National Institute of Technology Karnataka, Surathkal
Department of Mathematical and Computational Sciences
Discrete Mathematical Structures - MA602
Odd Semester (2018 - 2019)
Problem Sheet 1

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1. Construct the truth tables of the following formulas.
   (a) $(Q \wedge (P \to Q)) \to P$
   (b) $\neg(P \vee (Q \wedge R)) \leftrightarrows ((P \vee Q) \wedge (P \vee R))$
2. Given the truth values of P and Q as T and those of R and S as F, find the truth values of the following:
   (a) $(\neg(P \wedge Q) \vee \neg R) \vee ((Q \leftrightarrows \neg P) \to (R \vee \neg S))$
   (b) $(P \leftrightarrows R) \wedge (\neg Q \to S)$
   (c) $(P \vee (Q \to (R \wedge \neg P))) \leftrightarrows (Q \vee \neg S)$
3. From the formulas given below, select those which are well-formed and indicate which ones are tautologies or contradictions.
   (a) $(P \to (P \vee Q))$
   (b) $((P \to (\neg P)) \to \neg P)$
   (c) $((\neg Q \wedge P) \wedge Q)$
   (d) $((P \to (Q \to R)) \to ((P \to Q) \to (P \to R)))$
   (e) $((\neg P \to Q) \to (Q \to P)))$
   (f) $((P \wedge Q) \leftrightarrows P)$
4. Show the following implications without constructing the truth tables.
   (a) $P \to Q \Longrightarrow P \to (P \wedge Q)$
   (b) $(P \to Q) \to Q \Longrightarrow P \vee Q$
   (c) $((P \vee \neg P) \to Q) \to ((P \vee \neg P) \to R) \Longrightarrow (Q \to R)$
   (d) $(Q \to (P \wedge \neg P)) \to (R \to (P \wedge \neg P)) \Longrightarrow (R \to Q)$
5. Obtain the product-of-sums canonical forms of the following formulas.
   (a) $(P \wedge Q \wedge R) \vee (\neg P \wedge R \wedge Q) \vee (\neg P \wedge \neg Q \wedge \neg R)$
   (b) $(\neg S \wedge \neg P \wedge R \wedge Q) \vee (S \wedge P \wedge \neg R \wedge \neg Q) \vee (\neg S \wedge P \wedge R \wedge \neg Q) \vee (Q \wedge \neg P \wedge \neg R \wedge S) \vee (P \wedge \neg S \wedge \neg R \wedge Q)$
   (c) $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$

(d) $(P \wedge Q) \vee (\neg P \wedge Q \wedge R)$

6. Obtain the principal disjunctive and conjunctive normal forms of the following formulas.
   (a) $(\neg P \vee \neg Q) \to (P \leftrightarrows \neg Q)$
   (b) $Q \wedge (P \vee \neg Q)$
   (c) $P \vee (\neg P \to (Q \vee (\neg Q \to R)))$
   (d) $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$
   (e) $P \to (P \wedge (Q \to P))$
   (f) $(Q \to P) \wedge (\neg P \wedge Q)$
   Which of the above formulas are tautologies?
7. Show that the conclusion C follows from the premises $H_1, H_2 \ldots$ in the following cases.
   (a) $H_1 : \neg P \vee Q \quad H_2 : \neg(Q \wedge \neg R) \quad H_3 : \neg R \quad C\!: \neg P$
   (b) $H_1 : P \to Q \quad H_2 : Q \to R \quad C\!: P \to R$
8. Determine whether the conclusion C is valid in the following, when $H_1, H_2, \ldots$ are the premises.
   (a) $H_1 : P \vee Q \quad H_2 : P \to R \quad H_3 : Q \to R \quad C\!: R$
   (b) $H_1 : P \to (Q \to R) \quad H_2 : P \wedge Q \quad C\!: R$
9. Without constructing a truth table, show that $A \wedge E$ is not a valid consequence of

$$A \leftrightarrows B \quad B \leftrightarrows (C \wedge D) \quad C \leftrightarrows (A \vee E) \quad A \vee E$$

