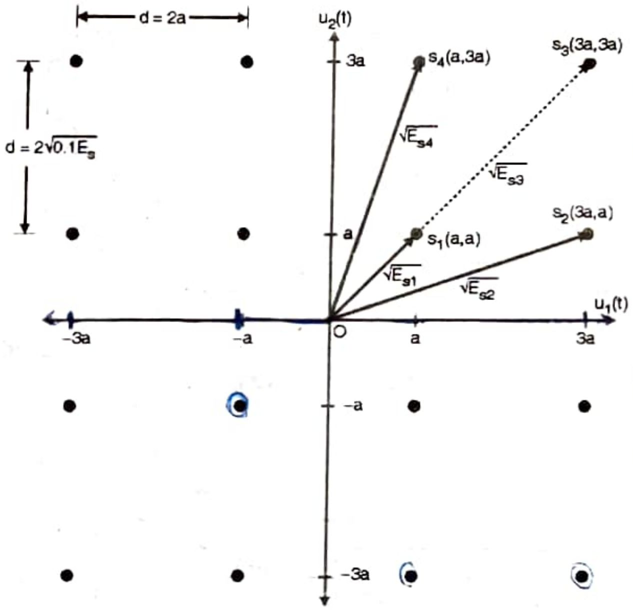
Chapter 4 : Digital Modulation-II

Q. 1 Draw signal space of 16-QAM system. **Dec. 16**

Ans. : Quadrature amplitude modulation

- The geometrical representation of 16 signals is shown in Fig. 4.1.



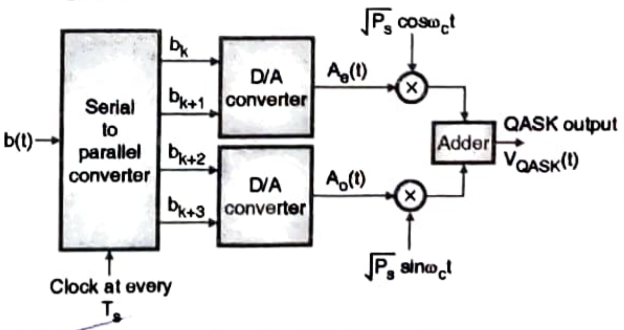
(E-402) Fig. 4.1 : Geometric representation of 16 signals in a QASK system (16 - QAM)

Q. 2 Explain M-ary QAM transmitter and receiver. **Dec. 19**

Ans. :

M-ary QAM transmitter :

- The block diagram of a QASK transmitter is shown in Fig. 4.2.



(E-405) Fig. 4.2 : QASK transmitter

Operation :

- The QASK signal shown in Fig. 4.2 can be mathematically represented as,

$$V_{QASK} = k_1\, a\, u_1(t) + k_2\, a\, u_2(t) \quad ...(1)$$

where k_1 and k_2 are each equal to ± 1 or ± 3.

- We know that, the basis functions are represented by,

$$u_1(t) = \sqrt{2/T_s}\cos\omega_c t$$

and $$u_2(t) = \sqrt{2/T_s}\sin\omega_c t \text{ and } a = \sqrt{0.1\,E_s}$$

- We can substitute these expressions into Equation (1) to get,

$$V_{QASK} = k_1 \times \sqrt{(0.2\,E_s/T_s)}\cos\omega_c t + k_2 \times \sqrt{(0.2\,E_s/T_s)}\sin\omega_c t \quad ...(2)$$

- But $E_s/T_s = P_s$ hence equation for QASK is given by,

$$V_{QASK} = k_1\sqrt{0.2\,P_s}\cos\omega_c t + k_2 \times \sqrt{0.2\,P_s}\sin\omega_c t \quad ...(3)$$

- The QASK generator is as shown in Fig. 4.2.
- The bit stream b (t) is applied to a serial to parallel converter operating on a clock which has a period of T_s sec. which is equal to the symbol duration.
- The bits b (t) are stored by the converter and then converted to the parallel form.
- The four bit symbol is $b_{k+3}\, b_{k+2}\, b_{k+1}\, b_k$.
- Out of these four bits, the first two bits are applied to the first D to A converter and the other two bits are applied to the second D to A converter.
- The output of the first D/A converter is $A_e(t)$ which is used to modulate the carrier $\sqrt{P_s}\cos\omega_c t$, whereas the output of the second D/A converter i.e.
- $A_o(t)$ is used to modulate the carrier $\sqrt{P_s}\sin\omega_c t$ with the help of the balanced modulators.
- The balance modulator outputs are added together to get the QASK output signal, which is expressed as follows :

$$V_{QASK}(t) = A_e(t) \times \sqrt{P_s}\cos\omega_c t + A_o(t) \times \sqrt{P_s}\sin\omega_c t \quad ...(4)$$

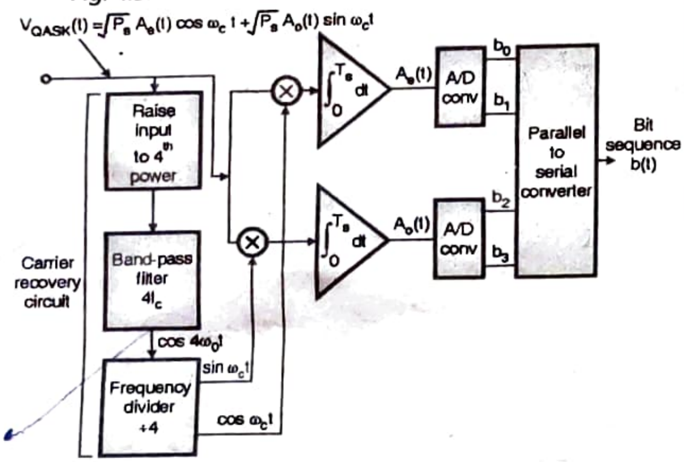
- Comparing Equations (4) and (3) we get,

$$A_e(t) \text{ and } A_o(t) = \pm\sqrt{0.2} \text{ or } \pm 3\sqrt{0.2} \quad ...(5)$$

depending on the input to D/A converter.

M-ary QAM receiver :

- The block diagram of QASK receiver is as shown in Fig. 4.3.



(E-406) **Fig. 4.3 : The QASK receiver**

Operation :

- Like QPSK this is also a synchronous demodulation which requires a locally generated set of quadrature (90° phase shifted) carriers i.e. $\cos \omega_c t$ and $\sin \omega_c t$.
- These quadrature carriers are recovered from the received QASK signal.
- The input QASK signal is first raised to the fourth power and using a bandpass filter with a centre frequency of $4 f_c$ alongwith a frequency divider (÷ 4), we can recover these quadrature carriers.
- Remember that the values of A_e and A_o are not constant and equal in the QASK system.
- Therefore it is not sure if we can really recover the quadrature carriers or not.
- Hence let us check whether we can really recover the carriers correctly.
- The input signal $V_{QASK}(t)$ is raised to fourth power as,

$$V^4_{QASK}(t) = P^2_s [A_e(t) \cos \omega_c t + A_o(t) \sin \omega_c t]^4$$

- This signal is then passed through a bandpass filter with a centre frequency $4 f_c$, therefore we neglect all the other terms except for those which have a frequency of $4 f_c$.

$$\therefore V^4_{QASK}(t) = \frac{P_s}{8} \left[A^4_e(t) + A^4_o(t) - 6 A^2_e(t) A^2_o(t)\right]$$

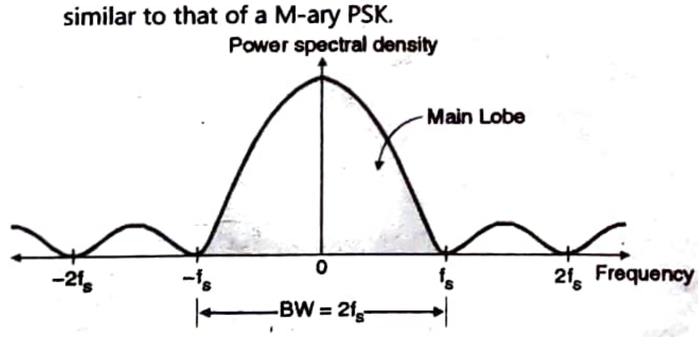
$$\cos 4\omega_c t + \frac{P_s}{2} \left[A_e(t) + A_o(t) [A^2_e(t) - A^2_o(t)]\right] \sin 4\omega_c t \quad ...(1)$$

- In Equation (1), the average value of the coefficient of $\cos 4\omega_c t$ is not zero but the average value of the coefficient of $\sin 4\omega_c t$ will be zero.
- Thus at the output of the bandpass filter and frequency divider combination we get the quadrature carrier components $\cos \omega_c t$ and $\sin \omega_c t$.
- Then two balanced modulators (multipliers) are used alongwith two integrators to recover the signals $A_e(t)$ and $A_o(t)$.
- Both the integrators integrate over one symbol period i.e. T_s.
- The symbol time synchronizer which is not shown in Fig. 4.3 is actually used alongwith each integrator.
- Finally the original bits are obtained from $A_e(t)$ and $A_o(t)$ by using two A to D converters.
- The outputs of the two A to D converters are then applied to a serial to parallel converter to obtain the sequence $b(t)$.

