ISyE 6739 Video Assignment 13

1. State two-sided null and alternative hypotheses for the test on the mean of a normal distribution(variance is known). Write the test statistic and the rejection region for the null hypothesis.

   Answer:

   $X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$

   $$H_0 : \mu = \mu_0, \quad H_1 : \mu \neq \mu_0$$

   Test statistic:

   $$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}},$$

   Rejection region:

   $$|Z_0| > Z_{\alpha/2}.$$

2. State one-sided null and alternative hypotheses for the test on the mean of a normal distribution(we want to check if $\mu > \mu_0$, variance is known). Write the confidence interval for the mean and explain how to use it to test the hypothesis.

   Answer:

   $X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$

   $$H_0 : \mu = \mu_0, \quad H_1 : \mu > \mu_0$$

   $100(1 - \alpha)$ one-sided confidence interval for the mean:

   $$\left[\bar{X} - Z_\alpha \sqrt{\frac{\sigma^2}{n}}, +\infty \right).$$

   If the confidence interval contains μ_0 then we fail to reject H_0.

3. State one-sided null and alternative hypotheses for the test on the mean of a normal distribution(we want to check if $\mu < \mu_0$, variance is known). What is a p-value for this test? How can we use it to test the hypothesis?

   Answer:

   $X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$

   $$H_0 : \mu = \mu_0, \quad H_1 : \mu < \mu_0$$

   $$p\text{-}value = P\{Z < Z_0\} = \Phi\left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \right).$$

   If $p\text{-}value < \alpha$ then H_0 is rejected on significance level of α.