

KENDRIYA VIDYALAYA SANGATHAN**ZIET, BHUBANESWAR****STUDY MATERIAL FOR CLASS XI SUBJECT : PHYSICS****CHAPTER -I****PHYSICAL WORLD****MAIN POINTS**

- Physics deals with the study of the basic laws of nature and their manifestation in different phenomena. The basic laws of physics are universal and apply in widely different contexts and conditions.
- The scope of physics is wide, covering a tremendous range of magnitude of physical quantities.
- Physics and technology are related to each other. Sometimes technology gives rise to new physics at other times physics generates new technology. Both have direct impact on society.
- There are four fundamental forces in nature that govern the diverse phenomena of the macroscopic and the microscopic world. These are the 'gravitational force', the electromagnetic force', 'the strong nuclear force', and the weak nuclear force'
- The physical quantities that remain unchanged in a process are called conserved quantities. Some of the general conservation laws in nature include the law of conservation of mass, energy, linear momentum, angular momentum, charge, parity, etc.
- Conservation laws have a deep connection with symmetries of nature. symmetries of space and time, and other types of symmetries play a central role in modern theories of fundamental forces in nature.
- Gravitational force is the force of mutual attraction between any two objects by virtue of their masses. It is always attractive
- Electromagnetic Force is the force between charged particles. It acts over large distances and does not need any intervening medium. Enormously strong compared to gravity. It can be attractive or repulsive.
- Strong nuclear force is the force that binds the nucleons together. It is the strongest of all the fundamental forces. It is charge independent. And very short range.
- Weak nuclear force appears only in certain nuclear processes such as β -decay. Weak nuclear force is not as weak as the gravitational force.
- In a chemical reaction if the total binding energy of the reacting molecules is less than that of the product molecules the difference appears as heat and the reaction is exothermic
- In a chemical reaction if the total binding energy of the reacting molecules is more than that of the product molecules the difference amount of energy is absorbed and the reaction is endothermic.
- In a nuclear process mass gets converted into energy. This is the energy which gets released in a nuclear power generation and nuclear explosions.

UNITS AND MEASUREMENT

CONCEPTS INVOLVED

- The International system of units
- Measurement of length
- Measurement of mass
- Measurement of Time
- Accuracy, Precision of instruments and errors in measurement
- Significant figures
- Dimensions of physical quantities
- Dimensional formulae and dimensional equations
- Dimensional analysis and its applications

Main points

- Physics is a quantitative science, based on measurement of physical quantities. Certain physical quantities have been chosen as fundamental or base quantities. The fundamental quantities that are chosen are Length, Mass, Time, electric current, thermodynamic temperature, amount of substance, and luminous intensity.
- Each base quantity is defined in terms of a certain basic arbitrarily chosen but properly standardised reference standard called unit (such as metre, kilogram, second, ampere, kelvin, mole, and candela). The units for the fundamental base quantities are called fundamental or base units and two supplementary units in relation to quantities plane angle and solid angle radian, Steradian..
- Other physical quantities derived from the base quantities can be expressed as a combination of the base units and are called derived units. A complete set of units both fundamental and derived units are called a system of units.
- The International System of units based on seven base units is at present internationally accepted unit system and is widely used throughout the world
- The SI units are used in all physical measurements, for both the base quantities and the derived quantities obtained from them. Certain derived units are expressed by means of SI units of special names such as joule, newton, watt etc.
- In computing any physical quantity the units for derived quantities involved in the relationships are treated as though they were algebraic quantities till the desired units are obtained
- In SI system that is System Internationale d' Units there are 7 base units' and two supplementary units.
- Direct and indirect methods can be used for the measurement of physical quantities. In measured quantities while expressing the result, the accuracy and precision of measuring instruments along with errors in measurement should be taken into account.

- In measured and computed quantities proper significant figures only should be retained.

Rules for determining the number of significant figures, carrying out arithmetic operations with them and rounding off the uncertain digits must be followed.

- The dimensions of base quantities and combination of these dimensions describe the nature of physical quantities. Dimensional analysis can be used to check the dimensional consistency of equations, deducing relations among physical quantities etc. A dimensionally consistent equation need not be actually an exact equation, but a dimensionally wrong or inconsistent equation must be wrong.
- The uncertainty in the measurement of a physical quantity is called an error.
- The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.
- Precision tells us to what limit the quantity is measured.
- The errors in measurement can be classified as
(i) Systematic errors and (ii) Random errors
- **SYSTEMATIC ERRORS:** These are the errors that tend to be either positive or negative. Sources of systematic errors are
(i) Instrumental errors
(ii) Imperfection in experimental technique or procedure
(iii) Personal errors
- **RANDOM ERRORS:** Those errors which occur irregularly. These errors arise due to unpredictable fluctuations in experimental conditions
- Least count error is the error associated with the resolution of the instrument.
- The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement.

Ex: $\Delta a = |a - a_{\text{mean}}|$

- The relative error or the percentage error is the ratio of the mean absolute error to the mean value of the quantity measured. When the relative error is expressed in per cent it is called the percentage error.

Ex: (i) Relative error = $\Delta a_{\text{mean}}/a_{\text{mean}}$ (ii) % error = $(\Delta a_{\text{mean}}/a_{\text{mean}}) \times 100$

COMBINATION OF ERRORS

✓ **ERROR OF A SUM OR A DIFFERENCE**

When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

IF $Z = A + B$ then the max possible error in Z, $\Delta Z = \Delta A + \Delta B$

IF $Z = A - B$ then the max possible error in Z, $\Delta Z = \Delta A + \Delta B$

✓ **ERROR OF A PRODUCT OR A QUOTIENT**

When two quantities are multiplied or divided the relative error is the sum of the relative errors in the multipliers

Suppose $Z = A \cdot B$ or $Z = A/B$ then the max relative error in 'Z' = $\Delta Z/Z = (\Delta A/A) + (\Delta B/B)$

✓ **ERROR IN CASE OF A QUANTITY RAISED TO A POWER**

The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

Suppose $Z = A^k$ then $\Delta Z/Z = K (\Delta A/A)$

SIGNIFICANT FIGURES

The reliable digits plus the first uncertain digit in a measurement are called Significant figures.

RULES FOR FINDING THE SIGNIFICANT FIGURES IN A MEASUREMENT

- ✓ All the non-zero digits are significant
- ✓ All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all
- ✓ If the number is less than 1, the zero(s) on the right side of decimal point but to the left of the first non-zero digit are not significant.
Ex: In 0.000 35 the underlined zeros are not significant.

- ✓ The terminal or trailing zeros in a number without a decimal point are not significant
Ex: 1795 m = 179500cm = 1795000mm has four significant figures.
- ✓ The trailing zeros in a number with a decimal point are significant
For ex: The numbers 75.00 or 0.06700 have four significant figures each.

RULES FOR ARITHMETIC OPERATIONS WITH SIGNIFICANT FIGURES.

- ✓ In multiplication or division, the final result should retain as many significant figures as there are in the original number with the least significant figures.

Suppose $F = 0.04 \text{ Kg} \times 0.452 \text{ m/sec}^2 = 0.0108 \text{ kg-m/sec}^2$

The final result is $F = 0.01 \text{ Kg-m/Sec}^2$

- ✓ In addition or subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.
For ex: $A = 334.5 \text{ kg}$; $B = 23.45 \text{ Kg}$ then $A + B = 334.5 \text{ kg} + 23.43 \text{ kg} = 357.93 \text{ kg}$
The result with significant figures is 357.9 kg

ROUNDING OFF:

While rounding off measurements the following rules are applied

- ✓ **Rule I:** If the digit to be dropped is smaller than 5, then the preceding digit should be left unchanged.
For ex: 9.32 is rounded off to 9.3
- ✓ **Rule II:** If the digit to be dropped is greater than 5, then the preceding digit should be raised by 1
For ex: 8.27 is rounded off to 8.3
- ✓ **Rule III:** If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit should be raised by 1

For ex: 9.351 on being rounded off to first decimal, becomes 9.4

- ✓ **Rule IV:** If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is not changed if it is even, is raised by 1 if it is odd.

For ex: 5.45, on being rounded off, become 5.4

5.450 on being rounded off, becomes 5.4

(ii) 7.35, on being rounded off, becomes 7.4

DIMENSIONS, DIMENSIONAL FORMULA AND DIMENSIONAL EQUATION

- Dimensions of a derived unit are the powers to which the fundamental units of mass, length and time etc. must be raised to represent that unit.

For ex: Density = Mass / Volume = $M/L^3 = M^1L^{-3}$

- Dimensional formula is an expression which shows how and which of the fundamental units are required to represent the unit of a physical quantity.

For Ex: $M^1 L^1 T^{-2}$ is the dimensional formula of Force.

CATEGORIES of PHYSICAL QUANTITIES

- Dimensional Constants: These are the quantities which possess dimensions and have a fixed value.
Ex: Gravitational Constant
- Dimensional Variables These are the quantities which possess dimensions and do not have a fixed value
For ex: velocity, acceleration etc.
- Dimensionless Constants: these are the quantities which do not possess dimensions and have a fixed value.
For ex: π etc.
- Dimensionless Variables: These are the quantities which are dimensionless and do not have a fixed value.
For ex: Strain, Specific Gravity etc.

PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

- A given physical relation is dimensionally correct if the dimensions of the various terms on either side of the relation are the same.

USES OF DIMENSIONAL EQUATIONS

- **TO CONVERT PHYSICAL QUANTITY FROM ONE SYSTEM OF UNITS TO ANOTHER**

- ✓ Consider a physical quantity whose dimensions are $M^a L^b T^c$. Let n_1 be its numerical value in a system of fundamental units M_1, L_1, T_1 . Then the magnitude of the physical quantity in this system is $n_1 [M_1^a L_1^b T_1^c]$.

Let n_2 be the numerical value in another system of fundamental units M_2 , L_2 and T_2 . The magnitude of the quantity in this system is $n_2 [M_2^a L_2^b T_2^c]$.