Q. 3 Draw the signal constellation of 16 QAM and find bandwidth requirement of M-QAM. May 18

Ans. :

- The spectrum of QASK is shown in Fig. 4.4 which is quite similar to that of a M-ary PSK.



(E-407) **Fig. 4.4 : Frequency spectrum of QASK**

- From the Fig. 4.4, it is evident that main lobe of the frequency spectrum extends from $-f_s$ to $+f_s$.
- Therefore the bandwidth of QASK is given by,

$$BW = f_s - (-f_s) = 2 f_s \quad ...(1)$$

$$= \frac{2}{T_s} \quad ... \text{As } f_s = \frac{1}{T_s}$$

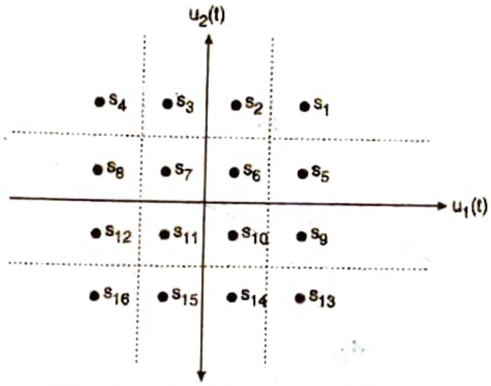
$$BW = \frac{2}{N T_b} \quad ... \text{As } T_s = N T_b$$

$$\therefore BW = \frac{2 f_b}{N} \quad ... \text{As } f_b = \frac{1}{T_b} \quad ...(2)$$

- Thus the bandwidth of QASK system is same as that of an M-ary PSK system.

Signal space of 16 QAM :

– The signal space diagram of 16 QAM is shown in Fig. 4.5.



(E-887) **Fig. 4.5 : Signal space of 16 QAM**

– Calculate the error probability for the symbol such as s_6 in Fig. 4.5 which is located at (a, a).

– This signal has the largest probability of error.

– The signals s_7, s_{10} and s_{11} also will have the largest error probability.

– The minimum distance is given by,

$$d = \sqrt{0.4\,E_s} = \sqrt{1.6\,E_b}$$

– And the error probability is given by,

$$P_e \le 4 \times \frac{1}{2}\,\text{erfc}\left[\frac{1.6\,E_b}{4\,N_0}\right]^{1/2} = 2\,\text{erfc}\left[0.4\,\frac{E_b}{N_0}\right]^{1/2}$$

Q. 4 Compare M-ary PSK and M-ary QAM. **May 19**

Ans. :

Comparison of M-ary PSK and M-ary QAM :

Table 4.1 : Comparison of 16 PSK with 16 QASK

Sr. No.	Parameter	16 PSK	16 QASK
1.	Type of modulation	M-ary PSK with M = 16	M-ary QAM with M = 16
2.	Location of signal points	On the circumference of a circle	4 points in each quadrant
3.	Distance between signal points	$d = 2\sqrt{4\,E_b}\,\sin(\pi / M)$	$d = 2\sqrt{0.15\,E_b}$
4.	Noise immunity	Poorer than 16 QASK	Better than 16 PSK
5.	Number of symbols	M = 16	M = 16
6.	Number of bits per symbol	N = 4	N = 4
7.	Detection method	Coherent	Coherent
8.	Symbol duration	$T_s = 4\,T_b$	$T_s = 4\,T_b$
9.	Bandwidth	$B = \frac{2\,f_b}{N} = f_b / 2$	$f_b / 2$
10.	System complexity	Less than 16 QASK	More than 16 PSK

Q. 5 Sketch the waveforms of MSK for the given bit stream 11001001. **May 15**

Ans. :

– The given bit stream : 11001001

$$C_H(t) = \frac{b_o(t) + b_e(t)}{2}$$

$$C_L(t) = \frac{b_o(t) - b_e(t)}{2}$$

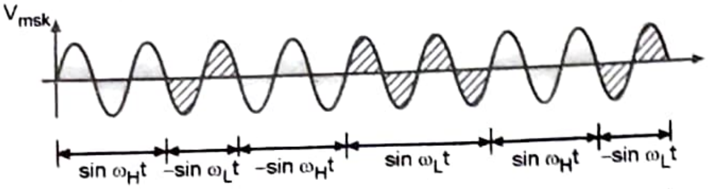
$$V_{MSK} = \sqrt{2\,P_s}\;C_H(t)\sin\omega_H(t) + \sqrt{2\,P_s}\;C_L(t)\sin\omega_L t.$$

– Prepare a table as follows to obtain V_{MSK} signal.

(E-2104) **Table 4.2**

Bit number	1	2	3	4	5	6	7	8
Bit sequence	1	1	0	0	1	0	0	1
$b_o(t)$	1	1	–1	–1	1	1	–1	–1
$b_e(t)$	–	1	1	–1	–1	–1	–1	1
$C_H(t)$	–	1	0	–1	0	0	–1	0
$C_L(t)$	–	0	–1	0	1	1	0	–1
V_{MSK}	–	$\sin\omega_H t$	$-\sin\omega_L t$	$-\sin\omega_H t$	$\sin\omega_L t$	$\sin\omega_L t$	$-\sin\omega_H t$	$-\sin\omega_L t$

- The MSK waveform is as shown in Fig. 4.6.



(E-1609) **Fig. 4.6 : MSK Waveform**

Chapter 5 : Pulse Shaping

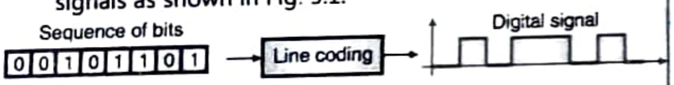
Q. 1 What are line codes ? Explain need of line coding. State its properties.

May 12. Dec. 13. May 15. May 18

Ans. :

Definition and need :

- The line coding is defined as the process of converting binary data, a sequence of bits to a digital signal.
- The digital data such as text, numbers, graphical images, audio and video are stored in computer memory in the form of sequences of bits.
- Line coding converts these sequences into digital signals as shown in Fig. 5.1.



(L-255) **Fig. 5.1 Line coding**

- Line codes are also called as **digital PAM formats.**
- There are various techniques used to convert the analog signal to digital signal.
- However, it is also possible to obtain digital data from the sources such as computers.
- The information from such a source is inherently discrete in nature.
- If such a discrete signal is transmitted over a band-limited channel, then the signal gets dispersed.
- That means the pulses spread out and overlap each other to cause distortion.
- Such a distortion is called as inter symbol interference (ISI).
- In order to avoid this we should not transmit the discrete signal as it is on the transmission medium.
- Instead this data is first converted into a PAM format or line code which is compatible with the base band channel and then transmitted over a base band communication channel.
- The various pulse formats used as per requirement are also called as **line codes.**

Properties of line codes :

- Following are some of the important properties of PAM signals (Line codes) :

1. No DC Component :

- All the cable systems and other communication systems, do not allow transmission of a dc signal over them.
- Therefore, the line signal must have a zero average (dc) value.
- NRZ bipolar formats usually satisfy this requirement.
- For this reason, long strings of element sequences having same polarity should not be transmitted.

2. Self clocking (synchronizing) capability :

- Any digital communication system needs symbol or bit synchronization.
- To ensure synchronization at the receiver the line code waveform must undergo a sufficient number of zero crossings. That means the waveform must always undergo transitions after regular intervals.
- This is known as the inherent synchronizing or clocking feature.
- Some of the codes such as the Manchester code have this feature inherently.

3. Bandwidth compression :

- The bandwidth of a line code should be as small as possible.
- The multilevel codes need less bandwidth as compared to the other codes. This happens due to effective utilization of bandwidth.

4. Differential encoding :

- The differential encoding is useful for those communication systems where the transmitted waveform sometimes experiences an inversion.

- In differential encoding, the polarity of encoded waveform is inverted without affecting the data detection.

5. Noise immunity :

- The selected line code should have a very high noise immunity (ability to minimize effects of noise).
- This is necessary to have minimum number of errors introduced due to noise.
- The NRZ formats have better noise immunity than that of the unipolar RZ format.

6. Minimum crosstalk :

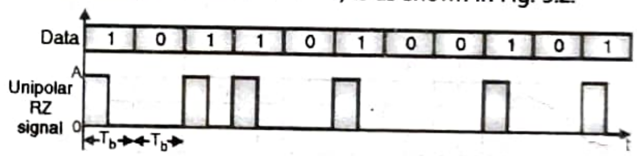
- The crosstalk between the adjacent channels that are being transmitted should be minimized.
- To achieve this the amount of energy in the signal at low frequencies should be small.

Q. 2 Explain various data formats. **May 15**

Ans. :

Unipolar RZ Line Code :

- The return to zero (RZ) unipolar line code for the data stream (1 0 1 1 0 1 0 0 1 0 1) is as shown in Fig. 5.2.



(L-262) **Fig. 5.2 : Unipolar RZ line code**

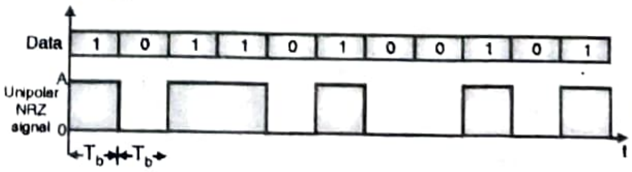
- The duration of each data bit (either 0 or 1) is equal to T_b as shown in Table 5.1.

