Bayes Classification

- Uncertainty & Probability
- Baye's rule
- Choosing Hypotheses- Maximum a posteriori
- Maximum Likelihood - Baye's concept learning
- Maximum Likelihood of real valued function
- Bayes optimal Classifier
- Joint distributions
- Naive Bayes Classifier

Uncertainty

- Our main tool is the probability theory, which assigns to each sentence numerical degree of belief between *0* and *1*
- It provides a way of summarizing the uncertainty

Variables

- Boolean random variables: cavity might be true or false
- Discrete random variables: weather might be sunny, rainy, cloudy, snow
  - *P(Weather=sunny)*
  - *P(Weather=rainy)*
  - *P(Weather=cloudy)*
  - *P(Weather=snow)*
- Continuous random variables: the temperature has continuous values

Where do probabilities come from?

- Frequents:
  - From experiments: form any finite sample, we can estimate the true fraction and also calculate how accurate our estimation is likely to be
- Subjective:
  - Agent's believe
- Objectivist:
  - True nature of the universe, that the probability up heads with probability 0.5 is a probability of the coin

- Before the evidence is obtained; prior probability
  - *P(a)* the prior probability that the proposition is true
  - *P(cavity)=0.1*
- After the evidence is obtained; posterior probability
  - *P(a|b)*
  - The probability of a given that all we know is b
  - *P(cavity|toothache)=0.8*

Axioms of Probability

(Kolmogorov’s axioms,
first published in German 1933)

- All probabilities are between 0 and 1. For any proposition *a* $0 \leq P(a) \leq 1$
- $P(true)=1,\ P(false)=0$
- The probability of disjunction is given by

$$P(a \vee b) = P(a) + P(b) - P(a \wedge b)$$

- Product rule

$$P(a \wedge b) = P(a \mid b)P(b)$$

$$P(a \wedge b) = P(b \mid a)P(a)$$

Theorem of total probability

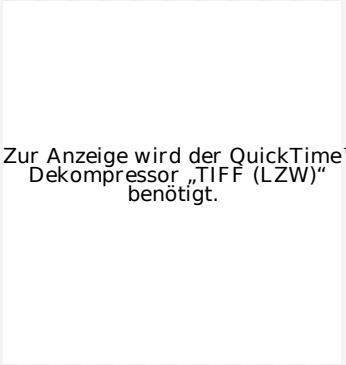
If events $A_1, \ldots, A_n$ are mutually exclusive with $\sum_{i=1}^{n} P(A_i) = 1$ then

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$$

$$P(B) = \sum_{i=1}^{n} P(B, A_i)$$

Bayes's rule

- (Reverent Thomas Bayes 1702-1761)
  - He set down his findings on probability in "Essay Towards Solving a Problem in the Doctrine of Chances" (1763), published posthumously in the *Philosophical Transactions of the Royal Society of London*



$$P(b \mid a) = \frac{P(a \mid b)P(b)}{P(a)}$$

Diagnosis

- What is the probability of meningitis in the patient with stiff neck?
  - A doctor knows that the disease meningitis causes the patient to have a stiff neck in 50% of the time -> $P(s|m)$
  - Prior Probabilities:
    - That the patient has meningitis is 1/50.000 -> $P(m)$
    - That the patient has a stiff neck is 1/20 -> $P(s)$

$$P(m \mid s) = \frac{P(s \mid m)P(m)}{P(s)}$$

$$P(m \mid s) = \frac{0.5 * 0.00002}{0.05} = 0.0002$$

Normalization

$$P(y \mid x) = \frac{P(x \mid y)P(y)}{P(x)}$$

$$P(\leftarrow y \mid x) = \frac{P(x \mid \leftarrow y)P(\leftarrow y)}{P(x)}$$

$$1 = \Pi(\psi \mid \xi) + \Pi(\leftarrow \psi \mid \xi)$$

$$\Pi(\Psi \mid \Xi) = \alpha \cdot \Pi(\Xi \mid \Psi)\Pi(\Psi)$$

$$\alpha \langle \Pi(\psi \mid \xi), \Pi(\leftarrow \psi \mid \xi) \rangle$$

$$\alpha \langle 0.12, 0.08 \rangle = \langle 0.6, 0.4 \rangle$$

Bayes Theorem

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- *P(h)* = prior probability of hypothesis h
- *P(D)* = prior probability of training data *D*
- *P(h|D)* = probability of *h* given *D*
- *P(D|h)* = probability of *D* given *h*