Since the value of the quantity is the same in all systems

$$n_2 [M_2^a L_2^b T_2^c] = n_1 [M_1^a L_1^b T_1^c]$$

$$n_2 = n_1 [M_1^a L_1^b T_1^c] / [M_2^a L_2^b T_2^c]$$

➤ **TO CHECK THE DIMENSIONAL CORRECTNESS OF A GIVEN PHYSICAL RELATION**

Ex: $v = u + at$, Here v represents the velocity of the body after t secs, a , is the acceleration and u the initial velocity of the body.

Dimensional formula of u is $M^0 L^1 T^{-1}$

Dimensional formula of V is $M^0 L^1 T^{-1}$

Dimensional formula of at is $\{M^0 L^1 T^{-2}\} \{T^1\} = \{M^0 L^1 T^{-1}\}$

The dimensions of every term in the given physical relation is same, hence according to principle of homogeneity the given physical relation is dimensionally correct.

➤ **TO ESTABLISH RELATION BETWEEN DIFFERENT PHYSICAL QUANTITIES**

To find an expression for the time period of a simple pendulum given that the time period(t) may depend upon (i) mass of the bob (ii) length of the pendulum (iii) acceleration due to gravity, (iv) angle of swing θ

$$\text{Let (i) } t \propto m^a \text{ (ii) } t \propto l^b \text{ (iii) } t \propto g^c \text{ (iv) } t \propto \theta^d$$

Or $t = K m^a l^b g^c \theta^d$ Where K is a Dimensionless constant of proportionality.

Writing down the dimensions on either side of the equation we get

$$[T] = [M^a][L^b][L T^{-2}]^c = [M^a L^{b+c} T^{-2c}]$$

Comparing the dimensions on either side

$$a=0; b+c=0; -2c=1 \text{ i.e., } c = -1/2, b = +1/2, a = 0$$

$$t = K l^{1/2} g^{-1/2} \text{ or } t = K \sqrt{l/g}$$

The value of K as found by experiment comes out to be 2π

$$\text{And hence } t = 2\pi \sqrt{l/g}$$

➤ **LIMITATIONS OF DIMENSIONAL ANALYSIS**

- It supplies no information about dimensionless constants. They have to be determined either by experiment or by mathematical investigation.
- This method applicable only in the case of power functions. It fails in case of exponential and trigonometric relations.
- It fails to derive a relation which contains two or more than two quantities of like nature.

- It can only check whether a physical relation is dimensionally correct or not. It cannot tell whether the relation is absolutely correct or not
- It cannot identify all the factors on which the given physical quantity depends upon.

ANSWER THE FOLLOWING EACH QUESTION CARRIES 1 MARK.

1. Define physical quantities
2. Distinguish between fundamental and derived quantities
3. Define one metre
4. Define one kilogram
5. Define one second
6. Define the SI unit of the following physical quantities
(i) Temperature (ii) Luminous intensity
7. Define one radian
8. Define one Steradian
9. Give the relation between light year and metre
10. Write two advantages in choosing the wavelength of light radiation as a standard of length
11. What is the difference between 5.0 and 5.000?
12. Write the uses of the dimensional analysis
13. Write the dimensional equation for force
14. Write the dimensional representation for torque
15. Give the relationship between calorie and joule
16. Write two advantages in choosing the wavelength of light radiation as a standard of length.
17. What is the difference between 4.0 and 4.0000?

18. Write the uses of Dimensional Analysis.

19. Define the term significant figures.

Answer the following. Each question carries 2 marks.

20. Write four limitations of dimensional analysis

21. If $(P + a/V^2)(V-b) = RT$, Where the difference symbols have their usual meaning then what are the dimensions of (a/V^2) and b .

22. Write the dimensions of the following

(i) Electric intensity (ii) Electric Potential (iii) E.M.F. of a cell (iv) Electrical resistance

23. Write the dimensions of the following

(i) Specific Resistance (II) Magnetic flux (III) Electric flux (IV) Magnetic Induction

24. Write the dimensions of the following

(i) Conductance (ii) Electric Permittivity (iii) Magnetic Permeability (iv) Coefficient of Self Inductance

23. Solve the following to correct significant figures

(i) $5.1 + 13.235$ (ii) $7.54 + 18.1295$ (iii) $14.632 - 5.52345$ (iv) 3.021×11

24. Define mean scalar second

25. Define second in terms of Cs-133 vibrations

Answer the following. Each question carries 3 marks.

26. Answer the following

(a) You are given a thread and a meter scale. How will you estimate the diameter of this thread?

(b) A screw gauge has a pitch of 1.00 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the gauge arbitrarily by increasing the number of divisions on the circular scale?

27. Explain briefly how you will estimate the size of the molecule of oleic acid.

28. Explain how will you estimate the distance of a planet or star by using parallax method.

29. Find the area of a circle of radius 3.458 cm up to correct significant figures.

30. If the % error in the measurement of the radius R of a sphere is 0.2%, then calculate the % error in its volume.

31. A laser light beamed towards the moon takes 2.56 sec to return to the earth after the reflection on the moon's surface. What is the distance of the moon from the earth?

3. Motion in a straight line

IMPORTANT POINTS

- Study of motion of objects along a straight line is known as rectilinear motion.
- If a body does not change its position with time it is said to be at rest. If it changes its position with time it is said to be in motion. The position of the object can be specified with reference to a conveniently chosen origin. For motion in a straight line, position to the right of the origin is taken as positive and to the left as negative.
- Path length is defined as the total length of the path traversed by an object.
- Displacement is the change in position : $\Delta x = x_2 - x_1$, Path length is greater than or equal to the magnitude of the displacement between the two positions
- An object is said to be in uniform motion in a straight line if its displacement is equal in equal intervals of time. Otherwise the motion is said to be non-uniform.
- Average velocity is the ratio of the displacement and time interval in which the displacement occurs.

$$\bar{V} = \Delta x / \Delta t$$

On an x-t graph, the average velocity over a time interval is the slope of the line connecting the initial and final positions corresponding to that interval.

- Average Speed is ratio of the total path length traversed and the corresponding time interval. The average speed of an object is greater than or equal to the magnitude of the average velocity over a given interval of time.
- Instantaneous velocity or simply velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$V_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \bar{V} = \lim_{\Delta t \rightarrow 0} \Delta x / \Delta t = dx/dt$$

The velocity at a particular instant is equal to the slope of the tangent drawn on position –time graph at that instant.

- Average acceleration is the change velocity divided by the time interval during which the change occurs.

$$a = \Delta V / \Delta t$$

- Instantaneous acceleration is defined as the limit of the average acceleration as the time interval Δt goes to zero.

$$a = \lim_{\Delta t \rightarrow 0} (a) = \lim_{\Delta t \rightarrow 0} (\Delta v / \Delta t) = dv/dt$$

$$\Delta t \rightarrow 0 \Delta t \rightarrow 0$$

The acceleration of an object at a particular time is the slope of the velocity- time curve at that instant of time. For uniform motion, acceleration is zero and x-t graph is a straight line inclined to the time axis. And v-t graph is a straight line parallel to the time axis. For motion with uniform acceleration, x-t graph is a parabola, while the v-t graph is a straight line inclined to the time axis.

- The area under the velocity- time curve between times t_1 and t_2 is equal to the displacement of the object during that interval of time.
- For objects in uniformly accelerated rectilinear motion, the five quantities, displacement x, time taken t, initial velocity u, final velocity v and acceleration are related by a set of simple equations called kinematic equations of motion.

(i) $V = u + at$

(ii) $X = ut + \frac{1}{2} a t^2$

(iii) $V^2 - u^2 = 2 a x$

Solve the following. Each question carries 1 mark.

1. Distinguish between scalar and vector quantities. Give an example in support of each.
2. Define speed. How is it different from velocity?
3. Define uniform velocity and variable velocity?
4. Plot the velocity- time graph for a uniform motion. What does the area under the graphs indicate?
5. Write any two equations of motion of a body moving with uniform acceleration.
6. Plot a velocity- time graph for a body moving with uniform acceleration.
7. Plot position – time graph for a body having uniformly retarded motion
8. What does the speedometer of car indicate?

9. Two cars are running at velocities of 60 km/hr and 45 km/hr respectively. Calculate the relative velocity of car A, if (i) they are both travelling eastwards; and (ii) car A is travelling eastwards and car B is travelling westwards.

10. A body goes from A to B with a velocity of 40 m/sec, and comes back from B to A with a velocity of 60 m/sec. What is the average velocity of the body during the whole journey.

Answer the following questions each question carries 2 marks

11. A player throws a ball upwards with an initial speed of 39.2 m/sec.

(a) What is the direction of acceleration during the upward motion?

(b) Find the velocity and acceleration of the ball at the highest point.

(c) Find the height through which the ball rises, and the time after which it returns to the player's hands.

12. From the top of a tower 100m height, balls is dropped, and at the same time another ball is projected vertically upwards from the ground with a velocity of 25 m/sec. find when and where the two balls meet. Take $g = 9.8 \text{ m/sec}^2$

13. The distance travelled by a body is found to be directly proportional to the square of time. Is the body moving with uniform velocity or with uniform acceleration?

14. The displacement (x) of a particle moving in one dimension, under the action of a constant force related to time t by the relation $t = \sqrt{x} + 3$ where x is in meters, and t is in seconds. Find the displacement of the particle when its velocity is zero.