(E-2095) **Table 5.1 : Unipolar RZ code**

Data bit	Amplitude	Duration
0	0	T_b
1	+A	$T_b/2$

Unipolar NRZ Format :

- A non-return to zero (NRZ) unipolar line code for the data stream (1 0 1 1 0 1 0 0 1 0 1) is as shown in Fig. 5.3.



(L-263) **Fig. 5.3 : Unipolar NRZ line code**

- In this format a logic "1" is represented by a pulse of full bit duration T_b and amplitude + A while a logic "0" is represented by an off pulse or zero amplitude of full bit duration, as shown in Table 5.2.

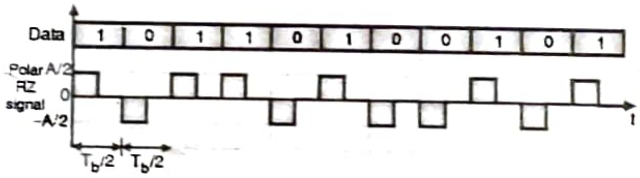
(E-2096) **Table 5.2 : Unipolar NRZ code**

Data bit	Amplitude	Duration
0	0	T_b
1	+A	T_b

- During the on time, the pulse does not return to zero after half bit period.
- Therefore the name NRZ format.
- As the pulses have either + A or 0 amplitude it is called as a unipolar format.
- Internal computer waveforms are usually of this type.
- Due to the unipolar nature, the unipolar NRZ format also will have a nonzero average (dc) value which does not carry any information.
- Due to longer pulse duration, the NRZ pulses carry more "energy" than the RZ pulses.
- But they need synchronization at the receiver as there is no separation between the adjacent pulses.

Polar RZ Line Code :

- The polar RZ format is as shown in Fig. 5.4. It shows that opposite polarity pulses of amplitude ± "A/2" are used to represent logic "1" and "0".



(L-264) **Fig. 5.4 : Polar RZ format**

- That means a logic 1 is represented by a pulse of amplitude "A/2" and duration "$T_b/2$", whereas, a logic 0 is represented by a pulse of amplitude "–A/2" and duration "$T_b/2$" as shown in Table 5.3.

(E-2097) **Table 5.3 : Polar RZ code**

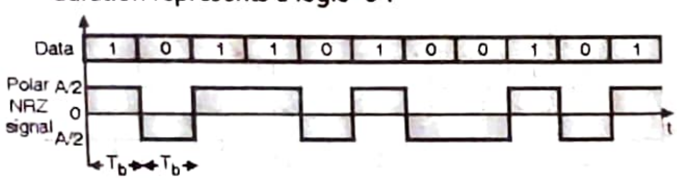
Data bit	Amplitude	Duration
0	+A/2	$T_b/2$
1	–A/2	$T_b/2$

- Therefore it is called as a "polar" format. As the pulses return to zero after half the bit duration "$T_b/2$" this format is a RZ format.
- Due to shorter pulse duration, and smaller amplitudes the polar RZ pulses carry much less energy.

- Due to the positive and negative amplitudes of the pulses, there is a possibility of reducing the dc component to zero.
- However, this line code will have a non-zero dc value for long strings of 0s and 1s.
- Due to the separation between the adjacent pulses, they can provide self synchronization at the receiver.

Polar NRZ Line Code :

- In the polar NRZ format, as shown in Fig. 5.5 a pulse of amplitude "+ A/2" of duration T_b is used to represent a logic "1" and a pulse of amplitude "– A/2" of the same duration represents a logic "0".



(L-265) **Fig. 5.5 : Polar NRZ code**

- This is as shown in Table 5.4.

(E-2098) **Table 5.4 : Polar NRZ code**

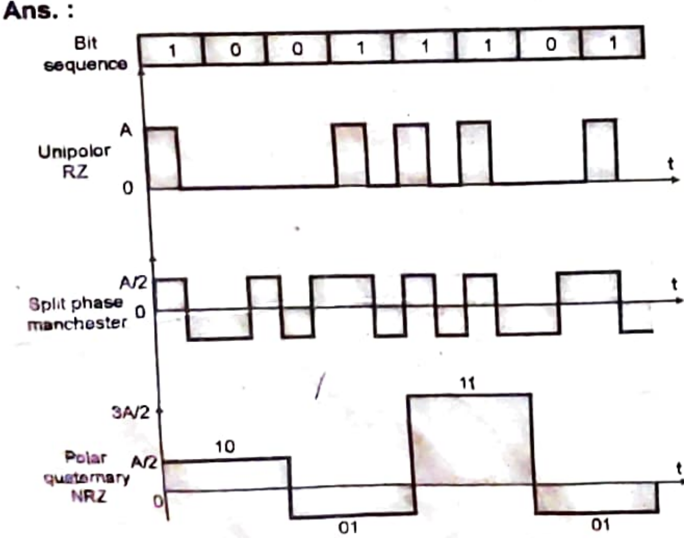
Data bit	Amplitude	Duration
0	+A/2	T_b
1	–A/2	T_b

- Unlike the unipolar waveform, a polar waveform has no dc component if the 0s and 1s in the input data occur in equal proportion.

Q. 3 **Represent the data 10011101 using following data formats :**

1. **Unipolar RZ.** 2. **Split phase Manchester.**
3. **M-ary format for M = 4.** May 19

Ans. :



(E-2047) **Fig. 5.6 : Required line codes**

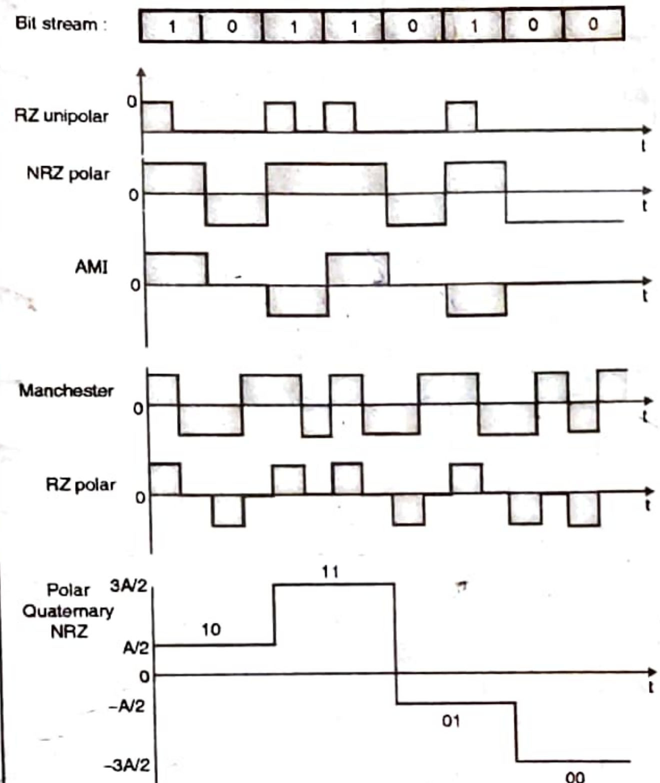
Q. 4 **Draw the line code formats for 10110100 :**

1. **RZ unipolar** 2. **NRZ polar**
3. **AMI** 4. **Manchester**
5. **RZ polar** 6. **Polar Quaternary (NRZ)**

May 03, May 06, Aug. 17

Ans. :

- The required line code formats are as shown in Fig. 5.7.



(E-807) **Fig. 5.7 : Required line codes**

Q. 5 **What absolute bandwidth is required to transmit an information rate of 8 kbps using 64 level baseband signaling over a raised cosine channel with roll off factor of 40 %.** Aug. 17

Ans. :

Given : $\alpha = 0.4$, $Q = 64$,

Information rate $= Nf_s = 8 \times 10^3$ bits/sec.

To find : B.W. of raised cosine channel.

Step 1 : Find the BW of an ideal channel :

- The relation between the number of levels Q and the number of bits per sample (N) is as follows :

$$Q = 2^N$$

$$\therefore\ 64 = 2^N$$

$$\therefore\ N = 6 \quad ...(1)$$

- The information rate $= 8 \times 10^3$ ...given

$$\therefore \; N f_s = 8 \times 10^3 \text{ bps}$$

– Therefore the B.W. of an ideal channel is given by,

$$B = \frac{1}{2} N f_s = 4 \text{ kbps} \qquad ...(2)$$

Step 2 : Find the BW of raised cosine channel :

