Unit - 1 - Random Process & Noise.

CO1 - Apply the statistical theory for describing various signals in a communication system.

* Importance of Random Processes :-

- Random processes / variables talk about quantities & signals which are unknown ;
- The data sent through a communication is modeld as random variable
- The noise, interference, & fading introduced by channel can all be modeld as random process.
- The measure of performance is expressed in terms of probiblity.

* Def :-

A random process is the natural extension of the concept of a random variable when dealing with signals.

- It is collection of time functions or signals corresponding to various random projects / experiments. & The random process represents the mathematical model of these random signals.
- The random process can be denoted by $x(t,s)$ or $x(t)$; where s is the sample point of the random experiment & t is the time.

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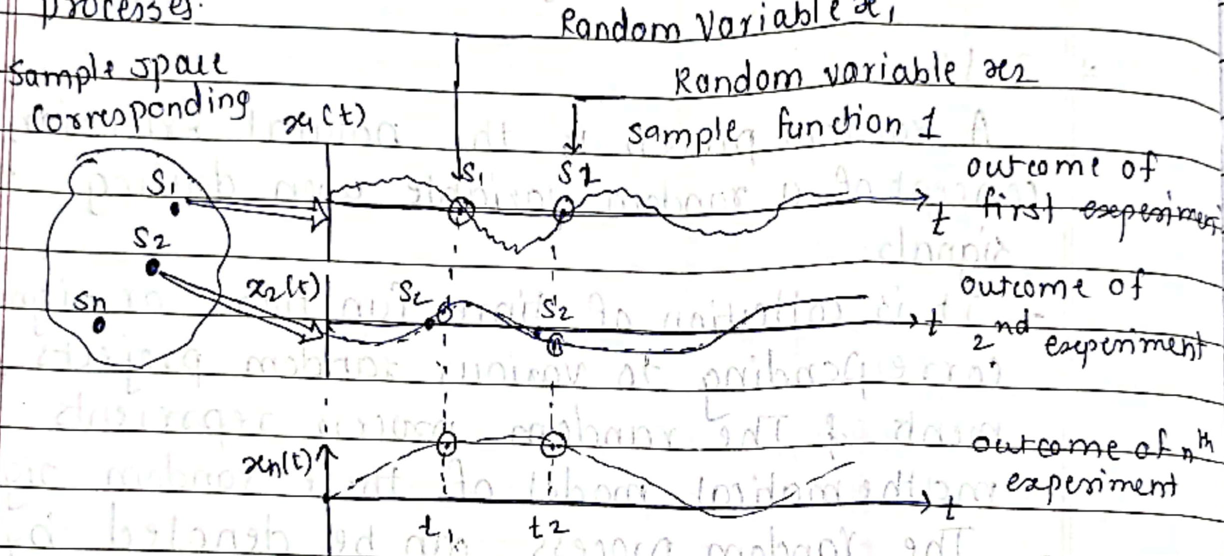
Ensemble. A group of variables / items viewed as whole rather than indiv

Random Variable	Random Process
1) It is set of numbers	1) It is a waveform
2) It need not be a function of time	2) It is a function of time
3) Random Variables are not further classified	3) It can be stationary or Ergodic.
4) Only ensembles average can be calculated	4) Ensemble as well as time can be calculated.

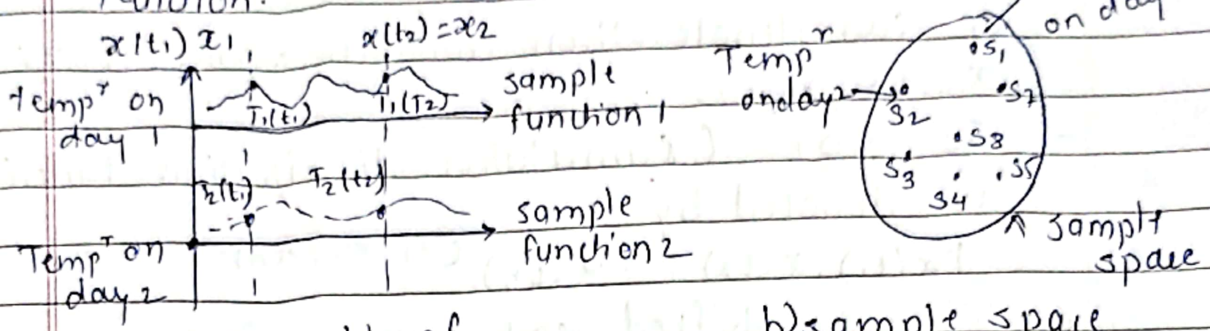
* Relation between a random variable & a random processes.



- Here we specify random variable x by repeating an experiments a large number of times & from the outcomes of the experiment we determine $f_X(x)$
- Here x_1 is a random variable generated by the amplitude of the random process at instant $t = t_1$ &
- x_2 is random variable generated by the amplitude of the random process at instant $t = t_2$

* Ensemble :-
- It means family or collection. So collection of all the possible sample function is called as an ensamble

* Relation betⁿ ensemble & sample space.
- Sample space is the collection of all possible sample points.
- Ensemble is the collection of all possible sample function.

