ISyE 6739 Homework 2

due Thursday, Feb 8

1. If X is normally distributed with mean μ and standard deviation four, and given that the probability that X is less than 32 is 0.0228, find the value of μ.

2. X and Y are independent and $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$ Find:

   (a) $\Pr\left\{\left(\frac{X-\mu_X}{\sigma_X}\right)^2 + \left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2 > 2\right\}$

   (b) $\Pr\left\{\frac{\left(\frac{Y-\mu_Y}{\sigma_Y}\right)}{\sqrt{\left(\frac{X-\mu_X}{\sigma_X}\right)^2}} > \frac{\sqrt{3}}{3}\right\}$

3. (7-10) Suppose that random variable X has the continuous uniform distribution

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise,} \end{cases}$$

   Suppose that a random sample of $n = 12$ observations is selected from this distribution. What is the approximate probability distribution of $\bar{X} - 6$? Find the mean and variance of this quantity.

4. (3-109) Because all airline passengers do not show up for their reserved seat, an airline sells 125 tickets for a flight that holds only 120 passengers. The probability that a passenger does not show up is 0.10, and the passengers behave independently.

   (a) What is the probability that every passenger who shows up can take the flight?

   (b) What is the probability that the flight departs with empty seats?

5. (3-129) A trading company uses eight computers to trade on the New York Stock Exchange (NYSE). The probability of a computer failing in a day is 0.005, and the computers fail independently. Computers are repaired in the evening, and each day is an independent trial. Note that if a computer fails on day k then the number of days until it fails is also k.

   (a) What is the probability that all eight computers fail in a day?

   (b) What is the mean number of days until a specific computer fails? What is the mean number of days until it fails three times?

   (c) What is the mean number of days until all eight computers fail on the same day?

6. A random sample of size $n_1 = 49$ is selected from a distribution with a mean of 75 and a standard deviation of 7. A second random sample of size $n_2 = 36$ is taken from another distribution with mean 70 and standard deviation 12. Let $\bar{X}_1$ and $\bar{X}_2$ be the two sample means. Find:

   (a) The probability distribution of $\bar{X}_1 - \bar{X}_2$. Explain.

   (b) The probability that $3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5$.

   (c) Suppose that n_1 and n_2 are unknown but it is given that $n_1 + n_2 = 150$. Find the possible values of n_1 and n_2 if $\mathrm{P}\{\bar{X}_1 - \bar{X}_2 < 6.3554\} = 0.7967$.

7. Let $Z_1, Z_2, \ldots, Z_{101}$ be a random sample from a standard normal distribution. Find:

   (a)
$$\mathrm{P}\{Z_1^2 + Z_{10}^2 + Z_{100}^2 > 7.815\}$$

   (b)
$$\mathrm{P}\left\{\frac{Z_1 + Z_{101}}{\sqrt{Z_2^2 + Z_4^2 + \ldots + Z_{100}^2}} < 0.2598\right\}$$

   (c)
$$\mathrm{P}\left\{\frac{Z_1^2 + Z_2^2}{Z_3^2 + Z_4^2 + Z_5^2} < 10.693\right\}$$

8. (a) $X_1, X_2, X_3 \sim NID(\mu, \sigma^2 = 3)$, what is $\Pr\{S^2 > 3\}$?

   (b) $X_1, X_2, \ldots, X_9 \sim NID(\mu = 100, \sigma^2)$ and $S^2 = 9$, what is $\Pr\{\bar{X} < 101.86\}$?

   (c) $X_1, X_2, \ldots, X_100 \sim NID(\mu = 0, \sigma^2)$ find α such that $\Pr\{\alpha\bar{X} > S\} = 0.05$.

   (d) $X_1, X_2, X_3 \sim NID(\mu_X, \sigma_X^2 = 8)$ and $Y_1, Y_2, Y_3 \sim NID(\mu_Y, \sigma_Y^2 = 4)$
   what is $\Pr\left\{sum_{i=1}^{3}(X_i - \bar{X})^2 > \sum_{i=1}^{3}(Y_i - \bar{Y})^2\right\}$?

9. **extra credit**
   $X_1, X_2, X_3 \sim NID(\mu = 0, \sigma^2)$ what is $\Pr\left\{\frac{X_1}{|X_2 - X_3|} > 1\right\}$?