Static Electric field (Electrostatics)

Coulomb's law, field Intⁿ, Elect. field due to continuous charge distⁿ, Elect. flux density, Gauss' law, Appⁿ of Gauss' law. Some Symmⁿ charge distributions & diffⁿ volⁿ element; Divergence, divergence Thm.

~~Cot~~ Coulomb's law.

$$F \propto \frac{Q_1 Q_2}{r^2} \Rightarrow F = K \frac{Q_1 Q_2}{r^2}$$

where $K = \frac{1}{4\pi\epsilon}$

$\epsilon = \epsilon_0 \epsilon_r$

$\epsilon_0 = 8.85 \times 10^{-12}$ F/m

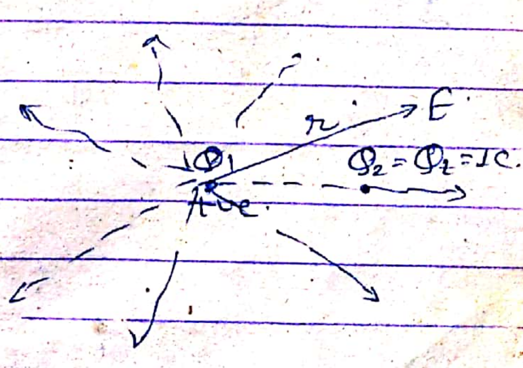
& $\epsilon_r = 1$ for vacuum

∴ for vaccume $\epsilon = \epsilon_0 = 8.85 \times 10^{-12}$ F/m

$= \frac{1}{36} \times 10^{-9}$ F/m.

$$F = (9 \times 10^9) \frac{Q_1 Q_2}{r^2}$$

Electric Field Intensity.



Let +ve charge Q_1 is placed at the origin & if another +ve charge $Q_2 = Q_t = 1C$ (test charge) is placed, then the force will radialy outward & ↑ the $Q_t = 1C$ approches to Q_1.

i.e Q_1 ~~ex~~ makes a field around it & exerts force on another chare. This field is called Electric field.

$$\therefore F_t = \frac{Q_1 Q_t}{4\pi\epsilon_0\epsilon_r r^2} \hat{a}_r$$

when $Q_t = 1\,C$ & $E_r = 1$.

Then force per unit charge is called an electric field intensity (E). i-e

$$\frac{F_t}{Q_t} = \frac{Q_1 Q_t}{4\pi\epsilon_0 r^2 Q_t} \cdot \hat{a_r} = E$$

$$\therefore \boxed{E = \frac{F_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 r^2} \hat{a_r}} \quad N/C \text{ or } V/m.$$

Charge Distribution.

1) Line charge distribution → If a charge is distributed in thin conductor or in a beam of CRT then is called line charge distribution.

i-e $\rho_L = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L}$ c/m

or, $\boxed{Q = \int \rho_L \, dL \text{ coulomb}}$

2] Surface charge distribution.

Total charge distributed on any conducting sheet.

$$\rho_s = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S} \; c/m^2$$

$$\therefore \boxed{Q = \int_S \rho_s \, ds \text{ Coulomb}}$$

3] Volume charge Distribution.

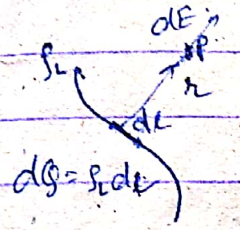
$$\rho_v = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} \; c/m^3$$

i-e $\boxed{Q = \int_V \rho_v \, dv \text{ colomb}}$ {amount of charge in a given volume}

Electric Field Intensity due to diffⁿ charges

(A) Electric field intensity due to line charge,

we know. $dQ = \rho_L \, dl$ & $dE = \frac{dQ}{4\pi\epsilon_0 r^2} \hat{a_r}$



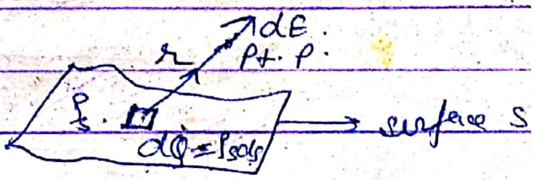
$\therefore E = \int \frac{\rho_L \, dl}{4\pi\epsilon_0 r^2} \hat{a_r} = \int_L \frac{\rho_L \hat{a_r}}{4\pi\epsilon_0 r^2} dl$

for infinite line charge $E = \frac{\rho_L}{2\pi\epsilon_0 r} \hat{a_\rho}$

(B) Electric field intensity due to surface charge.