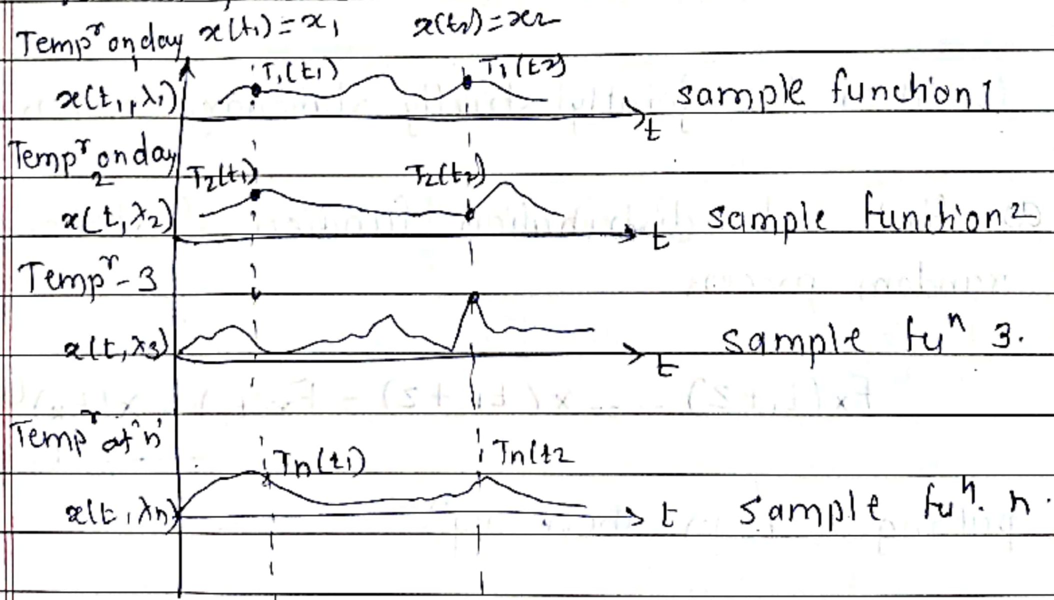


a) Ensamble of sample function.

b) sample space

* Classification of Random Processes.
1) Stationary & Non-stationary random process.
2) Wide-Sense (or weakly) stationary process.
3) Ergodic processes.

1) Stationary & Non-stationary
- A random process whose statistical characteristic do not change with time is known as stationary random process.



PDF- Probability Density function.

- let $x(t_1), x(t_2) ---- x(t_n)$ denote the random variable.
- The random process $x(t)$ at the different time instant $t = t_1, t_2, t_3 --- t_n$.
- We can denote these random variables as $x_1, x_2, x_3 ---- x_n$.
- So CDF (Cumulative distribution function) is denoted by

$$F_{x(t_1), x(t_2) --- x(t_n)}(x_1, x_2, x_n)$$

- If we shifted each instant of time with fixed duration - τ.

So,

- New set of variable

$$x(t_1+\tau), x(t_2+\tau), x(t_n+\tau).$$

* Strictly stationary Process:-
- The random process $x(t)$ is said to be stationary in strict sense if the joint CDF of the original set of random variable is equal to joint CDF of new set of random variable obtained after time shift of τ.

* Condition for jointly strictly stationary process

a) first order distribution function of stationary random process

$$F_{x(t_1+\tau) ---- x(t_k+\tau)} = F_{x(t_1) -- x(t_k)}(x_1, --- x_k) \quad (1)$$

putting $k=1$ in above eq^n.

$$F_{x(t)}(x) = F_{x(t+z)}(x)$$
$$= F_x(x) \quad \text{— for all } t \text{ \& } z.$$

- hence first order distribution fu^n of stationary random process is independent of time

b) second order distribution fu^n

Putting $k=2$ in eq^n (1)

$$F_{x(t_1),x(t_2)}(x_1,x_2) = F_{x(0)\,x(t_2-t_1)} \quad \text{- for all } t \text{ \& } z.$$

The above eq^n shows that, this random process depends only on the time difference bet^n the observation intervals. It does not depends on particular time.

* Mean, Correlation & Covarience function:-

1) Mean:-

Def^n:- Mean of the strictly stationary random process $x(t)$ is defined as the expectation of the random variable obtained over observation interval 't'

- It gives as,

$$\mu_x(t) = E[X(t)] \quad \text{———} \quad (1)$$

$$\therefore \mu_x(t) = \int_{-\infty}^{\infty} x f_{x(t)}(x)dx \quad \text{———} \quad (2)$$

$\because$ $F_{x(t)}(x) =$ 1st order probability density fu^n of $X(t)$

- Mean of strictly stationary random process

$$\mu_x(t) = \mu_x \quad \text{— for all } t \quad \text{———} \quad (3)$$

- In this $F_{x(t)}(x)$ is independent of t, Hence mean of strictly stationary process is constant

$$\therefore E[X(t_1)] = E[X(0)]$$
$$= \mu_x(0) = \text{constant.}$$

2) Autocorrelation function for $X(t)$.

- It is product of two random variables $X(t_1)$ & $X(t_2)$ obtained by process $X(t)$ at t_1 & t_2 rsp.

- $$R_X(t_1, t_2) = E[X(t_1) X(t_2)] \quad \text{--- (4)}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 \, f_{x(t_1), x(t_2)}(x_1, x_2)\, dx_1\, dx_2$$
$$= R_X(t_2 - t_1) \text{ - for all } t_1 \text{ \& } t_2 \quad \text{--- (5)}$$

- So, here we can say that autocorrelation function of strictly stationary process depends only on time difference $(t_2 - t_1)$

3) Auto-Covariance function for $x(t)$

$$C_X(t_1, t_2) = E[(X(t_1) - \mu_X)(X(t_2) - \mu_X)] \quad \text{--- (6)}$$

$$\therefore C_X(t_1, t_2) = E[X(t_1)X(t_2) - \mu_x X(t_1) - \mu_x X(t_2) + \mu_x^2]$$
$$= E[X(t_1) \cdot X(t_2)] - \mu_x E[X(t_1)] - \mu_x E[X(t_2)] + \mu_x^2$$

But $R_X(t_1, t_2) = E[X(t_1) \cdot X(t_2)]$ — from (4)

