ISyE 6739 – Statistical Methods

Confidence Intervals – One Population (Chapter 8)

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Outline

Confidence Interval for Mean (μ)

- Normally distributed data
  - with known σ
  - with unknown σ
- Non-normal data with large sample size (n>30)
  - with known/unknown σ

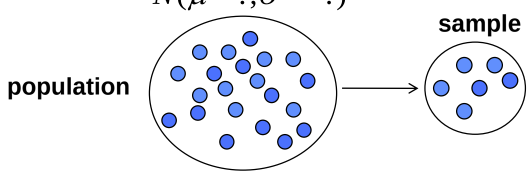
CI for Variance (σ^2)

- Normally distributed data

Point Estimation Vs. Confidence Intervals

- The population distribution parameters are unknown. How to estimate the population parameters from samples?
  - Point Estimation / Interval Estimation

$N(\mu = ?, \sigma^2 = ?)$



$X_1, X_2, \cdots, X_n$

Point Estimation

$\hat{\mu} = \overline{X}$

$\hat{\sigma}^2 = S^2$

Interval Estimation

$$\Pr\{L \le \mu \le U\} = 1 - \alpha \qquad \Pr\{L' \le \sigma^2 \le U'\} = 1 - \alpha$$

1-α : Confidence level

- Both lower and upper bounds are functions of a random sample

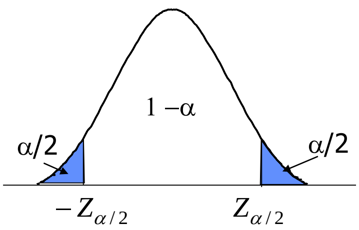
$$L = g(X_1, X_2, \cdots, X_n)$$
$$U = h(X_1, X_2, \cdots, X_n)$$

Confidence Interval (CI) for Mean

100(1-α)% CI for Mean of a Normal Distribution (two-sided, known variance)

- **For mean μ from a normal population with known σ,**

$$X_1, X_2, \cdots, X_n \sim NID(\mu, \sigma^2) \rightarrow \bar{X} \sim N(\mu, \frac{\sigma^2}{n}) \Rightarrow \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$$



$$\boxed{P\left\{-z_{\alpha/2} \le \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \le z_{\alpha/2}\right\} = 1-\alpha}$$

Confidence level

CI Lower **CI Upper**

$$\boxed{\bar{x} - Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}} \le \mu \le \boxed{\bar{x} + Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}}$$

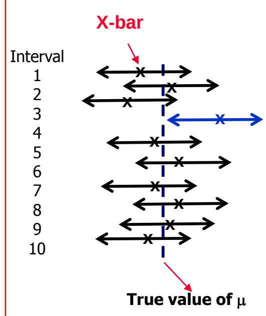
This CI can be used for mean of non-normal distributions when n>30

Example

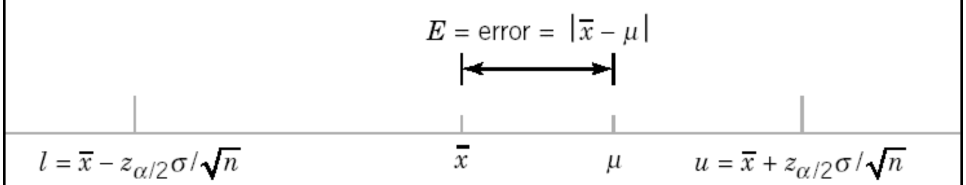
ASTM Standard E23 defines standard test methods for notched bar impact testing of metallic materials. The Charpy V-notch (CVN) technique measures impact energy and is often used to determine whether or not a material experiences a ductile-to-brittle transition with decreasing temperature. Ten measurements of impact energy (J) on specimens of A238 steel cut at 60°C are as follows: 64.1, 64.7, 64.5, 64.6, 64.5, 64.3, 64.6, 64.8, 64.2, and 64.3. Assume that impact energy is normally distributed with $\sigma = 1J$. We want to find a 95% CI for μ, the mean