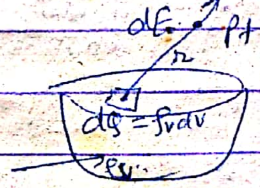
we know, $dQ = \rho_s \cdot ds$ & $dE = \frac{dQ}{4\pi\epsilon_0 r^2} \hat{a_r}$

$\therefore E = \int_S \frac{\rho_s \cdot \hat{a_r} \, ds}{4\pi\epsilon_0 \cdot r^2}$



for infinite sheet charge $E = \frac{\rho_s}{2\epsilon_0} \hat{a_n}$

(C) Electric field Intensity due to volume charge.



as, $dQ = \rho_v \, dv$ & $dE = \frac{dQ}{4\pi\epsilon_0 \cdot r^2} \hat{a_r}$

$\therefore E = \int_V \frac{\rho_v \cdot \hat{a_r} \, dv}{4\pi\epsilon_0 r^2}$

VVI

pg- 121, of sadiku*

$R^2 = r^2 + z^2$

$(r^2 + z^2)$ $^{1+1/2}$

Absent → 0

4, 9, 10, 15, 36, ~~45~~, 50, 53, 55, 58, 63, 70,
→ 4, 18, 26, ~~30~~, 35, ~~37~~, 39, 44, ~~49~~ 49, 50, 51, 53, 54, 55, 58, 62, 63, 69, 70, 72, 73, 77.

Electric flux

We know $E = \frac{Q}{4\pi\epsilon_0 r^2} a_r$ N/c or v/m.

Part above eq^n can be put up as :

$$\epsilon_0 E = \frac{Q}{4\pi r^2}$$

$\left\{ \text{As, } \rho_s = \frac{\Delta Q}{\Delta S} \right.$

$\rho_s = \frac{Q}{4\pi r^2}$

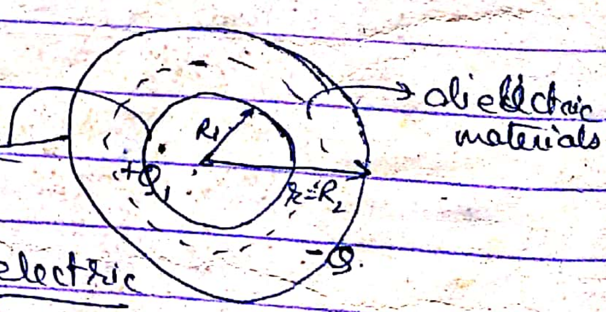
= charge per unit

⇒ surface charge density (ρ_s. ~~$4\pi r^2$~~)

& $4\pi r^2$ is the area of ~~surfa~~ sperical surface.

$\therefore$ $\boxed{D = \epsilon_0 E}$ Electric flux density.

Can be explained by faraday's observations.



From faraday's Exp^n →
i.e the "displacement" or the electric flux or displacement flux is directly proportional to the charge on the inner sphere.

$\boxed{+Q = \Psi}$ Coulomb

The path of Ψ → towards inner sphere to outer sphe
& the $Q = \Psi$ Coulomb is uniformly distributed over a surface having ~~4π~~ area $4\pi R_1^2$, (m^2)

CGSH

CGHS

$\therefore$ Flux density at this surface is

$\frac{\psi}{4\pi R_1^2}$ or $\frac{Q}{4\pi R_1^2}$ C/m^2

$\therefore$ $\frac{Q}{4\pi R_1^2} = \epsilon_0 E$

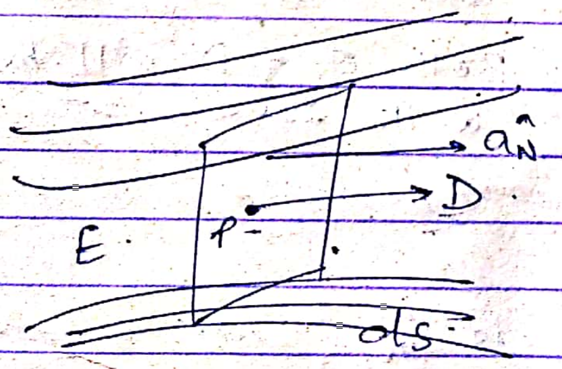
$$\boxed{D = \epsilon_0 E}$$

As $\boxed{E = \int_v \frac{\rho_v \, dv}{4\pi\epsilon_0 r^2} \, \hat{a}_r}$ —— (1)