– The BW of a raised cosine channel is given by,

$$B_T = B(1 + \alpha)$$

Where α = Rolling factor = 0.4

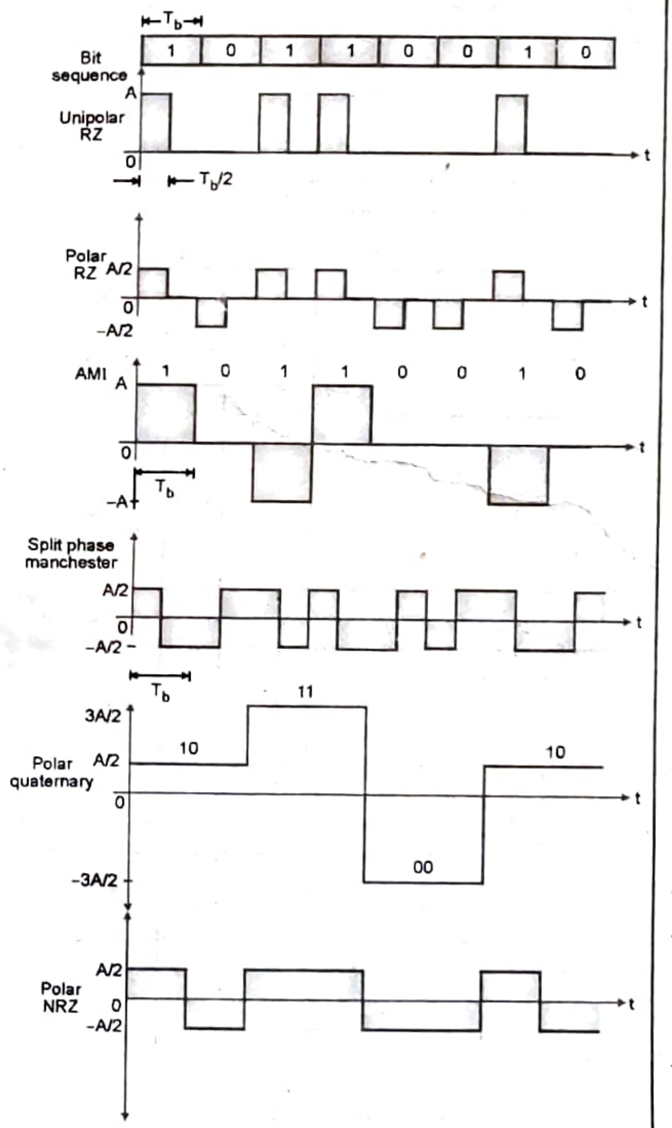
$$\therefore \; B_T = 4(1 + 0.4) = 5.6 \text{ kbps} \qquad \textbf{...Ans.}$$

Q. 6 Draw the following line codes for bit stream 10110010 :

1.	**Polar RZ**	**4.**	**AMI**
2.	**Polar NRZ**	**5.**	**Polar quaternary**
3.	**Manchester**	**6.**	**Unipolar RZ**

Dec. 19

Ans. :



(ET-10) Fig. 5.8 : Required line codes

Q. 7 What is intersymbol interference ? Explain its causes and remedies to avoid it.

May 12, Dec. 12, May 13, Dec. 13, Dec. 16

Ans. :

Definition :

– In a communication system when the data is being transmitted in the form of pulses (bits), the output produced at the receiver due to the other bits or symbols interferes with the output produced by the desired bit.

– This is called as inter symbol interference (ISI).

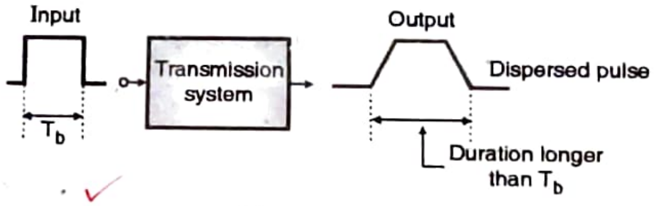
– The inter symbol interference will introduce errors in the detected signal at the receiver.

Causes of inter symbol interference :

– The ISI results because the overall frequency response of the system is never perfect and pulse spreading is bound to take place.

– When a short pulse of duration T_b seconds is transmitted through a band limited transmission system, then various frequency components present in the input pulse are **differentially attenuated** and more importantly **differentially delayed** by the system.

– Due to this the pulse appearing at the output of the system will be "dispersed" over an interval which is longer than "T_b" seconds as shown in Fig. 5.9.



(E-298) Fig. 5.9 : Cause of ISI

– Due to this dispersion, the adjacent symbols will interfere with each in time domain other when transmitted over the communication channel.

– This will result in the inter symbol interference (ISI).

– The transmitted pulse of duration T_b seconds and the dispersed pulse of duration more than T_b seconds are shown in Fig. 5.9.

– The four important causes for ISI are as follows :

1. Timing inaccuracies.
2. Insufficient bandwidth.

3. Amplitude distortion.
4. Phase distortion.

1. **Timing inaccuracies :**

- The ISI will take place if the transmitter rate of transmission is not same as the ringing frequency of the given channel.

2. **Insufficient bandwidth :**

- If the transmission rate is less than the channel bandwidth then there is a very small possibility of timing error.
- But if the channel bandwidth is reduced, then the possibility of timing error will increase and the possibility of ISI also will increase.

3. **Amplitude distortion :**

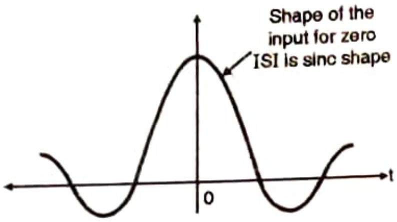
- Generally filters are used in the communication systems in order to band limit the signals and reduce the noise.
- But the frequency response of the communication channels cannot be accurately predicted.
- When the frequency characteristics of a communication channel differs from the expected one, the pulse distortion is likely to take place.
- The pulse distortion results in reduction of the peaks of the pulses i.e. amplitude distortion.
- In order to compensate for this, we have to use the amplitude equalization.

4. **Phase distortion :**

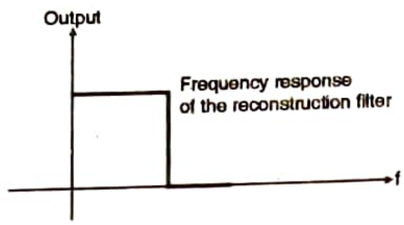
- If various frequency components in the input pulse undergo different amounts of time delay while travelling through the channel, then the phase distortion is bound to take place.
- This will cause the ISI. Special delay equalizers are required to be used to reduce the phase distortion and the associated ISI.

Remedy to reduce the ISI :

- It has been proved that the function which produces a zero inter symbol interference is a "sinc function".
- Thus instead of a rectangular pulse if we transmit a sinc pulse then the ISI can be reduced to zero.
- Using the since pulse for transmission is known as **"Nyquist Pulse Shaping"**.
- The sinc pulse transmitted to have a zero ISI is shown in Fig. 5.10(a).



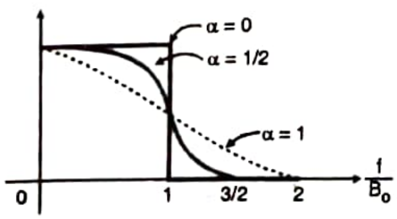
(a) Ideal pulse shape for zero ISI



(b) Frequency response of the filter

(E-299) **Fig. 5.10**

- We know that Fourier transform of a sinc pulse is a rectangular function.
- Therefore to preserve all the frequency components, the frequency response of the filter must be exactly flat in the pass band and zero in the attenuation band as shown in Fig. 5.10(b).
- This type of filter is practically not available.
- Therefore practically the frequency response of the filter is modified as shown in Fig. 5.11 with different roll off factors "α" to obtain the practically achievable filter response curves.



(E-300) **Fig. 5.11 : Practical filter characteristics**

The process in which narrowband signal is converted to wideband signal is called as spreading of spectrum

Chapter 6 : Spread Spectrum Modulation

Q. 1 What is spread spectrum technique ? How are they classified ?

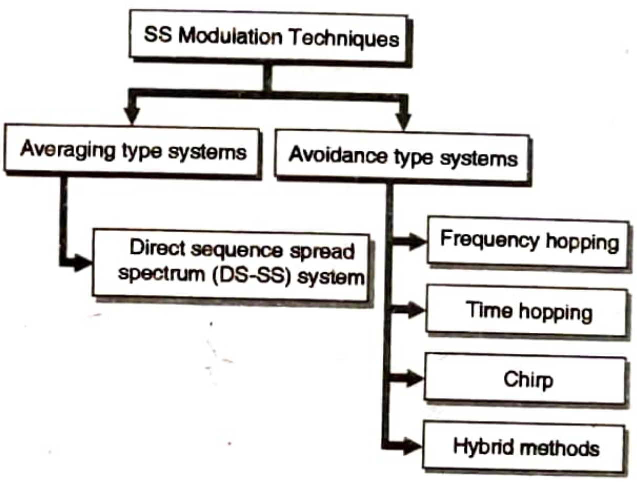
May 08, May 11, May 16, Dec. 18

Ans. :

Definition of spread spectrum :

- In the telecommunication or radio communication spread spectrum techniques are the methods by which a signal generated with a particular bandwidth is spread deliberately in the frequency domain to produce a signal with much wider bandwidth.

