ISyE 6739 – Group Activity 10
solutions

1. To find the CI the following formula was used

$$\bar{X} - \bar{Y} - Z\alpha/2\sqrt{\frac{\sigma_X^2}{n_x} + \frac{\sigma_Y^2}{n_y}} \le \mu_X - \mu_Y \le \bar{X} - \bar{Y} + Z\alpha/2S\sqrt{\frac{\sigma_X^2}{n_x} + \frac{\sigma_Y^2}{n_y}}, \tag{1}$$

   where $\sigma_X^2 = 1,000$, $\sigma_Y^2 = 1,200$.

$$\bar{X} = 71960.12, \quad \bar{Y} = 69127.3, \quad Z_{\alpha/2} = 1.96.$$

   Then the 95% CI for the mean difference is

$$2802.147 \le \mu_X - \mu_Y \le 2863.503.$$

   As we can see, on average attendance for home and away games are not the same: attendance for home games is higher.

2. (a) Use the following formula

$$\bar{X} - \bar{Y} - t_{\alpha/2,n_X+n_Y-2}S\sqrt{\frac{1}{n_x} + \frac{1}{n_y}} \le \mu_X - \mu_Y \le \bar{X} - \bar{Y} + t_{\alpha/2,n_X+n_Y-2}S\sqrt{\frac{1}{n_x} + \frac{1}{n_y}},$$

   where $S^2 = (n_X + n_Y - 2)^{-1}\left((n_X - 1)S_x^2 + (n_Y - 1)S_Y^2\right) = 77.18.$

$$\bar{X} = 22.5, \quad \bar{Y} = 20.9, \quad t_{\alpha/2,n_X+n_Y-2} = 2.12.$$

   Then the 95% CI for the mean difference is

$$-7.234 \le \mu_X - \mu_Y \le 10.434.$$

   On average, Falcons' scores for home and away games are the same with 95% confidence.

   (b) When variances are unknown and not equal then the CI for mean difference is

$$\bar{X} - \bar{Y} - t_{\alpha/2,\nu}\sqrt{\frac{S_X^2}{n_X} + \frac{S_Y^2}{n_Y}} \le \mu_X - \mu_Y \le \bar{X} - \bar{Y} + t_{\alpha/2,n_X+n_Y-2}S\sqrt{\frac{S_X^2}{n_X} + \frac{S_Y^2}{n_Y}},$$

   where

$$\nu = \frac{\left(\frac{S_X^2}{n_X} + \frac{S_Y^2}{n_Y}\right)^2}{\frac{(S_X^2/n_X)^2}{n_X-1} + \frac{(S_Y^2/n_Y)^2}{n_Y-1}} = 15.21.$$

$$t\alpha/2, \nu = 2.13.$$

   Then the 95% CI for the mean difference is

$$-7.261 \le \mu_X - \mu_Y \le 10.461.$$

   On average, Falcons' scores for home and away games are the same with 95% confidence.

3. The corresponding CI is following

$$\frac{S_X^2}{S_Y^2} F_{1-\alpha/2, n_Y-1, n_X-1} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2} F_{\alpha/2, n_Y-1, n_X-1}.$$

$$S_X^2 = 76.29, \;\; S_Y^2 = 77.88, \;\; F_{\alpha/2, n_Y-1, n_X-1} = 4.82, \;\; F_{1-\alpha/2, n_Y-1, n_X-1} = 0.238.$$

Then the 95% CI for the variance ratio is

$$0.233 \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq 4.725.$$

$\Rightarrow$ the Falcons' offense in home and away games is consistent.

4. (a) Check if we can assume the equality of variances:

$$\frac{S_X^2}{S_Y^2} F_{1-\alpha/2, n_Y-1, n_X-1} \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq \frac{S_X^2}{S_Y^2} F_{\alpha/2, n_Y-1, n_X-1}$$

$$S_X^2 = 35.357, \;\; S_Y^2 = 27.878.$$

Then 95% for the ratio of variances of home and away defense performance is:

$$0.302 \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq 6.117.$$

Now we are 95% confident that variances are equal. So we can use CI for equal variances:

$$\bar{X} - \bar{Y} - t_{\alpha/2, n_X+n_Y-2} S \sqrt{\frac{1}{n_x} + \frac{1}{n_y}} \leq \mu_X - \mu_Y \leq \bar{X} - \bar{Y} + t_{\alpha/2, n_X+n_Y-2} S \sqrt{\frac{1}{n_x} + \frac{1}{n_y}},$$

where $S^2 = (n_X + n_Y - 2)^{-1} \left((n_X - 1)S_x^2 + (n_Y - 1)S_Y^2\right) = 31.15.$

$$\bar{X} = 16.75, \;\; \bar{Y} = 20.9, \;\; t_{\alpha/2, n_X+n_Y-2} = 2.12.$$

Then the 95% CI for the mean difference of defense performance is

$$-9.76 \leq \mu_X - \mu_Y \leq 1.462.$$

$\Rightarrow$ means are equal with 95% confidence.

(b) As it is shown above, the CI for the variances ratio is

$$0.302 \leq \frac{\sigma_X^2}{\sigma_Y^2} \leq 6.117.$$

So variances of home and away defense performance are equal with confidence 95%.

5. $100(1-\alpha)$% CI for a population proportion is following:

$$\hat{p}_X - \hat{p}_Y - Z_{\alpha/2} \sqrt{\frac{\hat{p}_X(1-\hat{p}_X)}{n_X} + \frac{\hat{p}_Y(1-\hat{p}_Y)}{n_Y}} \leq p \leq \hat{p}_X - \hat{p}_Y + Z_{\alpha/2} \sqrt{\frac{\hat{p}_X(1-\hat{p}_X)}{n_X} + \frac{\hat{p}_Y(1-\hat{p}_Y)}{n_Y}}$$

$$\hat{p}_{MR} = 0.5 \quad , \hat{p}_{DF} = 0.222.$$

Then the CI is

$$-0.0226 \leq p_{MR} - p_{DF} \leq 0.578.$$