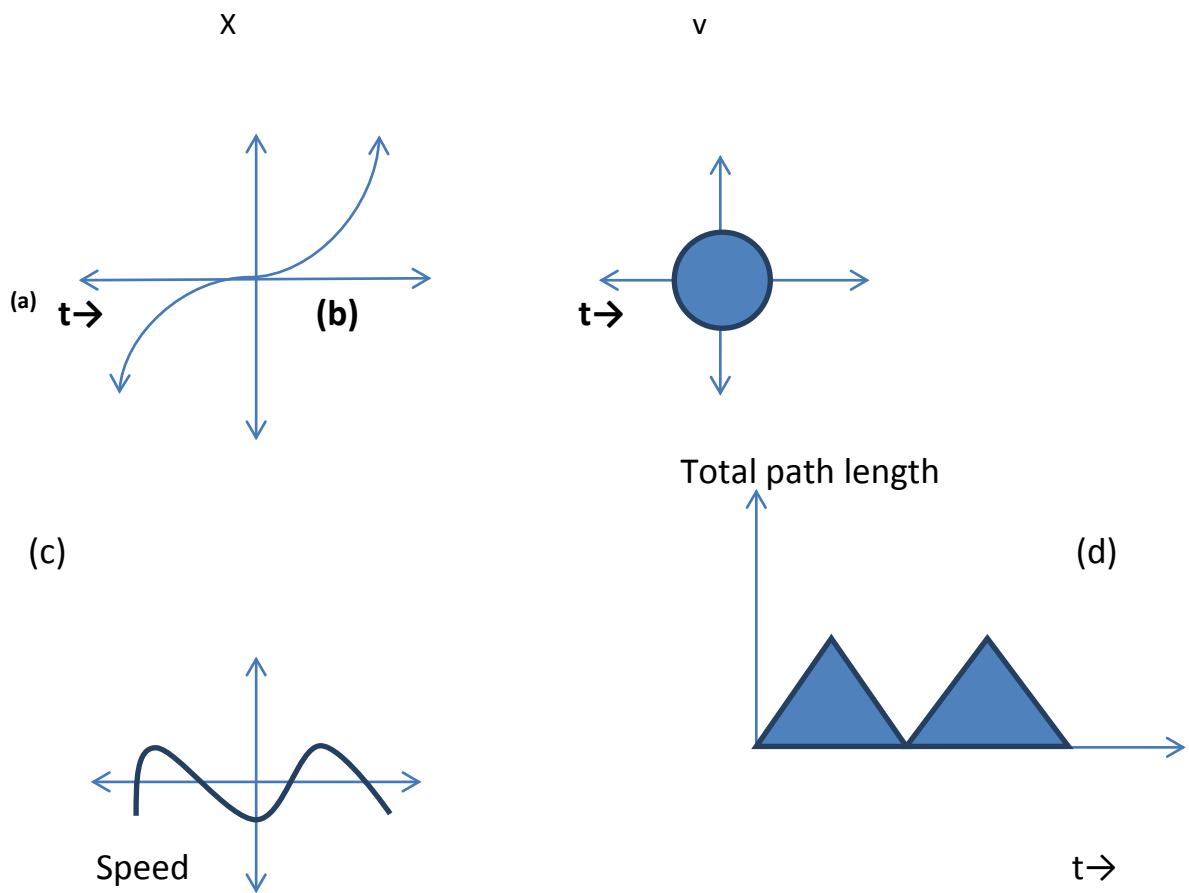
15. Can a body have zero velocity, and finite acceleration?

16. Can a body have constant speed, but a varying velocity?

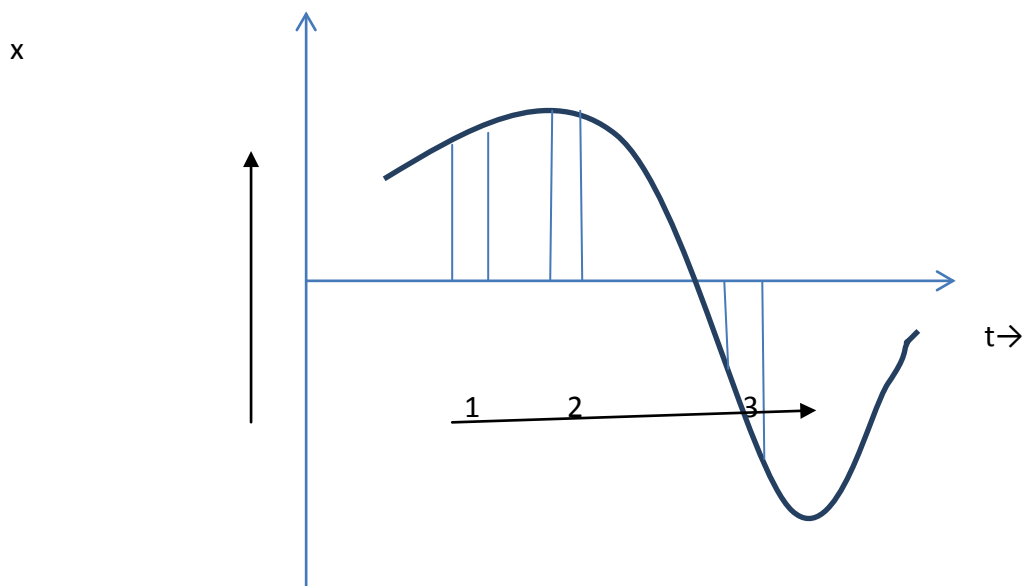
17. Can a body have constant velocity, but a varying speed?

18. Why is it that a parachute descends slowly whereas a stone dropped from the same height falls rapidly?

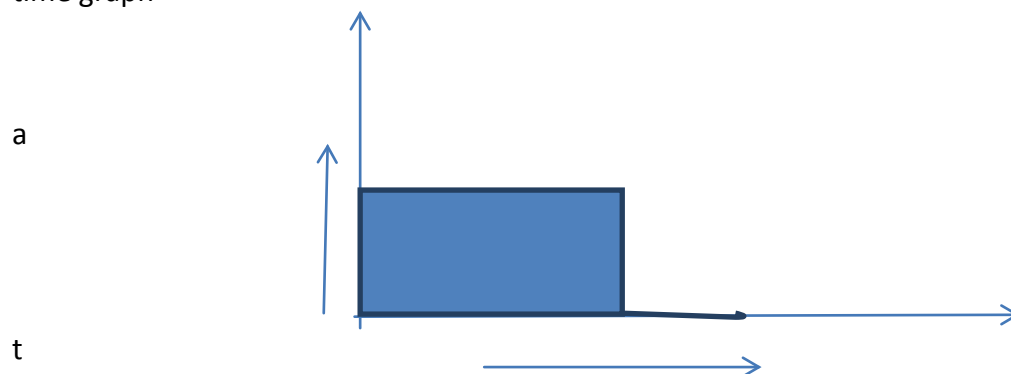
19. Look at the graphs (a) to (d) in fig carefully and state with reasons which of these cannot possibly represent one dimensional motion of a particle.



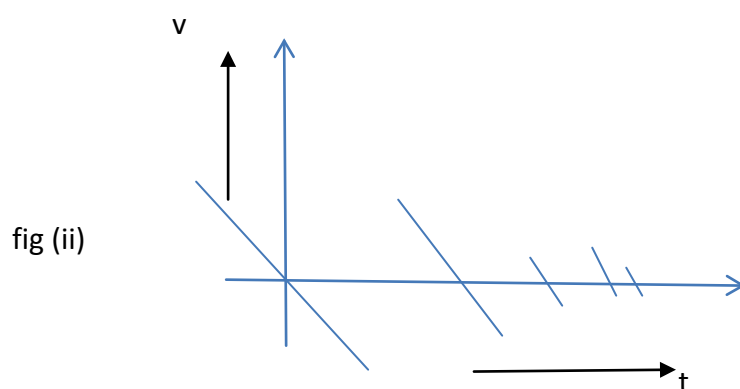
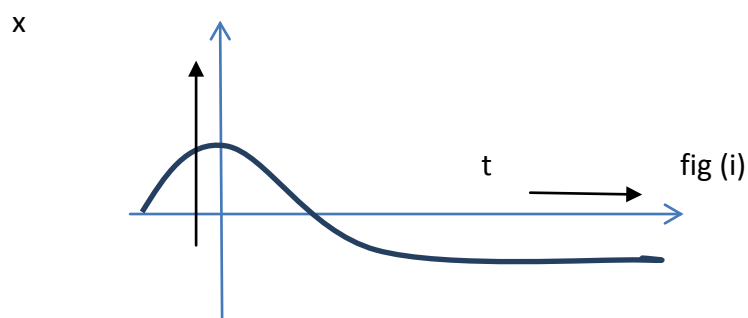
20. Fig gives the $x-t$ plot of a particle in one dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval



21. The acceleration –time graph for a body is shown below. Plot the corresponding velocity-time graph



21. Suggest a suitable physical situation for each of the following graphs



22. Give the equations of motion of a body falling under gravity. Also give the graphs showing the variation of (i) acceleration of a body with time

(ii) Velocity of a body with time

(iii) Distance with time in case of a freely falling body.

23. Discuss the motion of an object under free-fall.

24. Derive a relation between the position and time for a particle moving with uniform acceleration.

25. Derive a relation for the distance covered in the n th second by a uniformly accelerated body.

26. Show that when a body has uniformly accelerated motion, the distance covered in a certain interval is equal to the area under the velocity-time graph for that time interval.

27. A body is moving with uniform acceleration its velocity after 5 seconds is 25 m/sec and after 8 seconds is 34 m/sec. Calculate the distance it will cover in the 10th second.

28. The speed of a train increases at a constant rate α from zero, to v , and then remains constant for an interval, and finally decreases to zero at a constant rate β . If L be the total distance described, prove that the total time taken is

$$(L/v) + (v/2) (1/\alpha + 1/\beta)$$

29. A body moving with a uniform acceleration describes 12 m in the third second of its motion and 20 m in the fifth second. Find the velocity after 10 seconds.

Description of Motion in Two and Three Dimensions

Main Points:

1. Scalar quantities with magnitudes only. Examples are distance, speed mass and temperature.

2. Vector quantities with magnitude and direction both. Examples are displacement, velocity and acceleration. They obey special rules of vector algebra.

3. A vector ' \mathbf{A} ' multiplied by a real number ' λ ' is also a vector, whose magnitude is ' λ ' times the magnitude of the vector ' \mathbf{A} ' and whose direction is same or opposite depending upon whether ' λ ' is positive or negative.

