ISyE 6739 Video Assignment 3

1. Suppose X is a random variable. Write the PMF, the mean, and the variance if the distribution of X :

   (a) Poisson with the rate λ.

   (b) Geometric with probability of success p.

   (c) Negative-Binomial with parameters (r, p).

   Answer:

   (a) PMF:
   $$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, \ldots,$$
   Mean:
   $$\mathrm{E}[X] = \lambda,$$
   Variance:
   $$\mathrm{Var}(X) = \lambda.$$

   (b) PMF:
   $$P(k) = p(1-p)^{k-1}, \quad k = 1, 2 \ldots,$$
   Mean:
   $$\mathrm{E}[X] = \frac{1}{p},$$
   Variance:
   $$\mathrm{Var}(X) = \frac{1}{p}\left(\frac{1}{p} - 1\right).$$

   (c) PMF:
   $$P(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}, \quad k = 0, 1, \ldots,$$
   Mean:
   $$\mathrm{E}[X] = \frac{r}{p},$$
   Variance:
   $$\mathrm{Var}(X) = \frac{r}{p}\left(\frac{1}{p} - 1\right).$$

2. Write the memoryless property. What distribution does have this property?

   Answer:
   $$\Pr(X > a + b | x > a) = \Pr(X > b).$$

   Both geometric and exponential distributions have this property.