Classification of the spread spectrum modulation techniques :



(E-470) **Fig. 6.1 : Classification of spread spectrum technique**

Q. 2 Draw the block diagram of spread spectrum digital communication and explain the various blocks.

Dec. 17

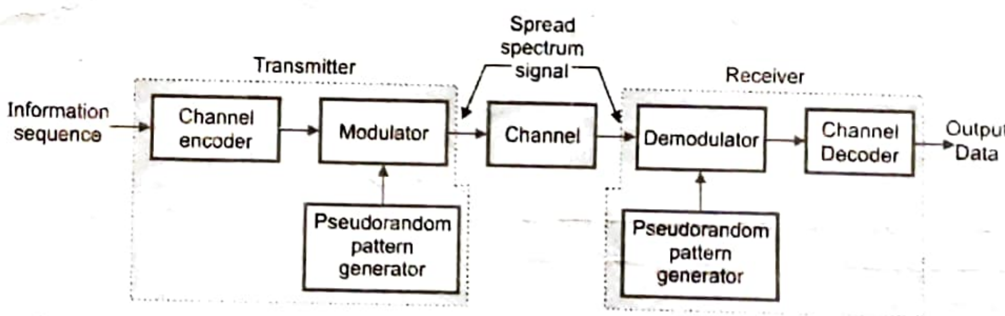
Ans. :

Block diagram :

- The block diagram shown in Fig. 6.2 illustrates the basic elements of a spread spectrum digital communication system.
- It consists of a transmitter, a communication channel and a receiver.

Operation of transmitter :

- The information sequence at the input of the system is a binary information sequence.
- The same signal is recovered at the output of the system as output data signal.
- In addition to these basic building blocks of a digital communication system, two additional blocks called "pseudo-random pattern generator" are used as shown in Fig. 6.2.



(E-471) **Fig. 6.2 : Model of spread spectrum digital communication system**

- One of them is connected to the modulator on the transmitter side whereas the other is connected to the demodulator on the receiving side. Both these generators are identical to each other.
- These generators generate a pseudorandom or pseudonoise (PN) binary sequence.
- It is impressed on the transmitted signal at the modulator.
- Thus the modulated signal along with the pseudorandom sequence travels over the communication channel.
- This sequence spreads the signal randomly over a wide frequency band.
- Thus the output of the modulated signal is a spread spectrum signal.

PN code is the deterministic & periodic binary sequence generated by feedback logic circuit It provides noise like waveform at the output

Operation of the receiver :

- The pseudorandom sequence is removed from the received signal, by the other "pseudorandom generator" operating at the receiver.
- Thus the pseudorandom pattern generators operate in synchronization with each other.
- The synchronization between these pattern generators is achieved before the beginning of the signal transmission.
- This is done by transmitting a fixed pseudorandom bit pattern which a receiver can recognize even in presence of interference.
- Once this synchronization is established, it is possible to begin the transmission.
- Thus in the spread spectrum receiver, the receiver can demodulate the transmitted signal if and only if a known pseudo-noise sequence has been transmitted along with the information signal.

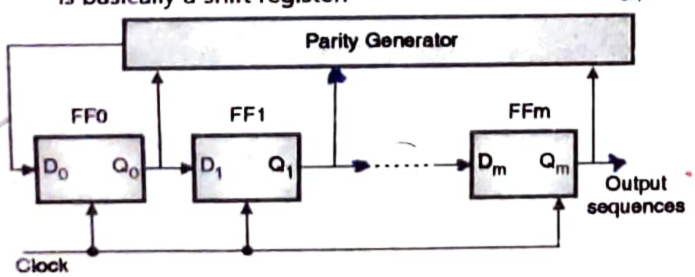
Q. 3 Write a short note on : PN sequence generator.

Dec. 08, Dec. 12, May 19

Ans. :

PN sequence generator :

- The PN sequence generator is as shown in Fig. 6.3. This is basically a shift register.



(E-472) Fig. 6.3 : A pseudo-random sequence generator

- Type D flip-flops are being connected such that the D input to a flip-flop is connected to the Q output of the previous flip-flop.
- The input D_0 of the first D flip-flop has been connected to the output of the parity generator. A parity generator generally consists of exclusive - OR gates.
- Therefore the output of the parity generator is equal to zero when an even number of its inputs Q_0, Q_1 ... are at 0 level. And its output is equal to 1 when an odd number of its inputs are at logic 1 level.
- The inputs to the parity generator are the outputs from the flip-flops i.e. Q_0, Q_1 ... Q_m.
- However it is not necessary to connect all the Q outputs to the input of parity generator.

Output of Parity Generator	Status of Inputs
Logic "0"	Even number of inputs are at logic "0"
Logic "1"	Odd number of inputs are at logic "1"

- The character generated by a PN sequence generator (Q_0, Q_1 ... Q_m) depends on the number of flip-flops used (m) and on the selection of which flip-flop outputs are connected to the inputs of parity generator.
- The state of each flip-flop changes and gets shifted to the next flip-flop corresponding to each pulse of the clock.
- A shift register of "m" flip-flops will have 2^m number of states; i.e.

  $Q_0\, Q_1 \ldots Q_{m-1} = 000 \ldots 0$ to $Q_0\, Q_1 \ldots Q_{m-1} = 111 \ldots 1$.

  Thus the output sequence will repeat itself after every 2^m bits.
- The PN generator of Fig. 6.3 cannot generate a truly random sequence because this structure is a deterministic structure. This is the reason why, the sequence repeats itself.
- In order to make the random sequence "look like" truly random, its length should be sufficiently large i.e. a large number of flip flops should be included. Typically upto 2000 flip flops are used.
- The maximum length of the sequence will be $2^m - 1$. This is because the state 000...0 is not to be considered.
- If all zero state is allowed to exist then the EX-OR gates used in the parity generator will produce a zero output all the time. To avoid this the all zero state should be excluded.

Q. 4 What is PN sequence ? Explain its properties with 4-stage shift register.

Dec. 12, Dec. 14, May 15, Dec. 16, Dec. 17, Dec. 18, Dec. 19

Ans. :

PN sequence :

- A Pseudo-Noise (PN) sequence is defined as a coded sequence of 1s and 0s with certain auto-correlation properties.

Properties of maximum-length sequences :

– Maximum length sequences have many properties possessed by a truly random sequence. Some of the important properties are :

1. Balance property.
2. Run property.
3. Correlation property.

Balance property :

– In each period of a maximum-length sequence, i.e. N the number of 1s is always one more than the number of 0s.

– This is called as the balance property. So if there are four 0's then there will be five 1's.

Run property :

– This property states that among the "runs" of 1s and 0s in each period of a maximum length sequence, one half the runs of a each kind are of length one (only one 0 or only one 1), one-fourth are of length two, (i.e.00 or 11) one eighth are of length three, and so on.

– In this statement, the word "run" means a subsequence of identical symbols (1s or 0's) within one period of the sequence. That means 000, 111 or 00, 11 or only 0, 1.... etc.

– And the length subsequence is the length of the run. The total number of runs will be (m + 1) / 2 if an m stage feedback shift register is being used.

Correlation property :

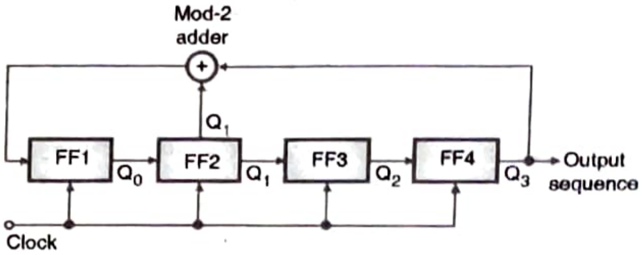
– This property states that the auto-correlation of a maximum length sequence is periodic and has two possible values (i.e. it is binary valued).

Q. 5 **A PN sequence is generated using a feedback shift register of length 4. Find the generated output sequence if the initial contents of the shift register are 1000. If the chip rate is 10^7 chips/sec., caloulate the chip and PN sequence duration and period of output sequence. Draw its scheme arrangement.**

May 08, May 09, May 11, Dec. 11, Dec. 15

Ans. :

– One of the possible schematic diagram of the PN generator is as shown in Fig. 6.4.



(E-490) **Fig. 6.4 : A four stage shift register to generate PN sequence**

– The PN sequence generated by the above generator is shown in Table 6.1.

(E-575) **Table 6.1 : PN sequence generated by the generator shown in Fig. 6.4**

Clock Pulse Number	Shift register outputs Q_3	Q_2	Q_1	Q_0	EX-OR gate output $Q_3 \oplus Q_2$	PN sequence Q_3
0	0	0	0	1	$0 \oplus 0 = 0$	0
1	0	0	1	0	$0 \oplus 1 = 1$	0
2	0	1	0	1	$0 \oplus 0 = 0$	0
3	1	0	1	0	$1 \oplus 1 = 0$	1
4	0	1	0	0	$0 \oplus 0 = 0$	0
5	1	0	0	0	$1 \oplus 0 = 1$	1
6	0	0	0	1	$0 \oplus 0 = 0$	0
7	0	0	1	0	$0 \oplus 1 = 1$	0
8	0	1	0	1	$0 \oplus 0 = 0$	0
9	1	0	1	0	$1 \oplus 1 = 0$	1
10	0	1	0	0	$0 \oplus 0 = 0$	0
11	1	0	0	0	$1 \oplus 0 = 1$	1
12	0	0	0	1	$0 \oplus 0 = 0$	0
13	0	0	1	0	$0 \oplus 1 = 1$	0
14	0	1	0	1	$0 \oplus 0 = 0$	0
15	1	0	1	0	$1 \oplus 1 = 0$	1
The sequence repeats after this						

Chip duration :

– The chip rate $R_c = 1 \times 10^7$ chips/sec. Hence the chip duration T_c is given by,

$$T_c = \frac{1}{R_c} = \frac{1}{1 \times 10^7} = 0.1\ \mu\text{sec.} \quad \text{..Ans.}$$

Length of the PN sequence :

– The length of the PN sequence is given by,

$$N = 2^m - 1$$

But m = 4 $\therefore N = 2^4 - 1 = 15$ digits ...**Ans.**

Duration of the PN sequence :

– The duration of the PN sequence is given by,

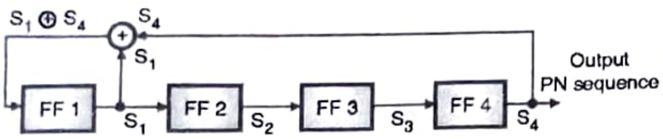
$$T_b = N\,T_c$$

$$= 15 \times 0.1\ \mu s = 1.5\ \mu s \quad \text{...Ans.}$$

Q. 6 For a 4 stage shift with feedback combination of (4, 1) demonstrate the balance property and run property of PN sequence, also calculate and plot the autocorrelation function of PN sequence produced by this shift register. Dec. 07, May 15

Ans. :

Step 1 : Draw the 4 stage shift register :



(E-1382) **Fig. 6.5(a) : 4 stage shift register**

Step 2 : Obtain the PN sequence :

- Assume that initially $S_1\ S_2\ S_3\ S_4$ = 1000. Table 6.1(a) shows the PN sequence generated by this circuit.