4. Two vectors \mathbf{A} and \mathbf{B} may be added graphically using head to tail method or parallelogram method.

5. Vector addition is **commutative**:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

It also obeys the **associative law**:

$$(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$$

6. A null or zero vector is a vector with zero magnitude. Since the magnitude is zero, we don't have to specify its direction. It has the properties:

$$\mathbf{A} + \mathbf{0} = \mathbf{A}$$

$$\lambda \mathbf{0} = \mathbf{0}$$

$$\overline{\mathbf{0}} \cdot \overline{\mathbf{A}} = \mathbf{0}$$

7. The subtraction of vector **B** from **A** is defined as the sum of **A** and **-B**:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

8. A vector **A** can be resolved into components along two given vectors **a** and **b** lying in the same plane:

$$\mathbf{A} = \lambda \mathbf{a} + \mu \mathbf{b} \text{ where } \lambda \text{ and } \mu \text{ are real numbers.}$$

9. A unit vector associated with a vector **A** has magnitude one and is along the vector **A** :

$\hat{n} = \mathbf{A} / |\mathbf{A}|$ The unit vectors $\hat{i}, \hat{j}, \hat{k}$ are vectors of unit magnitude and point in the direction of x, y, and z- axes respectively in a right handed coordinate system.

10. A vector '**A**' can be expressed as

$\mathbf{A} = A_x \hat{i} + A_y \hat{j}$ where A_x, A_y are components along x- and y- axes . If vector **A** makes an angle Θ with the x- axis, then $A_x = A \cos \Theta$, $A_y = A \sin \Theta$ and $A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2}$,

$$\tan \Theta = A_y / A_x.$$

11. Vectors can be conveniently added using analytical method. If sum of two vectors '**A**' and '**B**', that lie in x-y plane is '**R**', then:

$$\mathbf{R} = R_x \hat{i} + R_y \hat{j}, \text{ where } R_x = A_x + B_x \text{ and } R_y = A_y + B_y$$

12. The position vector of an object in x-y plane is given by $\mathbf{r} = x \hat{i} + y \hat{j}$ and the displacement from position **r** to position **r'** is given by

$$\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$$

$$= (x' - x) \hat{i} + (y' - y) \hat{j}$$

$$= \Delta x \hat{i} + \Delta y \hat{j}$$

13. If an object undergoes a displacement Δr in time Δt , its average velocity is given by

$V = \Delta r / \Delta t$. The velocity of an object at time t is the limiting value of the average velocity as Δt tends to zero..

$V = \lim_{\Delta t \rightarrow 0} \Delta r / \Delta t = dr / dt$.It can be written in unit vector notation as

$$\Delta t \rightarrow 0$$

$$V = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \quad \text{where } V_x = dx/dt, V_y = dy/dt, V_z = dz/dt$$

When position of an object is plotted on a coordinate system v is always tangent to the curve representing the path of the object.

14. If the velocity of an object changes from v to v' in time Δt , then its average acceleration is given by $\underline{a} = (v - v') / \Delta t = \Delta v / \Delta t$

The acceleration a at any time t is the limiting value of \underline{a} as $\Delta t \rightarrow 0$

$$a = \lim_{\Delta t \rightarrow 0} \Delta v / \Delta t = dv / dt$$

$$\Delta t \rightarrow 0$$

In component form, we have $a = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

Where $a_x = dv_x/dt$, $a_y = dv_y/dt$, $a_z = dv_z/dt$

15. If an object is moving in a plane with constant acceleration $a = \underline{a} = \sqrt{a_x^2 + a_y^2}$

And its position vector at time $t = 0$ is r_0 , then at any other time t , it will be at a point given by

$$r = r_0 + V_0 t + \frac{1}{2} a t^2$$

and its velocity is given by :

$$V = V_0 + at \quad \text{where } V_0 \text{ is the velocity at time } t = 0$$

In component form

$$X = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$$

$$Y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2$$

$$V_x = V_{0x} + a_x t$$

$$V_y = V_{0y} + a_y t$$

Motion in a plane can be treated as superposition of two separate simultaneous one dimensional motions along two perpendicular directions.

16. An object that is in flight after being projected is called a projectile. If an object is projected with an initial velocity V_0 making an angle Θ_0 with x-axis and if assume its initial position to coincide with the origin of the coordinate system, then the position and velocity of the projectile at time t are given by

$$X = (V_0 \cos \Theta_0) t$$

$$Y = (V_0 \sin \Theta_0) t - \frac{1}{2} g t^2$$

$$V_x = V_{0x} = V_0 \cos \Theta_0$$

$$V_y = V_0 \sin \Theta_0 - gt$$

The path of a projectile is parabolic and is given by

$$Y = (\tan \Theta_0) x - \frac{gx^2}{2V_0^2 \cos^2 \Theta_0}$$

The maximum height that a projectile attains is

$$H_m = \frac{(V_0 \sin \Theta_0)^2}{2g}$$

The time taken to reach this height is

$$t_m = \frac{V_0 \sin \Theta_0}{g}$$

The horizontal distance travelled by a projectile from its initial position to the position it passes $y = 0$ during its fall is called the RANGE, R of the projectile. It is:

$$R = \frac{V_0^2 \sin 2\Theta_0}{g}$$

Projectile motion

Motion along vertical direction

Motion along horizontal direction

force of gravity acting on the body

Equations of motion of a body moving with constant acceleration

No force in the horizontal direction and hence horizontal velocity remains constant.

17. When an object follows a circular path at constant speed. The motion of the object is called uniform circular motion. The magnitude of its acceleration is $a_c = v^2/R$. The direction of a_c is always towards the centre of the circle.

The angular speed is the rate of change of angular distance. It is related velocity v by

$V = \omega R$. The acceleration is $a_c = \omega^2 R$.

- If T is the time period of revolution of the object in circular motion and v is the frequency then we have $\omega = 2\pi v$, $V = 2\pi v R$ $a_c = 4\pi^2 v^2 R$
- Centripetal force is the name given to the force that provides inward radial acceleration to a body in circular motion. We should always look for some material force like tension, gravitational force, electrical force, friction etc. as the centripetal force.

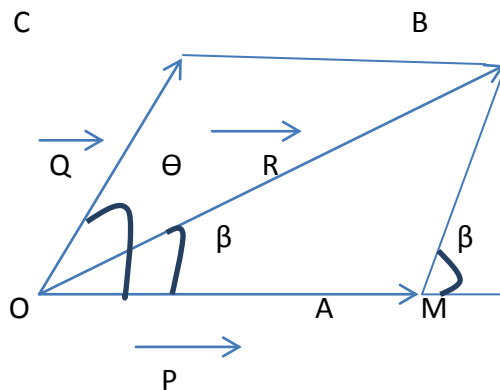
18. The path length traversed by an object between two points is not the same as the magnitude of displacement always. The displacement depends only on the end points; whereas the path length depends on the actual path. The two quantities are equal only if the object does not change its direction during the course of its motion. In all other cases, the path length is greater than the magnitude of displacement.

19. The average speed of an object is greater than or equal to the magnitude of the average velocity over a given interval of time. The two are equal only if the path length is equal to the magnitude of the displacement.

Answer the following

1. When are two vectors equal?
2. What are co-initial and collinear vectors?
3. When are two vectors equal?
4. State the triangle law of vector addition
5. State the parallelogram law of vector addition.
6. Using the parallelogram law of vectors, find the magnitude and direction of the resultant R in the following fig

Discuss cases for (i) $\theta = 0^\circ$ (ii) $\theta = 90^\circ$ (iii) $\theta = 180^\circ$



7. State three important properties of vector addition.
8. What do you mean by the null vector or zero vector.
9. Write properties of zero vector.
10. Define scalar product of two vectors. Give one example.
11. Define vector product of two vectors? Give one example.
12. Express the area of a parallelogram in terms of cross product of two vectors.
13. Define the following terms associated with projectile motion.
 - (i) Projectile (ii) trajectory (iii) time of flight (IV) maximum height (v) Horizontal Range
14. Give two examples of projectile motion.

15. Write two applications of projectile motion
16. At what angle with the vertical should a projectile be fired so that its range is maximum?
17. A projectile is fired with 'u' at an angle of projection Θ . Write
 - (i) The equation of trajectory
 - (ii) Expression for the maximum height reached
 - (iii) Expression for the time of flight
 - (iv) Expression for the time taken to reach the maximum height.
 - (v) expression for the horizontal range.
 - (vi) the other angle of projection for the same horizontal range.
18. Define average angular velocity. Write its units and dimensions.
19. Write the relation between angular velocity (ω), time period (T) and frequency (ν).
20. Write relation between angular acceleration and linear acceleration.
21. Find the min velocity for which the horizontal range is 39.2m
22. Prove that the Max horizontal range is 4 times the max height attained by a projectile.
23. Calculate the magnitude of the vector $\vec{r} = (3\hat{i} + 4\hat{j} + 7\hat{k})$
24. Prove that the vectors $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $(2\hat{i} - \hat{j})$ are perpendicular to each other.
25. A string can withstand a tension of 25 N. What is the greatest speed at which a body of mass 1 kg can be whirled in a horizontal circle using a 1m length of the string?
26. A bullet is fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away? Assume the muzzle speed to be fixed, and neglect air resistance. Take $g = 10 \text{ m/sec}^2$.
27. Show that the trajectory of a projectile is a parabola.
28. From the top of a building 19.6 m high, a ball is projected horizontally, after how long does it strike the ground? If the line joining the point of projection to the point, where it hits the ground makes an angle of 45° with the horizontal, what is the initial velocity of the ball?

29. A projectile is fired at an angle θ with the horizontal. Obtain the expressions for (i) the maximum height attained (ii) the time of flight (iii) the horizontal range.
30. What do you mean by centripetal force ? Derive an expression for the centripetal force.
31. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 seconds. (a) What is the angular speed, and the linear speed of the motion? (b) Is the acceleration vector a constant vector? What is its magnitude?
32. A cricket ball is thrown at a speed of 28 m/sec in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level and (c) the distance from the thrower to the point where the ball returns to the same level.
33. A particle starts from origin at $t=0$ with a velocity of $5.0 \hat{i}$ m/sec and moves in x-y plane under the action of a force which produces a constant acceleration of $(3.0 \hat{i} + 2.0 \hat{j})$ m/sec². What is the y-coordinate of the particle at the instant its x – coordinate is 84 m? (b) What is the speed of the particle at this time?

