ISyE 6739 – Group Activity 13
solutions

1.

$$\bar{X} = 7.55, \quad n = 10, \quad S = 0.103, \quad t_{0.05/2,9} = 2.26$$

(a)

$$H_0: \ \mu = 7.5, \ ; \ \ H_1: \ \mu \neq 7.5$$

Find a test statistics:

$$t_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{7.55 - 7.5}{0.103/\sqrt{10}} = 1.535 < 2.26 = t_{0.05/2,9}$$

$\Rightarrow$ we fail to reject the null hypothesis.
Find the confidence interval:

$$\left[\bar{X} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \ , \ \bar{X} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}\right] = \left[7.55 - 2.36\frac{0.103}{\sqrt{10}} \ , \ 7.55 + 2.36\frac{0.103}{\sqrt{10}}\right] =$$

$$= [7.473 \ , \ 7.627]$$

$7.5 \in [7.473 \ , \ 7.627] \Rightarrow$ we fail to reject the null.
Find the p-value for the test:

$$\text{p-value} = 2\left[1 - T_{n-1}\left(\left|\frac{\bar{X} - \mu_0}{S/\sqrt{n}}\right|\right)\right] = 2\left[1 - T_{n-1}(1.535)\right] = 0.159 > 0.05 = \alpha$$

$\Rightarrow$ we fail to reject H_0.

(b)

$$H_0: \ \mu = 7.5, \quad H_1: \ \mu < 7.5$$

Find a test statistics:

$$t_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{7.55 - 7.5}{0.103/\sqrt{10}} = 1.535 > -1.833 = -t_{0.05,9}$$

$\Rightarrow$ we fail to reject the null hypothesis.

2.

$$S = 0.08912, \quad n = 8, \quad \sigma_0^2 = 0.01, \quad \chi^2_{1-0.05,7} = 2.167$$

(a)

$$H_0: \ \sigma^2 = 0.01, \quad H_1: \ \sigma^2 < 0.01$$

Find the test statistic:

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{7 \cdot 0.08912^2}{0.01} = 5.56 > 2.167 = \chi^2_{1-0.05,7}$$

$\Rightarrow$ we fail to reject H_0.
Find the confidence interval:

$$\left(0 \ , \ \frac{(n-1)S^2}{\chi^2_{1-\alpha,n-1}}\right] = \left(0 \ , \ \frac{7 \cdot 0.08912^2}{2.167}\right] = (0 \ , \ 0.0257]$$

$\sigma_0^2 = 0.01 \in (0 \ , \ 0.0257] \Rightarrow$ fail to reject the null.

(b)

$$H_0:\ \sigma = 0.1, \quad H_1:\ \sigma < 0.1$$

Find the confidence interval:

$$\left(0\ ,\ \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\alpha,n-1}}}\right] = \left(0\ ,\ \sqrt{\frac{7\cdot 0.08912^2}{2.167}}\right] = (0\ ,\ 0.16]$$

$\sigma_0 = 0.1 \in (0\ ,\ 0.16] \Rightarrow$ fail to reject the null.

3.

$$\hat{p} = \frac{12}{60} = 0.2, \quad n = 60, \quad p_0 = 0,25, \quad Z_{0.05/2} = 1.96$$

Find the test statistic:

$$|Z_0| = \left|\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}\right| = \left|\frac{0.2-0.25}{\sqrt{\frac{0.25\cdot 0.75}{60}}}\right| = 0.8944 < 1.96 = Z_{0.025}$$

$\Rightarrow$ fail to reject the null.
Find the confidence interval:

$$\left[\hat{p} - Z_{\alpha/2}\sqrt{\frac{p_0(1-p_0)}{n}}\ ,\ \hat{p} + Z_{\alpha/2}\sqrt{\frac{p_0(1-p_0)}{n}}\right] =$$

$$= \left[0.2 - 1.96\sqrt{\frac{0.25\cdot 0.75}{60}}\ ,\ 0.2 + 1.96\sqrt{\frac{0.25\cdot 0.75}{60}}\right] = [0.0904\ ,\ 0.31]$$

$p_0 = 0.25 \in [0.0904\ ,\ 0.31] \Rightarrow$ we fail to reject the null.
Find the p-value for the test:

$$\text{p-value} = 2\left[1 - \Phi\left(\left|\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}\right|\right)\right] = 2\left[1-\Phi\left(0.8944\right)\right] = 0.3711 > 0.05 = \alpha$$

$\Rightarrow$ fail to reject the null hypothesis.

4. We know that MLE for parameter λ of Poisson distribution is $\bar{X}$. For large sample size n MLE follows normal distribution. Then we can define the following test:

$$H_0:\ \lambda = 9, \quad H_1:\ \lambda > 9$$

The test statistics in this case is:

$$Z_0 = \frac{\bar{X}-\lambda_0}{\sqrt{\lambda_0/n}} = \frac{\bar{X}-9}{\sqrt{9/n}}$$

if $Z_0 < Z_\alpha$ then we fail to reject H_0.