ISyE 6739 –Statistical Methods
Review of Probability (Chapters 3, 4 & 7)

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List of Topics

- Review of Probability
- Discrete Distributions
  – Binomial
  – Poisson
  – Geometric
  – Negative-Binomial
- Continuous Distributions
  – Exponential
  – Normal
  – Chi-square
  – Student's t
  – F

Review of Probability Distributions

Random Variables

- Random Experiment: An experiment that can result in different outcomes, even if repeated in the same manner every time, e.g., rolling a dice, producing a part.

- Random Variable: A variable that associates a number with the outcome of a random experiment is called a random variable.

  X: Compressive strength of each specimen

  Y: Number of defectives in a sample of *n* produced parts

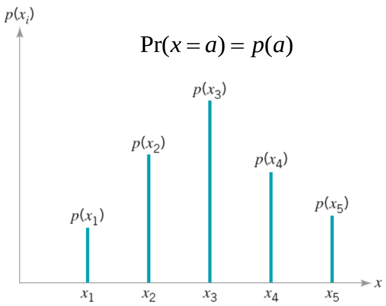
- Type of RVs:
  - A discrete random variable is a rv with a finite (or countably infinite) range. They are usually integer counts, e.g., number of errors or number of bit errors per 100,000 transmitted (rate).
  - A continuous random variable is a rv with an interval (either finite or infinite) of real numbers for its range. Its precision depends on the measuring instrument.

Discrete Distributions

Discrete distributions. When the parameter being measured can only take on certain values, such as the integers 0, 1, 2, . . . , the probability distribution is called a *discrete distribution.*

Examples:Quality of a part (Ok = 0 , Not ok = 1), Number of typos on a typed page (0,1,2, …)

Probability mass function (pmf): $p(x) \text{ is pmf} \leftrightarrow \begin{cases} p(x) \geq 0 \\ \sum_{-\infty}^{+\infty} p(x) = 1 \end{cases}$



Examples: X:Number of observed 6 in 2 rolls of a dice

a	0	1	2
Pr(X=a)	$(5/6)^2$	$2(1/6)^1(5/6)^1$	$(1/6)^2$

$$\Pr(X = a) = \binom{2}{a}\left(\frac{1}{6}\right)^a\left(\frac{5}{6}\right)^{2-a} \;;\; a = 0,1,2$$

Cumulative Distribution Function (CDF)

Cumulative Distribution Function (CDF): The CDF of a discrete random variable X is denoted by F(X=a) and is calculated by

$$F(X = a) = F(a) = \Pr(x \leq a) = \sum_{i=-\infty}^{a} p(i)$$

Some Properties:

1) $0 \leq F(a) \leq 1$

2) if $x \leq y, \text{then } F(x) \leq F(y)$

3) $\Pr(a < x \leq b) = \Pr(x \leq b) - \Pr(x \leq a) = F(b) - F(a)$

4) $\Pr(a \leq x \leq b) = \Pr(x \leq b) - \Pr(x \leq a-1) = F(b) - F(a-1)$

Continuous Distributions

Continuous distributions. When the variable being measured is expressed on a continuous scale, its probability distribution is called a *continuous distribution.*

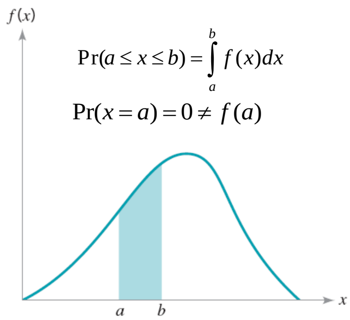
Examples: Diameter of piston rings, viscosity of oil, customers' Waiting time

Probability density function (pdf):

$$f(x) \text{ is pdf} \leftrightarrow \begin{cases} f(x) \ge 0 \\ \int_{-\infty}^{+\infty} f(x)dx = 1 \end{cases}$$

$$\Pr(a \le x \le b) = \int_a^b f(x)dx$$

$$\Pr(x = a) = 0 \ne f(a)$$



Cumulative disruption function (cdf):

$$F(X = a) = F(a) = \Pr(x \le a) = \int_{-\infty}^{a} f(x)dx$$

Example: The probability distribution of X is

$$f(x) = 2e^{-kx}, \;\; x \ge 0;$$

(a) Find the appropriate value of k;

(b) Find the Pr(X>0.5).

