ISyE 6739 Video Assignment 11

1. Write the expressions for two-sided, one-sided (upper and lower bounds) confidence intervals for mean of a normal distribution (variance is unknown). How can the percentage points of the t-distribution be approximated if the sample size is large?

   Answer:

   $100(1-\alpha)\%$ CI for Mean of a Normal Distribution(two-sided, unknown variance):

   $$\left(\bar{X} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right)$$

   $100(1-\alpha)\%$ CI for Mean of a Normal Distribution(one-sided, unknown variance):

   $$\mu \geq \bar{X} - t_{\alpha,n-1}\frac{s}{\sqrt{n}} \quad \text{(lower bound)}$$

   $$\mu \leq \bar{X} + t_{\alpha,n-1}\frac{s}{\sqrt{n}} \quad \text{(upper bound)}$$

   $t_{\alpha,n-1} \approx Z_\alpha$ when $n > 30$.

2. Write the expressions for two-sided, one-sided (upper and lower bounds) confidence intervals for variance of a normal distribution.

   Answer:

   $100(1-\alpha)\%$ CI for Variance of a Normal Distribution(two-sided):

   $$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

   $100(1-\alpha)\%$ CI for Variance of a Normal Distribution(one-sided):

   $$\sigma^2 \geq \frac{(n-1)s^2}{\chi^2_{\alpha,n-1}} \quad \text{(lower bound)}$$

   $$\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}} \quad \text{(upper bound)}$$

3. Suppose there is a large sample from the distribution that depends on parameter θ. Let $\hat{\theta}$ be an MLE of θ. What is the CI for the parameter θ?

   Answer:

   $100(1-\alpha)\%$ CI for θ:

   $$\hat{\theta} - Z_{\alpha/2}\sqrt{\text{Var}(\hat{\theta})} \leq \theta \leq \hat{\theta} + Z_{\alpha/2}\sqrt{\text{Var}(\hat{\theta})}.$$