Choosing Hypotheses

- Generally want the most probable hypothesis given the training data

- **Maximum a posteriori** hypothesis h_{MAP}:

$$h_{MAP} = \arg\max_{h \in H} P(h|D)$$

$$h_{MAP} = \arg\max_{h \in H} P(h|D)$$

$$= \arg\max_{h \in H} \frac{P(D|h)P(h)}{P(D)}$$

$$= \arg\max_{h \in H} P(D|h)P(h)$$

- If assume $P(h_i)=P(h_j)$ for all h_i and h_j, then can further simplify, and choose the

- **Maximum likelihood** (ML) hypothesis

$$h_{ML} = \arg\max_{h_i \in H} P(D|h_i)$$

Example

- Does patient have cancer or not?

A patient takes a lab test and the result comes back positive. The test returns a correct positive result (+) in only 98% of the cases in which the disease is actually present, and a correct negative result (-) in only 97% of the cases in which the disease is not present

Furthermore, 0.008 of the entire population have this cancer

Suppose a positive result (+) is returned...

$$P(cancer) = 0.008 \qquad P(\neg cancer) = 0.992$$

$$P(+|cancer) = 0.98 \qquad P(-|cancer) = 0.02$$

$$P(+|\neg cancer) = 0.03 \qquad P(-|\neg cancer) = 0.97$$

$$P(+|cancer) \cdot P(cancer) = 0.98 \cdot 0.008 = 0.0078$$

$$P(+|\neg cancer) \cdot P(\neg cancer) = 0.03 \cdot 0.992 = 0.0298$$

$$h_{MAP} = \neg cancer$$

Normalization

$$\frac{0.0078}{0.0078 + 0.0298} = 0.20745 \qquad \frac{0.0298}{0.0078 + 0.0298} = 0.79255$$

- The result of Bayesian inference depends strongly on the prior probabilities, which must be available in order to apply the method

Brute-Force Bayes Concept Learning

- For each hypothesis *h* in *H*, calculate the posterior probability

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- Output the hypothesis h_{MAP} with the highest posterior probability

$$h_{MAP} = _{h \in H} P(h|D)$$

- Given no prior knowledge that one hypothesis is more likely than another, what values should we specify for $P(h)$?
- What choice shall we make for $P(D|h)$?

- Choose *P(h)* to be uniform distribution
- $P(h) = \frac{1}{|H|}$ for all *h* in *H*

- *P(D|h)=1* if *h* consistent with *D*
- *P(D|h)=0* otherwise

P(D)

$$P(D) = \sum_{h_i \in H} P(D \mid h_i) P(h_i)$$

$$P(D) = \sum_{h_i \in VS_{H,D}} 1 \cdot \frac{1}{|H|} + \sum_{h_i \notin VS_{H,D}} 0 \cdot \frac{1}{|H|}$$

$$P(D) = \frac{|VS_{H,D}|}{|H|}$$

- Version space $VS_{H,D}$ is the subset of consistent Hypotheses from H with the training examples in D

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

$$P(h|D) = \frac{0 \cdot P(h)}{P(D)} = 0$$

if h is inconsistent with D

$$P(h|D) = \frac{1 \cdot \frac{1}{|H|}}{\frac{|VS_{H,D}|}{|H|}} = \frac{1}{|VS_{H,D}|}$$

if h is consistent with D

$$P(h|D) = \begin{cases} \frac{1}{|VS_{H,D}|} & \text{if } h \text{ is consistent with } D \\ 0 & \text{otherwise} \end{cases}$$

