ISyE 6739 Video Assignment 9

1. Write the main properties of the maximum likelihood estimator.

   Answer:

   Let $\hat{\theta}$ be the MLE of the parameter θ. Than under very general and not restrictive conditions, when the sample size n is large,

   (a) $\hat{\theta}$ is an approximately unbiased estimator for θ,.

   (b) the variance of $\hat{\theta}$ is nearly as small as the variance that could be obtained with any other estimator,

   (c) $\hat{\theta}$ has an approximate normal distribution.

2. Suppose $\hat{\theta}$ is the MLE of the parameter $\theta = (\theta_1, \theta_2)^T \in \mathbb{R}^2$. What is the maximum likelihood estimator of $\Theta = \theta_1 + \theta_2$?

   Answer:

   $\hat{\Theta} = \mathbb{I} \cdot \hat{\theta}$, where $\mathbb{I} \in \mathbb{R}^2$ is the vector of 1's.

3. What is the MLE of

   (a) the rate λ of the exponential distribution,

   (b) the parameter a of the uniform distribution on the interval 0 to a?

   Answer:

   (a) $\hat{\lambda} = \bar{X}^{-1}$,

   (b) $\hat{a} = \max_i(X_i)$.