ISyE 6739 Video Assignment 12

1. Write the expressions for two-sided confidence interval for difference in means for two normal distributions (variance is known).

   Answer:

   $100(1-\alpha)\%$ CI for Mean of a Normal Distribution(two-sided, unknown variance):

   $$(\bar{X}-\bar{Y}) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \leq \mu_1-\mu_2 \leq (\bar{X}-\bar{Y}) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

2. Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be two independent samples from normal distributions with means μ_1 and μ_2 and known variances σ_1^2 and σ_2^2 respectively.L and U are lower and upper bounds for $100(1-\alpha)\%$ CI for difference in means $\mu_1-\mu_2$. What conclusion can be made if $L>0$, $U>0$? What if $L<0$ and $U>0$?

   Answer:

   If $L>0$ and $U>0$ then $\mu_1>\mu_2$ with probability $1-\alpha$.
   If $L<0$ and $U>0$ then $\mu_1=\mu_2$ with probability $1-\alpha$.

3. Write the expressions for two-sided confidence intervals for variances ratio of two normal distributions.

   Answer:

   $100(1-\alpha)\%$ CI for Variance ratio of two normal distributions (two-sided):

   $$\frac{s_1^2}{s_2^2}F_{1-\alpha/2,n_2-1,n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2}{s_2^2}F_{\alpha/2,n_2-1,n_1-1}$$

4. Suppose $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ are random samples from the binomial distribution with parameter p_1 and p_2 respectively and sample sizes n_1 and n_2 are large enough. Write a two-sided confidence interval for difference p_1-p_2.
   Answer:

   $100(1-\alpha)\%$ CI for difference in population proportions with large sample size:

   $$(\hat{p}_1-\hat{p}_2)-Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \leq p_1-p_2 \leq (\hat{p}_1-\hat{p}_2)+Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$