Similarly. $\boxed{D = \int_v \frac{\rho_v \, dv}{4\pi r^2} \, \hat{a}_r}$ —— (2)

$\therefore$ $\boxed{\text{Total flux} = \psi = \int_s D \cdot ds}$ —— (3)

As,



then $D = \frac{d\psi}{ds} \hat{a}_N$ C/m^2

$\therefore$ $d\psi = D \cdot ds$

$\therefore$ $\psi = \int_S D \cdot ds$

where D is called displacement flux density or displacement density.

Gauss Law.

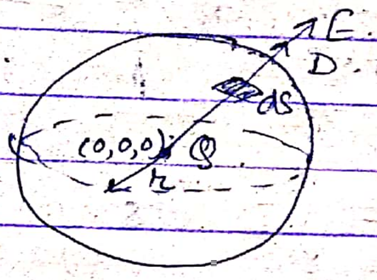
Faraday's experiment leads to the Gauss law.

⟹ "Electric flux passing through any closed surface is equal to the charge enclosed by that surface.

∴ Total flux $\psi = Q = \oint_S D \cdot ds$.

Again it is stated that "The total flux passing through the closed surface is obtained by adding the differential contributions crossing each surface element ds".

Hence.



E due to pt. charge Q will be.

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \cdot \hat{a}_r$$ { Both E & D will be in the same directions.

$$\therefore D = \frac{Q}{4\pi r^2} \cdot \hat{a}_r$$

$$\therefore D \cdot ds = \frac{Q}{4\pi r^2} \cdot \hat{a}_r \cdot ds \cdot \hat{a}_n$$ { ds is incremental surface }

$$= \frac{Q}{4\pi r^2} \cdot \hat{a}_r \cdot ds\, \hat{a}_r$$ { $\hat{a}_n = \hat{a}_r$

$$= \frac{Q}{4\pi r^2} \cdot ds.$$

$$\therefore \oint_S D \cdot ds = \oint_S \frac{Q}{4\pi r^2} \cdot ds.$$

$$= \frac{Q}{4\pi r^2} \cdot \oint_S ds.$$ { $\oint_S ds = 4\pi r^2$ for sphere }

$$= Q.$$

$$\therefore \boxed{\psi = \oint D \cdot ds = Q}$$

As Q is the total charge enclosed by the closed surface which can be further considered as volume charge density ρ_v distributed within the sphere. Then the volume $\int$ of ρ_v will also result charge Q.

$$\therefore \quad Q = \oint_V \rho_v \, dv = \oint_S D \cdot ds .$$

Hence Gauss law. $\Rightarrow$ $\boxed{\oint_S D \cdot ds = \oint_V \rho_v \cdot dv = Q .}$

Gauss Law in Differential form:

This law states that the electric flux per unit volume leaving an infinitesimal small volume unit is exactly equal to volume charge density exist in the volume. (Gauss law in a pt. form)

As, $$Q = \psi = \oint_S D \cdot ds = \int_V \rho_v \, dv .$$

Also, it is proved $\nabla \cdot D = \rho_v$. Hence

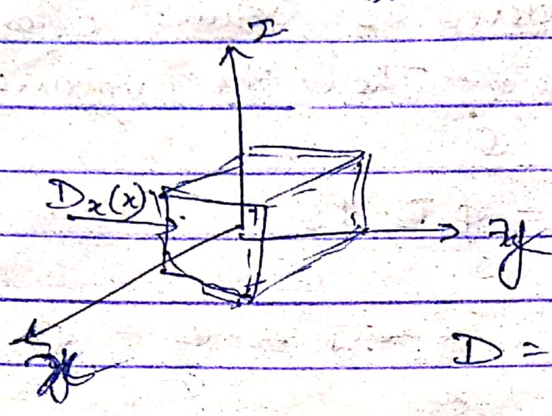
$$\oint_S D \cdot ds = Q = \int_V \rho_v \, dv = \int_V (\nabla \cdot D) \, dv .$$

$$\therefore \boxed{\oint_S D \cdot ds = \int_V (\nabla \cdot D) \, dv}$$

$\hookrightarrow$ Divergence Thm.