CI Interpretation

- **Assume to calculate a 95% CI for μ based on a sample of 4 observations.**
- **If 100 samples are collected, and those 100 sample means are computed. Then, 100 computed CIs are obtained correspondingly,**
  - **(0.85,1.15), (0.8, 1.1), (0.9,1.0),…**
  - **Some of those computed CIs may contain the true mean, and some may not contain the true mean.**
- **"95% confidence" means that in the long run, 95% of all computed CIs will contain the true mean μ.**
  - **In other words, 5% of these computed CIs will not trap the true mean.**



Choice of Sample Size



If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error $|\bar{x}-\mu|$ will not exceed a specified amount E when the sample size is

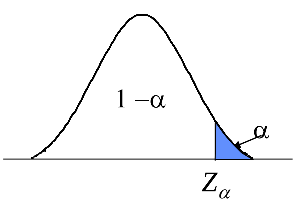
$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 \tag{8-8}$$

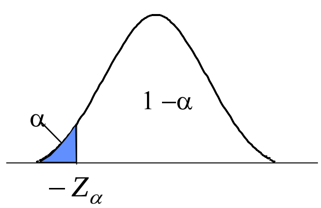
Choice of Sample Size

Example 8-2

To illustrate the use of this procedure, consider the CVN test described in Example 8-1, and suppose that we wanted to determine how many specimens must be tested to ensure that the 95% CI on μ for A238 steel cut at 60°C has a length of at most 1.0J.

100(1-α)% CI for Mean of a Normal Distribution (one-sided, known variance)





$$\Pr\left\{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le Z_\alpha\right\} = 1-\alpha$$

$$\boxed{\mu \ge \bar{X} - Z_\alpha \times \frac{\sigma}{\sqrt{n}}}$$

Lower bound

$$\Pr\left\{-Z_\alpha \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\right\} = 1-\alpha$$

$$\boxed{\mu \le \bar{X} + Z_\alpha \times \frac{\sigma}{\sqrt{n}}}$$

Upper bound

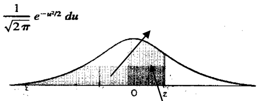
This CI can be used for mean of non-normal distributions when n>30

Example

Example: The strength of a disposable plastic beverage container is being investigated. The strengths are normally distributed, with a known standard deviation of 15 psi. A sample of 20 plastic containers has a mean strength of 246 psi. Compute the 95% lower bound CI for the process mean.

Appendix II Cumulative Standard Normal Distribution

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$$



z	0.00	0.01	0.02	0.03	0.04	z
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.0
0.1	0.53983	0.54379	0.54776	0.55172	0.55567	0.1
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.2
0.3	0.61791	0.62172	0.62551	0.62930	0.63307	0.3
0.4	0.65542	0.65910	0.62276	0.66640	0.67003	0.4
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.5
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.6
0.7	0.75803	0.76115	0.76424	0.76730	0.77035	0.7
0.8	0.78814	0.79103	0.79389	0.79673	0.79954	0.8
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.9
1.0	0.84134	0.84375	0.84613	0.84849	0.85083	1.0
1.1	0.86433	0.86650	0.86864	0.87076	0.87285	1.1
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	1.2
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	1.3
1.4	0.91924	0.92073	0.92219	0.92364	0.92506	1.4
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	1.5
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	1.6
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	1.7
1.8	0.96407	0.96485	0.96562	0.96637	0.96711	1.8
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	1.9
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	2.0
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	2.1
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	2.2
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	2.3
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	2.4
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	2.5
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	2.6
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	2.7
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	2.8
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	2.9
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	3.0
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	3.1
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	3.2
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	3.3
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	3.4
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	3.5
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	3.6
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	3.7
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	3.8
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	3.9