& $\mu_X(t) = E[X(t)]$

$$\therefore C_X(t_1, t_2) = R_X(t_1, t_2) - \mu_x^2 - \mu_x^2 + \mu_x^2$$
$$= R_X(t_2 - t_1) - \mu_x^2$$

Example:-

$X(t) = A\cos(\omega t + \phi)$, ϕ is uniform $[-\pi, \pi]$

find $m_x(t)$, $R_x(t_1, t_0)$

1) $m_x(t) = E[x(t)]$

$$= A\int_{-\pi}^{\pi} \cos(\omega t + \phi) \cdot \frac{d\phi}{2\pi} = 0$$

$= 0$

2) $R_x(t_1, t_0) = E[x(t_1), x(t_0)]$

$$= \int_{-\pi}^{\pi} A\cos(\omega t_1 + \phi) \cdot A\cos(\omega t_0 + \phi) \frac{d\phi}{2\pi}$$

$$= \frac{A^2}{2\pi} \cdot \frac{1}{2} \int_{-\pi}^{\pi} \cos(\omega(t_1 - t_0))\, d\phi + \frac{A^2}{2\pi} \cdot \frac{1}{2} \int_{-\pi}^{\pi} \cos(\omega(t_1 + t_0) + 2\phi)\, d\phi$$

$$= \frac{A^2}{2} \cos(\omega(t_1 - t_0))$$ fun of $t_1 - t_0$

for $t_1 - t_0 = \tau$

$$R_x(t_1, t_0) = \frac{A^2}{2} \cos[\omega(t_1 - t_0)]$$

$$= \frac{A^2}{2} \cos(\omega\tau)$$ ie. $R_x(\tau)$.

* Properties of autocorrelation function

Autocorrelation of stationary process is given as,

$$R_x(\tau) = E[X(t+\tau)X(t)]$$ for all t.

Property 1.

The mean square value of the process $\tau = 0$

$$R_x(0) = E[x(t+0) \cdot x(t)]$$
$$R_x(0) = E[x^2(t)].$$

Property-2 :-

The autocorrelation fu^n $R_x(\tau)$ is an even function of τ.

$$R_x(\tau) = R_x(-\tau)$$

Proof :-

Putting $t = (t-\tau)$

$$R_x(\tau) = E[x[(t-\tau)+\tau] \cdot x(t-\tau)].$$
$$= E[x(t) \cdot x(t-\tau)]$$

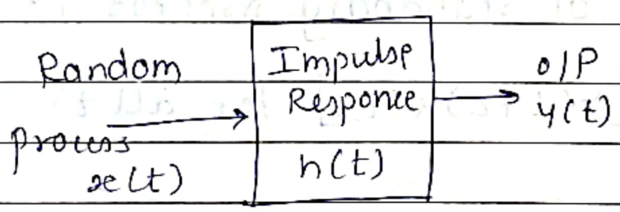
$x(t-\tau)$ - delayed version of $x(t)$
$x(t+\tau)$ = Advance version of $x(t)$
$\therefore R_x(\tau) = R_x(-\tau).$

Property 3 :-

- The autocorrelation function $R_x(\tau)$ has its maximum magnitude at $\tau=0$

$$|R_x(\tau)| \leq R_x(0)$$

* Transmission of signal through linear filter



- let a random process $x(t)$ is applied as input to linear time Invarient filter (LTI)
- impulse response assumed to be $h(t)$

- $x(t)$ - it is stationary process.

o/p

$$y(t) = \int_{-\infty}^{\infty} h(\tau_1)\, x(t-\tau_1)\, d\tau_1 \quad \text{--- (1)}$$

τ_1 = integration variable.

- Mean of $Y(t)$ [output process]

$$\mu_y(t) = E[Y(t)]$$

$$= E\left[\int_{-\infty}^{\infty} h(\tau_1)\, x(t-\tau_1)\, d\tau_1\right] \quad \text{--- (2)}$$

$$= \int_{-\infty}^{\infty} h(\tau_1)\, E[x(t-\tau_1)]\, d\tau_1$$

But $\mu_x(t-\tau_1) = E[x(t-\tau_1)]$

$$\therefore \mu_y(t) = \int_{-\infty}^{\infty} h(\tau_1)\cdot \mu_x(t-\tau_1)\, d\tau_1$$

$$\mu_y = \mu_x \int_{-\infty}^{\infty} h(\tau_1)\, d\tau_1$$

$$\therefore \mu_y = \mu_x H(0)$$

$H(0)$ = zero frequency Response of system

IMP - 1) Mean of random process $Y(t)$ produced at the output of LTI system in response to input random process $X(t)$ equal to the mean of $X(t)$ multiplied by the dc response of the system.
2) Autocorrelation function of random process $Y(t)$ is a constant.

* Power Spectral Density:-

Def^n - The distribution of average power of a signal $x(t)$ in the frequency domain is called power spectral density.

- A wide-sensed stationary random process is characterized in the frequency domain with the help of its PSD. - which is defined as the F.T of its auto-correlation function

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau \quad \text{——} \ ①$$

$X(t)$ calculated at $f = f_c$.

$$E[Y^2(t)] = \int_{-\infty}^{\infty} R \, |H(f)|^2 S_x(f) \, df \quad \text{——} \ ②$$

where

$$E[Y^2(t)] = \int_{-\infty}^{\infty} df |H(f)|^2 \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

Here $E[Y^2(t)]$ - it is square value of function $Y(t)$

eq^n ② represents that the mean square value $E[Y^2(t)]$ of the output of LTI filter for wide sense stationary process $x(t)$ applied at its input, is integration of

PSD of square magnitude of the transfer function in the frequency domain.