Newton's Laws of Motion

(10 Marks)

Main points

1. **Newton's first law of motion** states that everybody continues to be in its state of rest or of uniform motion unless it is acted by an external force or it can also be stated as "If external force on a body is zero, its acceleration is zero."

2. Momentum (p) of a body is the product of mass (m) and velocity (v):

$$P=mv$$

3. **Newton's Second law of motion** : It states that the rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which force acts.

$$\text{Thus } F = k \frac{dp}{dt} = k ma$$

The SI unit of force is: $1N = 1\text{kgms}^{-2}$

- The second law is consistent with the First law ($F=0$ implies $a=0$)
- It is a vector equation
- It is applicable to a particle, and also to a body or a system of particles, provided F is the total external force on the system and a is the acceleration of the system as a whole.
- F at a point at a certain instant determines acceleration at the same point at that instant. Acceleration at an instant does not depend on the history of motion'
- Force is not always in the direction of motion. Depending on the situation F may be along V , opposite to v , normal to v , or may make some other angle with v . In every case it is parallel to acceleration.
- If $v=0$ at an instant, i.e., if a body is momentarily at rest, it does not mean that force or acceleration are necessarily zero at that instant. For ex: When a ball thrown upward reaches its maximum height, but the force continues to be its weight ' mg ' and the acceleration is ' g ' the acceleration due to gravity.

4. Impulse is the product of force and time which equals change in momentum. The notion of impulse is useful when a large force acts for a short time to produce a measurable change in momentum. Since the time of action of the force is very short, one can assume that there is no appreciable change in the position of the body during the action of the impulsive force.

(i) **Impulse = $F \times \Delta t = m (v - u)$**


(ii) Unit of measurement of Impulse is Ns

5. Newton's third law of motion:

"To every action, there is an equal and opposite reaction."

The law can be stated:

Forces in nature always occur between pairs of bodies. Force on a body A by a body B is equal and opposite to the force on the body B by A.

- Action and reaction forces are simultaneous forces.
- There is no cause-effect relation between action and reaction.
- Any of the two mutual forces can be called action and the other reaction.
- Action and reaction can act on different bodies and so they cannot be cancelled out.
- The internal action and reaction forces between parts of a body do however sum to zero.
- The equation $mg = R$ for a body on a table is true only if the body is in equilibrium. The two forces mg and R can be different. The equality of mg and R has no connection with the third law of motion.
- $F_{AB} = -F_{BA}$ 

6. Law of conservation of momentum:

"The total momentum of an isolated system of particles is conserved." This law follows from the second law of motion.

7. Friction

- Frictional force opposes relative motion between two surfaces in contact.
- It is the component of the contact force along the common tangent to the surface in contact.
- **Static frictional force** ' f_s ' oppose impending relative motion: **kinetic frictional force** ' f_k ' opposes actual relative motion.
 - They are independent of the area of contact.
 - They satisfy the following approximate laws:

$$f_s \leq (f_s)_{\max} = \mu_s R$$

$$f_k = \mu_k R$$

where μ_s (coefficient of static friction) and μ_k (Coefficient of kinetic friction) are constants characteristic of the pair of surfaces in contact.

- $\mu_k < \mu_s$
- Static frictional force is a self-adjusting force up to its limit $\mu_s N$ ($f_s \leq \mu_s N$)

Answer the following questions. Each question carries 1 mark.

1. State the Galileo's law of inertia?
2. Define force? Give the SI unit of force.
3. Give the Dimensional formula of force.
4. State Newton's first law of motion.
5. Define linear Momentum of a body. Give the SI unit of it.
6. Give the Dimensional formula of linear momentum. Is it a scalar or a Vector quantity?
7. State the law of Conservation of linear momentum.
8. Express Newton's second law of motion in the vector form
9. What is meant by Impulse? Give the SI unit of it.
10. Give the Dimensional formula of Impulse.
11. Is weight a force or a mass of a body? Name the unit in which force is measured.
12. Name some basic forces in nature.
13. Constant force acting on a body of mass 3 kg changes its speed from 2 m/sec to 3.5 m/sec. If the direction of motion of the body remains unchanged, what is the magnitude and direction of motion of the force?
14. A charged comb is able to attract bits of paper against the huge gravitational pull of the earth. What does it prove?
15. What is centripetal Force? Give the expression for finding the centripetal force.
16. Define force of friction. What is the cause friction?
17. Define coefficient of kinetic friction
18. Define angle of friction
19. Define angle of Repose
20. Compare μ_k with μ_s . Is it reasonable to expect the value of coefficient of friction to exceed unity?
21. State Newton's third law of motion.

Answer the following. Each question carries 2 marks

1. What do you mean by inertial frame of reference? Why is it so important? Give an example of inertial frame of reference.

2. A force acts for 20 sec on a body of mass 10 kg after which the force ceases and the body describes 50 m in the next 10 secs. Find the magnitude of the force.
3. A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Find the magnitude and direction of the acceleration.
4. A constant retarding force of 50 N is applied to a body of mass 10 kg moving initially with a speed of 15 m/sec. How long the body does take to stop.
5. What is linear momentum of a body how is it related to impulse?
6. Write two applications of the concept of Impulse.
7. What do you mean by free-body diagram?
8. Why does a cricket player lower his hands while catching a ball?
9. How does the spring balance weigh the weight of a body?
10. Two masses m_1 and m_2 are connected at the ends of a light inextensible string that passes over a frictionless pulley. Find the acceleration, tension in the string and thrust on the pulley when the masses are released.

Answer the following. Each question carries 3 marks.

1. Show that Newton's second law is the real law of motion
2. State and prove the law of conservation of linear momentum
3. A force 10 N acts on a body for 3×10^{-6} sec calculate the impulse. If mass of the body is 5 g. Calculate the change in velocity.
4. A man weighs 60 kg. He stands on a weighing machine in a lift, which is moving
 - (a) Upwards with a uniform speed of 6 m/s
 - (b) Downward with a uniform acceleration of 6 m/s^2 .
 - (c) Upwards with a uniform acceleration of 6 m/s^2

What would be the reading of the scale in each case? What would be the reading if the lift mechanism fails, and fall freely?

5. A shell of mass 20 g is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m/s. What is the recoil speed of the gun? What is the recoil speed of the gun?
6. Given the magnitude and direction of the net force acting on
 - (a) A drop of rain falling down with a constant speed
 - (b) A cork of mass 10 g floating on water
 - (c) A kite skilfully held stationary in the sky
 - (d) A car moving with a constant velocity of 30 km/h on a rough road.

- (e) A high-speed electron in space far from all gravitating objects and free of electric and magnetic fields.

7. A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble

- During its upward motion
- During its downward motion
- At the highest point where it is momentarily at rest. Do your answers alter if the pebble were thrown at an angle of 45° with the horizontal direction?

8. What are the laws of limiting friction?

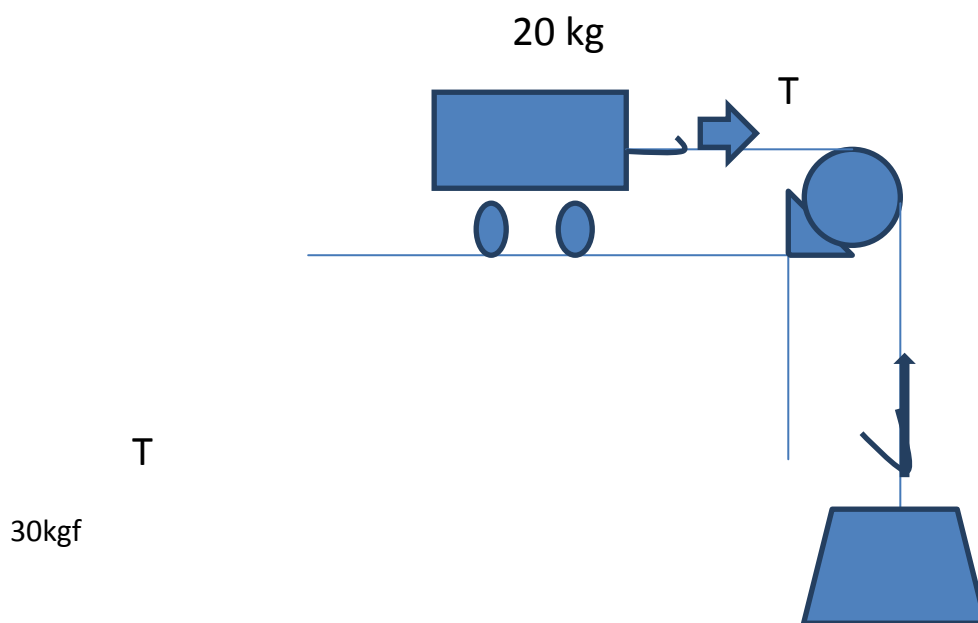
9. State and prove the law of conservation of linear momentum

10. Discuss the variation of frictional force with the applied force on a body.

Also show the variation by plotting a graph between them.

Answer the following questions. Each question carries 5 marks.

- Show that Newton's second law is the real law of motion.
- State Newton's second law of motion.
 - What is the acceleration of the block and trolley system shown in fig. if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? Take $g = 10 \text{ m/s}^2$. Neglect the mass of the string.



3. (a) State Newton's laws of motion
 (b) A machine gun has a mass of 20 kg. It fires 35 g bullets at the rate of 400 bullets per second with a speed of 400 m/s. What force must be applied to the gun to keep it in position?
4. (a) State the principle of conservation of momentum.
 (b) A hammer of mass 1 kg moving with a speed of 6 m/s strikes a wall and comes to rest in 0.1 s, Calculate
 - (i) The impulse of force
 - (ii) The retardation of the hammer, and
 - (iii) The retarding force that stops the hammer.
5. What is meant by banking of roads? Discuss the motion of a car on a banked road.

