

ISyE 6739 Video Assignment 6

1. Suppose X_1, X_2, \dots, X_n is a sample from the distribution with pdf $f(x, \theta)$. What is a point estimator for the parameter θ ? What is an unbiased point estimator for θ ?

Answer:

A point estimator of θ is the statistic $\hat{\Theta} = h(X_1, X_2, \dots, X_n)$. The point estimator $\hat{\Theta}$ is an unbiased estimator for the parameter θ if $E[\hat{\Theta}] = \theta$.

2. X_1, X_2, \dots, X_n are independent and identically distributed with unknown mean μ and variance σ^2 . Write unbiased estimators for μ and σ^2 . What is the unbiased estimator for σ^2 if μ is known?

Answer:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n X_i,$$

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \hat{\mu})^2,$$

if μ is known:

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \mu)^2.$$

3. Suppose $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators for some parameter θ . It is given that $\text{Var}(\hat{\theta}_1) > \text{Var}(\hat{\theta}_2)$. Which estimator would you prefer?

Answer:

$\hat{\theta}_2$, because it has smaller variance.

4. Suppose X_1, X_2, \dots, X_n is a sample from the distribution with pdf $f(x, \theta)$. Write the definition of the minimum variance unbiased estimator for θ .

Answer:

$\hat{\Theta}$ is the MVUE for θ if:

(a) $\hat{\Theta}$ is unbiased.

(b)

$$\text{Var}(\hat{\Theta}) = \frac{1}{nE \left[\left(\frac{\partial \ln f(x, \theta)}{\partial \theta} \right)^2 \right]}$$