* Properties of PSD :-

1) Property 1 :- [Zero frequency value of PSD]
- The value of PSD of wide sense stationary process at $f=0$ is equal to the total area under the curve of autocorrelation fun.
that is

$$S_x(0) = \int_{-\infty}^{\infty} R_x(\tau) d\tau$$

We know that

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) \cdot e^{-j2\pi f\tau} d\tau.$$

Here $f=0$.

$$S_x(0) = \int_{-\infty}^{\infty} R_x(\tau) \cdot e^{0} d\tau$$

$$S_x(0) = \int_{-\infty}^{\infty} R_x(\tau) d\tau$$

2) Property 2 :- [Mean Square Value of process]
- The mean square value of wide-sense stationary random process is equal to the area under the curve of PSD.
ie.

$$E[x^2(t)] = \int_{-\infty}^{\infty} S_x(f) df.$$

We know that

$$R_x(\tau) = \int_{-\infty}^{\infty} S_x(f) \cdot e^{j2\pi f\tau} d\tau.$$

Here substitute $\tau=0$

$$R_x(0) = \int_{-\infty}^{\infty} S_x(f) e^{0} df$$

$$R_x(0) = \int_{-\infty}^{\infty} S_x(f) df$$

We know that

$$E[x^2(t)] = R_x(0)$$

$$\therefore \quad E[x^2(t)] = \int_{-\infty}^{\infty} S_x(f) df.$$

3) Property 3 :- [PSD is always Positive]

- The PSD of a wide sense random process will always be non-negative for all value of 'f'

$$S_x(f) \geq 0 \text{ for all } f.$$

Proof :-

The PSD of $S_x(f) = E[y^2(t)]$

ie. the mean square value of $E[y^2(t)]$ will be non-negative.

4) Property 4 :- [PSD is even function]

- The PSD of a real valued random process is an even function of frequency

ie. $S_x(-f) = S_x(f)$.

Proof :-

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) \cdot e^{-j2\pi f\tau} d\tau.$$

put $-f$ for f value.

$$S_x(-f) = \int_{-\infty}^{\infty} R_x(\tau) \cdot e^{j2\pi f\tau} d\tau.$$

Now $-\tau$ for τ

$$S_x(-f) = \int_{-\infty}^{\infty} R_x(-\tau)\, e^{-j2\pi f\tau}\, d\tau.$$

But $R_x(-\tau) = R_x(\tau)$ — coz it is a even fu^n.

$$\text{So } S_x(-f) = \int_{-\infty}^{\infty} R_x(\tau) \cdot e^{-j2\pi f\tau}\, d\tau.$$

$$\therefore S_x(f) = S_x(-f).$$

5) Property 5:-

If the PSD is properly normalized, then it will have all the properties that are usually associated with a probability density fu^n (PSD).

* Examples:-

1) Show that the random process

$x(t) = A\cos(\omega_c t + \theta) \rightarrow \theta$ is a random variable uniformly distributed in the range $(0, 2\pi)$ is wide-sense stationary process.

→ for wide-sense stationary process it is necessary to show that

-1) Mean of sample function amplitude at any 't' is same

2) Autocorrelation

$R_x(t_1, t_2) = R_x(t_2 - t_1)$

As θ is uniformly distributed the PDF is given by $f_\theta(\theta) = 1/2\pi \quad 0 < \theta < 2\pi$.

$= 0$

1) Mean of the process is given by

$$m_x(t) = \int_{-\infty}^{\infty} x f_x(x,t)\, dx \quad \text{— (2)}$$

$$x = x(t)$$
$$= A\cos(\omega_c t+\theta) \;\&\; f_x(x,t)$$
$$= f_\theta \theta = \frac{1}{2\pi}$$

$$m_x(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi} \cdot A\cos(\omega_c t+\theta)\, d\theta$$

$$= \frac{A}{2\pi}\left[\sin(\omega_c t+\theta)\right]_0^{2\pi}$$

$$m_x(t) = \underline{\underline{0}}.$$

2) Autocorrelation function:-

$$R_x(t_1,t_2) = E[x(t_1)\cdot x(t_2)]$$
$$= E[A\cos(\omega_c t_1+\theta)\cdot A\cos(\omega_c t_2+\theta)]$$

$$R_x(t_1,t_2) = E[A^2\cos(\omega_c t_1+\theta)\cdot\cos(\omega_c t_2+\theta)]$$

but

$$\cos A\cdot\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$$

$$\therefore\; R_x(t_1,t_2) = E\left[\frac{A^2}{2}\{\cos[\omega_c(t_1-t_2)] + \cos[\omega_c(t_1+t_2) + 2\theta)]\right]$$

$$R_x(t_1,t_2) = \frac{A^2}{2}E\{\cos[\omega_c(t_1-t_2)]\} + \frac{A^2}{2}E\{\cos[\omega_c(t_1+t_2+2\theta)]\}$$

the first term does not contain random variable of θ.

So Here only we consider the 2^{nd} term

$$= \int_{-\infty}^{\infty} \cos[w_c(t_1+t_2+2\theta)] \cdot \frac{1}{2\pi}\, d\theta$$

But $E[X] = \int_{-\infty}^{\infty} x f_x(x)\, dx.$

$$= \int_{0}^{2\pi} \frac{1}{2\pi} \cos[w_c(t_1+t_2+2\theta)]\, d\theta$$

$$= \frac{1}{2\pi}\left\{\sin w_c(t_1+t_2+2\theta)\right\}_0^{2\pi}$$

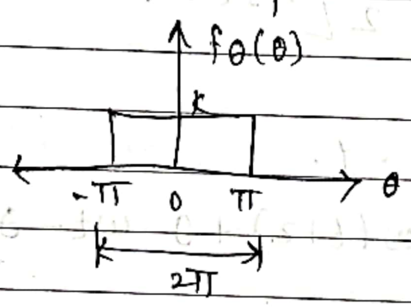
$$= 0$$

So Here we consider the final eq^n

$$R_z(t_1,t_2) = \underbrace{\frac{A^2}{2}\left[\cos[w_c(t_1-t_2)]\right]}_{1^{st}\ \text{term}}$$

2) Show that the random process $x(t) = A\cos(w_c$ $\theta)$ where θ is uniformly distributed random variable in the range $(-\pi, +\pi)$ is a wide sense stationary process.