(E-1383) **Table 6.1(a)**

Clock number	Shift register state S_4	S_3	S_2	S_1	MOD2 adder output - $S_1 \oplus S_4$	PN sequence S_4
0	0	0	0	1	$0 \oplus 1 = 1$	0
1	0	0	1	1	$0 \oplus 1 = 1$	0
2	0	1	1	1	1	0
3	1	1	1	1	0	1
4	1	1	1	0	1	1
5	1	1	0	1	0	1
6	1	0	1	0	1	1
7	0	1	0	1	1	0
8	1	0	1	1	0	1
9	0	1	1	0	0	0
10	1	1	0	0	1	1
11	1	0	0	1	0	1
12	0	0	1	0	0	0
13	0	1	0	0	0	0
14	1	0	0	0	1	1
15	0	0	0	1	1	0

Repeat after this

$\therefore$ PN sequence = [000 1111 01 01 1001]

Step 3 : Properties of PN sequence :

1. **Balance property :**

- In one period of PN sequence there are seven 0s and eight 1s.
- As number of 1s is greater than number of 0s the balance property is satisfied.

2. **Run property :**

- As per this property there should be 2^{m-1} runs where m = Number of stages (4 here).

$\therefore$ Number of runs = $2^{4-1} = 2^3 = 8$

- In the actual PN sequence generated the runs are identified as follows :

$$\text{PN sequence : } \underbrace{0\ 0\ 0}_{1}\ \underbrace{1\ 1\ 1\ 1}_{2}\ \underbrace{0}_{3}\ \underbrace{1}_{4}\ \underbrace{0}_{5}\ \underbrace{1\ 1}_{6}\ \underbrace{0\ 0}_{7}\ \underbrace{1}_{8}$$

Run : 1 2 3 4 5 6 7 8

- Thus the run property also is verified.

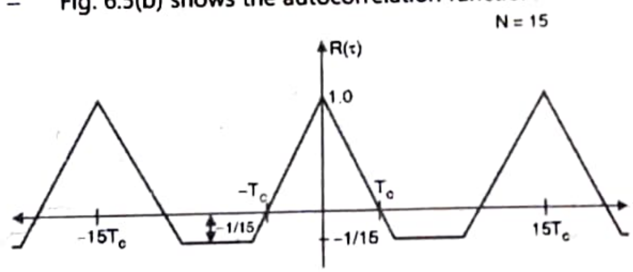
Step 4 : Autocorrelation function :

$$R(\tau) = \begin{cases} 1 - \dfrac{N+1}{N T_c} |\tau| & \text{for } |\tau| < T_c \\ -\dfrac{1}{N} & \text{elsewhere} \end{cases}$$

Here $N = 2^m - 1 = 2^4 - 1 = 15$

$$\therefore \quad R(\tau) = \begin{cases} 1 - \dfrac{16|\tau|}{15 T_c} & \text{...for } |\tau| < T_c \\ -\dfrac{1}{15} & \text{...elsewhere} \end{cases}$$

- Fig. 6.5(b) shows the autocorrelation function.



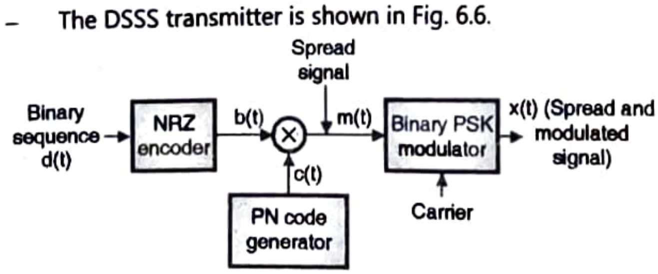
(E-1388) **Fig. 6.5(b) : Auto-correlation of a PN sequence**

Q. 7 Explain direct sequence spread - spectrum baseband transmitter and receiver with neat waveform. May 12, Dec. 14, May 18, May 19, Dec. 19

Ans. :

1. **DSSS transmitter :**

- The DSSS transmitter is shown in Fig. 6.6.



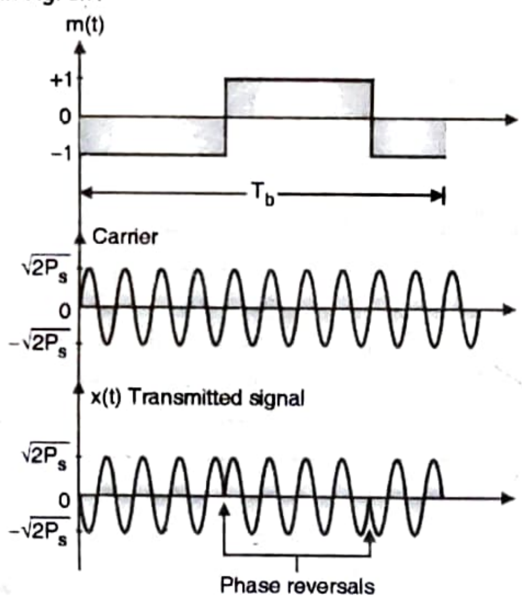
(E-480) **Fig. 6.6 : DSSS transmitter**

Operation :

- The binary sequence d (t) is converted into NRZ signal b (t) by applying d(t) to the NRZ encoder.
- The NRZ signal b (t) at the output of the NRZ encoder is then used to modulate the PN sequence c (t) generated by the PN code generator.
- The transmitter of Fig. 6.6 uses two stages of modulation.
- The first stage uses a product modulator or multiplier with b (t) and c (t) as its inputs and the second stage consists of a BPSK modulator.
- The modulated signal at the output of the product modulator i.e. m (t) is the spread version of the original input and it is used to modulate the carrier for BPSK modulation.
- Thus the BPSK modulator will modulate the spread signal (SS) to produce a DS-SS BPSK signal.
- The transmitted signal x (t) is thus a direct sequence spread BPSK i.e. DS-BPSK signal.

Waveforms :

- The waveforms for the DS-BPSK transmitter are shown in Fig. 6.7.



(E-481) Fig. 6.7 : Waveforms of DS-BPSK transmitter

- The carrier signal applied to the BPSK modulator is given by,

$$V_{carrier}(t) = \sqrt{2 P_s} \sin (2 \pi f_c t) \quad ...(1)$$

- The output of BPSK modulator i.e. x(t) is transmitted. x(t) is given mathematically as -

$$x(t) = m(t) \times V_{carrier}(t)$$

$$= m(t) \times \sqrt{2 P_s} \sin (2 \pi f_c t)$$

But $m(t) = \pm 1$

$$\therefore \; x(t) = \pm \sqrt{2 P_s} \sin (2 \pi f_c t) \quad ...(2)$$

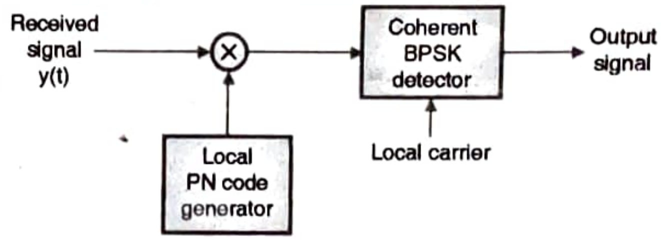
$$= + \sqrt{2 P_s} \sin (2 \pi f_c t) \quad \text{...Positive } m(t)$$

$$= - \sqrt{2 P_s} \sin (2 \pi f_c t) \quad \text{...Negative } m(t)$$

- Thus the phase shift of x(t) is 0° corresponding to a positive m (t) and it is 180° corresponding to a negative m (t).

2. DS-SS Receiver :

- The DS-BPSK receiver is as shown in Fig. 6.8.



(E-482) Fig. 6.8 : The DS-SS receiver

Operation :

- At the receiver we have to generate the replica of the original PN-sequence used at the transmitter.
- The received signal y(t) and the locally generated replica of the PN - sequence are applied to a multiplier.
- This is the first stage of multiplication. The multiplier performs the de-spreading operation.
- Output of multiplier is then applied to a coherent BPSK detector with a locally generated synchronous carrier applied to it.
- At the output of the coherent BPSK detector we get back the original data sequence i.e. d(t).