Maximum Likelihood of real valued function

$$h_{ML} = arg\max_{h \in H} p(D|h)$$

$$= arg\max_{h \in H} \prod_{i=1}^{m} p(d_i|h)$$

$$= arg\max_{h \in H} \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{d_i - h(x_i)}{\sigma}\right)^2}$$

- Maximize natural log of this instead...

$$h_{ML} = arg\max_{h \in H} \sum_{i=1}^{m} \ln \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{1}{2}\left(\frac{d_i - h(x_i)}{\sigma}\right)^2$$

$$= arg\max_{h \in H} \sum_{i=1}^{m} -\frac{1}{2}\left(\frac{d_i - h(x_i)}{\sigma}\right)^2$$

$$= arg\max_{h \in H} \sum_{i=1}^{m} -(d_i - h(x_i))^2$$

$$= arg\min_{h \in H} \sum_{i=1}^{m} (d_i - h(x_i))^2$$

Bayes optimal Classifier

A weighted majority classifier

- What is he most probable classification of the new **instance** given the training data?
  - The most probable classification of the new instance is obtained by combining the prediction of *all hypothesis*, weighted by their *posterior probabilities*
- If the classification of new example can take any value v_j from some set V, then the probability $P(v_j|D)$ that the correct classification for the new **instance is v_j**, is just:

$$P(v_j|D) = \sum_{h_i \in H} P(v_j|h_i)P(h_i|D)$$

Bayes optimal classification:

$$\arg\max_{v_j \in V} \sum_{h_i \in H} P(v_j|h_i)P(h_i|D)$$

$$P(h_1|D) = .4, P(-|h_1) = 0, P(+|h_1) = 1$$

$$P(h_2|D) = .3, P(-|h_2) = 1, P(+|h_2) = 0$$

$$P(h_3|D) = .3, P(-|h_3) = 1, P(+|h_3) = 0$$

$$\sum_{h_i \in H} P(+|h_i)P(h_i|D) = .4$$

$$\sum_{h_i \in H} P(-|h_i)P(h_i|D) = .6$$

$$\arg\max_{v_j \in V} \sum_{h_i \in H} P(v_j|h_i)P(h_i|D) = -$$

Gibbs Algorithm

- Bayes optimal classifier provides best result, but can be expensive if many hypotheses

- Gibbs algorithm:
  - Choose one hypothesis at random, according to $P(h|D)$
  - Use this to classify new instance

- Suppose correct, uniform prior distribution over *H*, then
  - Pick any hypothesis at random..
  - Its expected error no worse than twice Bayes optimal

Joint distribution

- A joint distribution for toothache, cavity, catch, *dentist‘s probe catches in my tooth :-(*
- We need to know the conditional probabilities of the conjunction of toothache and cavity
- What can a dentist conclude if the probe catches in the aching tooth?

$$P(cavity \mid toothache \wedge catch) = \frac{P(toothache \wedge catch \mid cavity)P(cavity)}{P(toothache \wedge cavity)}$$

- For *n* possible variables there are 2^n possible combinations

Conditional Independence

- Once we know that the patient has cavity we do not expect the probability of the probe catching to depend on the presence of toothache

$$P(catch \mid cavity \wedge toothache) = P(catch \mid cavity)$$
$$P(toothache \mid cavity \wedge catch) = P(toothache \mid cavity)$$

- Independence between a and b

$$P(a \mid b) = P(a)$$
$$P(b \mid a) = P(b)$$

$$P(a \wedge b) = P(a)P(b)$$

$$P(toothache, catch, cavity, Weather = cloudy) =$$

$$= P(Weather = cloudy)P(toothache, catch, cavity)$$

- The decomposition of large probabilistic domains into weakly connected subsets via conditional independence is one of the most important developments in the recent history of AI

- This can work well, even the assumption is not true!