(1)

Gauss law in differential volⁿ element



$$D = D_x \hat{a_x} + D_y \hat{a_y} + D_z \hat{a_z}$$

A small volume

$\Delta x \Delta y \Delta z = \Delta V$.

$D_x(x) \longrightarrow D_x(x+\Delta x)$.

$D_y(y) \longrightarrow D_y(y+\Delta y)$

$D_z(z) \longrightarrow D_z(z+\Delta z)$.

$\therefore$ The flux out of the one surface will will be

$$\frac{\partial D_y}{\partial y} \cdot \Delta x \Delta y \Delta z$$

Same mathematical apporch for the remaining surface

$$\int_S D \cdot ds = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \Delta x \Delta y \Delta z.$$

Divide by ΔV, where $\Delta V \to 0$.

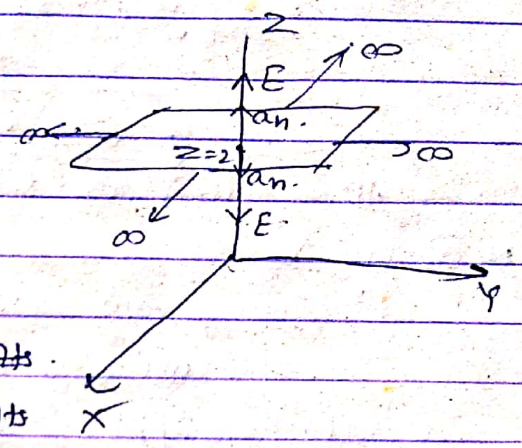
$$\lim_{\Delta V \to 0} \frac{\int_S D \cdot ds}{\Delta V} = \frac{\partial}{\partial x} D_x + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}.$$

$$\rho_v = \nabla \cdot D = \text{div } D.$$

↳ Maxwell's 1st Eqⁿ

28/1 → 4, 17, 20, 23, 27, 32, 36, ~~33~~, 41, 44, 53, 54, 55, ~~58~~, 63,

Q. A surface charge $\rho_s = 5\ \mu C/m^2$ is uniformly distributed over a plane $z=2$ in free space. Find the electric field intensity E.



$\Rightarrow$ $E = \frac{\rho_s}{2\epsilon_0} \hat{a_N}$.

$\therefore |E| = \frac{\rho_s}{2\epsilon_0}$ only

$= \frac{5\times10^{-6}}{2\left(\frac{10^{-9}}{36\pi}\right)} = 0.282 \times 10^{6}$ volts.

$= 282358$ volts

Q. The flux density $\bar{D} = \frac{r}{3}\bar{a_r}$ nC/m^2 is in the free space. a) ...

b)

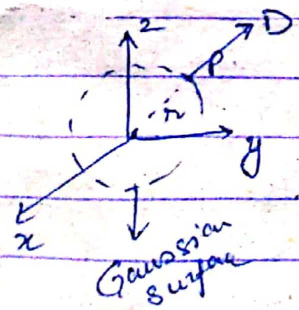
c) ...

(2-53) G&B.

Q. Ex (2.13) G&B. pg (2-44).

· Application of Gauss law. $\left\{ \begin{array}{l} D \cdot ds = D\,ds \rightarrow \text{when D is normal} \\ D \cdot ds = 0 \rightarrow \text{when D is tangential to the surface} \end{array} \right\}$

1) Pt. charge



$Q = \oint D\cdot ds = D_r \cdot 4\pi r^2$ ($= D_r \cdot \oint ds$) surface area of Gaussian surface

$\therefore D = \frac{Q}{4\pi r^2} \cdot \hat{a_r}$.

As $\oint D\cdot ds = D \cdot \oint ds$.

$= D_r \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} r^2 \sin\theta\, d\theta\, d\phi$

$= D_r r^2[-\cos\theta]_0^{\pi}[\phi]_0^{2\pi} = 4\pi r^2 D_r$.

$\therefore D_r = \frac{Q}{4\pi r^2} \cdot \hat{a_r}$.

