

ISyE 6739 – Group Activity 14

solutions

- Find the proportion of the numbers that exceed 5

$$\hat{p} = \frac{16}{38} = 0.4211$$

and test the following hypothesis:

$$H_0 : p = \frac{1}{2}, \quad H_1 : p \neq \frac{1}{2}$$

Find the statistic:

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.4211 - 0.5}{\sqrt{\frac{0.5 \cdot 0.5}{38}}} = -0.9733,$$

$$|Z_0| < 1.96 = Z_{0.025}$$

\Rightarrow we fail to reject the null.

- Find proportions of even number for female and male students:

$$p_F = \frac{3}{7} = 0.4286, \quad p_M = \frac{12}{31} = 0.3871,$$

$$p = \frac{3 + 12}{7 + 31} = \frac{15}{38} = 0.3947$$

Then test the following hypothesis:

$$H_0 : p_F = p_M, \quad H_1 : p_F \neq p_M$$

Find the statistic:

$$Z_0 = \frac{p_F - p_M}{\sqrt{p(1-p) \left(\frac{1}{n_F} + \frac{1}{n_M} \right)}} = 0.2028 < 1.96 = Z_{0.025}$$

\Rightarrow fail to reject the null which means the proportion of even numbers does not depend on gender.

Find p -value:

$$p\text{-value} = 2(1 - \Phi(|Z_0|)) = 0.839$$

- First, test if the variances are equal:

$$H_0 : \sigma_{GA}^2 = \sigma_{SC}^2, \quad H_1 : \sigma_{GA}^2 \neq \sigma_{SC}^2$$

Find F -statistic:

$$F_0 = \frac{S_{GA}^2}{S_{SC}^2} = \frac{447.43}{254.23} = 1.76 > 1.628 = F_{0.025, 116, 52}$$

\Rightarrow We reject the null.

Then test the following hypothesis:

$$H_0 : \mu_{GA} = \mu_{SC}, \quad H_1 : \mu_{GA} \neq \mu_{SC}$$

Here we use the statistic for two normal samples with unknown and different variances.

$$\bar{X}_{GA} = 31.95, \quad \bar{X}_{SC} = 33, \quad S_{GA}^2 = 447.428, \quad S_{SC}^2 = 254.231,$$

$$t_0 = \frac{\bar{X}_{GA} - \bar{X}_{SC}}{\sqrt{\frac{S_{GA}^2}{n_{GA}} + \frac{S_{SC}^2}{n_{SC}}}} = -0.358,$$

$$\nu = \frac{\left(\frac{S_{GA}^2}{n_{GA}} + \frac{S_{SC}^2}{n_{SC}}\right)^2}{\left(\frac{S_{GA}/n_{GA}}{n_{GA}-1}\right)^2 + \left(\frac{S_{SC}/n_{SC}}{n_{SC}-1}\right)^2} = 130.72$$

$$t_0 = -0.358 > -1.657 = -t_{0.05, \nu}$$

\Rightarrow We fail to reject the null.