A single cause directly influence a number of effects, all of which are conditionally independent

$$P(cause, effect_1, effect_2, ...effect_n) = P(cause)\prod_{i=1}^{n} P(effect_i \mid cause)$$

Naive Bayes Classifier

- Along with decision trees, neural networks, nearest nbr, one of the most practical learning methods

- When to use:
  - Moderate or large training set available
  - Attributes that describe instances are conditionally independent given classification
- Successful applications:
  - Diagnosis
  - Classifying text documents

Naive Bayes Classifier

- Assume target function *f: X* ➔ *V*, where each instance *x* described by attributes $a_1, a_2 .. a_n$
- Most probable value of *f(x)* is:

$$v_{MAP} = \arg\max_{v_j \in V} P(v_j | a_1, a_2 \ldots a_n)$$

$$v_{MAP} = \arg\max_{v_j \in V} \frac{P(a_1, a_2 \ldots a_n | v_j) P(v_j)}{P(a_1, a_2 \ldots a_n)}$$

$$= \arg\max_{v_j \in V} P(a_1, a_2 \ldots a_n | v_j) P(v_j)$$

V_{NB}

- Naive Bayes assumption:

$$P(a_1, a_2 \ldots a_n | v_j) = \prod_i P(a_i | v_j)$$

- which gives

$$\text{Naive Bayes classifier: } v_{NB} = \arg\max_{v_j \in V} P(v_j) \prod_i P(a_i | v_j)$$

Naive Bayes Algorithm

- For each target value v_j
- $\hat{P}(v_j)$ ← estimate $P(v_j)$
- For each attribute value a_i of each attribute a
- $\hat{P}(a_i|v_j)$ ← estimate $P(a_i|v_j)$

$$v_{NB} = \arg\max_{v_j \in V} \hat{P}(v_j) \prod_{a_i \in x} \hat{P}(a_i|v_j)$$

Training dataset

Class:
C1:buys_computer='yes'
C2:buys_computer='no'

Data sample:

X =
(age<=30,
Income=medium,
Student=yes
Credit_rating=Fair)

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
30…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Naïve Bayesian Classifier: Example

- Compute $P(X|C_i)$ for each class

P(age="<30" | buys_computer="yes") = 2/9=0.222
P(age="<30" | buys_computer="no") = 3/5 =0.6
P(income="medium" | buys_computer="yes")= 4/9 =0.444
P(income="medium" | buys_computer="no") = 2/5 = 0.4
P(student="yes" | buys_computer="yes)= 6/9 =0.667
P(student="yes" | buys_computer="no")= 1/5=0.2
P(credit_rating="fair" | buys_computer="yes")=6/9=0.667
P(credit_rating="fair" | buys_computer="no")=2/5=0.4

P(buys_computer=„yes“)=9/14

P(buys_computer=„no“)=5/14

- X=(age<=30 ,income =medium, student=yes,credit_rating=fair)

$P(X|C_i)$: P(X|buys_computer="yes")= 0.222 x 0.444 x 0.667 x 0.0.667 =0.044
P(X|buys_computer="no")= 0.6 x 0.4 x 0.2 x 0.4 =0.019

$P(X|C_i)*P(C_i)$: P(X|buys_computer="yes") * P(buys_computer="yes")=0.028
P(X|buys_computer="no") * P(buys_computer="no")=0.007

- X belongs to class "buys_computer=yes"

- Conditional independence assumption is often violated
- ...but it works surprisingly well anyway

Estimating Probabilities

- We have estimated probabilities by the fraction of times the event is observed to n_c occur over the total number of opportunities n
- It provides poor estimates when n_c is very small

- If none of the training instances with target value v_j have attribute value a_i?
  - n_c is 0

- When n_c is very small:

$$\hat{P}(a_i|v_j) = \frac{n_c + mp}{n + m}$$

- n is number of training examples for which $v=v_j$
- n_c number of examples for which $v=v_j$ and $a=a_i$
- p is **prior** estimate
- m is weight given to prior (i.e. number of ``virtual'' examples)

$$v_{NB} = _{v_j \in V} P(v_j) \prod_i \hat{P}(a_i|v_j)$$

Naïve Bayesian Classifier: Comments

- Advantages :
  - Easy to implement
  - Good results obtained in most of the cases
- Disadvantages
  - Assumption: class conditional independence , therefore loss of accuracy
  - Practically, dependencies exist among variables
  - E.g., hospitals: patients: Profile: age, family history etc