Example: The product quality characteristic measurement is a random variable X with probability distribution

$$f(x) = \begin{cases} 4(x - 11.75) & 11.75 \le x \le 12.25 \\ 4(12.75 - x) & 12.25 < x \le 12.75 \end{cases}$$

If the quality specification limit is that X must be less than12.50, what is the percentage of products to be nonconforming?

a) 0.125

b) 0.5

c) 0.875

Mean, Variance and Moments

- Mean is a measure of the center of a probability distribution.

$$E(x) = \mu = \sum_{-\infty}^{+\infty} xp(x) \qquad \text{Discrete RVs}$$

$$E(x) = \mu = \int_{-\infty}^{+\infty} xf(x)dx \qquad \text{Continuous RVs}$$

- Moments are measures related to the location/shape of probability distribution.

$$E(x^r) = \sum_{-\infty}^{+\infty} x^r p(x) \qquad \text{Discrete RVs}$$

$$E(x^r) = \int_{-\infty}^{+\infty} x^r f(x)dx \qquad \text{Continuous RVs}$$

- Variance is a measure of the dispersion or variability of a probability distribution.

$$V(x) = \sigma^2 = \sum_{-\infty}^{+\infty} (x-\mu)^2 p(x) \qquad \text{Discrete RVs}$$

$$V(x) = \sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx \qquad \text{Continuous RVs}$$

$$V(x) = E(x^2) - [E(x)]^2$$

Example: Find mean and variance for the following RVs

a) X: Number of observed 6 in 2 rolls of a dice

a	0	1	2
Pr(X=a)	$(5/6)^2$	$2(1/6)^1(5/6)^1$	$(1/6)^2$

b) X: exam time limited between 0 and 1 hour.

$f(x) = \alpha(x)^{\alpha-1}; 0 < x < 1$

Useful Results on Mean and Variance

If x is a random variable and a is a constant, then

$E(a+x)=a+E(x)$

$E(a\times x)=aE(x)$

$V(a+x)=V(x)$

$\boxed{V(a\times x)=a^2V(x)}$

If $x_1, x_2, \ldots, x_n$ are random variables,

$\boxed{E(a_1x_1+\ldots+a_nx_n)=a_1E(x_1)+\ldots+a_nE(x_n)}$

If they are **mutually independent**, and $a_1, \ldots, a_n$ are constants

$\boxed{V(a_1x_1+\ldots+a_nx_n)=a_1^2V(x_1)+\ldots+a_n^2V(x_n)}$

Review of Discrete Distributions

Bernoulli Distribution

$X \sim Bernoulli(p) \quad x = 0,1$ $\quad$ *p*: probability of success

Application: To model the outcome of a Bernoulli trial with two possible outcomes (Success = 1, Failure = 0).

Example: The outcome of inspecting part quality;
Nonconforming (Success=1), Conforming (Failure=0)

pmf, mean and variance:

$$P(x) = p^x (1-p)^{1-x}; \ x = 0,1$$

$$E(x) = p$$

$$Var(x) = p(1-p)$$

Binomial Distribution

$X \sim Bin(n,p) \quad x = 0,1,\ldots,n$ *p*: probability of success

Application: To model the number of successes in ***n*** independent Bernoulli trials.

Example: number of nonconforming parts in a random sample of parts of size *n*.

pmf, mean and variance:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x}; \; x = 0,1,\ldots,n$$

$$E(x) = np \qquad Var(x) = np(1-p)$$

Fact: sum of *n* independent Bernoulli variables (*p*) is Binomial (*n*,*p*)

Examples

(1) In a welding process, 10 different points of a battery tab are welded. If the probability of making an acceptable weld is 0.99, calculate :

a) the probability of making a battery tab with 6 failed welds.

b) the probability of making a battery with at least 2 failed welds.