→ PSD of θ represented as



He $k \times 2\pi = 1$ → {area of PDF is always 1}

$k = \frac{1}{2\pi}$

$$f_\Theta(\theta) = \frac{1}{2\pi} \quad -\pi \le \theta \le \pi$$
$$= 0$$

1) Mean $x(t)$ calculation

$m_x(t)$ is process of $x(t)$ is given as

$$m_x(t) = E[x(t)]$$
$$= \int_{-\infty}^{\infty} A\cos(\omega t + \theta) \cdot f_\theta(\theta)\, d\theta.$$
$$m_x(t) = \frac{A}{2\pi}\int_{-\pi}^{\pi} \cos(\omega t + \theta)\, d\theta.$$
$$= 0$$

2) Auto correlation $R_x(t, t+\tau)$ → Here π & $-\pi$ is the fu^n of two different time intervals. so
$R_{xx}(t, t+\tau) = E[x(t) \cdot x(t+\tau)]$

$$\text{step-1} = \int_{-\infty}^{\infty} (A\cos(\omega t + \theta) \cdot A\cos[\omega(t+\tau) + \theta]\, f_\theta(\theta)\, d\theta$$
$$= \frac{A^2}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2}\left[\cos(\omega t) + \cos(2\omega t + 2\theta + \omega t)\right] d\theta.$$

$distribut^n$ of step-1.

$$= \frac{A^2}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2}\cos[\omega(t+\tau) + \theta - \omega t - \theta] + \frac{1}{2}\cos[\omega(t+\tau) + \theta + \omega t + \theta]\, d\theta$$

integration of 2^{nd} term is zero.

$$\text{So } R_{xx}(t, t+\tau) = \frac{A^2}{2\pi} \times \frac{1}{2}[\cos\omega t][\theta]_{-\pi}^{\pi}$$

$$= \frac{A^2}{2\pi} \times \frac{1}{2}\cos\omega t \times [2\pi]$$

$$= \frac{A^2}{2}\cos\omega t$$

* Concept of Noise:-

It is a random unwanted energy that interferes with desired signal.

* Classification of Noise:-

Noise

- Internal Noise
  - Thermal noise
  - Shot Noise
  - Transit time Noise
  - Partition Noise
  - Miscellaneous Noise
    - Filker
    - Resistance
    - Noise in mixer.
- External Noise
  - Atmospheric
  - Industrial
  - Extraterrestrial
    - Solar
    - Cosmic.

I) External Noise:-
- It is originated from the sources outside the receiver.

a) Atmospheric Noise:- (static)
- It is caused by lightening discharges in thunderstroms & other natural eletric disturbance occurring in atmosphere
- It consist of fake radio signals & propagates over earth as normal frequency signals. Hence they are picked up by antenna
- It affects more with radio reception reception than TV reception. As a result, we can here strange sounds.

b) Extraterrestrial Noise- (space noise)
- This is the noise originated outside the earth.

* Solar Noise :- Noise that originates from sun is called solar noise.
* Cosmic Noise :- The noise received from stars is called thermal or cosmic noise.
* Industrial Noise:- This is manmade noise. It is significant at frequencies of 1-600 MHz in urban, suburban & other industrial area.

II) Internal Noise:-
- It is originated by any of the active or passive devices found in the communication devices.

* Thermal Noise:-

- It arises due to random motion of electrons in a conductor
- free electrons in a conductor possess kinetic energy. This kinetic energy gives motion to electrons
- This motion is randomized due to collision of electrons. This is due to thermal energy.

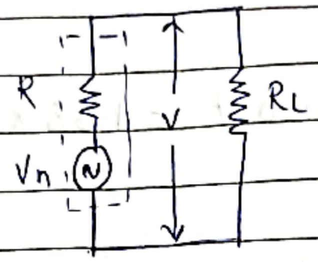


fig. current equivalent ckt for thermal noise.

- Thermal noise power thus produces is proportional to temperature & it is given by,

$$P_n = KTB$$

$\therefore$ $k = 1.38 \times 10^{-23} J/°k.$

$B = B.W.$ in Hz

$T =$ Absolute tempr in °k.

$$P_n = \frac{V^2}{R_L}$$

if $R = R_L$ then $V_n = \frac{V}{2}$

$$\therefore \quad P_n = \frac{\left(\frac{V_n}{2}\right)^2}{R} = \frac{V_n^2}{4R}$$

$$V_n^2 = 4RP_n$$

$$\underline{V_n = \sqrt{4KTBR}}$$

Example

1) Calulate the thermal noise power available from any resistor at room temperature ($290^\circ K$) for a B.W. of 1MHz. Calulate also the corresponding noise voltage given that $R = 50\,\Omega$.

