

ISyE 6739 Homework 2

due Thursday, Feb 8

1. If X is normally distributed with mean μ and standard deviation four, and given that the probability that X is less than 32 is 0.0228, find the value of μ .
2. X and Y are independent and $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$ Find:

(a) $\Pr \left\{ \left(\frac{X - \mu_X}{\sigma_X} \right)^2 + \left(\frac{Y - \mu_Y}{\sigma_Y} \right)^2 > 2 \right\}$

(b) $\Pr \left\{ \frac{\left(\frac{Y - \mu_Y}{\sigma_Y} \right)}{\sqrt{\left(\frac{X - \mu_X}{\sigma_X} \right)^2}} > \frac{\sqrt{3}}{3} \right\}$

3. (7-10) Suppose that random variable X has the continuous uniform distribution

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise,} \end{cases}$$

Suppose that a random sample of $n = 12$ observations is selected from this distribution. What is the approximate probability distribution of $\bar{X} - 6$? Find the mean and variance of this quantity.

4. (3-109) Because all airline passengers do not show up for their reserved seat, an airline sells 125 tickets for a flight that holds only 120 passengers. The probability that a passenger does not show up is 0.10, and the passengers behave independently.
 - (a) What is the probability that every passenger who shows up can take the flight?
 - (b) What is the probability that the flight departs with empty seats?
5. (3-129) A trading company uses eight computers to trade on the New York Stock Exchange (NYSE). The probability of a computer failing in a day is 0.005, and the computers fail independently. Computers are repaired in the evening, and each day is an independent trial. Note that if a computer fails on day k then the number of days until it fails is also k .
 - (a) What is the probability that all eight computers fail in a day?
 - (b) What is the mean number of days until a specific computer fails? What is the mean number of days until it fails three times?
 - (c) What is the mean number of days until all eight computers fail on the same day?
6. A random sample of size $n_1 = 49$ is selected from a distribution with a mean of 75 and a standard deviation of 7. A second random sample of size $n_2 = 36$ is taken from another distribution with mean 70 and standard deviation 12. Let \bar{X}_1 and \bar{X}_2 be the two sample means. Find:
 - (a) The probability distribution of $\bar{X}_1 - \bar{X}_2$. Explain.
 - (b) The probability that $3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5$.
 - (c) Suppose that n_1 and n_2 are unknown but it is given that $n_1 + n_2 = 150$. Find the values of n_1 and n_2 if $P\{\bar{X}_1 - \bar{X}_2 < -2.5536\} = 0.7967$.

7. Let Z_1, Z_2, \dots, Z_{101} be a random sample from a standard normal distribution. Find:

(a)

$$P\{Z_1^2 + Z_{10}^2 + Z_{100}^2 > 7.815\}$$

(b)

$$P\left\{\frac{Z_1 + Z_{101}}{\sqrt{Z_2^2 + Z_4^2 + \dots + Z_{100}^2}} < 0.2598\right\}$$

(c)

$$P\left\{\frac{Z_1^2 + Z_2^2}{\sqrt{Z_3^2 + Z_4^2 + Z_5^2}} < 10.693\right\}$$

8. (a) $X_1, X_2, X_3 \sim NID(\mu, \sigma^2 = 3)$, what is $\Pr\{S^2 > 3\}$?

(b) $X_1, X_2, \dots, X_9 \sim NID(\mu = 100, \sigma^2)$ and $S^2 = 9$, what is $\Pr\{\bar{X} < 101.86\}$?

(c) $X_1, X_2, \dots, X_{100} \sim NID(\mu = 0, \sigma^2)$ find α such that $\Pr\{\alpha\bar{X} > S\} = 0.05$.

(d) $X_1, X_2, X_3 \sim NID(\mu_X, \sigma_X^2 = 8)$ and $Y_1, Y_2, Y_3 \sim NID(\mu_Y, \sigma_Y^2 = 4)$

what is $\Pr\left\{\sum_{i=1}^3 (X_i - \bar{X})^2 > \sum_{i=1}^3 (Y_i - \bar{Y})^2\right\}$?

9. **extra credit**

$X_1, X_2, X_3 \sim NID(\mu = 0, \sigma^2)$ what is $\Pr\left\{\frac{X_1}{|X_2 - X_3|} > 1\right\}$?