& $E = \frac{Q}{4\pi\epsilon_0 r^2} \cdot \hat{a_r}$.

$$Q = \Psi = \oint_S D \cdot ds = \int \rho_v \cdot dv \quad \text{—} (1)$$

$$\& \quad \oint_S D \cdot ds = \int_V \nabla \cdot D \, dv \quad (\text{by divergence Thm})$$

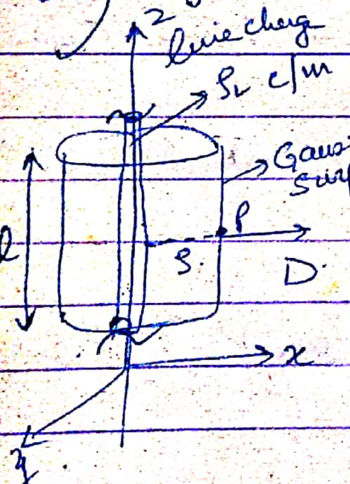
$$\therefore \boxed{\rho_v = \nabla \cdot D} \quad \text{—} (2)$$

→ Also called as Maxwell's first Eq^n.

Observation.

1. Gauss law shows eq (1) in integral form & eq (2) in differential or pt. form.
2. Proper application of divergence Thm to Coulomb's law results in Gauss's law.
3. Gauss law provides easy mean of finding E or D for symmetrical charge distributions only.

2) Infinite line charge.



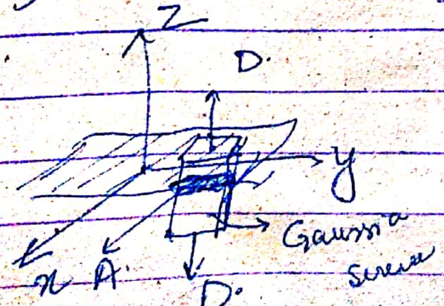
$$\rho_L l = Q = \oint D \cdot ds = D_\rho \oint ds$$

$$= D_\rho \cdot 2\pi \rho l$$

$$\therefore D = \frac{\rho_L}{2\pi\rho} a_\rho$$

for top & bottom $ds = 0$ as D is tangential

3) Infinite sheet of charges



$$\rho_s \int ds = Q = \oint D \cdot ds$$

$$= D_z \left[\int_{top} ds + \int_{bottom} ds \right]$$

$$\therefore \rho_s A = D_z (A + A)$$

$$\therefore D = \frac{\rho_s}{2} a_z$$

Applications of Gauss's Law.

① for pt. charge → already done.

$$D = \frac{Q}{4\pi r^2} \cdot a_r \quad \& \quad E = \frac{Q}{4\pi\epsilon_0 r^2} \cdot a_r$$

② for Infinite line charge.

$$D = \frac{\rho_L}{2\pi\rho} \cdot a_\rho$$

③ for infinite sheet charge. $D = \frac{\rho_s \cdot a_z}{2}$ {∵ take sheet at $z=0$}

from Sadiku.

Gaussian Surface → To apply the Gauss law in order to calculate $\vec{E}$, ~~...~~ knowing whether symmetry exist i-e charge distribution should be symetric & this a closed mathematical surface is chosen which is called Gaussian Surface.

Surface is chosen such that D is either normal or tangential to the gaussian surface.

when $D \cdot ds = D\, ds$ when D is normal to Gaussian

$\oint D \cdot ds = 0$, when D is tangential — " —

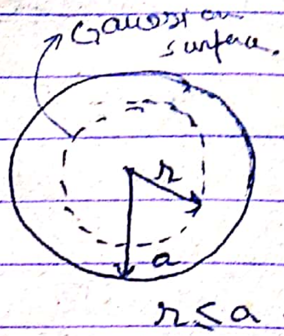
④ Uniformly charged sphere

Consider a uniformly charged sphere of radius 'a' with charge density ρ_v C/m3.

To determine D everywhere, construct a Gaussian surface for cases $r \le a$ & $r \ge a$.

Case I) for $r \le a$,

The total charge enclosed by the spherical surface of radius r will be as shown in fig given.



$$\therefore Q_{enc} = \int \rho_v \cdot dv = \rho_v \int_0^{2\pi}\int_0^{\pi}\int_0^{r} r^2 \sin\theta \, dr \, d\theta \, d\phi.$$

$$= \rho_v \frac{4}{3}\pi r^3.$$

& $$\oint D \cdot ds = D_r \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta \, d\theta \, d\phi.$$

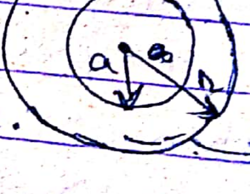
$$= D_r 4\pi r^2.$$

$$\therefore \rho_v \frac{4}{3}\pi r^3 = D_r 4\pi r^2.$$

$$\therefore \boxed{D = \frac{r}{3} \cdot \rho_v \cdot a_r}. \qquad 0 < r \le a.$$

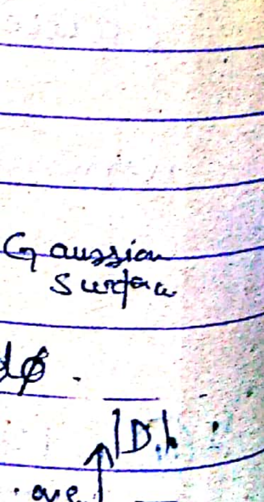
Case II). for $r \ge a$.

