

ISyE 6739 Video Assignment 17

1. Write the expressions for total variations, between-group and within-group variations. What is the dependence between them?

Answer:

$$\text{Total variations} = SST = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{..})^2$$

$$\text{Between-group variations} = SSB = n \sum_{i=1}^a (\bar{y}_{i.} - \bar{y}_{..})^2$$

$$\text{Within-group variations} = SSW = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{i.})^2$$

$$SST = SSB + SSW$$

2. State the null and alternative hypotheses for the ANOVA. Write the test statistic and the rejection region.

Answer:

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_a, \quad H_1 : \text{at least one mean differs from others}$$

Test statistic:

$$F_0 = \frac{SSB/(a-1)}{SSW/(an-a)} = \frac{n \sum_{i=1}^a (\bar{y}_{i.} - \bar{y}_{..})^2 / (a-1)}{\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{i.})^2 / (an-a)},$$

Rejection region:

$$F_0 < -F_{\alpha, a-1, a(n-1)}.$$

3. Suppose we have treatment i and treatment j , and sample sizes in each treatment are n_i and n_j , respectively. Write the formula for the least significant difference for this pair. How can we use it to compare means of treatments i and j ?

Answer:

$$LSD = t_{\alpha/2, N-a} \sqrt{MSE \left(\frac{1}{n_i} + \frac{1}{n_j} \right)},$$

where $N = n_1 + \dots + n_a$.

If $\bar{y}_{i.} - \bar{y}_{j.} > LSD$ then mean treatment i differs from mean treatment j .

4. Explain how to use a normal plot to check the normality.

Answer:

If points lay on a straight line we can assume that observations are normal.