Main points

1. Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of force.
2. $W = FS \cos\theta$
3. Unit of measurement Work done is Joule or Nm
4. Work done is a scalar quantity.
5. If a graph is constructed of the components $F \cos\theta$ of a variable force, then the work done by the force can be determined by measuring the area between the curve and the displacement axis.
6. Energy of a body is defined as the capacity of the body to do work.
7. Energy is a scalar quantity.
8. The Dimensional formula of Energy is same as Work and is given by ML^2T^{-2}
9. The SI unit of Energy is same as that of Work. i.e., Joule.

10. The work-Energy theorem states that the change in kinetic energy of a body is the work done by the net force on the body.

$$K_f - K_i = W_{\text{net}}$$

11. Momentum of a body is related to Kinetic Energy by

$$P = \sqrt{2m E_k}$$

12. A force is said to be conservative if work done by or against the force in

Moving a body depends only on the initial and final positions of the body

not on the nature of the path followed between the initial and final positions.

For ex: Gravitational Force, Elastic Force, Electrostatic Force etc.

13. A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

14. For a conservative force in one dimension, we may define potential energy

Function $U(x)$ such that

$$F(x) = -dU(x)/dx$$

$$x_f$$

$$\text{Or } U_f - U_i = \int_{x_i}^{x_f} F(x) dx$$

$$x_i$$

15. The principle of conservation of mechanical energy states that the total mechanical energy of a body remains constant if the only forces that act on the body are conservative.

16. The gravitational potential energy of a particle of mass m at a height x above the earth's surface is $U(x) = mgx$

Where the variation of g with height is ignored.

17. The elastic potential energy of a spring of force constant k and extension x is

$$U(x) = \frac{1}{2} k x^2$$

18. Power is defined as the time rate of doing work.

19. Average Power is given by $P_{\text{av}} = W/t$

20. Power is a scalar quantity, $P = \mathbf{F} \cdot \mathbf{v} = Fv \cos \theta$, $P = dW/dt$

21. SI unit of Power is Watt

22. The Dimensional Formula of Power is $M^1 L^2 T^{-3}$

23. The Scalar or dot product of two vectors A and B is written as $A \cdot B$ and is a scalar quantity given by : $A \cdot B = AB \cos \theta$ where θ is the angle between A and B . It can be positive, negative or zero depending upon the value of the scalar product of the two vectors can be interpreted as the product of magnitude of one vector and the component of the other vector in the direction of the first vector.

For unit vector $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ and $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

24. For two bodies, the sum of the mutual forces exerted between them is zero from Newton's third law, $F_{12} + F_{21} = 0$

But the sum of the work done by the two forces need not always cancel. i.e.,

$$W_{12} + W_{21} \neq 0$$

However, it may sometimes be true.

25. Elastic collision is a collision in which both momentum and kinetic energy are conserved

26. In inelastic collision momentum is conserved but kinetic energy is not conserved.

27. In a collision between two bodies the Coefficient of restitution can be defined as the ratio between the velocity of separation to the velocity of approach i.e.

$$e = \text{velocity of separation} / \text{velocity of approach}$$

If $e = 1$ then the collision is perfectly elastic, if $e = 0$, then the collision is perfectly inelastic.

Answer the following questions. Each question carries one mark.

1. Define work. How can you measure the work done by a force?

2. Write the Dimensional formula of work. Give its SI unit of measurement

3. Give examples of zero work done

4. A body is compelled to move along the x-direction by a force given by

$$F = (2\hat{i} - 2\hat{j} + \hat{k}) \text{ N}$$

What is the work done in moving the body?

(i) a distance of 2m along x-axis

(ii) a distance of 2m along y-axis.

5. What do you mean by a conservative force? Is frictional force a conservative force.
6. Define power. Is it a scalar or vector?
7. Define one electron volt
8. State work – energy theorem
9. Compare 1kwh with electron volt
10. Two bodies of mass 1 kg and 4 kg have equal linear momentum. What is the ratio of their kinetic energies?
11. Distinguish between elastic and inelastic collision.
12. A light and a heavy body have the same momentum. Which one will have the greater kinetic energy?
13. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the law of conservation of momentum?
14. Define coefficient of restitution.
15. What is the amount of work done, when a body of mass m moves with a uniform speed v in a circle?

Answer the following questions. Each question carries 2 marks.

1. Plot a graph showing the variation of Force (F) with displacement(x), if its force equation is given by $F = -kx$ Name at least four commonly used units of energy.
2. Differentiate between a conservative force and a non-conservative force.
3. What is Einstein's energy-mass equivalence relation?
4. A man whose mass is 75kg walks up 10 steps each 20 cm high, in 5 sec. Find the power he develops Take $g = 10 \text{ m/s}^2$.
5. A block of mass 2 kg moving at a speed of 10 m/s accelerates at 3 m/s^2 for 5 sec. Compute the final kinetic energy.
6. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative.
 - (a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
 - (b) Work done by gravitational force in the above case.

(c) Work done by friction on a body sliding down an inclined plane

(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.

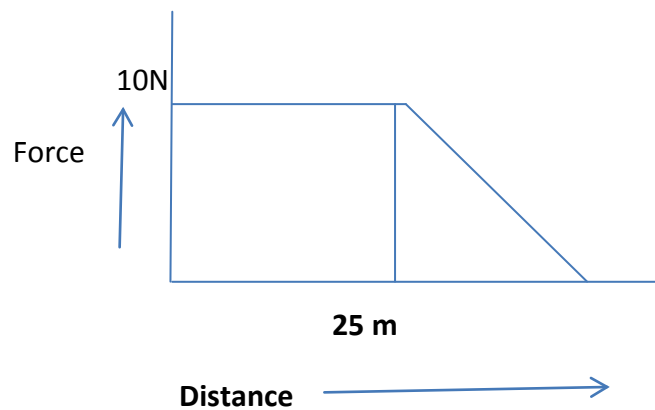
7. What happens to the potential energy of a body when the work done by the conservative force is positive?

Answer the following. Each question carries 3 marks.

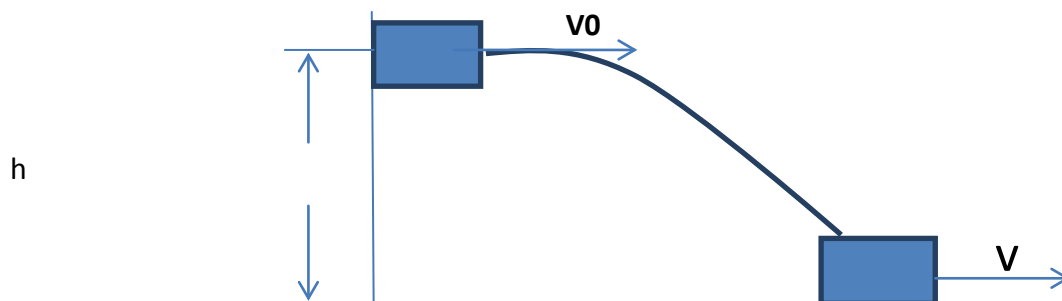
1. State and prove work-energy theorem
2. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force equal to $-k/r^2$, where k is a constant. What is the total energy of the particle?
3. State and prove the law of conservation of energy.
4. Show that Potential energy $y = mgh$
5. Show that kinetic energy $= \frac{1}{2} mv^2$
6. Prove the law of conservation of energy at every point of the motion of a body under freefall.
7. Derive an expression to find the potential energy of a spring.
8. Plot graphs between K.E, P.E, and total energy of an elastic spring with displacement (x).
9. What is the power output of the sun if 4×10^9 kg of matter per second is converted into energy in the sun?
10. A body of mass 5 kg initially at rest is moved by a horizontal force of 20 N on a frictionless table. Calculate the work done by the force in 10 seconds and prove that this equals the change in kinetic energy.
11. An engine pumps out 40 kg of water per second. If water comes out with a velocity of 3 m/sec, then what is the power of the engine?

Answer the following questions. Each question carries 5 marks.

1. Discuss elastic collision in one dimensional motion.
2. Discuss the elastic collision in two dimension
3. Derive the expression for the work done by (i) Constant force and (ii) a variable force.
4. (a) What are conservative and non-conservative forces? Give examples.
(b) A body of mass 5 kg is acted upon by a variable force. The force varies with the distance covered by the body as shown in fig. What is the speed of the body when the body has covered 25 m. Assume that the body starts from rest?



5. A 10 kg ball and a 20 kg ball approach each other with velocities 20m/s and 10 m/s respectively. What are their velocities after collision if the collision is perfectly elastic. Also show that kinetic energy before collision is equal to kinetic energy after collision.
6. Starting with an initial speed v_0 , a mass m slides down a curved frictionless track. Arriving at the bottom with a speed v . From what height did it start?



7. A ball dropped from a height of 8m hits the ground and bounces back to a height of 6m only. Calculate the frictional loss in kinetic energy.

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

MAIN POINTS

1. A rigid body is one for which the distance between different particles of the body do not change, even though there is a force acting on it.
2. A rigid body fixed at one point or along a line can have only rotational motion. A rigid body not fixed in some way can have either pure rotation or a combination of translation and rotation.
3. In pure translation every particle of the body moves with the same velocity at any instant of time.
4. In rotation about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis. Every point in the rotating rigid body has the same angular velocity at any instant of time.
5. Angular velocity is a vector. Its magnitude is $\omega = d\theta/dt$ and it is directed along the axis of rotation. For rotation about a fixed axis, this vector ω has a direction.
6. The vector or a cross product of two vectors A and B is a vector written as $A \times B$. The magnitude of this vector is $AB \sin \theta$ and its direction is given by right handed screw or the right hand rule.
7. The linear velocity of a particle of rigid body rotating about a fixed axis is given by $V = \omega \times r$ where r is the position vector of the particle with respect to an origin along the fixed axis. The relation applies even to more general rotation of a rigid body with one point fixed. In that case r is the position vector of the particle with respect to the fixed point taken as the origin.
8. The centre of mass of the system particles is defined as the point whose position vector is $R = (\sum m_i r_i) / M$
9. Velocity of the centre of mass of a system of particles is given by $V = p / M$, where P is the linear momentum of the system. The centre of mass moves as if all the mass of the system is concentrated at this point and all the external forces act at it. If the total external force is zero, then the total linear momentum of the system is constant.
10. The angular momentum of a system of n particles about the origin is

$$L = \sum_{i=1}^n (r_i \times p_i)$$

11. The Torque or moment of force on a system of n particles about the origin is given by

$$T = \sum_{i=1}^n (r_i \times F_i)$$

The force F_i acting on the i^{th} particle includes the external as well as the internal forces.