The total charge enclosed by the surface is the entire charge in this case.



$$\therefore Q_{enc} = \int \rho_v \cdot dv = \rho_v \int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2 \sin\theta \, dr \, d\theta \, d\phi.$$

$$= \rho_v \frac{4}{3}\pi a^3.$$



& $$\oint D \cdot ds = D \oint ds = D_r 4\pi r^2$$

gaussian surface instead of a now 'r'

$$\therefore \rho_v \frac{4}{3}\pi a^3 = D_r 4\pi r^2$$

$$\therefore D = \frac{a^3}{\ldots}$$

Q1] The flux density $\bar{D} = \frac{r}{3}\bar{a_r}$ nC/m^2 is in the free space
a) find E ? at $r = 0.2$ m.
b) find total $\vec{\Phi}$ leaving the sphere of $r=0.2$ m.
c) find total Q within the sphere of $r=0.3$ m.

Solⁿ — a). $E = \frac{D}{\epsilon_0} = \frac{r}{3\epsilon_0}\bar{a_r}$ and $r = 0.2$ m.

$\therefore E = \frac{0.2 \times 10^{-9}}{3 \times 8.85 \times 10^{-12}} = 7.52\, \bar{a_r}$ V/m.

b) As $Q = \psi = \oint D \cdot ds$.

{Consider the diffⁿ area is ds & which is normal to a_r}

$\therefore d\bar{s} = r^2 \sin\theta\, d\theta\, d\phi\, \bar{a_r}$.

$\bar{D} = r/3\ \bar{a_r}$.

$$\therefore Q = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{r^3}{3}\sin\theta\, d\theta\, d\phi$$

$$= \frac{r^3}{3}\left[-\cos\theta\right]_0^{\pi}\left[\phi\right]_0^{2\pi}$$

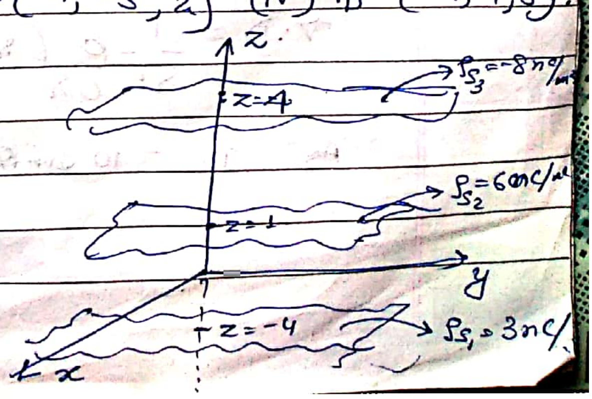
$$= \frac{4}{3}\pi r^3 \text{ nC}$$

$\therefore$ at $r = 0.2$ m, $Q = \psi = \frac{4}{3}\pi(0.2)^3 = 0.0335$ nC
$= 33.51$ pC.

c) at $r = 0.3$ m, $Q = \frac{4}{3}\pi(0.3)^3 = 0.113$ nC $= 113.097$ pC

Q2]. Three infinite uniform sheets of charge are located in free space as follows 3 nC/m^2 at $z=-4$, 6 nC/m^2 at $z=1$ and -8 nC/m^2 at $z=4$.
Find $\vec{E}$ at the pt (i) $P_A = (2,5,-5)$
(ii) $P_B = (4,2,-3)$ (iii) $P_C = (-1,-5,2)$ (iv) $P_D = (-2,4,5)$.

Solⁿ: $\vec{E} = \frac{\rho_s}{2\epsilon_0}\cdot \hat{a_n}$.



