ISyE 6739 Video Assignment 16

1. State two-sided null and alternative hypotheses for the test on the difference in means of two normal distributions (variances are unknown and equal).Write the confidence interval for $\mu_1 - \mu_2$.

   Answer:

   $X_1, X_2, \ldots, X_{n_1} \sim N(\mu_1, \sigma^2)$, $Y_1, Y_2, \ldots, Y_{n_2} \sim N(\mu_2, \sigma^2)$

   $$H_0:\ \mu_1 - \mu_2 = \Delta_0, \quad H_1:\ \mu_1 - \mu_2 \neq \Delta_0$$

   $$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

   CI::

   $$(\bar{X} - \bar{Y}) - t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{X} - \bar{Y}) + t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$

2. State two-sided null and alternative hypotheses for the test on the difference in means of two normal distributions (variances are unknown and not equal, the alternative is $\mu_1 - \mu_2 < \Delta_0$). Write the test statistic and the rejection region.

   Answer:

   $X_1, X_2, \ldots, X_{n_1} \sim N(\mu_1, \sigma_1^2)$, $Y_1, Y_2, \ldots, Y_{n_2} \sim N(\mu_2, \sigma_2^2)$

   $$H_0:\ \mu_1 - \mu_2 = \Delta_0, \quad H_1:\ \mu_1 - \mu_2 < \Delta_0$$

   Test statistic:

   $$t_0 = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}},$$

   Rejection region:

   $$t_0 < -t_{\alpha,\nu}$$

   where

   $$\nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}}.$$

3. In what case do we use paired t-test instead of regular t-test for means of two samples from normal distribution? State null and alternative hypotheses and write the CI for paired t-test with two-sided alternative.

   Answer:

   We use paired t-test when two populations are collected in pairs and observations within a pair are dependent.

   $$H_0:\ \mu_d = \mu_1 - \mu_2 = \Delta_0, \quad H_1:\ \mu_d \neq \Delta_0$$

The CI for hypothesis testing:

$$\left(\bar{d} - t_{\alpha/2,n-1}\frac{S_d}{\sqrt{n}}, \bar{d} + t_{\alpha/2,n-1}\frac{S_d}{\sqrt{n}}\right)$$

where $d_i = X_i - Y_i$.

4. Write the formula for Type II error for the test on two population proportion if the alternative is $p_1 > p_2$.

Answer:

$$\beta = \Phi\left(\frac{-(p_1 - p_2) + z_\alpha\sqrt{p(1-p)(1/n_1 + 1/n_2)}}{\sigma_{\hat{p}_1 - \hat{p}_2}}\right).$$

where $p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$