ISyE 6739 Video Assignment 10

1. Write the expressions for two-sided and one-sided (upper and lower bounds) confidence intervals for mean of a normal distribution (variance is known). When can we use them for mean of non-normal distributions?

   Answer:

   $100(1-\alpha)\%$ CI for Mean of a Normal Distribution(two-sided, known variance):

   $$\left(\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

   $100(1-\alpha)\%$ CI for Mean of a Normal Distribution(one-sided, known variance):

   $$\mu \ge \bar{X} - Z_{\alpha}\frac{\sigma}{\sqrt{n}} \quad \text{(lower bound)}$$

   $$\mu \le \bar{X} + Z_{\alpha}\frac{\sigma}{\sqrt{n}} \quad \text{(upper bound)}$$

   This CI's can be used for mean of non-normal distributions when $n > 30$.

2. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a normal distribution with mean μ and known variance σ^2. What sample size should we use to be $100(1-\alpha)\%$ confident that the error $|\bar{X} - \mu|$ will not exceed a specified amount E (we consider two-sided estimation)?

   Answer:

   $$n = \left(\frac{Z_{\alpha/2}\sigma}{E}\right)^2.$$

3. What is the interpretation of words "$100(1-\alpha)\%$ confidence" in terms of CI's?

   Answer:
   That means that in the long run, $100(1-\alpha)\%$ of all computed CI's will contain the true value of an estimated parameter.