Poisson Distribution

$X \sim Poisson(\lambda) \quad x = 0,1,2,\ldots$

λ: **average** number of events in a unit (a.k.a. rate)

Application: To model the number of events in a desired unit

Example: number of $\underbrace{\text{scratches}}_{\textbf{events}}$ on $\underbrace{\text{the surface of a refrigerator}}_{\textbf{unit}}$.

pmf, mean and variance:

$$P(x) = \frac{\lambda^x \exp(-\lambda)}{x!}; \quad x = 0,1,2,\ldots \qquad E(x) = Var(x) = \lambda$$

Fact: sum of *n* independent Poisson variables (λ) is Poisson with the rate of ($n\lambda$)

Examples

(2) In a painting shop, on average, there are 5 defects on the body of a painted car. Calculate

a) the probability of finding a painted body with 10 defects.

b) the probability of finding 10 defects on 2 painted bodies.

Geometric Distribution

$X \sim Geo(p) \quad x = 1,2,\ldots$ *p*: probability of success

Application: To model the number of Bernoulli trails until the first success

Example: number of inspected part until the first nonconforming part is found

pmf, mean and variance:

$$P(x) = p(1-p)^{x-1}; \ x = 1,2,\ldots$$

$$E(x) = \frac{1}{p}$$

$$Var(x) = \frac{1}{p}\left(\frac{1}{p} - 1\right)$$

Fact: the number of Bernoulli trails between two successes also follows a Geometric distribution (because of Memoryless property)

Negative-Binomial (Pascal) Distribution

$X \sim NB(r,p) \quad x = r, r+1,\ldots$ *p*: probability of success

Application: To model the number of Bernoulli trails until ***r*** successes occur

Example: number of inspected part until the *r*th nonconforming part is found

pmf, mean and variance:

$$P(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}; \ x = r, r+1,\ldots$$

$$E(x) = \frac{r}{p}$$

$$Var(x) = \frac{r}{p}\left(\frac{1}{p} - 1\right)$$

Fact: sum of *r* independent Geometric variables (*p*) is Negative-Binomial (*r*,*p*)

Examples

In a welding process, probability of making an acceptable weld is 0.99, calculate :

a) the probability of making at least 5 welds until a defective weld is made.

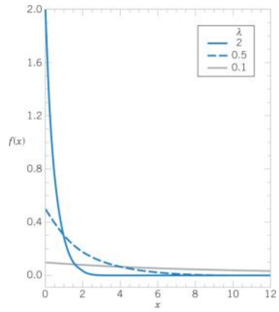
b) the probability of making the 3^{rd} defective weld in the 5^{th} battery tab.

Review of Continuous Distributions

Exponential Distribution

$X \sim Exp(\lambda) \quad x \geq 0$

$f(x) = \lambda \exp(-\lambda x); \; x \geq 0$



Example: waiting time of customers, time to failure of a machine

CDF, mean and variance:

$$E(x) = \frac{1}{\lambda}$$

$$Var(x) = \frac{1}{\lambda^2}$$

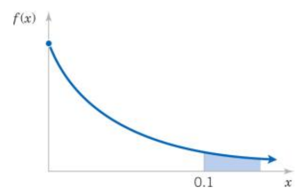
$$F(a) = \Pr(X \leq a) = 1 - \exp(-a\lambda)$$

Memoryless property: $\Pr(x > a + b \mid x > a) = \Pr(x > b)$

Example 4-21: Computer Usage-1

In a large corporate computer network, time to user log-ons to the system can be modeled as exponential distribution with a mean of 0.04 hours. What is the probability that there are no log-ons in the next 6 minutes (0.1 hours)?

Let *X* denote the time in hours from the start of the interval until the first log-on.

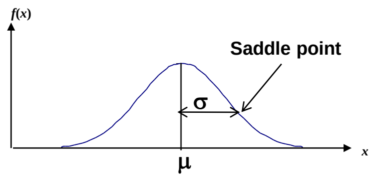


what is the interval of time such that the probability that no log-on occurs during the interval is 0.90?

Normal Distribution

$X \sim N(\mu, \sigma^2); \qquad -\infty < x < +\infty$

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

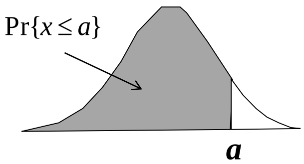


$E(x) = \mu \qquad Var(x) = \sigma^2$

Fact: If x_1, x_2 are independently normally distributed variables, then $y=x_1+x_2$ also follows the normal distribution, i.e. $y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

$$\Pr\{x \le a\} = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} dx$$

?



