ISyE 6739 – Group Activity 6

Names: Group Number:

Problem 1.

The lifetime (in hours) Y_i of electronic components are independently exponentially distributed random variables with λ= 1/2. If a component is failed, it is replaced with a new one. What is the probability that the 50th component is failed after 200 hours?

Problem 2.

In a pipe milling process, the eccentricity of inner circle and outer circle, respectively denoted by X and Y, follow normal distributions with X ~ N(0, $\sigma^2 = 0.01$) and Y ~ N(0, $\sigma^2 = 0.04$). A standard requires that the absolute ratio of the eccentricity of the inner circle to outer circle to be less than 0.1. What is the percentage of defective pipes?

If 5 parts are randomly selected, find $\Pr\left(|\frac{\bar{X}}{\bar{Y}}| > 0.1\right)$?

Problem 3. Is $\bar{X}$ an unbiased estimator for the parameter of a Geometric distribution. What is the variance of $\bar{X}$? It is known that the mean and variance of a Geometric random variable are $(\frac{1}{p})$ and $(\frac{1}{p})(\frac{1}{p} - 1)$, respectively.

Problem 4. Suppose $X \sim Uniform(\theta, 3\theta)$. Is $\bar{X}$ an unbiased estimator for θ? If not, what is the bias? Suggest an unbiased estimator for θ? What is the variance of your proposed estimator? It is known that the mean and variance of a uniform random variable are (2θ) and $(\frac{\theta^2}{3})$.

Problem 5. Let $X_1, X_2, \ldots X_7$ denote a random sample from a population with mean μ and variance σ^2. Calculate the bias and variance of the following estimators of μ. Which estimator is more efficient?

$$\hat{\Theta}_1 = \frac{\sum_{i=1}^{7} X_i}{7}$$

$$\hat{\Theta}_2 = \frac{2X_1 - X_6 + X_4}{2}$$

Problem 6. Steel rods in a large batch have lengths with mean μ and variance σ^2. Two trainees are set the task of estimating μ. Trainees A and B take random samples of 20 rods and 5 rods, respectively. Based on these samples, Trainee A calculates the mean length to be $\bar{X}_A$ and Trainee B calculates the mean length to be $\bar{X}_B$. However, the trainees decide they may get a better estimate by combining their individual estimates. They consider two different estimators:

$$\hat{\mu}_1 = \frac{1}{2}(\bar{X}_A + \bar{X}_B)$$

$$\hat{\mu}_2 = \frac{4}{5}\bar{X}_A + \frac{1}{5}\bar{X}_B$$

(a) Are these estimators of μ unbiased? Show your work.

(b) Calculate the bias and variance of these estimators. Which estimator is more efficient?

Problem 7. Show that $\bar{X}$ is an MVUE for p of a Bernoulli distribution.

Problem 8. Show that $\bar{X}$ is an MVUE for λ of a Poisson distribution.

Problem 9. Is S^2 an MVUE for σ^2 of a normal distribution.