

(e) The significance level of this test is $\alpha = 0.05$.