Standard Normal Distribution

$X \sim N(\mu, \sigma^2); \qquad -\infty < x < +\infty$

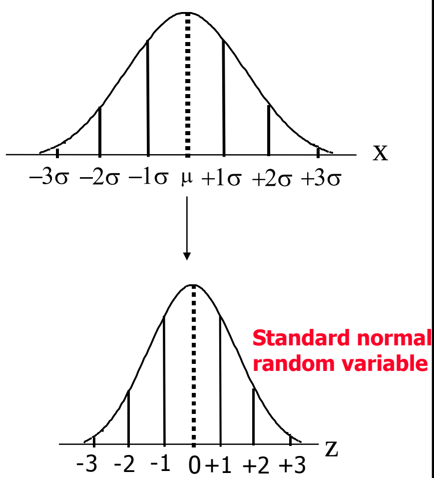
$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

Map any X into Z

$$\boxed{Z = \frac{X-\mu}{\sigma}}$$

$Z \sim N(0, 1^2); \qquad -\infty < z < +\infty$

$$f(z; \mu=0, \sigma^2=1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$$

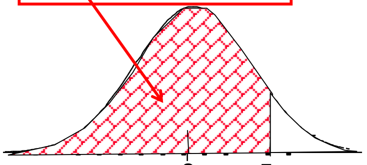


Excel Function: F(x)=NORMDIST(x,μ,σ,true)

Pr(μ−σ≤x≤μ+σ)=68.26%
Pr(μ−2σ≤x≤μ+2σ)=95.46%
Pr(μ−3σ≤x≤μ+3σ)=99.73%

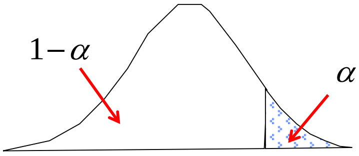
Z-Values and CDF

$\boxed{\Phi(z) = P(Z \le z)}$



Input: real number (z)
Output: probability $\Phi(z)$

$\boxed{Z_\alpha = \Phi^{-1}(1-\alpha)} \Leftrightarrow P(Z > Z_\alpha) = \alpha$



Input: probability α
Output: real number Z_α

$\Phi(1.645) = P(Z < 1.645) = 0.95 \Leftrightarrow z_{0.05} = \Phi^{-1}(0.95) = 1.645$

Useful equations:

$\Phi(Z_\alpha) = P(Z < Z_\alpha) = 1-\alpha \Rightarrow \alpha = 1 - \Phi(Z_\alpha)$

$\Phi(-z) = 1 - \Phi(z)$

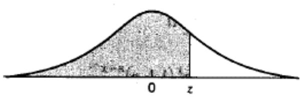
$Z_{1-\alpha} = -Z_\alpha$

Example $X \sim N(40, 5^2)$

$p(x \le 37.9) =$

$p(x \ge a) = 0.3783$

$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$



Appendix A: Table III

z	0.00	0.01	0.02	0.03	0.04	z
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.0
0.1	0.53983	0.54379	0.54776	0.55172	0.55567	0.1
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.2
0.3	0.61791	0.62172	0.62551	0.62930	0.63307	0.3
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.4
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.5
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.6
0.7	0.75803	0.76115	0.76424	0.76730	0.77035	0.7

Example: Three shafts are made and assembled in a linkage. The length of each shaft, in centimeters, is distributed as follows:

Shaft 1: N ~ (75, 0.09)
Shaft 2: N ~ (60, 0.16)
Shaft 3: N ~ (25, 0.25)

Assume the shafts' length are independent to each other:
(a) What is the distribution of the linkage?
(b) What is the probability that the linkage will be longer than 160.5 cm?

Chi-square Distribution

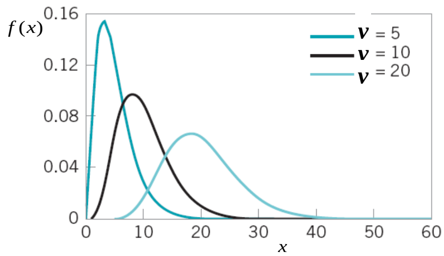
$$X \sim \chi^2(\nu)$$

Degrees of freedom

$$E(X) = \nu$$
$$Var(X) = 2\nu$$

$$f(x;\nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \exp\left(-\frac{x}{2}\right) x^{\left(\frac{\nu}{2}-1\right)}$$

$$\Gamma(a) = (a-1)!;\ a \in Z^+$$



The Chi-square pdf is not easily integrable (except for *v*=2). The CDF table (Appendix A, Table IV) is usually used.