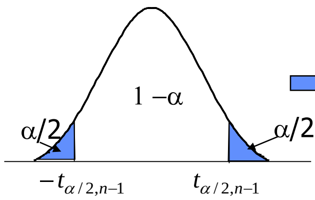
Appendix II *(Continued)*

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$$

z	0.05	0.06	0.07	0.08	0.09	z
0.0	0.51994	0.52392	0.52790	0.53188	0.53586	0.0
0.1	0.55962	0.56356	0.56749	0.57142	0.57534	0.1
0.2	0.59871	0.60257	0.60642	0.61026	0.61409	0.2
0.3	0.63683	0.64058	0.64431	0.64803	0.65173	0.3
0.4	0.67364	0.67724	0.68082	0.68438	0.68793	0.4
0.5	0.70884	0.71226	0.71566	0.71904	0.72240	0.5
0.6	0.74215	0.74537	0.74857	0.75175	0.75490	0.6
0.7	0.77337	0.77637	0.77935	0.78230	0.78523	0.7
0.8	0.80234	0.80510	0.80785	0.81057	0.81327	0.8
0.9	0.82894	0.83147	0.83397	0.83646	0.83891	0.9
1.0	0.85314	0.85543	0.85769	0.85993	0.86214	1.0
1.1	0.87493	0.87697	0.87900	0.88100	0.88297	1.1
1.2	0.89435	0.89616	0.89796	0.89973	0.90147	1.2
1.3	0.91149	0.91308	0.91465	0.91621	0.91773	1.3
1.4	0.92647	0.92785	0.92922	0.93056	0.93189	1.4
1.5	0.93943	0.94062	0.94179	0.94295	0.94408	1.5
1.6	0.95053	0.95154	0.95254	0.95352	0.95448	1.6
1.7	0.95994	0.96080	0.96164	0.96246	0.96327	1.7
1.8	0.96784	0.96856	0.96926	0.96995	0.97062	1.8
1.9	0.97441	0.97500	0.97558	0.97615	0.97670	1.9
2.0	0.97982	0.98030	0.98077	0.98124	0.98169	2.0
2.1	0.98422	0.98461	0.98500	0.98537	0.98574	2.1
2.2	0.98778	0.98809	0.98840	0.98870	0.98899	2.2
2.3	0.99061	0.99086	0.99111	0.99134	0.99158	2.3
2.4	0.99286	0.99305	0.99324	0.99343	0.99361	2.4
2.5	0.99461	0.99477	0.99492	0.99506	0.99520	2.5
2.6	0.99598	0.99609	0.99621	0.99632	0.99643	2.6
2.7	0.99702	0.99711	0.99720	0.99728	0.99736	2.7
2.8	0.99781	0.99788	0.99795	0.99801	0.99807	2.8
2.9	0.99841	0.99846	0.99851	0.99856	0.99861	2.9
3.0	0.99886	0.99889	0.99893	0.99897	0.99900	3.0
3.1	0.99918	0.99921	0.99924	0.99926	0.99929	3.1
3.2	0.99942	0.99944	0.99946	0.99948	0.99950	3.2
3.3	0.99960	0.99961	0.99962	0.99964	0.99965	3.3
3.4	0.99972	0.99973	0.99974	0.99975	0.99976	3.4
3.5	0.99981	0.99981	0.99982	0.99983	0.99983	3.5
3.6	0.99987	0.99987	0.99988	0.99988	0.99989	3.6
3.7	0.99991	0.99992	0.99992	0.99992	0.99992	3.7
3.8	0.99994	0.99994	0.99995	0.99995	0.99995	3.8
3.9	0.99996	0.99996	0.99996	0.99997	0.99997	3.9

100(1-α)% CI for Mean of a Normal Distribution (two-sided, unknown variance)

$$X_1, X_2, \cdots, X_n \sim NID(\mu, \sigma^2) \rightarrow \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$$



$$P\left\{-t_{\alpha/2,n-1} \le \frac{\bar{x}-\mu}{s/\sqrt{n}} \le t_{\alpha/2,n-1}\right\} = 1-\alpha$$

$$\boxed{\bar{x} - t_{\alpha/2,n-1} s/\sqrt{n} \le \mu \le \bar{x} + t_{\alpha/2,n-1} s/\sqrt{n}}$$

Note that $t_{\alpha/2,n-1} \approx Z_{\alpha/2}$; for $n > 30$

Example

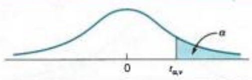
An article in the journal *Materials Engineering* (1989, Vol. II, No. 4, pp. 275–281) describes the results of tensile adhesion tests on 22 U-700 alloy specimens. The load at specimen failure is as follows (in megapascals):

19.8	10.1	14.9	7.5	15.4	15.4
15.4	18.5	7.9	12.7	11.9	11.4
11.4	14.1	17.6	16.7	15.8	
19.5	8.8	13.6	11.9	11.4	

The sample mean is $\bar{x} = 13.71$, and the sample standard deviation is $s = 3.55$.

