

ISyE 6739 Homework 4

due Thursday, Mar 15

1. **(8-22)** Ishikawa et al. (*Journal of Bioscience and Bioengineering*, 2012) studied the adhesion of various biofilms to solid surfaces for possible use in environmental technologies. Adhesion assay is conducted by measuring absorbance at A_{590} . Suppose that for the bacterial strain *Acinetobacter*, five measurements gave readings of 2.69, 5.76, 2.67, 1.62 and 4.12 dyne-cm². Assume that the standard deviation is known to be 0.66 dyne-cm².
 - (a) Find a 95% confidence interval for the mean adhesion.
 - (b) If the scientists want the confidence interval to be no wider than 0.55 dyne-cm², how many observations should they take?
2. **(8-23)** Dairy cows at large commercial farms often receive injections of bST (Bovine Somatotropin), a hormone used to spur milk production. Bauman et al. (*Journal of Dairy Science*, 1989) reported that 12 cows given bST produced an average of 28.0 kg/d of milk. Assume that the standard deviation of milk production is 2.25 kg/d.
 - (a) Find a 99% confidence interval for the true mean milk production.
 - (b) If the farms want the confidence interval to be no wider than ± 1.25 kg/d, what level of confidence would they need to use?
3. Assume X_1, X_2, \dots, X_n is a random sample from exponential distribution with rate λ . If n is large, find a 95% approximate (general) confidence interval for λ .
4. **(8-38)** A particular brand of diet margarine was analyzed to determine the level of polyunsaturated fatty acid (in percentages). A sample of six packages resulted in the following data: 16.8, 17.2, 17.4, 16.9, 16.5, 17.1.
 - (a) Check the assumption that the level of polyunsaturated fatty acid is normally distributed.
 - (b) Calculate a 99% confidence interval on the mean μ . Provide a practical interpretation of this interval.
 - (c) Calculate a 99% lower confidence bound on the mean. Compare this bound with the lower bound of the two-sided confidence interval and discuss why they are different.
5. **(8-57)** From the data on the pH of rain in Ingham County, Michigan:
5.47 5.37 5.38 4.63 5.37 3.74 3.71 4.96 4.64 5.11 5.65 5.39 4.16 5.62 4.57 4.64 5.48 4.57 4.57
4.51 4.86 4.56 4.61 4.32 3.98 5.70 4.15 3.98 5.65 3.10 5.04 4.62 4.51 4.34 4.16 4.64 5.12 3.71
4.64
Find a two-sided 95% confidence interval for the standard deviation of pH.
6. **(8-60)** An article in *Knee Surgery, Sports Traumatology, Arthroscopy* ["Arthroscopic Meniscal Repair with an Absorbable Screw: Results and Surgical Technique" (2005, Vol. 13, pp. 273–279)] showed that only 25 out of 37 tears (67.6%) located between 3 and 6 mm from the meniscus rim were healed.
 - (a) Calculate a two-sided 95% confidence interval on the proportion of such tears that will heal.
 - (b) Calculate a 95% lower confidence bound on the proportion of such tears that will heal.

7. **(9-10)** The heat evolved in calories per gram of a cement mixture is approximately normally distributed. The mean is thought to be 100, and the standard deviation is 2. You wish to test $H_0: \mu = 100$ versus $H_1: \mu \neq 100$ with a sample of $n = 9$ specimens.
 - (a) If the acceptance region is defined as $98.5 \leq \bar{x} \leq 101.5$, find the type I error probability α .
 - (b) Find β for the case in which the true mean heat evolved is 103.
 - (c) Find β for the case where the true mean heat evolved is 105. This value of β is smaller than the one found in part (b). Why?
8. **(9-14)** In Exercise 9-10, calculate the P-value if the observed statistic is
 - (a) $\bar{x} = 98$
 - (b) $\bar{x} = 101$
 - (c) $\bar{x} = 102$
9. **(9-44)** A melting point test of $n = 10$ samples of a binder used in manufacturing a rocket propellant resulted in $\bar{x} = 154.2^\circ\text{F}$. Assume that the melting point is normally distributed with $\sigma = 1.5^\circ\text{F}$.
 - (a) Test $H_0: \mu = 155$ versus $H_1: \mu \neq 155$ using $\alpha = 0.01$.
 - (b) What is the P-value for this test?
 - (c) What is the β -error if the true mean is $\mu = 150$?
 - (d) What value of n would be required if we want $\beta < 0.1$ when $\mu = 150$? Assume that $\alpha = 0.01$.
10. **(9-50)** Humans are known to have a mean gestation period of 280 days (from last menstruation) with a standard deviation of about 9 days. A hospital wondered whether there was any evidence that their patients were at risk for giving birth prematurely. In a random sample of 70 women, the average gestation time was 274.3 days.
 - (a) Is the alternative hypothesis one- or two-sided?
 - (b) Test the null hypothesis at $\alpha = 0.05$.
 - (c) What is the P-value of the test statistic?