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(i) $P_A = (2, 5, -5)$. { As $\hat{a_n}$ is below the sheets. $-$ve of a_z }

$$\therefore E_t = \frac{\rho_{s1}}{2\epsilon_0}(-a_z) + \frac{\rho_{s2}}{2\epsilon_0}(-a_z) + \left(\frac{\rho_{s3}}{2\epsilon_0}\right)(-a_z).$$

$$= -56.47\ \bar{a_z}\ V/m .$$

(ii) $P_B = (4, 2, -3)$.

$$\therefore E_t = \frac{3\times10^{-9}}{2\epsilon_0}(\bar{a_z}) + \frac{6\times10^{-9}}{2\epsilon_0}(-\bar{a_z}) + \left(\frac{-8\times10^{-9}}{2\epsilon_0}\right)(-a_z).$$

$$= 282.358\ \bar{a_z}\ V/m .$$

(iii) $E_t = (\ )\bar{a_z} + (\ )\bar{a_z} + (\ )(-a_z) = 960.018\ \bar{a_z}\ V/m$

(iv) $E_t = (\ )\bar{a_z} + (\ )\bar{a_z} + (\ )(a_z) = +56.47\bar{a_z}\ V/m$

Divergence. { in diff^n co-ordinate systems }

(1) $$\nabla\cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

(2) $$\nabla\cdot A = \frac{1}{r}\frac{\partial}{\partial r}(r A_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}.$$

(3) $$\nabla\cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}.$$

Q. Given $\bar{D} = 10\sin\theta\ \bar{a_r} + 2\cos\theta\ \bar{a_\theta}\ C/m^2$
Prove that charge density

$$\rho_v = \frac{\sin\theta}{r}\left[18 + 2\cot^2\theta\right] C/m^3 .$$

$\Rightarrow$ By Gauss law in pt. form $\nabla\cdot\bar{D} = \rho_v$.
Given $\bar{D}$ in sph. co-ordinate hence

$$\nabla\cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}.$$

As $D_r = 10\sin\theta$, $D_\theta = 2\cos\theta$, $D_\phi = 0$.

$$\therefore \nabla \cdot D = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 10 \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(2\sin\theta\cos\theta\right) + 0$$

$$= \frac{1}{r^2} 10\sin\theta\,(2r) + \frac{1}{r\sin\theta}\left(2\cos 2\theta\right)$$

$$\therefore \rho_v = \frac{20\sin\theta}{r} + \frac{2}{r}\frac{\cos 2\theta}{\sin\theta} \quad C/m^3.$$

As, $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

$$\therefore \rho_v = \frac{20\sin\theta}{r} + \frac{2(\cos^2\theta - \sin^2\theta)}{r\sin\theta}$$

$$= \frac{20\sin^2\theta + 2\cos^2\theta - 2\sin^2\theta}{r\sin\theta} = \frac{18\sin^2\theta + 2\cos^2\theta}{r\sin\theta}$$

$$= \frac{\sin\theta}{r}\left[18 + \frac{2\cos^2\theta}{\sin^2\theta}\right] = \frac{\sin\theta}{r}\left[18 + 2\cot^2\theta\right] \quad C/m^3.$$

Q. A pt. charge of 5 nC is located at origin. If $V = 2V$ at $(0, 6, -8)$. Find

(i) The potential at $A(-3, 2, 6)$.

(ii) The potential at $B(1, 5, 7)$.

(iii) Potential diff^n V_{AB}.

$\Rightarrow$ (i) $A(-3, 2, 6)$. $\quad r_A = \cancel{\sqrt{9+4+}}\ -3a_x + 2a_y + 6a_z$.

$$\therefore r_A = \sqrt{9+4+36} = 7$$

$$\therefore V_A = \frac{Q}{4\pi\epsilon_0 r_A} + C.$$

$V_{Ref} = 2\,V$ at $(0, 6, -8)$ hence.

$$\vec{r}_R = 6\bar{a}_y - 8\bar{a}_z$$

$$r_R = \sqrt{6^2 + 8^2} = 10.$$

$$\therefore V_R = \frac{Q}{4\pi\epsilon_0 r_R} + C.$$

$$2 = \frac{5\times10^{-9}}{4\pi\epsilon_0 \times 10} + C \qquad \therefore C = -2.4938.$$

$$\therefore V_A = \frac{Q}{4\pi\epsilon_0 r_A} + C = \frac{5\times10^{-9}}{4\pi\epsilon_0\times 7} - 2.4938$$