    Symptoms: fever, cough etc., Disease: lung cancer, diabetes etc
  - Dependencies among these cannot be modeled by Naïve Bayesian Classifier
- How to deal with these dependencies?
  - Bayesian Belief Networks

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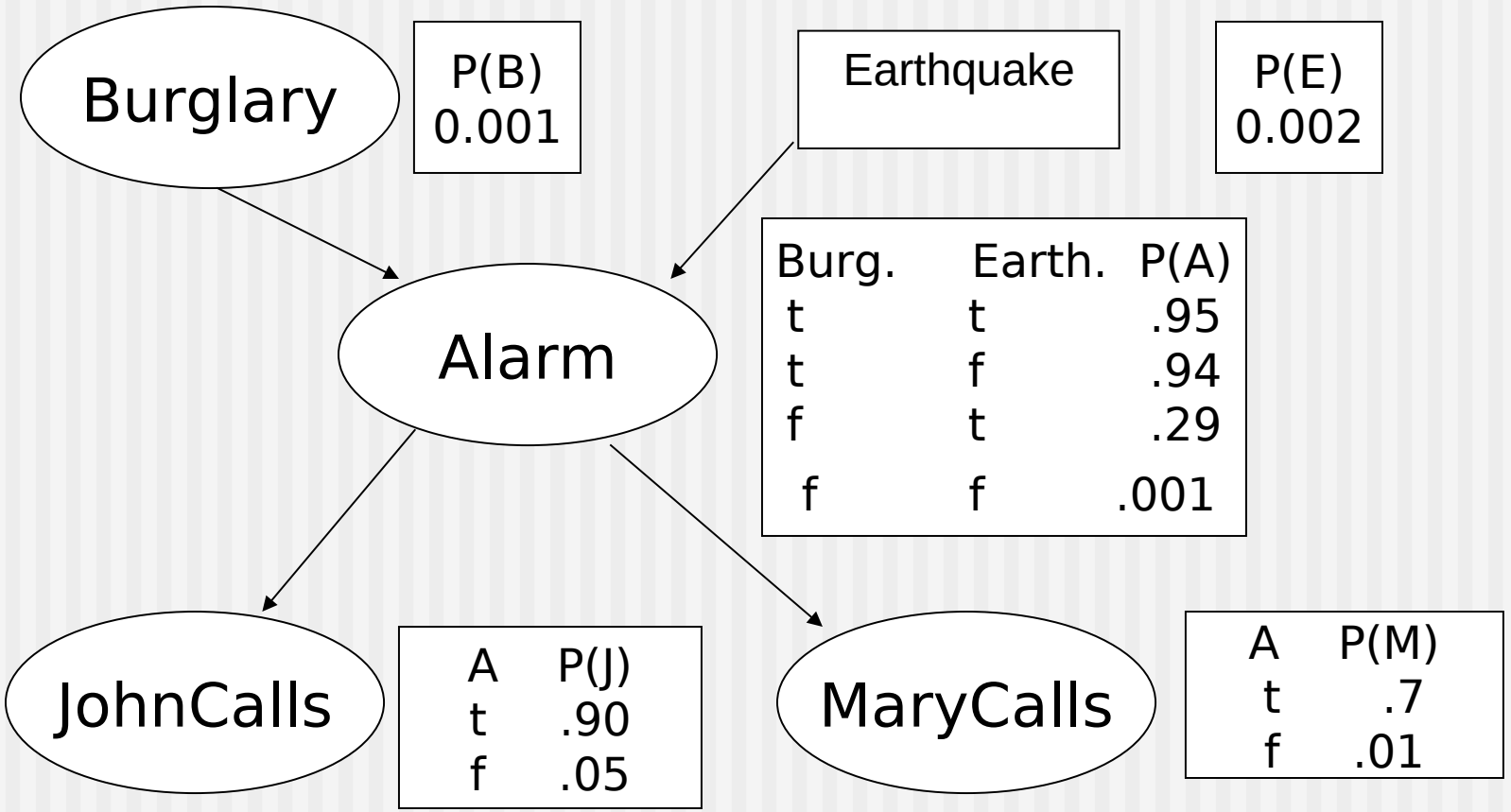
Bayesian Belief Networks