→ Given

$T = 290^\circ K$

$B = 1\,MHz$

$R = 50\,\Omega$

k = Boltzmann's constant $= 1.38 \times 10^{-23}\, J/^\circ K$.

i) Thermal noise

$P_n = kTB$

$= 1.38 \times 10^{-23} \times 290 \times 1 \times 10^{6}$

$= 4 \times 10^{-15}$ Watt

ii) Thermal noise voltage

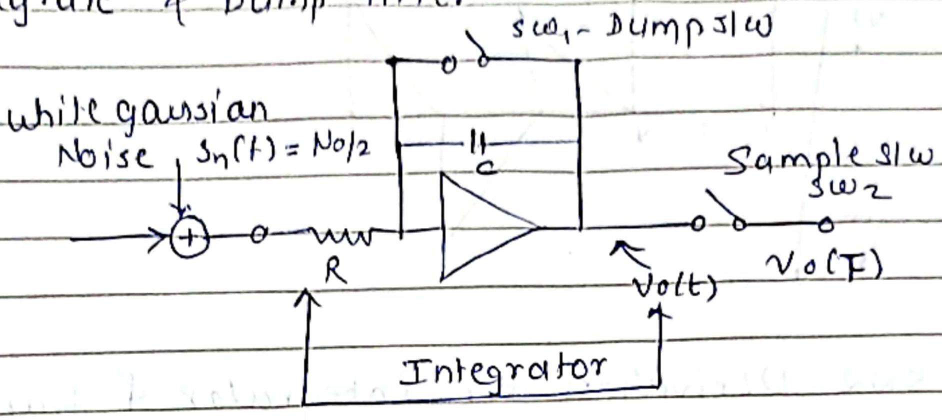
$V_n = \sqrt{4kTBR}$

$= \sqrt{4 \times 1.38 \times 10^{-23} \times 290 \times 1 \times 10^{6} \times 50}$

$V_n = 0.895\,\mu V$.

Unit-2: Digital Modulation.

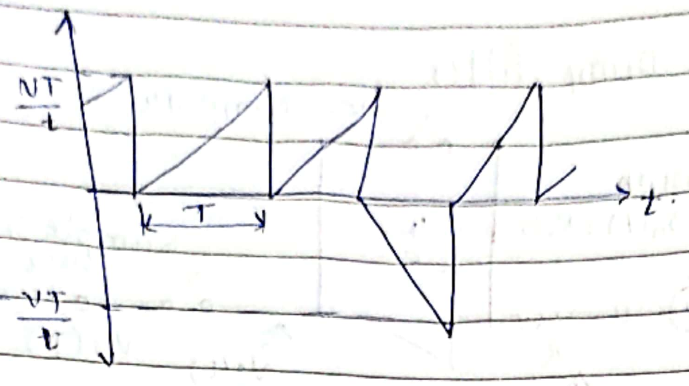
* Integrate & Dump filter -



- It consist of two major circuits:-
- a) Adder
 b) Integrate & dump s/w combination.
- The transmitted signal $x(t)$ is added with noise $n(t)$. This $[x(t) + n(t)]$ is the input provided to the integrator & dump filter.
- The PSD of $n(t) = \left(\frac{N_0}{2}\right)$ → as it is white gaussian noise

Gaussian Noise - It is a stationary & ergodic random process with zero mean.

* Working:-
- Initially sw_2 is closed. Due to this, capacitor discharges completely at the begining of the bit interval.
- Voltage at $C = 0$ at starting of bit.
- Then integrator integrates over one bit periode T second
- sw_1 is kept open for the bit period 'T' sec.
- at the end of the bit period. sw_2, the sampling s/w is closed for a moment.



* SNR Derivation for integrator & Dump filter.

- The input of integrator is $[x(t) + n(t)]$
- output of integrator is given by,

$$V_o(t) = \frac{1}{RC}\int_0^T [x(t) + n(t)]\, dt$$

- Integration operation over a one bit interval

$$V_o(t) = \frac{1}{RC}\int_0^T x(t)\, dt + \frac{1}{RC}\int_0^T n(t)\, dt.$$

Now the sample voltage due to signal is

$$x_o(T) = \frac{1}{RC}\int_0^T x(t)\, dt$$

let $x(t) = +V$

$$x_o(T) = \frac{1}{RC}\int_0^T V\, dt.$$

$$= \frac{V}{RC}[t]_0^T$$

$$= \frac{VT}{RC}$$

let $\tau = RC$, determine the normalized output of signal power

$$x_o^2(T) = \frac{V^2T^2}{\tau^2}$$

* Output noise power calculation.

$$P = \frac{N_0 T}{2\tau^2}$$

* Signal to noise ratio of integrate & dump receiver

$$SNR = \frac{V^2T^2/\tau^2}{N_0T/2\tau^2} = \frac{2V^2T}{N_0}$$

Important Formulae:-

1) $P_e = \frac{1}{2} \, \text{erfc} \sqrt{\frac{E}{N_0}}$

$E = V^2T$ - signal energy bit
V = signal amplitude
T = bit duration.

Probability of error

erfc - complementry error fn. $\frac{N_0}{2}$ = PSD of white noise.