Assuming Newton's third law and that forces between any two particles act along the

line joining the particles, we can show that $T_{\text{int}} = 0$ and $dL/dt = T_{\text{ext}}$.

12. A rigid body is in mechanical equilibrium if

(i) It is in translational equilibrium, i.e., the total external force on it is zero:

$$\sum F_i = 0 \text{ and}$$

(ii) It is in rotational equilibrium, i.e., the total external torque on it is zero:

$$\sum \tau_i = \sum r_i \times F_i = 0$$

13. The centre of gravity of an extended body is that point where the total gravitational torque on the body is zero.

14. The moment of inertia of a rigid body is defined by the formula $I = \sum m_i r_i^2$

Where r_i is the perpendicular distance of the i^{th} point of the body from the axis. The kinetic energy of rotation is $K = \frac{1}{2} I \omega^2$

15. The theorem of parallel axes: $I' = I_G + M a^2$ where I_G moment of inertia about the axis passing through centre of gravity.

Allows us to determine the moment of inertia of a rigid body about an axis as the sum

of the moment of inertia of the rigid body about a parallel axis passing through its centre of mass and the product of its mass and the square of the perpendicular distance between the two parallel axes.

16. Rotation about a fixed axis is directly analogous to linear motion in respect of kinematics and dynamics.

17. For a rigid body rotating about a fixed axis of rotation $L_z = I_z \omega$ where I_z is the moment of inertia about z-axis. In general, the angular momentum about the axis of rotation, L is along the axis of rotation. In that case $|L| = L_z = I\omega$. The angular acceleration of a rigid body rotating about a fixed axis is given by $I\alpha = \tau$. If the external torque acting on the body $\tau = 0$, the component of angular momentum about the fixed axis of such a rotating body is constant,

18. For rolling motion without slipping $V_{\text{cm}} = r\omega$, where V_{cm} is the velocity of translation. r is the radius and m is the mass of the body. The kinetic energy of such rolling motion of the body is the sum of kinetic energies of translation and rotation.

$$K = \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} I \omega^2$$

19. To determine the motion of the centre of mass of a system, we need to know external forces acting on the body.
20. The time rate of change of angular momentum is the Torque acting on the body,
21. The total torque on a system is independent of the origin, if the total external force is zero.
22. The centre of gravity of a body coincides with its centre of mass only if the gravitational field does not vary from one part of the body to the other part of the body.

23. Principle of conservation of angular momentum:

It states that if there is no external torque acting on the system the total angular momentum of the system remains constant.

i.e. If $\tau_{\text{ext}} = 0$, $dL/dt = 0$, Hence $L = \text{constant}$.

24. Kepler's laws:

- (i) All planets revolve around the sun in elliptical orbits with sun at one of its foci.
- (ii) The line joining the sun to the planet sweeps out equal areas in equal intervals of time
That is the areal velocity of a planet remains constant.
- (iii) The square of the timeperiod of revolution of the planet is proportional to the cube

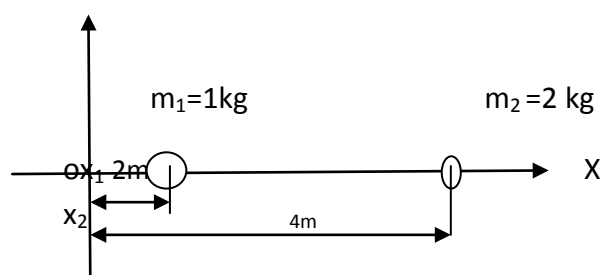
of the semi major axis.

$$T^2 \propto R^3$$

Answer the following questions. Each question carries one mark

1. Define the term 'Centre of mass of a system of particles.
2. What will be the centre of mass of the pair of particles described below in fig on the x-axis?

Y



3. If two masses are equal where does their centre of mass lie?
4. Define a rigid body?

5. Is centre of mass same as the centre of gravity of a body? How can a rigid body be balanced?
6. Write an expression for the the velocity of the centre of mass of particles.
7. Does the total momentum of a system of particles depend upon the velocity of the centre of mass?
8. Write the expression for the acceleration of the centre of mass of particles. A projectile fired into the air suddenly explodes into several fragments. What can you say about motion of the fragments after collision?
9. Briefly explain about the centre of mass of the earth- moon system.
10. Define one radian.
11. Convert one radian into degrees.
12. Define angular velocity? What is its SI unit?
13. Define angular acceleration. What are its SI units?
14. Write the dimensional formula of angular acceleration.
15. Write the dimensional formula of angular velocity.
16. Considering rotational motion about some fixed axis, write equations corresponding to
 - (i) $x(t) = x(0) + v(0)t + \frac{1}{2}at^2$
 - (ii) $v^2(t) = v^2(0) + 2a[x(t) - x(0)]$
 - (iii) $v(t) = v(0) + at$
 - (iv) $v(t) = \frac{[v(t) + v(0)]}{2}$
17. Define angular momentum. Write the SI unit of angular momentum.
18. Name the dimensional constant whose dimensions are same as that of angular momentum.
19. Does the magnitude and direction of angular momentum \vec{L} depend on the choice of the origin?
20. Express torque in terms of the rate of change of linear momentum.
21. Define moment of inertia of a body.
22. Is moment of inertia scalar or vector physical quantity? Write the SI unit of moment of inertia
23. Why is the most of the mass of a fly wheel placed on the rim?
24. Why are the spokes fitted in a cycle wheel?
25. The cap of pen can be opened with help of two fingers than with one finger. Explain why?
26. State the Work-Energy theorem for rotational motion.
27. State the law of conservation of angular momentum.
28. For an isolated system plot a graph between moment of inertia (I) and angular velocity (ω)
29. What is the law of rotation?
30. State the theorem of parallel axes
31. State the theorem of perpendicular axes.

Answer the following questions. Each question carries 2 marks.

1. Write an expression for the moment of inertia of a ring of mass M and radius R ,
 - (i) About an axis passing through the centre , and perpendicular to its plane
 - (ii) About a diameter
 - (iii) About a tangent to its plane
 - (iv) About a tangent perpendicular to the plane of the ring.

2. Write an expression for the moment of inertia of a circular disc of mass M and radius R ,

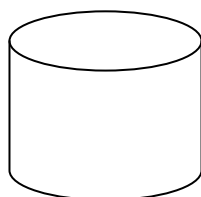
- (i) About an axis passing through the centre , and perpendicular to its plane
- (ii) About a diameter
- (iii) About a tangent to its plane
- (iv) About a tangent perpendicular to the plane of the disc.

3. Calculate the angular momentum of the earth rotating about its own axis.

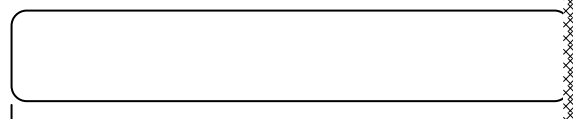
Mass of the earth = 5.98×10^{27} kg, radius of the earth = 6.37×10^6 m.

4. A thin metal hoop of radius 0.25 m and mass 2 kg starts from rest, and rolls down an inclined plane. Its linear velocity on reaching the foot of the plane is 4 ms^{-1} . What is the rotational kinetic energy when it reaches the foot of the inclined plane?
5. Three mass points m_1 , m_2 , m_3 are located at the vertices of an equilateral triangle of length 'a'. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1 ?
6. If the angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved?
7. A sphere of radius 10 cm weighs 1 kg. Calculate the moment of inertia
 - (i) About the diameter
 - (ii) about the tangent
8. A wheel rotates with a constant angular acceleration of 3.6 rad/s^2 . If the angular velocity of the wheel is 4.0 rad/s at $t = 0$. What angle does the wheel rotate in 1 s? What will be its angular velocity at $t = 1 \text{ s}$?
9. Mark the centre of mass of the following figures.

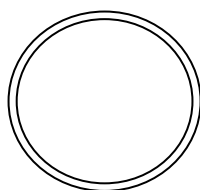
- (i) Right circular cylinder



- (ii) Cylindrical rod

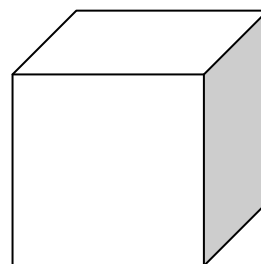


(iii)



Circular ring

(iv)



Symmetrical cube.

10. To maintain a rotor at a uniform angular speed of 100 s^{-1} an engine needs to transmit a torque of 200 Nm . What is the power of the engine required?
11. Two cars are going around two concentric circular paths at the same angular speed. Does the inner or the outer car have the larger speed?

Answer the following questions. Each question carries 3 marks.

1. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 \AA ($1 \text{ \AA} = 10^{-10} \text{ m}$). Find the approximate location of the centre of mass of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom, and nearly all the mass of an atom is concentrated in its nucleus.
2. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 s^{-1} . The radius of the cylinder is 0.25 m . What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?
3. A Long playing record revolves with a speed of $33\frac{1}{3} \text{ rev/min}$, and has a radius of 15 cm . Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and the record is 0.15 , which of the two coins will revolve with the record.
4. State and prove the law of conservation of angular momentum.
5. Derive an expression for Torque acting on a body.
6. Explain the motion of centre of mass of a body with examples.
7. Find the torque of a force $7\hat{i} + 3\hat{j} - 5\hat{k}$ about the origin. The force acts on a particle whose position vector is $\hat{i} - \hat{j} + \hat{k}$?
8. What constant torque should be applied to a disc of mass 16 kg and diameter 0.5 m ; so that it acquires an angular velocity of $4\pi \text{ rad/s}$ in 8 s ? The disc is initially at rest, and rotates about an axis through the centre of the disc in a plane perpendicular to the disc.
9. A uniform ring of radius 0.5 m has a mass of 10 kg . A uniform circular disc of same radius has a mass of 10 kg . Which body will have the greater?

moment of inertia? Justify your answer.