Naive Bayes Classifier

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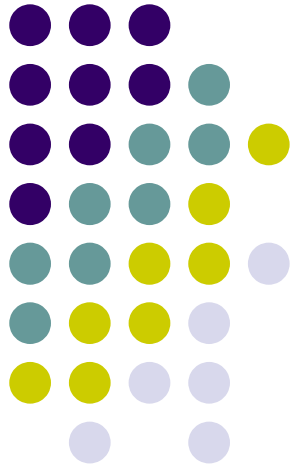
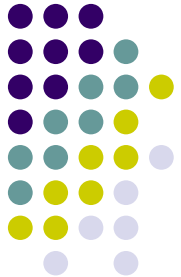


Bayesian Methods



- Learning and classification methods based on probability theory.
- Bayes theorem plays a critical role in probabilistic learning and classification.
- Uses *prior* probability of each category given no information about an item.
- Categorization produces a *posterior* probability distribution over the possible categories given a description of an item.

Basic Probability Formulas

- Product rule

$$P(A \wedge B) = P(A \mid B)P(B) = P(B \mid A)P(A)$$

- Sum rule

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B)$$

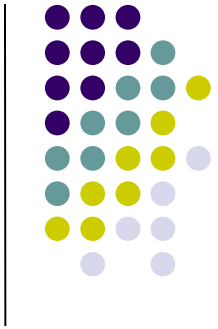
- Bayes theorem

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}$$

- Theorem of total probability, if event *Ai* is mutually exclusive and probability sum to 1

$$P(B) = \sum_{i=1}^{n} P(B \mid A_i)P(A_i)$$

Bayes Theorem

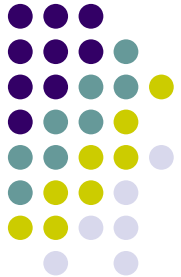


- Given a hypothesis *h* and data *D* which bears on the hypothesis:

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)}$$

- *P(h)*: independent probability of *h*: *prior probability*
- *P(D)*: independent probability of *D*
- *P(D|h)*: conditional probability of *D* given h: *likelihood*
- *P(h|D)*: conditional probability of *h* given *D*: *posterior probability*

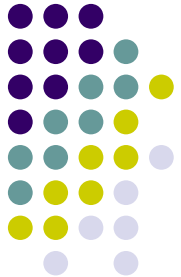
Does patient have cancer or not?



- A patient takes a lab test and the result comes back positive. It is known that the test returns a correct positive result in only 99% of the cases and a correct negative result in only 95% of the cases. Furthermore, only 0.03 of the entire population has this disease.

  1. What is the probability that this patient has cancer?
  2. What is the probability that he does not have cancer?
  3. What is the diagnosis?

Maximum A Posterior



- Based on Bayes Theorem, we can compute the *Maximum A Posterior* (MAP) hypothesis for the data
- We are interested in the best hypothesis for some space H given observed training data D.

$$h_{MAP} \equiv \underset{h \in H}{\operatorname{argmax}} P(h \mid D)$$

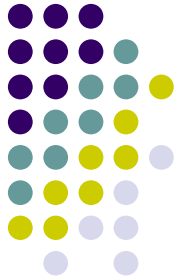
$$= \underset{h \in H}{\operatorname{argmax}} \frac{P(D \mid h) P(h)}{P(D)}$$

$$= \underset{h \in H}{\operatorname{argmax}} P(D \mid h) P(h)$$

H: set of all hypothesis.

Note that we can drop *P(D)* as the probability of the data is constant (and independent of the hypothesis).