Q. 8 Explain the performance parameters of DS-SS system. Dec. 06, Dec. 18, Dec. 19

Ans. :

Performance Parameters of a DS-SS System :

1. Processing gain.
2. Probability of error.
3. Jamming margin.

1. Processing gain :

- The processing gain of a DS-SS system represents the extent of spreading in the frequency domain applied to an unspread signal.
- The processing gain PG as the ratio of the bandwidth of the spread spectrum signal to the bandwidth of the unspread signal.

$\therefore$ Processing gain $= \dfrac{\text{BW of spread spectrum signal}}{\text{BW of unspread signal}}$

2. Probability of error :

– The error probability P_e of a coherent BPSK system is given by,

$$P_e = \frac{1}{2}\,\text{erfc}\sqrt{E_b / N_0} \qquad \text{...(1)}$$

Where E_b = Energy per bit

and $N_0/2$ = Power spectral density of white noise.

– In a direct-sequence spread binary PSK system the interference may be treated as a wideband noise signal with a power spectral density of $N_0/2$.

$$\therefore \quad \frac{N_0}{2} = \frac{J\,T_c}{2} \qquad \text{...(2)}$$

OR $N_0 = J\,T_c$ ...(3)

where J = Average interference power

and T_c = Chip duration

– Substituting this expression for N_0 into Equation (2) we get error probability as,

$$P_e = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{J\,T_c}} \qquad \text{...(4)}$$

– This is the required expression for the error probability.

– The complementary error function is a monotonically decreasing function.

– Hence the error probability decreases as the value of $\sqrt{E_b / J\,T_c}$ increases.

– But P_e increases if $\sqrt{E_b / J\,T_c}$ decreases due to increase in the average interference power J or the chip period T_c.

3. Jamming Margin :

– Since the energy per bit i.e. $E_b = P_s\,T_b$ where, P_s is the average signal power and T_b is the bit duration, we can express the bit energy to noise density ratio as follows :

$$\frac{E_b}{N_0} = \frac{P_s\,T_b}{N_0} \qquad \text{...(5)}$$

– But $N_0 = J\,T_c$ ...Referring to Equation (3)

$$\therefore \quad \frac{E_b}{N_0} = \frac{P_s\,T_b}{J\,T_c}$$

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P_s}{J}\right) \qquad \text{...(6)}$$

– We can write the process gain PG as,

$$PG = \frac{T_b}{T_c}$$

$$\therefore \quad \frac{E_b}{N_0} = PG\left(\frac{P_s}{J}\right)$$

$$\therefore \quad \frac{J}{P_s} = \frac{PG}{E_b / N_0} \qquad \text{...(7)}$$

– The ratio J/P_s is called as the **jamming margin**. Hence the jamming margin is defined as the ratio of average interference power J and the signal power P_s.

– The jamming margin has an ideal value equal to zero and practically it should be as small as possible.

– If the jamming margin and the process gain both are expressed in dB then,

$$(\text{Jamming margin})\,\text{dB} = (\text{Processing Gain})\,\text{dB} - 10\log_{10}\,[\,E_b / N_0\,]_{min} \qquad \text{...(8)}$$

– Where $(E_b/N_0)_{min}$ is the minimum bit energy to noise density ratio needed to support a prescribed average error probability.

Q. 9 Enlist the disadvantages of DS-SS. Dec. 18

Ans. :

Disadvantages of DS-SS :

1. With the serial search system, the acquisition time is too large. This makes the DS-SS system slow.
2. The sequence generated at the PN code generator output must have a high rate. The length of such a sequence needs to be long enough to make the sequence truly random.
3. The channel bandwidth required, is very large. But this bandwidth is less than that of a FH-SS system.
4. The synchronization is affected by the variable distance between the transmitter and receiver.

Q. 10 The information bit duration in DS-BPSK spread spectrum communication system is 4 mS while the chipping rate is 1 MHz. Assuming an average error probability of 10^{-5} for proper defection of message signal, calculate the jamming margin. Interpret your result. Given Q (4.25) = 10^{-5} Dec. 10, May 16

Ans. :

Given :

1. Chipping rate = 1 MHz

$\therefore$ Chip duration $T_c = \dfrac{1}{1 \times 10^6} = 1\,\mu$ sec.

2. Information bit duration T_b = 4 ms 3. $P_e = 10^{-5}$

Information bitduration Tb = ms
(1) Information
(2) Processing gain $PG = \frac{Tb}{Tc}$
$Pe = \frac{1}{2} erfc \sqrt{Eb/K}$

To find : Jamming margin

1. Processing gain PG :

$$PG = \frac{T_b}{T_c} = \frac{4 \times 10^{-3}}{1 \times 10^{-6}} = 4000$$

$$P_e = \frac{1}{2} \text{erfc} \sqrt{E_b / N_o}$$

$$= \frac{1}{2} \times 2Q\left[\sqrt{2E_b / N_o}\right]$$

$$10^{-5} = Q\left[\sqrt{2E_b / N_o}\right]$$

$$\therefore 4.25 = \left[\sqrt{2E_b / N_o}\right]$$

$$\therefore E_b / N_o = 9.03$$

2. Jamming margin :

$$\text{Jamming margin } (J/P_s) = \frac{PG}{E_b / N_o} = \frac{4000}{9.03} = 442.90$$

$$\text{Jamming margin dB} = 10 \log 442.90$$

$$= 26.463 \text{ dB} \quad \text{...Ans.}$$

Q. 11 A spread spectrum communication system is characterised by the following parameters. Duration of each information bit, T_b = 4.095 ms. Chip duration of a PN sequence, T_c = 1 μs. Calculate the processing gain and jamming margin if (E_b / N_0) = 10 and the average probability of error $P_e = 0.5 \times 10^{-5}$. Dec. 16, Dec. 17, Dec. 19

Ans. :

– It has been given that,

T_b = 4.095 ms, T_c = 1 μs, (E_b / N_0) = 10,

$P_e = 0.5 \times 10^{-5}$

Processing gain PG :

$$P.G. = \frac{T_b}{T_c} = \frac{4.095 \times 10^{-3}}{1 \times 10^{-6}} = 4095 \quad \text{...Ans.}$$

We know that $T_b = N\,T_c$.

$$PG = \frac{N\,T_c}{T_c} = N \qquad \therefore N = 4095$$

Jamming margin :

$$(\text{Jamming Margin})_{dB} = (PG)_{dB} - 10 \log_{10} [E_b / N_0]$$

$$\therefore (\text{Jamming Margin})_{dB} = 10 \log_{10} 4095 - 10 \log_{10} [10]$$

$$= 36.1 - 10 = 26.1 \text{ dB} \quad \text{...Ans.}$$

Q. 12 The information bit duration in DS-BPSK spread spectrum communication system is 5 mS while the chipping rate is 1 MHz. Assuming an average error probability of 10^{-5} for proper detection of message signal, calculate Jamming margin. Given Q (4.25) = 10^{-5}. May 18

Ans. :

Given :

1. Chipping rate = 1 MHz

$$\therefore \text{Chip duration } T_c = \frac{1}{1 \times 10^6} = 1\ \mu\text{ sec.}$$

2. Information bit duration T_b = 5 mS

3. $P_e = 10^{-5}$

To find : Jamming margin

1. Processing gain PG :

$$PG = \frac{T_b}{T_c} = \frac{5 \times 10^{-3}}{1 \times 10^{-6}} = 5000$$

$$P_e = \frac{1}{2} \text{erfc} \sqrt{E_b / N_o}$$

$$= \frac{1}{2} \times 2Q\left[\sqrt{2E_b / N_o}\right]$$

$$10^{-5} = Q\left[\sqrt{2E_b / N_o}\right]$$

$$\therefore 4.25 = \left[\sqrt{2E_b / N_o}\right]$$

$$\therefore E_b / N_o = 9.03$$

2. Jamming margin :

$$\text{Jamming margin } (J/P_s) = \frac{PG}{E_b / N_o} = \frac{5000}{9.03} = 553.7$$

$$\text{Jamming margin dB} = 10 \log 553.7$$

$$= 27.43 \text{ dB} \quad \text{...Ans.}$$

Q. 13 A BPSK-DSSS system, using coherent detection, is used to transmit data at 250 bps and system has to work in a hostile jamming environment with minimum error performance of one error in 20, 000 bits. Determine the minimum chipping rate, if the jamming signal is 300 times stronger than the received signal. May 19

Ans. :

Given : A BPSK-DSSS system, Data rate R_b = 250 bps,

Error performance = 1 error in 20×10^3 bits,

$\therefore P_e = 5 \times 10^{-5}$

Jamming signal strength = 300 × received signal.

