Problem 1:

7.2 Let $X_1, \ldots, X_n$ be a random sample from a gamma(α, β) population.

(a) Find the MLE of β, assuming α is known.

Problem 2:

7.6 Let $X_1, \ldots, X_n$ be a random sample from the pdf

$$f(x|\theta) = \theta x^{-2}, \quad 0 < \theta \leq x < \infty.$$

(a) What is a sufficient statistic for θ?
(b) Find the MLE of θ.
(c) Find the method of moments estimator of θ.

Problem 3:

7.10 The independent random variables $X_1, \ldots, X_n$ have the common distribution

$$P(X_i \leq x|\alpha, \beta) = \begin{cases} 0 & \text{if } x < 0 \\ (x/\beta)^\alpha & \text{if } 0 \leq x \leq \beta \\ 1 & \text{if } x > \beta, \end{cases}$$

where the parameters α and β are positive.

(a) Find a two-dimensional sufficient statistic for (α, β).
(b) Find the MLEs of α and β.
(c) The length (in millimeters) of cuckoos' eggs found in hedge sparrow nests can be modeled with this distribution. For the data

22.0, 23.9, 20.9, 23.8, 25.0, 24.0, 21.7, 23.8, 22.8, 23.1, 23.1, 23.5, 23.0, 23.0,

find the MLEs of α and β.

(Part (a) is not required)

Problem 4:

7.1 One observation is taken on a discrete random variable X with pmf $f(x|\theta)$, where $\theta \in \{1, 2, 3\}$. Find the MLE of θ.

x	$f(x\|1)$	$f(x\|2)$	$f(x\|3)$
0	$\frac{1}{3}$	$\frac{1}{4}$	0
1	$\frac{1}{3}$	$\frac{1}{4}$	0
2	0	$\frac{1}{4}$	$\frac{1}{4}$
3	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$
4	$\frac{1}{6}$	0	$\frac{1}{4}$

Extra Problem:

7.13 Let $X_1, \ldots, X_n$ be a sample from a population with double exponential pdf

$$f(x|\theta) = \frac{1}{2}e^{-|x-\theta|}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty.$$

Find the MLE of θ. (*Hint*: Consider the case of even n separate from that of odd n, and find the MLE in terms of the order statistics. A complete treatment of this problem is given in Norton 1984.)