2) Complementry error fn:-

$$\text{Erfc}(u) = \int_u^\infty e^{-x^2} dx$$

3) signal to Noise ratio of integrate & dump receiver = $\frac{2V^2T}{N_0}$

Problems:-

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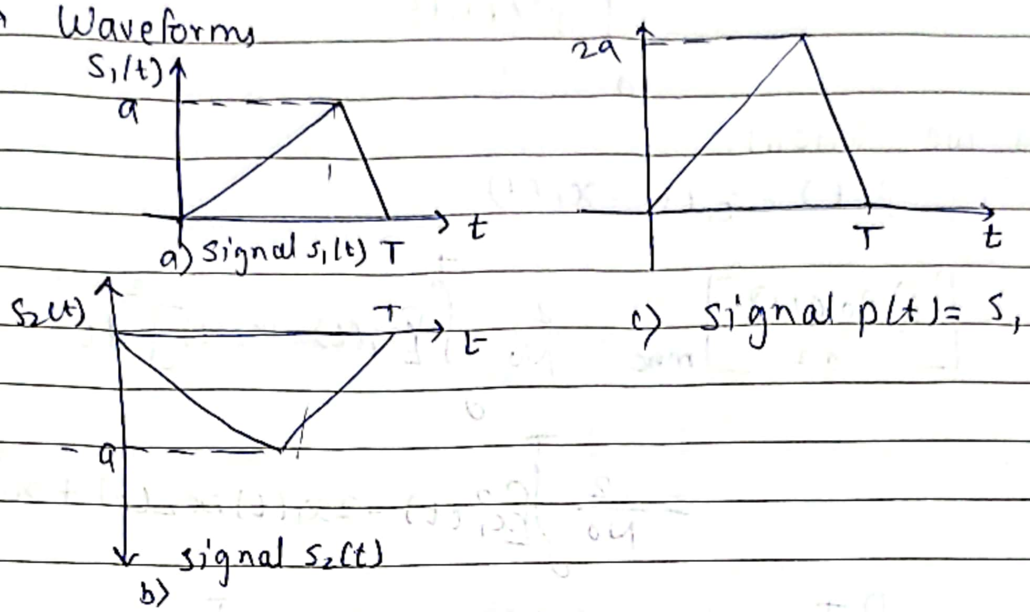
Matched filter:-

Def^n :- An optimum filter which yields a max. ratio $\left[\frac{[p_o(T)]^2}{\sigma^2}\right]$ is known as matched filter.

→ Waveforms



c) signal $p(t) = s_1(t) - s_2(t)$.

Probability of Error 'P_e' of the Matched filter

- It can be obtained by evaluating maximum signal to noise ratio

$$\left[\frac{[p_o(T)]^2}{\sigma^2}\right]_{max}$$

Input $S_{ni}(f) = \frac{N_0}{2}$

$$\left[\frac{[p_o(T)]^2}{\sigma^2}\right]_{max} = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{S_{ni}(f)} df$$

$$= \int_{-\infty}^{\infty} \frac{|P(f)|^2}{N_0/2} df$$

$$= \frac{2}{N_0} \int_{-\infty}^{\infty} |P(f)|^2 df.$$

but

$$\int_{-\infty}^{\infty} |P(f)|^2 df = \int_{-\infty}^{\infty} p^2(t)\, dt$$

$$= \int_{0}^{T} p^2(t)\, dt.$$

as we know

$$p(t) = x_1(t) - x_2(t)$$

$$\left[\frac{[p_o(T)]^2}{\sigma^2}\right]_{max} = \frac{2}{N_0} \int_0^T [x_1(t) - x_2(t)]^2 dt$$

$$= \frac{2}{N_0} \int_0^T [x_1^2(t) - 2x_1(t)x_2(t) + x_2^2(t)]\, dt$$

$$= \frac{2}{N_0}\left[\int_0^T x_1^2(t)\,dt + \int_0^T x_2^2(t)\,dt - 2\int_0^T x_1(t)\cdot x_2(t)\,dt\right]$$

$$= \frac{2}{N_0}\left[E_{S_1} + E_{S_2} - 2E_{S_{12}}\right].$$

$\because$ $E_{S_1} = \int_0^T x_1^2(t)\, dt$ = signal energy of $x_1(t)$

$E_{S_2} =$

$E_{S21} = \int_0^T x_1(t)\cdot x_2(t)\, dt$ - signal energy due to correlation of $x_1(t)$ & $x_2(t)$.

with the correlation property

$$x_1(t) = -x_1(t)$$

$$E_{S_1} = E_{S_2} = E_{S12} = E_S$$

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So $\left[\frac{[P_o(T)]^2}{\sigma^2}\right]_{max} = \frac{2}{N_0}\left[E_s + E_s - (-2E_s)\right]$

$$= \frac{8E_s}{N_0}.$$

Now the error probability of optimum filter

$$\left[\frac{[P_o(T)]^2}{\sigma^2}\right]_{max} = P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{x_{o1}(T) - x_{o2}(T)}{2\cdot\sqrt{2}\cdot\sigma}\right]$$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{P_o^2(T)}{8\sigma}\right]^{1/2}.$$

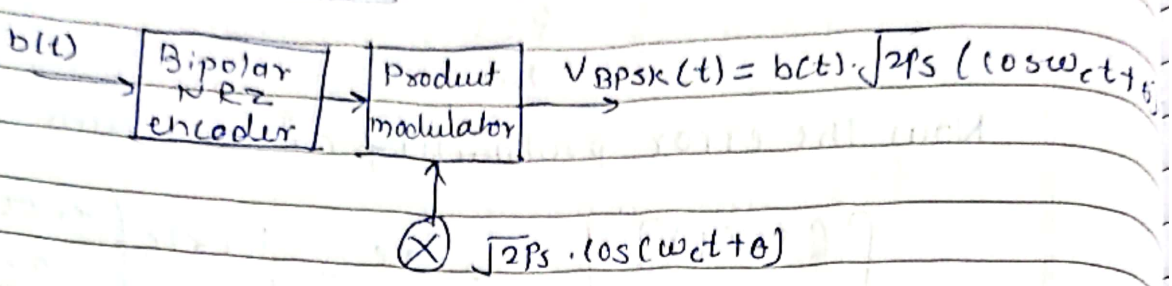
$$P_{e(min)} = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{1}{8}\left[\frac{P_o^2(T)}{\sigma^2}\right]_{max}\right\}^{1/2}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{8}\left(\frac{8E_s}{N_0}\right)\right]^{1/2}.$$

$$\boxed{P_{e(min)} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{E_s}{N_0}\right)^{1/2}}.$$

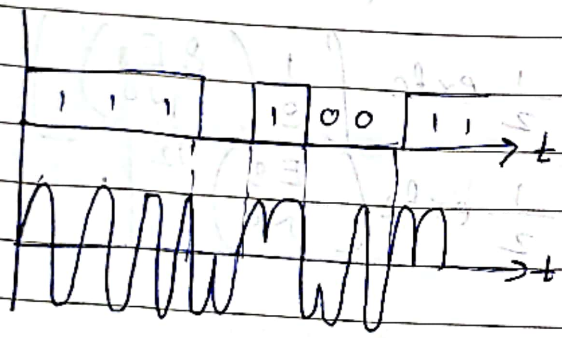
* Binary Phase shift keying (BPSK) :-

* BPSK transmitter:-



1) Polar NRZ:-

- It convert the input binary data sequence into polar NRZ format.
- In this binary symbol 1 & 0 are represented by $\pm\sqrt{2P_s}$. P_s



2) Product modulator:-

- The polar NRZ symbol are multiplied with sinusoidal carrier.
- The desired BPSK output has fixed phase when the data is at one level.
- It changes the phase by 180° for other level change.