   Also show that $A \vee C$ is not a valid consequence of

$$A \leftrightarrows (B \to C) \quad B \leftrightarrows (\neg A \vee \neg C) \quad C \leftrightarrows (A \vee \neg B) \quad B$$

10. Show the validity of the following arguments, for which the premises are given on the left and the conclusion on the right.
    (a) $(A \to B) \wedge (A \to C), \neg(B \wedge C), D \vee A \quad D$
    (b) $\neg J \to (M \vee N), (H \vee G) \to \neg J, H \vee G \quad M \vee N$
    (c) $B \wedge C, (B \leftrightarrows C) \to (H \vee G) \quad G \vee H$
11. Derive the following, using contrapositive, if necessary.
    (a) $\neg P \vee Q, \neg Q \vee R, R \to S \Longrightarrow P \to S$
    (b) $P \to (Q \to R), Q \to (R \to S) \Longrightarrow P \to (Q \to S)$
12. Show the following (use indirect method if needed).

(a) $(R \to \neg Q), R \vee S, S \to \neg Q, P \to Q \implies \neg P$

(b) $S \to \neg Q, S \vee R, \neg R, \neg R \leftrightarrows Q \implies \neg P$

(c) $\neg(P \to Q) \to \neg(R \vee S), ((Q \to P) \vee \neg R), R \implies P \leftrightarrows Q$

13. Which of the following are statements?

    (a) $\forall x\ (P(x) \vee Q(x)) \wedge R$

    (b) $\forall x\ (P(x) \wedge Q(x)) \wedge (\exists x)\ S(x)$

    (c) $\forall x\ (P(x) \wedge Q(x)) \wedge S(x)$

14. Show that $\exists z\ (Q(z) \wedge R(z))$ is not implied by the formulas $\exists x\ (P(x) \wedge Q(x))$ and $\exists y\ (P(y) \wedge R(y))$, by assuming a universe of discourse which has two elements.

15. Explain why the following steps in the derivations are not correct.

    (a) $\forall x\ P(x) \to Q(x) \implies P(x) \to Q(x)$

    (b) $\forall x\ P(x) \to Q(x) \implies P(y) \to Q(x)$

    (c) $\forall x\ (P(x) \vee Q(x)) \implies P(a) \vee Q(b)$

    (d) $\forall x(P(x) \vee \exists x\ (Q(x) \wedge R(x))) \implies P(a) \vee \exists x\ (Q(x) \wedge R(a))$

16. Demonstrate the following implications.

    (a) $\neg(\exists x\ P(x) \wedge Q(a)) \implies \exists x\ P(x) \to \neg Q(a)$

    (b) $\forall x\ (P(x) \to Q(x)), \forall x\ (Q(x) \to R(x)) \implies P(x) \to R(x)$

17. Are the following conclusions validly derivable from the premises given?

    (a) $\forall x\ ((P(x) \to Q(x)), \exists y\ P(y)$ C: $\exists z\ Q(z)$

    (b) $\exists x\ P(x), \exists x\ Q(x)$ C: $\exists x\ (P(x) \wedge Q(x))$

18. Using contrapositive or otherwise, show the following implication.
    $\forall x\ (P(x) \to Q(x)), \forall x\ (R(x) \to \neg Q(x)) \implies \forall x\ (R(x) \to \neg P(x))$

19. Construct the derivation of the following equivalence.
    $P \to \exists x\ Q(x) \iff \exists x\ (P \to Q(x))$

20. Show the following by constructing derivation.
    $\exists x\ P(x) \to \forall x\ ((P(x) \vee Q(x)) \to R(x)), \exists x\ P(x),$
    $\exists x\ Q(x) \implies \exists x\ \exists y\ (R(x) \wedge R(y))$