ISyE 6739 Video Assignment 18

1. Consider the randomized complete block design. Write the expressions for total variations, between-treatment, between-block, and within-group variations. What is the dependence between them?

   Answer:

$$\text{Total variations} = SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij} - \bar{y}_{\cdot\cdot})^2$$

$$\text{Between-treatment variations} = SS_{Treatments} = b\sum_{i=1}^{a}(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$$

$$\text{Between-block variations} = SS_{Blocks} = a\sum_{j=1}^{b}(\bar{y}_{\cdot j} - \bar{y}_{\cdot\cdot})^2$$

$$\text{Within-group variations} = SS_E = \sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij} - \bar{y}_{\cdot j} - \bar{y}_{i\cdot} + \bar{y}_{\cdot\cdot})^2$$

$$SS_T = SS_{Treatments} + SS_{Blocks} + SS_E.$$

2. State the null and alternative hypotheses for the two-way ANOVA (randomized complete blocking design). Write the test statistic and the rejection region.

   Answer:

$$H_0: \ \tau_1 = \tau_2 = \cdots = \tau_a = 0, \quad H_1: \ \tau_i \neq 0 \text{ for at least one } i$$

   Test statistic:

$$F_0 = \frac{SS_{Treatments}/(a-1)}{SS_E/(a-1)(b-1)},$$

   Rejection region:

$$F_0 > F_{\alpha,a-1,(a-1)(b-1)}.$$

3. Complete the two-way ANOVA table:

Source of Variation	Sum of squares	Degrees of freedom	Mean Square	F_0
A	(**1**)	$a-1$	$MS_A = \frac{SS_A}{a-1}$	(**6**)
B	$SS_B = \sum_{j=1}^{b} \frac{y_{\cdot j\cdot}^2}{an} - \frac{y_{\cdots}^2}{abn}$	$b-1$	$MS_B = \frac{SS_B}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
$Intaraction$	(**2**)	$(a-1)(b-1)$	(**5**)	$F_0 = \frac{MS_{AB}}{MS_E}$
$Error$	(**3**)	(**4**)	$MS_E = \frac{SS_E}{ab(n-1)}$	
$Total$	$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} \frac{y_{ijk}^2}{n} - \frac{y_{\cdots}^2}{abn}$	$abn-1$		

   Answer:

$$(\mathbf{1}): \quad SS_A = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$(\mathbf{2}): \quad SS_{AB} = \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn} - SS_A - SS_B$$

$$(\mathbf{3}): \quad SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

$$(\mathbf{4}): \quad ab(n-1)$$

$$(\mathbf{5}): \quad MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$$

$$(\mathbf{6}): \quad F_0 = \frac{MS_A}{MS_E}$$