T table

Percentage Points of the t Distribution[a]



ν	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.727	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.49	4.019	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.20	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.992
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792

100(1-α)% CI for Mean of a Normal Distribution (one-sided, unknown variance)

- **x-bar and s are based on sample data from a normal population.**
- **Upper confidence bound for μ:** $\mu \le \bar{x} + t_{\alpha,n-1} \cdot \frac{s}{\sqrt{n}}$
- **Lower confidence bound for μ:** $\mu \ge \bar{x} - t_{\alpha,n-1} \cdot \frac{s}{\sqrt{n}}$

Note that $t_{\alpha,n-1} \approx Z_\alpha ; \text{for}\, n > 30$

Example

- **Suppose mean load at failure from a tensile adhesion test is 13.71 MPa. *s* = 3.55; n=22. Construct a one sided 95% lower CI for μ**

$$\mu \geq \bar{x} - t_{\alpha,n-1} \cdot \frac{s}{\sqrt{n}}$$

$$L = 13.71 - 1.721 \times \frac{3.55}{\sqrt{22}} = 12.41$$

$$t_{0.05,22-1} = 1.721$$

100(1-α)% CI for Variance of a Normal Distribution

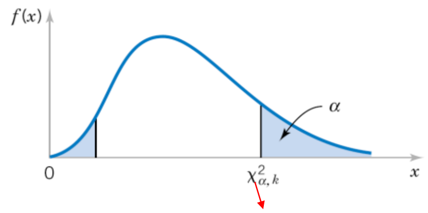
- Using chi-square, we can estimate a CI for the variance, σ^2, of a normal population.

Two-sided CI

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1} \Rightarrow P\left\{\chi^2_{1-\alpha/2,n-1} \leq \frac{(n-1)s^2}{\sigma^2} \leq \chi^2_{\alpha/2,n-1}\right\} = 1-\alpha \Rightarrow \boxed{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}}$$

One-sided Upper bound $\boxed{\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}}$

One-sided Lower bound $\boxed{\frac{(n-1)s^2}{\chi^2_{\alpha,n-1}} \leq \sigma^2}$



chi-squared critical Value

Example

An automatic filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ (fluid ounces)^2. If the variance of fill volume is too large, an unacceptable proportion of bottles will be under- or overfilled. We will assume that the fill volume is approximately normally distributed. A 95% upper-confidence interval is found from Equation 8-22 as follows:

$$\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{0.95,19}}$$

or

$$\sigma^2 \leq \frac{(19)0.0153}{10.117} = 0.0287 \text{ (fluid ounce)}^2$$

This last expression may be converted into a confidence interval on the standard deviation σ by taking the square root of both sides, resulting in

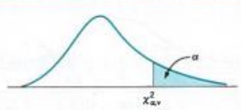
$$\sigma \leq 0.17$$

Therefore, at the 95% level of confidence, the data indicate that the process standard deviation could be as large as 0.17 fluid ounce.

