ISyE 6739 Video Assignment 15

1. State two-sided null and alternative hypotheses for the test on the variance of a normal distribution. Write the test statistic and the rejection region for the null hypothesis.

   Answer:

   $X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$

   $$H_0: \ \sigma^2 = \sigma_0^2, \quad H_1: \ \sigma^2 \neq \sigma_0^2$$

   Test statistic:

   $$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2},$$

   Rejection region:

   $$\chi_0^2 > \chi_{\alpha/2,n-1}^2 \ \text{ or } \ \chi_0^2 < \chi_{1-\alpha/2,n-1}^2.$$

2. State one-sided null and alternative hypotheses for the test on the population proportion (we want to check if $p < p_0$, variance is known). Write the confidence interval for p and explain how to use it to test the hypothesis.

   Answer:

   $$H_0: \ p = p_0, \quad H_1: \ p < p_0$$

   $100(1-\alpha)\%$ one-sided confidence interval for the population proportion:

   $$\left[-\infty, \hat{p} + Z_\alpha \sqrt{\frac{p_0(1-p_0)}{n}} \right).$$

   If the confidence interval contains p_0 then we fail to reject H_0.

3. Write the formula for Type II error for the test on the population proportion if the alternative is $p > p_0$.

   Answer:

   $$\beta = \Phi\left(\frac{p_0 - p + z_\alpha \sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}} \right).$$

4. What property of MLE can we use to develop a hypothesis test for any distribution?

   Answer:
   We can use the fact that MLE follow normal distribution when sample size is large.