ISyE 6739 – Group Activity 3
solution

1. Denote a number of nonconforming units from a sample of 50 units by X. The fraction of nonconforming units is 0.02 therefore $X \sim Binomial(50, 0.02)$. Then:

$$\Pr\{\hat{p} \le 0.04\} = \Pr\{X/50 \le 0.04\} = \Pr\{X \le 2\} =$$

$$= \Pr\{X = 0\} + \Pr\{X = 1\} + \Pr\{X = 2\} =$$

$$= \binom{50}{0} 0.02^0 (1 - 0.02)^{50} + \binom{50}{1} 0.02^1 (1 - 0.02)^{49} + \binom{50}{2} 0.02^2 (1 - 0.02)^{48} = 0.922$$

2. Denote X as a number of patients that must be seen to find one with high blood pressure. X is geometrically distributed with probability of success 0.15, which means that $\Pr\{X = k\} = 0.15(1 - 0.15)^{k-1}$, $k = 1, 2, \ldots$.

   (a) $\Pr\{X = 3\} = 0.15 \cdot 0.85^2 = 0.1084$.

   (b) $\mathrm{E}[X] = \frac{1}{0.15} = 0.667$.

   (c) Let Y denote a number of patients with high blood pressure on a specific day. $Y \sim Binomial(50, 0.15)$. Then:

$$\Pr\{Y = 10\} = \binom{50}{10} 0.15^{10} (1 - 0.15)^{40} = 0.089.$$

3. Denote a number of nonconforming units in a sample of five by X.
   With replacement:

$$\Pr\{X = 1\} = 5 \frac{3 \cdot 27^4}{30^5} = \frac{6561}{100,000} = 0.328,$$

$$\Pr\{X \ge 1\} = 1 - \Pr\{X < 1\} = 1 - \Pr\{X = 0\} = 1 - \frac{27^5}{30^5} = \frac{40951}{100,000} = 0.40951.$$

   Without replacement:

$$\Pr\{X = 1\} = \frac{\binom{3}{1}\binom{27}{4}}{\binom{30}{5}} = \frac{75}{203} = 0.37,$$

$$\Pr\{X \ge 1\} = 1 - \Pr\{X < 1\} = 1 - \Pr\{X = 0\} = 1 - \frac{\binom{3}{0}\binom{27}{5}}{\binom{30}{5}} = \frac{88}{203} = 0.43.$$

4. Let X and Y denote numbers of errors in different bills. X and Y are independent and Poisson distributed with parameter $\lambda = 0.01$.

$$\Pr\{X = 1\} = \frac{\lambda e^{-\lambda}}{1!} = 0.01 \cdot e^{-0.01} = 0.01$$

   X and Y are independent thus $X + Y \sim Poisson(2\lambda)$

$$\Pr\{X + Y \ge 1\} = 1 - \Pr\{X + Y = 0\} = 1 - \frac{e^{-2\lambda}}{0!} = e^{-0.02} = 0.0198.$$

5. Let L, W and P denote the length, the width and the perimeter of a steel sheet, respectively. $L \sim N(10, 1)$, $W \sim N(15, 1) \Rightarrow P = 2(L + W) \sim N(2(10 + 15), 4(1 + 1)) = N(50, 8)$.

$$\Pr\{P \leq 47, P \geq 52\} = 1 - \Pr\{47 \leq P \leq 52\} = 1 - \Pr\{\frac{47-50}{2\sqrt{2}} \leq Z \leq \frac{52-50}{2\sqrt{2}}\} =$$

$$= 1 - \Phi(0.707) + \Phi(-1.061) = 1 - 0.76115 + 0.14457 = 0.383$$

The percentage of waste product is 38.3%.
Denote the number of defective sheets out of 5 by X. $X \sim Binomial(5, 1 - \Pr\{47 \leq P \leq 52\}) = Binomial(5, 0.383)$.

$$\Pr\{X \geq 1\} = 1 - \Pr\{X = 0\} = 1 - \binom{5}{0} 0.383(1 - 0.383)^5 = 0.91.$$