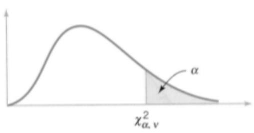


Table IV Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Squared Distribution

ν \ α	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19

Chi-square Distribution

- Fact 1: If $Z_1, \ldots, Z_k$ are *independent*, *standard normal* random variables, then the sum of their squares is distributed according to the chi-square distribution with k degrees of freedom.

$$X = \sum_{i=1}^{k} Z_i^2 \sim \chi^2(k)$$

- Fact 2: If X_i's (i = 1,2,…,n) are *independent*, *Chi-square* random variables with v_i's degrees of freedom, then

$$Y = \sum_{i=1}^{n} X_i \sim \chi^2\left(\sum_{i=1}^{n} \nu_i\right)$$

Example

- Calculate the following quantities:

1. $\chi^2_{0.05,5}$

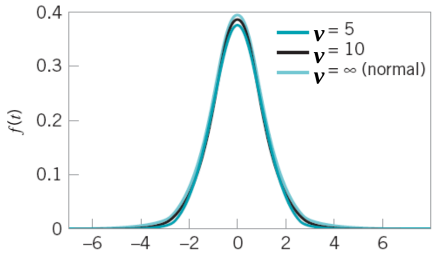
2. $\Pr\left\{\sum_{i=1}^{8}\left(\frac{X_i - \mu}{\sigma}\right)^2 > 1.34\right\}$

Student's t Distribution

$$f(t;\nu)=\frac{1}{\sqrt{\pi\nu}}\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\nu/2)}\left(1+\frac{t^2}{\nu}\right)^{-\left(\frac{\nu+1}{2}\right)}$$

$$E(t)=0$$

$$Var(t)=\frac{\nu}{\nu-2};\nu>2$$



The student's *t* pdf is not easily integrable (except for *v* = 1). The CDF table (Appendix A, Table V) is usually used.

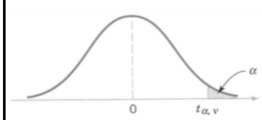


Table V Percentage Points $t_{\alpha,\nu}$ of the t Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587

Student's t Distribution

- Fact 1: If
  - *Z* is a *standard normal* (N(0,1)) random variable
  - *V* has a *chi-square distribution* with *v* degrees of freedom;
  - *Z* and *V* are *independent*.

  Then the following ratio follows a student's *t*

$$\frac{Z}{\sqrt{V/\nu}}\sim t(\nu)$$

- Fact 2: the student's t distribution tends to standard normal distribution as *v* increases (*v* > 30)

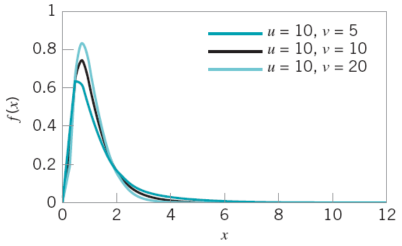
Example

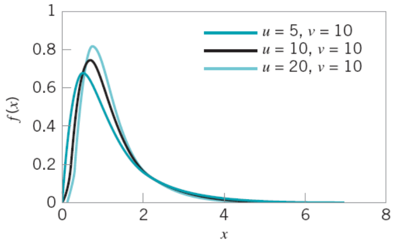
- Calculate the following quantities:

1. $t_{0.05,5}$

2. $\Pr\left(\frac{Z}{\sqrt{\left(\frac{X-\mu_x}{\sigma_x}\right)^2}} > 1\right)$ Z follows a standard normal dist. and X follows a normal dist. and they are independent

F Distribution





$$f(x) = \frac{\Gamma\left(\frac{u+v}{2}\right)\left(\frac{u}{v}\right)^{u/2}}{\Gamma\left(\frac{u}{2}\right)\Gamma\left(\frac{v}{2}\right)} \frac{x^{(u/2)-1}}{\left[\left(\frac{u}{v}\right)x+1\right]^{(u+v)/2}}, 0 < x < \infty$$

F Distribution

The *F* pdf is not easily integrable. The CDF table (Appendix A, Table VI) is usually used.