Maximum Likelihood

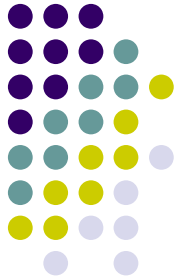


- Now assume that all hypotheses are equally probable a priori, i.e., *P(hi) = P(hj)* for all *hi, hj* belong to *H*.
- This is called assuming a *uniform prior*. It simplifies computing the posterior:

$$h_{ML} = \arg\max_{h \in H} P(D \mid h)$$

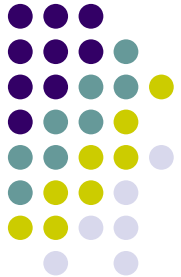
- This hypothesis is called the *maximum likelihood hypothesis.*

Desirable Properties of Bayes Classifier



- *Incrementality:* with each training example, the prior and the likelihood can be updated dynamically: flexible and robust to errors.
- *Combines prior knowledge and observed data:* prior probability of a hypothesis multiplied with probability of the hypothesis given the training data
- *Probabilistic hypothesis:* outputs not only a classification, but a probability distribution over all classes

Bayes Classifiers



Assumption: training set consists of instances of different classes described *cj* as conjunctions of attributes values

Task: Classify a new instance *d* based on a tuple of attribute values into one of the classes $cj \in C$

Key idea: assign the most probable class c_{MAP} using Bayes Theorem.

$$c_{MAP} = \underset{c_j \in C}{\operatorname{argmax}} P(c_j \mid x_1, x_2, \ldots, x_n)$$

$$= \underset{c_j \in C}{\operatorname{argmax}} \frac{P(x_1, x_2, \ldots, x_n \mid c_j) P(c_j)}{P(x_1, x_2, \ldots, x_n)}$$

$$= \underset{c_j \in C}{\operatorname{argmax}} P(x_1, x_2, \ldots, x_n \mid c_j) P(c_j)$$

Parameters estimation

- $P(c_j)$
  - Can be estimated from the frequency of classes in the training examples.
- $P(x_1,x_2,\ldots,x_n|c_j)$
  - O($|X|^n \bullet |C|$) parameters
  - Could only be estimated if a very, very large number of training examples was available.
- Independence Assumption: attribute values are conditionally independent given the target value: ***naïve Bayes***.

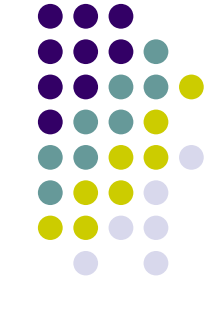
$$P(x_1, x_2, \ldots, x_n \mid c_j) = \prod_i P(x_i \mid c_j)$$

$$c_{NB} = \arg\max_{c_j \in C} P(c_j) \prod_i P(x_i \mid c_j)$$

Properties

- Estimating $P(x_i \mid c_j)$ instead of $P(x_1, x_2, \ldots, x_n \mid c_j)$ greatly reduces the number of parameters (and the data sparseness).
- The learning step in Naïve Bayes consists of estimating $P(x_i \mid c_j)$ and $P(c_j)$ based on the frequencies in the training data
- An unseen instance is classified by computing the class that maximizes the posterior
- When conditioned independence is satisfied, Naïve Bayes corresponds to MAP classification.

Example. 'Play Tennis' data



Day	Outlook	Temperature	Humidity	Wind	Play Tennis
Day1	Sunny	Hot	High	Weak	*No*
Day2	Sunny	Hot	High	Strong	*No*
Day3	Overcast	Hot	High	Weak	Yes
Day4	Rain	Mild	High	Weak	Yes
Day5	Rain	Cool	Normal	Weak	Yes
Day6	Rain	Cool	Normal	Strong	*No*
Day7	Overcast	Cool	Normal	Strong	Yes
Day8	Sunny	Mild	High	Weak	*No*
Day9	Sunny	Cool	Normal	Weak	Yes
Day10	Rain	Mild	Normal	Weak	Yes
Day11	Sunny	Mild	Normal	Strong	Yes
Day12	Overcast	Mild	High	Strong	Yes
Day13	Overcast	Hot	Normal	Weak	Yes
Day14	Rain	Mild	High	Strong	*No*

Question: For the day <sunny, cool, high, strong>, what's the play prediction?