10. Obtain a relation between torque applied to a body and angular acceleration produced. Hence define moment of inertia.
11. If the earth were to suddenly contract to half of its present size without change in its mass, what will be the duration of the new day.
12. Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground? with maximum velocity?

Gravitation

MAIN POINTS

(06 Marks)

1. Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has a magnitude

$$F = G \frac{m_1 m_2}{r^2} \text{ where } G \text{ is the Universal gravitational constant, which has the value } 6.672 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}.$$

2. In considering motion of an object under the gravitational influence of another object the following quantities are conserved,
 - (i) Angular momentum
 - (ii) Total mechanical energy.

3. If we have to find the resultant gravitational force acting on the particle m due to a number of masses M_1, M_2, \dots, M_n each given by the law of gravitation, From the principle of superposition each force acts independently and uninfluenced by the other bodies. The resultant force F_n is then found by vector addition

$$F_n = F_1 + F_2 + F_3 + \dots F_n = \sum_{i=1}^n F_i$$

4. Kepler's laws of planetary motion state that
 - (i) All planets move in elliptical orbits with the Sun at one of the focal points
 - (ii) The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time. This follows from the fact that the force of gravitation on the planet is central and hence angular momentum is conserved.
 - (iii) The square of the orbital period of a planet is proportional to the cube of the semi major axis of the elliptical orbit of the planet.

The period T and radius R of the circular orbit of a planet about the Sun are related by

$T^2 = (4\pi^2 / GM_s) R^3$ where M_s is the mass of the Sun. Most planets have nearly circular orbits about the Sun. For elliptical orbits, the above equation is valid if R is replaced by the semi-major axis ' a '.

5. Angular momentum conservation leads to Kepler's second law. It holds for any central force.
6. According to Kepler's third law $T^2 = K R^3$. The constant K is same for all planets in circular orbits. This applies to satellites orbiting the earth.
7. An astronaut experiences weightlessness in a space satellite. This is because the gravitational force is small at that location in space. It is because both the astronaut and the satellite are in 'free fall' towards the Earth.
8. The acceleration due to gravity

(a) At a height 'h' above the Earth's surface

$$g(h) = GM_E / (R_E + h)^2$$

$$= (GM_E / R_E^2) [1 - (2h/R_E)] \text{ for } h \ll R_E$$

$$g(h) = g(0) [1 - (2h/R_E)] \quad \text{where } g(0) = GM_E / R_E^2$$

(b) At a depth 'd' below the Earth's surface is

$$g(d) = GM_E / R_E^2 [1 - (d/R_E)] = g(0) [1 - (d/R_E)]$$

9. The gravitational force is a conservative force, and therefore a potential energy function can be defined. The gravitational potential energy associated with two particles separated by a distance r is given by

$$V = - (Gm_1m_2/r) + \text{constant}$$

The constant can be given any value. The simplest choice is to take the value of it to be zero. With this V becomes

$$V = -Gm_1m_2/r \quad \text{where } V \text{ is taken to be zero at } r \rightarrow \infty.$$

10. The total potential energy for a system of particles is the sum of the energies for all pairs of particles, with each pair represented by a term of the form given by above equation. This follows from the principle of superposition. If an isolated system consists of a particle of mass m moving with a speed v in the vicinity of a massive body of mass M, the total mechanical energy of the particle is given by

$$E = \frac{1}{2} mv^2 - \left(\frac{GMm}{r} \right)$$

That is the total mechanical energy is the sum of kinetic and potential energies.

The total energy is a constant of motion.

11. If m moves in a circular orbit of radius a about M , where $M \gg m$, the total energy of the system is

$$E = -\frac{GMm}{2a}$$

The total energy is negative for any bound system, that is, one in which the orbit is closed, such as an elliptical orbit. The kinetic and potential energies are

$$K = \frac{GMm}{2a}$$

$$V = -\frac{GMm}{a}$$

12. The escape speed from the surface of the earth is

$$v_e = \sqrt{2GM_E/R_E} = \sqrt{2g R_E}$$

and has a value of 11.2 km s^{-1} .

13. If a particle is outside a uniform spherical shell or solid sphere with a spherically symmetric internal mass distribution, the sphere attracts the particle as though the mass of the sphere or shell were concentrated at the centre of the sphere.
14. If a particle is inside a uniform spherical shell, the gravitational force on the particle is zero. If a particle is inside a homogeneous solid sphere, the force on the particle acts toward the centre of the sphere. This force is exerted by the spherical mass interior to the particle.
15. A geostationary satellite moves in a circular orbit in the equatorial plane at an approximate distance of $4.22 \times 10^4 \text{ km}$ from the Earth's surface.

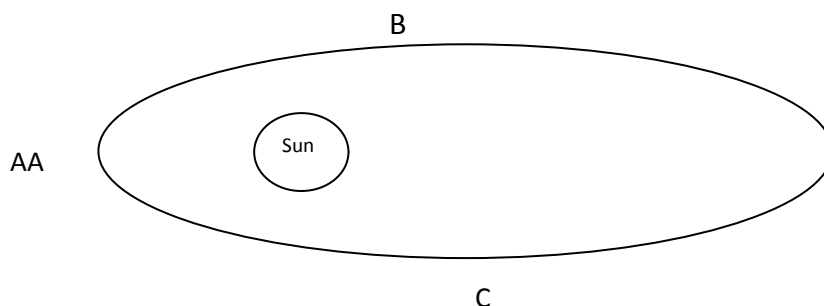
Answer the following questions. Each question carries 1 mark

1. State Newton's law of Gravitation
2. Write the SI unit of Gravitational constant. Also give its dimensional formula
3. What is the effect of medium on the value of G ?
4. Define gravitational potential.
5. Name the physical quantity whose dimensional formula is same as that of the gravitational potential.
6. Define gravitational potential energy.
7. Define Orbital velocity.
8. Define escape velocity. Give its value for the earth.
9. What is the value of gravitational potential energy at infinity?
10. What do you mean by the earth's satellite?

11. Write an expression for the escape velocity of a body from the surface of the earth. Also give the factors on which it depends,
12. What does a low value of escape velocity indicates?
13. What is the period of the moon?
14. What is geocentric theory? Who propounded it?
15. What is geostationary satellite? Is it same as synchronous satellite?
16. What is heliocentric theory? Who propounded it?
17. State Kepler's laws of planetary motion.
18. Name the force that provides the necessary centripetal force for a planet to move around the sun in a nearly circular orbit.
19. Does speed increase, decrease, or remain constant when a planet comes closer to the sun?
20. Where does a body weigh more near the poles or the equator? Why?
21. The value of G on the earth is $6.7 \times 10^{-11} \text{ Nm}^{-2} \text{ kg}^{-2}$. What is its value on the moon?

Answer the following questions. Each question carries 2 marks.

1. Give the differences between weight and mass
2. You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
3. An astronaut inside a small space-ship orbiting around the Earth has a large size. Can he hope to detect gravity?
4. The mass and diameter of a planet have twice the value of the corresponding parameters of the earth. What is the acceleration due to gravity on the surface of the planet?
5. The planet earth is revolving in an elliptical orbit around sun as shown in the fig. At what point on the orbit will the kinetic energy be (i) minimum (ii) maximum?



6. Does a rocket really need the escape velocity of 11.2 km/s initially to escape from the earth? Plot a graph showing the variation of gravitational force (F) with square of the distance, Plot a graph showing the variation of acceleration due to gravity with height or depth.

7. Two planets A and B have their radii in a ratio 'r'. The ratio of the acceleration due to gravity on the planets is 'x'. What is the ratio of the escape velocity from the two planets?
8. Write formula for the variation of g with (i) height above the surface of the earth (ii) depth below the surface of the earth (iii) rotation of the earth.
9. Give the differences between inertial mass and gravitational mass.
10. Does the escape velocity of a body from the earth depend on :
 - (i) The mass of the body,
 - (ii) The location from where it is projected,
 - (iii) The direction of projection,
 - (iv) The height of the location from where the body is launched.

Answer the following questions. Each question carries 3 marks.

1. Derive an expression for finding the escape velocity of a body from the surface of the earth.
2. Deduce an expression for the velocity required by a body so that it orbits around the earth.
3. At what height above the surface of the earth will the acceleration due to gravity become 1% of its value at the earth's surface. Take the radius of the earth, $R = 6400\text{km}$.
4. The mass and diameter of a planet are twice those of the earth. What will be the period of oscillation of a pendulum on this planet, If it is a second's pendulum on the earth?
5. Two masses 100kg and 10000kg are at a distance 1m from each other. At which point on the line joining them, the intensity of the gravitational field will be zero.
6. Assuming the earth to be a sphere of uniform density, what is the value of g in a mine 100 km below the earth's surface? Given $R = 6380\text{ km}$ and $g = 9.8\text{ms}^{-2}$.
7. A body of mass m is raised to a height h above the earth's surface. Show that the loss in weight due to variation in g is approximately $2mgh/R$.
8. Calculate the orbital velocity for a satellite revolving near the earth's surface. Radius of the earth's surface is $6.4 \times 10^6\text{m}$ and $g = 10\text{ ms}^{-2}$.
9. An artificial satellite circles around the earth at a distance of 3400km. Calculate the mass of the sun. Given 1 year = 365 days and $G = 6.7 \times 10^{-11}\text{Nm}^2\text{ kg}^{-2}$.
10. Show that the gravitational potential at a point of distance r from the mass M is given by $V = - (GM/r)$.
11. A satellite orbits the earth at a height of 500 km from its surface. Calculate the kinetic energy, potential energy and total energy of the satellite.
 Given: Mass of the satellite = 300kg
 Mass of the earth = $6 \times 10^{24}\text{ kg}$
 Radius of the earth = $6.4 \times 10^6\text{m}$
 $G = 6.67 \times 10^{-11}\text{ Nm}^2\text{ kg}^{-2}$