

## ISyE 6739 Video Assignment 5

1. Let  $X_1, X_2, \dots, X_n$  be normally distributed with mean  $\mu$  and variance  $\sigma^2$ . Let  $S^2$  denote the sample variance. What is the distribution of  $\frac{(n-1)S^2}{\sigma^2}$ ?

*Answer:*

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1).$$

2. What is the relationship between  $\chi^2$ -distribution with 2 degrees of freedom and the exponential distribution?

*Answer:*

$$\chi^2(2) = \text{Exp}(0.5).$$

3. Suppose  $X_1, X_2, \dots, X_{n_1} \sim NID(\mu_1, \sigma_1^2)$ ,  $Y_1, Y_2, \dots, Y_{n_2} \sim NID(\mu_2, \sigma_2^2)$ .

(a) What is the distribution of  $\frac{\bar{X}-\mu}{S/\sqrt{n_1}}$ ?

(b) What is the estimate  $\hat{\sigma}^2$  of the sample variance if we assume that  $\sigma_1^2 = \sigma_2^2$ ?

*Answer:*

(a)  $\frac{\bar{X}-\mu}{S/\sqrt{n_1}} \sim t(n_1-1)$

(b)  $\hat{\sigma}^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1) + (n_2-1)}$