Naive Bayes solution

Classify any new datum instance $\mathbf{x}=(a_1,\ldots a_T)$ *as:*

$$h_{Naive\ Bayes} = \arg\max_h P(h)P(\mathbf{x}\mid h) = \arg\max_h P(h)\prod_t P(a_t \mid h)$$

- To do this based on training examples, we need to estimate the parameters from the training examples:
  - For each target value (hypothesis) *h*

    $$\hat{P}(h) := \text{estimate } P(h)$$

  - For each attribute value a_t of each datum instance

    $$\hat{P}(a_t \mid h) := \text{estimate } P(a_t \mid h)$$

Based on the examples in the table, classify the following datum ***x***:

x=(Outl=Sunny, Temp=Cool, Hum=High, Wind=strong)

- That means: Play tennis or not?

$$h_{NB} = \arg\max_{h \in [yes,no]} P(h)P(\mathbf{x} \mid h) = \arg\max_{h \in [yes,no]} P(h) \prod_t P(a_t \mid h)$$

$$= \arg\max_{h \in [yes,no]} P(h)P(Outlook = sunny \mid h)P(Temp = cool \mid h)P(Humidity = high \mid h)P(Wind = strong \mid h)$$

- Working:

$$P(PlayTennis = yes) = 9/14 = 0.64$$

$$P(PlayTennis = no) = 5/14 = 0.36$$

$$P(Wind = strong \mid PlayTennis = yes) = 3/9 = 0.33$$

$$P(Wind = strong \mid PlayTennis = no) = 3/5 = 0.60$$

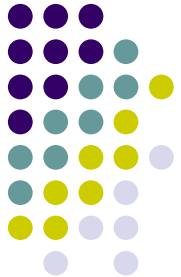
etc.

$$P(yes)P(sunny \mid yes)P(cool \mid yes)P(high \mid yes)P(strong \mid yes) = 0.0053$$

$$P(no)P(sunny \mid no)P(cool \mid no)P(high \mid no)P(strong \mid no) = \mathbf{0.0206}$$

$$\Rightarrow answer : PlayTennis(x) = no$$

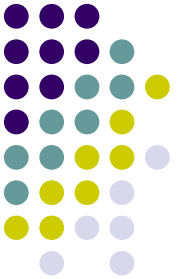
Underflow Prevention



- Multiplying lots of probabilities, which are between 0 and 1 by definition, can result in floating-point underflow.
- Since *log(xy) = log(x) + log(y),* it is better to perform all computations by summing logs of probabilities rather than multiplying probabilities.
- Class with highest final un-normalized log probability score is still the most probable.

$$c_{NB} = \underset{c_j \in C}{\operatorname{argmax}} \log P(c_j) + \sum_{i \in positions} \log P(x_i \mid c_j)$$

Maximum likelihood of real valued function



$$h_{ML} = arg\max_{h \in H} p(D|h)$$

$$= arg\max_{h \in H} \prod_{i=1}^{m} p(d_i|h)$$

$$= arg\max_{h \in H} \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{d_i - h(x_i)}{\sigma}\right)^2}$$

Maximum log likelihood

$$h_{ML} = arg\max_{h \in H} \sum_{i=1}^{m} \ln \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{1}{2}\left(\frac{d_i - h(x_i)}{\sigma}\right)^2$$

$$= arg\max_{h \in H} \sum_{i=1}^{m} -\frac{1}{2}\left(\frac{d_i - h(x_i)}{\sigma}\right)^2$$

$$= arg\max_{h \in H} \sum_{i=1}^{m} -(d_i - h(x_i))^2$$

$$= arg\min_{h \in H} \sum_{i=1}^{m} (d_i - h(x_i))^2$$