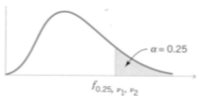


Table VI Percentage Points f_{α,ν_1,ν_2} of the F Distribution

$f_{0.25,\nu_1,\nu_2}$

ν_2 \ ν_1	Degrees of freedom for the numerator (ν_1)												
	1	2	3	4	5	6	7	8	9	10	12	15	20
1	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.41	9.49	9.58
2	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.41	3.43
3	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.46	2.46
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.88
6	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.76	1.76
7	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.70	1.69	1.68	1.68	1.67
8	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.62	1.62	1.61
9	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.57	1.56
10	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.53	1.52

- A random variate of the F-distribution arises as the ratio of two chi-squared variates: $\frac{U_1/d_1}{U_2/d_2}$
  where
  - U_1 and U_2 have *chi-square distributions* with d_1 and d_2 degrees of freedom respectively, and
  - U_1 and U_2 are independent.

Example

- Calculate the following quantities:

1. $F_{0.25,6,8}$

2. $\Pr\left(\frac{\sum_{i=1}^{2}\left(\frac{X_i-\mu_x}{\sigma_x}\right)^2}{\sum_{i=1}^{2}\left(\frac{Y_i-\mu_y}{\sigma_y}\right)^2} > 3\right)$

X and Y are independent and follow a normal dist.

Excel Function: f(f0)=fdist(f0,df1,df2)

Summary

Standard Normal

Chi-square

$X_i \sim N(\mu, \sigma^2) \rightarrow Z_i = \dfrac{X_i - \mu}{\sigma} \rightarrow V = Z_1^2 + Z_1^2 + \ldots + Z_v^2$

Student's *t*

$\dfrac{Z}{\sqrt{\dfrac{V}{v}}}$

$\begin{matrix} Z_i \sim N(0,1) \\ W_i \sim N(0,1) \end{matrix} \rightarrow \begin{matrix} U_1 = Z_1^2 + Z_2^2 + \ldots + Z_{d_1}^2 \sim \\ U_2 = W_1^2 + W_2^2 + \ldots + W_{d_2}^2 \end{matrix} \rightarrow \dfrac{U_1 / d_1}{U_2 / d_2} \sim$

F-distribution

Useful Fact About Sampling Distributions

- Fact 1: A Chi-square random variable with 2 dof is equivalent to an exponential random variable with λ = 0.5.

$$f(x) = \lambda e^{-\lambda x},\ x \ge 0;$$

- Fact 2: A *t* random variable with 1 dof is equivalent to a Cauchy random variable

$$f(t) = \frac{1}{\pi} \times \frac{1}{1+t^2}$$

- Fact 3: An *F* random variable with 2 and 2 dof's

$$f \sim F_{2,2} \rightarrow g(f) = \frac{1}{(1+f)^2}$$

- Fact 4: if $f \sim F(v_1, v_2)$, then $1/f \sim F(v_2, v_1)$ $\quad F_{1-\alpha,v_1,v_2} = \dfrac{1}{F_{\alpha,v_1,v_2}}$

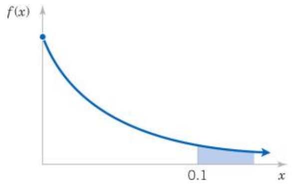
- Fact 5: if $t \sim$ students'-$t(v)$, then $t^2 \sim F(1, v)$ $\quad F_{\alpha,1,v} = t^2_{\alpha/2,v}$

Example 4-21: Computer Usage-1

In a large corporate computer network, time to user log-ons to the system can be modeled as exponential distribution with a mean of 0.04 hours. What is the probability that there are no log-ons in the next 6 minutes (0.1 hours)?

Let *X* denote the time in hours from the start of the interval until the first log-on.

$$P(X > 0.1) = \int_{0.1}^{\infty} 25e^{25x}dx = e^{-25(0.1)}$$
$$= 1 - F(0.1) = 0.082$$



what is the interval of time such that the probability that no log-on occurs during the interval is 0.90?

$$P(X > x) = e^{-25x} = 0.90, \quad -25x = \ln(0.90)$$
$$x = \frac{-0.10536}{-25} = 0.00421 \text{ hour} = 0.253 \text{ minute}$$