ISyE 6739 Video Assignment 4

due Thursday, Jan 25

1. What is the following expression equal to:

$$\Pr\{\chi^2_{\alpha,\nu} < X\},$$

where $X \sim \chi^2(\nu)$?

Answer:

$$\Pr\{\chi^2_{\alpha,\nu} < X\} = \alpha.$$

2. What is the relationship between the normal and χ^2- distributions? Write the mean and the variance of $\chi^2(\nu)$.

Answer: If $Z_1, Z_2, \ldots, Z_\nu \sim N(0,1)$ and are independent then

$$X = \sum_{i=1}^{\nu} Z_i^2 \sim \chi^2(\nu),$$

$$\mathrm{E}[X] = \nu, \quad \mathrm{Var}(X) = 2\nu.$$

3. Suppose $X \sim \chi^2(\nu)$, $Z \sim N(0,1)$ are independent random variables, $t = \frac{Z}{\sqrt{X/\nu}}$.

   (a) What is the distribution of t?

   (b) What is the distribution of t as $\nu \to \infty$?

   Answer:

   (a) $t \sim t(\nu)$.

   (b) $t \sim N(0,1)$ as $\nu \to \infty$.

4. What is the relationship between $F-$ and χ^2- distributions?

Answer: Suppose $X_1 \sim \chi^2(\nu_1)$, $X_2 \sim \chi^2(\nu_2)$ and X_1 and X_2 are independent. Then

$$\frac{X_1/\nu_1}{X_2/\nu_2} \sim F_{(\nu_1,\nu_2)}.$$

5. Formulate the Central Limit Theorem.

Answer: If $X_1, X_2, \ldots, X_n$ are i.i.d., $\mathrm{E}[X_i] = \mu$, $\mathrm{Var}(X_i) = \sigma^2 < \infty$ then

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1),$$

as $n \to \infty$.