Chi-square Table

■ APPENDIX III

Percentage Points of the χ^2 Distribution[a]



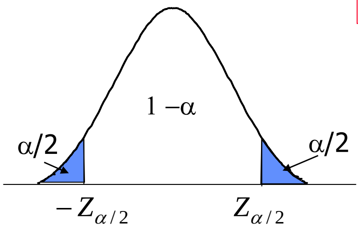
	α								
ν	0.995	0.990	0.975	0.950	0.500	0.050	0.025	0.010	0.005
1	0.00 +	0.00 +	0.00 +	0.00 +	0.45	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	1.39	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	2.37	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	3.36	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	4.35	11.07	12.38	15.09	16.75
6	0.68	0.87	1.24	1.64	5.35	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	6.35	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	7.34	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	8.34	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	9.34	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	10.34	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	11.34	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	12.34	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	13.34	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	14.34	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	15.34	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	16.34	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	17.34	28.87	31.53	34.81	37.16
19	6.884	7.63	8.91	10.12	18.34	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	19.34	31.41	34.17	37.57	40.00
25	10.52	11.52	13.12	14.61	24.34	37.65	40.65	44.31	46.93

100(1-α)% CI for a Population Proportion with large sample size

$$X_1, X_2, \cdots, X_n \sim Bernoulli(p)$$

- Using CLT, we can estimate a CI for the population proportion.

$$\hat{p} = \frac{\sum_{i=1}^{n} X_i}{n} \sim N\left(p, \frac{p(1-p)}{n}\right) \Rightarrow \boxed{\Pr\left\{-z_{\alpha/2} \le \frac{\hat{p}-p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} \le z_{\alpha/2}\right\} = 1-\alpha}$$



$$\boxed{\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

Two-sided CI

Example

In a random sample of 85 automobile engine crankshaft bearings, 10 have a surface finish that is rougher than the specifications allow. Therefore, a point estimate of the proportion of bearings in the population that exceeds the roughness specification is $\hat{p} = x/n = 10/85 = 0.12$. A 95% two-sided confidence interval for p is computed from Equation 8-25 as

Choice of Sample Size

The sample size for a specified value *E* is given by

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) \qquad (8\text{-}26)$$

An upper bound on *n* is given by (i.e., at least (1-α)100% confidence)

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (0.25) \qquad (8\text{-}27)$$

Example

Consider the situation in Example 8-7. How large a sample is required if we want to be 95% confident that the error in using $\hat{p}$ to estimate p is less than 0.05? Using $\hat{p} = 0.12$ as an initial estimate of p, we find from Equation 8-26 that the required sample size is

$$n = \left(\frac{z_{0.025}}{E}\right)^2 \hat{p}(1-\hat{p}) = \left(\frac{1.96}{0.05}\right)^2 0.12(0.88) \cong 163$$

If we wanted to be *at least* 95% confident that our estimate $\hat{p}$ of the true proportion p was within 0.05 regardless of the value of p, we would use Equation 8-27 to find the sample size

$$n = \left(\frac{z_{0.025}}{E}\right)^2 (0.25) = \left(\frac{1.96}{0.05}\right)^2 (0.25) \cong 385$$

Notice that if we have information concerning the value of p, either from a preliminary sample or from past experience, we could use a smaller sample while maintaining both the desired precision of estimation and the level of confidence.

One-sided 100(1-α)% CI for a Population Proportion with large sample size

$X_1, X_2, \cdots, X_n \sim Bernoulli(p)$

- Using CLT, we can estimate a CI for the population proportion.

$$\hat{p} = \frac{\sum_{i=1}^{n} X_i}{n} \sim N(p, \frac{p(1-p)}{n})$$

$$\boxed{\hat{p} - Z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p}$$

One-sided
Lower bound

$$\boxed{p \le \hat{p} + Z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

One-sided
Upper bound

$$\boxed{\hat{p} - Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

A General Large Sample Confidence Interval

If $\hat{\theta}$ is an MLE estimator of θ, then for large samples

$$\hat{\theta} \approx N\left(\theta, \mathrm{var}(\hat{\theta})\right) \rightarrow Z = \frac{\hat{\theta} - \theta}{\sqrt{\mathrm{var}(\hat{\theta})}}$$

$\sqrt{\mathrm{var}(\hat{\theta})}$

$$\hat{\Theta} - z_{\alpha/2}\sigma_{\hat{\Theta}} \le \theta \le \hat{\Theta} + z_{\alpha/2}\sigma_{\hat{\Theta}} \qquad (8\text{-}14)$$

Example: Poisson(λ)