# ISyE 6739 – Group Activity 5
# **solutions**

1. (a) $\Pr\{\bar{X} < S\} = \Pr\left\{\frac{\bar{X}-0}{S/\sqrt{2}} < \sqrt{2}\right\}$,
   $\frac{\bar{X}-0}{S/\sqrt{2}} \sim t(1)$
   $\Pr\{\bar{X} < S\} = 0.804.$
   $\Pr\{\bar{X} < \sigma\} = \Pr\left\{\frac{\bar{X}-0}{\sigma/\sqrt{2}} < \sqrt{2}\right\}$,
   $\frac{\bar{X}-0}{\sigma/\sqrt{2}} \sim N(0,1)$
   $\Pr\{\bar{X} < \sigma\} = 0.921.$

   (b) $\Pr\{X_1 + X_2 > X_3\} = \Pr\{X_1 + X_2 - X_3 > 0\}$,
   $X_1 + X_2 - X_3 \sim N(1+1-1, 1+1+1) = N(1,3)$
   $\Pr\{X_1 + X_2 > X_3\} = 1 - 0.282 = 0.718.$

   (c) i. $\Pr\{2\sum_{i=1}^{2}(X_i-\bar{X})^2+\sum_{i=1}^{2}(Y_i-\bar{Y})^2 > \sigma^2\} = \Pr\left\{\sum_{i=1}^{2}\frac{(X_i-\bar{X})^2}{\sigma^2} + \sum_{i=1}^{2}\frac{(Y_i-\bar{Y})^2}{2\sigma^2} > \frac{1}{2}\right\}$,
   $\sum_{i=1}^{2}\frac{(X_i-\bar{X})^2}{\sigma^2} \sim \chi^2(1)$
   $\sum_{i=1}^{2}\frac{(Y_i-\bar{Y})^2}{2\sigma^2} \sim \chi^2(1)$
   $\Pr\{2\sum_{i=1}^{2}(X_i-\bar{X})^2+\sum_{i=1}^{2}(Y_i-\bar{Y})^2 > \sigma^2\} = 0.779.$

   ii. $\Pr\{2\sum_{i=1}^{2}(X_i-0)^2+\sum_{i=1}^{2}(Y_i-0)^2 > \sigma^2\} = \Pr\left\{\sum_{i=1}^{2}\frac{(X_i-0)^2}{\sigma^2} + \sum_{i=1}^{2}\frac{(Y_i-0)^2}{2\sigma^2} > \frac{1}{2}\right\}$,
   $\sum_{i=1}^{2}\frac{(X_i-0)^2}{\sigma^2} \sim \chi^2(2)$
   $\sum_{i=1}^{2}\frac{(Y_i-0)^2}{2\sigma^2} \sim \chi^2(2)$
   $\Pr\{2\sum_{i=1}^{2}(X_i-0)^2+\sum_{i=1}^{2}(Y_i-0)^2 > \sigma^2\} = 0.974.$

   iii. $\Pr\{2\sum_{i=1}^{2}(X_i-\bar{X})^2 > \sum_{i=1}^{2}(Y_i-\bar{Y})^2\} = \Pr\left\{\frac{\sum_{i=1}^{2}(X_i-\bar{X})^2/\sigma^2}{\sum_{i=1}^{2}(Y_i-\bar{Y})^2/2\sigma^2} > 1\right\}$,
   $\frac{\sum_{i=1}^{2}(X_i-\bar{X})^2/\sigma^2}{\sum_{i=1}^{2}(Y_i-\bar{Y})^2/2\sigma^2} \sim F(1,1)$
   $\Pr\{2\sum_{i=1}^{2}(X_i-\bar{X})^2 > \sum_{i=1}^{2}(Y_i-\bar{Y})^2\} = 0.5.$

   iv. $\Pr\{2\sum_{i=1}^{2}(X_i-0)^2 > \sum_{i=1}^{2}(Y_i-0)^2\} = \Pr\left\{\frac{\sum_{i=1}^{2}(X_i-0)^2/\sigma^2}{\sum_{i=1}^{2}(Y_i-0)^2/2\sigma^2} > 1\right\}$,
   $\frac{\sum_{i=1}^{2}(X_i-0)^2/\sigma^2}{\sum_{i=1}^{2}(Y_i-0)^2/2\sigma^2} \sim F(2,2)$
   $\Pr\{2\sum_{i=1}^{2}(X_i-0)^2 > \sum_{i=1}^{2}(Y_i-0)^2\} = 0.5.$

2. Let $X_i$, $i = 1, \ldots, 200$ denote the indicator of the event $i^th$ order was delayed, $X_i \sim Bernoulli(0.05)$. Then $Y = \sum_{i=1}^{2}00X_i/200$ is the proportion of delayed packages and by CLT $Y \sim N(\mathrm{E}[X_i], \mathrm{Var}(X_i)) = N(0.05, 0.05 * 0.95)$.

   (a) $\mathrm{E}[Y] = 0.05$,
   $\mathrm{Var}(Y) = 0.05 \cdot 0.95/200 = 0.0002375.$

   (b) $\Pr\{Y \leq 0.1\} = 1$

   (c) $200Y \sin Binomial(200, 0.05)$
   $\Pr\{Y \leq 0.1 \cdot 200\} = 0.999$