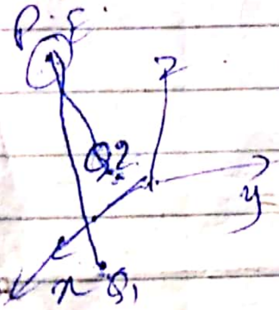
$$= 3.926\,V$$

T.E. (Electronics) (EWP)

Unit test - In Sem Exam (2014-15)

Q1). a) The pt. charges Q_1 & Q_2 are located at $(4,0,-3)$ & $(2,0,1)$ resp^n. If $Q_2 = 4nC$, find Q_1 such that the E at $(5,0,6)$ has no z comp.

$$\Rightarrow \bar{E}(5,0,6) = \frac{Q_1[(5,0,6)-(4,0,-3)]}{4\pi\epsilon_0 [(5,0,6)-(4,0,-3)]^{3/2}} + \frac{Q_2((5,0,6)-(2,0,1))}{4\pi\epsilon_0 r^2}$$



$$= \frac{Q_1(1,0,9)}{4\pi\epsilon_0(82)^{3/2}} + \frac{Q_2(3,0,5)}{4\pi\epsilon_0(34)^{3/2}}$$

if $E_z = 0$ then:

$$\therefore \frac{9Q_1}{4\pi\epsilon_0(82)^{3/2}} + \frac{5Q_2}{4\pi\epsilon_0(34)^{3/2}} = 0$$

$\therefore$ Putting $Q_2 = 4nC$. $Q_1 = -8.32\ nC$.

b) Determine the total charge if:

(i) line $0 < x < 5m$ if $\rho_L = 12x^2\ mC/m$.

(ii) On the cylinder with $\rho = 3m$, $0 < z < 4m$ if $\rho_s = \rho z^2\ C/m^2$.

$$\Rightarrow (i)\ Q = \int \rho_L \cdot dl = \int_0^5 12x^2\,dx = \left[4x^3\right]_0^5 = 0.5\,C.$$

$$(ii)\ Q = \int \rho_s \cdot ds = \int_0^{2\pi}\int_0^4 \rho z^2 \rho\, d\phi\, dz \Big|_{\rho=3}$$

$$= 9(2\pi)\left[\frac{z^3}{3}\right]_0^4\ C$$

$$= 1206.5\ C.$$

Q3(a) If $V = r^2 z \sin\phi$, calculate the energy within the region defined by $1 < r < 4$, $-2 < z < 2$, $0 < \phi < \pi/3$.

$\Longrightarrow$ $V = r^2 z \sin\phi$. As $W_E = \frac{1}{2}\int_V \epsilon_0 |E|^2 dv$.

$$\therefore \bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}a_\phi + \frac{\partial V}{\partial z}\bar{a}_z\right]$$

$$= -\left[2rz\sin\phi\, \bar{a}_r + \frac{1}{r} r^2 z\cos\phi\, a_\phi + r^2 \sin\phi\, a_z\right]$$

$$\therefore |\bar{E}| = \sqrt{4r^2 z^2 \sin^2\phi + r^2 z^2 \cos^2\phi + r^4 \sin^2\phi}.$$

$$\therefore W_E = \frac{\epsilon_0}{2}\int_{z=-2}^{2}\int_{0}^{\pi/3}\int_{r=1}^{4} \left[4r^2 z^2 \sin^2\phi + r^2 z^2\cos^2\phi + r^4 \sin^2\phi\right] dv$$

$dv = r\,dr\,d\phi\,dz$

$$\therefore W_E = \frac{\epsilon_0}{2}\int_{-2}^{2}\int_{0}^{\pi/3}\int_{1}^{4} r^3\left[4z^2\sin^2\phi + z^2\cos^2\phi + r^2\sin^2\phi\right] dr\,d\phi\,dz.$$

$$= \frac{\epsilon_0}{2}\int_{z=-2}^{2}\int_{\phi=0}^{\pi/3}\left\{4z^2\sin^2\phi\left[\frac{r^4}{4}\right]_1^4 + z^2\cos^2\phi\left[\frac{r^4}{4}\right]_1^4 + \left[\frac{r^6}{6}\right]_1^4 \sin^2\phi\right\} d\phi\, dz$$

$\int \sin^2\phi = \frac{1-\cos 2\phi}{2}$

$\int \cos^2\phi = \frac{1+\cos 2\phi}{2}$

$= 6.6735\ nJ$.