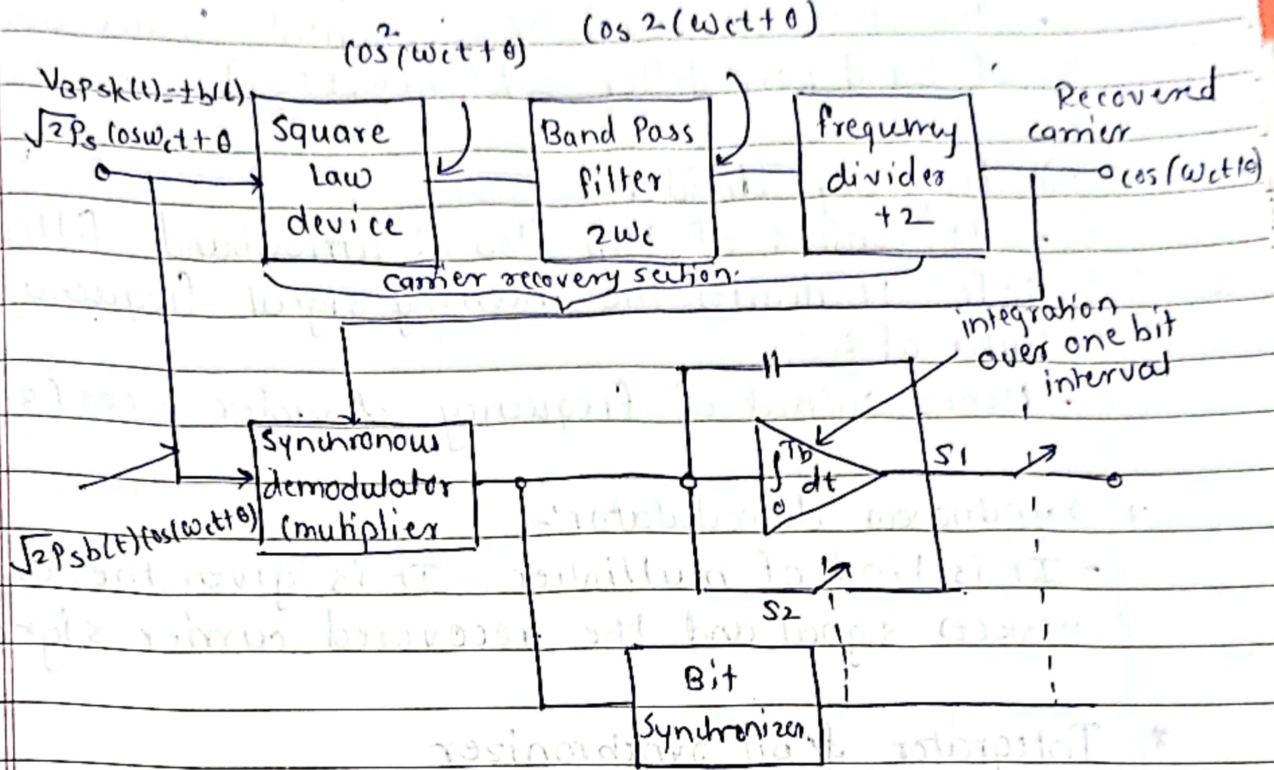
Mathematical eq^n:-

$$V_{BPSK}(t) = \sqrt{2P_s} \cdot \cos(\omega_c t)$$

$\searrow 0^\circ$ phase

$$V_{BPSK}(t) = \sqrt{2P_s} \cdot \cos(\omega_c t + \pi)$$

* BPSK Receiver:-



- The received signal at BPSK receiver is,

$$V_{BPSK}(t) = b(t)\sqrt{2P_s}\cdot\cos(\omega_c t+\theta)$$
$$= b(t)\sqrt{2P_s}\cdot\cos\left[\omega_c\left(t+\frac{\theta}{\omega_c}\right)\right]$$

θ is a fixed phase shift. It refers to time delay $\left(\frac{\theta}{\omega_c}\right)$

- The time delay (θ/ω_c) depends on majorly two factors
  i) Distance betn Transmitter & receiver

* Carrier Recovery Section:-
1) Square Law device:-
   - Input of square law device = $V_{BPSK}(t)$
   - It will square the received signal. Hence O/P of square law device
     $= \cos^2(\omega_c t+\theta)$
2) Band pass filter:-
   - The band pass filter used has the

Cut-off frequency = $2f_c$

- The filter removes DC component. Hence output of band pass filter = $[\cos^2(\omega_c t + \theta)]$

3) frequency divider:-

- It consists of flip-flop & narrowband filter tuned to f_c. It divides the incoming signal frequency by a factor of 2
- Hence output of frequency divider = $\cos(\omega_c t + \theta)$.

* Synchronous demodulator:-

- It is kind of multiplier. It is given the original BPSK(t) signal and the recovered carrier signal.

* Integrator & bit synchronizer

- Bit synchronizer is used to detect the begining & end of the bit interval