$\therefore J/P_s$ = 300 (Jamming margin)

To find : Minimum chip rate

– We are supposed to find $R_{c(min)} = \dfrac{1}{T_{c(max)}}$

1. **Find E_b / N_0 :**

$$P_e = \frac{1}{2} \text{erfc} \sqrt{E_b / N_0}$$

$$\therefore \quad 5 \times 10^{-5} = \frac{1}{2} \text{erfc} \sqrt{E_b / N_0}$$

$$\therefore \quad 10^{-4} = \text{erfc} \sqrt{E_b / N_0}$$

$$\therefore \quad \frac{E_b}{N_0} \approx 2.75$$

2. **Find Processing Gain (PG) :**

$$\text{Jamming margin } (J/P_s) = \frac{PG}{E_b / N_0}$$

$$\therefore \quad 300 = \frac{PG}{E_b / N_0}$$

$$\therefore \quad PG = 825$$

3. **Find minimum chip rate (R_c) :**

$$PG = \frac{T_b}{T_c}$$

– But $T_b = \frac{1}{R_b}$ and $T_c = \frac{1}{R_c}$

$$\therefore \quad PG = \frac{R_c}{R_b}$$

$$\therefore \quad R_c = PG \times R_b = 825 \times 250$$

$$\therefore \quad Rc_{(min)} = 2.063 \times 105 \text{ per second}$$ ...Ans.

Q. 14 The information bit duration in DS-BPSK SS system is 4 msec. while the clipping rate is 1 MHz. Assuming an average error probability of 10^{-5}, calculate the jamming margin. Interpret the result. Given : Q(4.25) = 10^{-5}. May 19

Ans. :

Given :

1. Chipping rate = 1 MHz

$$\therefore \quad \text{Chip duration } T_c = \frac{1}{1 \times 10^6} = 1\ \mu\text{ sec.}$$

2. Information bit duration T_b = 4 mS

3. $P_e = 10^{-5}$

To find : Jamming margin

1. **Processing gain PG :**

$$PG = \frac{T_b}{T_c} = \frac{4 \times 10^{-3}}{1 \times 10^{-6}} = 4000$$

$$P_e = \frac{1}{2} \text{erfc} \sqrt{E_b / N_0} = \frac{1}{2} \times 2Q \left[\sqrt{2E_b / N_0} \right]$$

$$10^{-5} = Q \left[\sqrt{2E_b / N_0} \right]$$

$$\therefore \quad 4.25 = \left[\sqrt{2E_b / N_0} \right]$$

$$\therefore \quad E_b / N_0 = 9.03$$

2. **Jamming margin :**

$$\text{Jamming margin } (J/P_s) = \frac{PG}{E_b / N_0} = \frac{4000}{9.03} = 443$$

Jamming margin dB = 10 log 443 = 26.46 dB ...Ans.

Q. 15 Draw and explain FHSS spread spectrum system with transmitter and receiver section. Dec. 14, May 15, May 19, Dec. 19

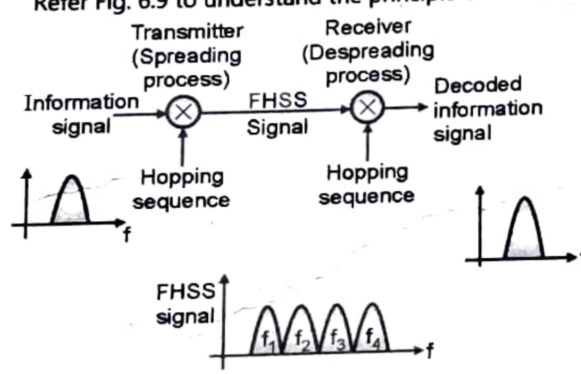
Ans. :

Definition :

– FHSS is a method of transmitting radio signals by rapidly switching a carrier among many frequency channels using a PN sequence known to both transmitter and receiver.

Operation of FHSS :

– Refer Fig. 6.9 to understand the principle of FHSS.



(O-1105) Fig. 6.9 : Basic concept of FHSS technique

– The frequency hopped multiple access technique is based on the frequency hopping spread spectrum (FHSS) modulation scheme.

– The SS signal with a wideband frequency spectrum is generated in a different manner in a frequency hopping technique.

– In frequency hopping, the transmission frequency is periodically changed over a wide band.

– The rate of hopping from one frequency to another depends on the information rate.

– Whereas the specific order in which the signal occupies the frequencies is a function of a code sequence.

– The set of possible carrier frequencies say f_1, f_2, f_3, f_4 is called the **hop-set**.

- Hopping occurs over a frequency band that includes a number of channels.
- We may define each channel as a spectral band with a central frequency in the hop-set and a sufficient bandwidth.
- The bandwidth of a channel is called the instantaneous bandwidth whereas, the bandwidth of the spectrum over which the hopping takes place is called the total hopping bandwidth.
- The FHSS transmitter sends data by changing (hopping) the transmitter carrier frequencies from frequency to the other in a seemingly random manner, which is known only to the desired receiver.
- As shown the bandwidth of a frequency-hopping signal is 'W' times the number of frequency slots available, and the bandwidth of each hop channel is equal to 4 W.

FHSS transmitter and receiver :

- In an FHSS transmitter and receiver a pseudorandom (PN) frequency hopping sequence is used as shown in Fig. 6.9.
- At the transmitter it is used for changing the radio signal frequency randomly across a broad frequency band in a random manner.
- In this way in FHSS, the radio transmitter frequency hops from one channel to the other channel in a predetermined but pseudorandom sequence.
- The received signal at the receiver is despread by using a frequency synthesizer which is controlled by a pseudorandom sequence generator.
- The PN sequence generator is synchronized to the transmitter's pseudorandom sequence generator.

Q. 16 Write short note on : Fast and slow frequency hopping. May 19, Dec. 19

Ans. :

1. Slow frequency hopping :

- The slow frequency hopping is the type of FHSS in which the **symbol rate R_s** of the MFSK signal is an integer multiple of the hop rate R_h.
- That means several symbols are transmitted corresponding to each frequency hop.

∴ Each frequency hop ⇒ Several symbols.

- Thus frequency hopping takes place slowly.

2. Fast frequency hopping :

- It is the type of FHSS in which the hop rate R_h is an integer multiple of the MFSK **symbol rate R_s**.
- That means during the transmission of one symbol, the carrier frequency will hop several times.

∴ Each symbol transmission ⇒ Several frequency hops.

- Thus the frequency hopping takes place at a fast rate.

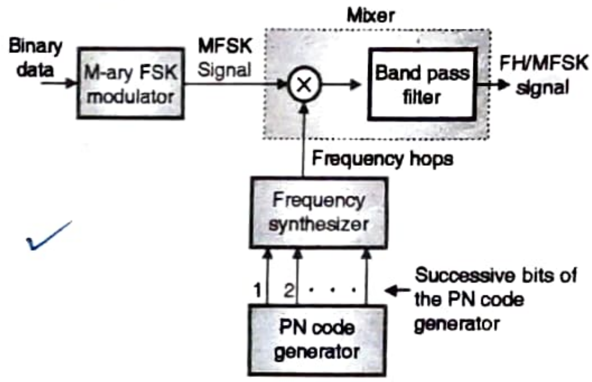
Q. 17 Draw the block diagram of FHSS transmitter and receiver.

May 15, Dec. 16, Dec. 17, Dec. 18, May 19, Dec. 19

Ans. :

FHSS transmitter :

- Fig. 6.10 shows the block diagram of a slow-frequency hopping FH/MFSK transmitter.



(E-484) **Fig. 6.10 : Frequency hop spread M-ary FSK transmitter**

Operation :

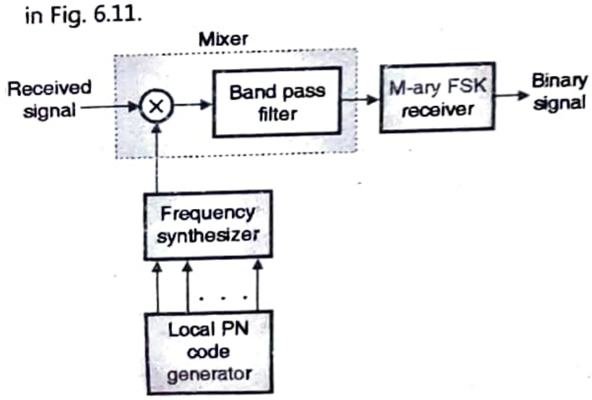
- The binary data sequence b (t) is applied to the M-ary FSK modulator the output of which goes to the input of the mixer.
- The other input to the mixer block is obtained from a digital frequency synthesizer. The mixer consists of a multiplier followed by a band pass filter.
- At the multiplier output we get the two input frequencies, their sum and their difference frequency components.
- The band-pass filter is designed to select only the sum frequency component rejecting all other components. This sum components of frequency is then transmitted.
- Successive K-bits of the input binary data sequence will form one symbol. M such symbols can be transmitted using the M-ary FSK system with

$M = 2^k$.

- The M-ary FSK modulator will assign a distinct frequency for each of these M symbols.
- Thus the frequency of mixer input obtained from MFSK modulator is changing continuously.
- The other input to the mixer is obtained from the digital frequency synthesizer. The synthesizer output at a given instant of time is the "frequency hop".
- Each frequency hop is mixed with the MFSK signal to produce the transmitted signal.
- The frequency hops at the output of the synthesizer are controlled by the successive bits at the output of the PN code generator.
- The output bits of the PN generator change randomly.
- Therefore the synthesizer output frequency will also change randomly. Hence the frequency hops produced will vary in a random manner.
- If the number of successive bits at the output of PN generator is "n", then the total number of frequency hops will be 2^n. The total bandwidth of the transmitted FH/MFSK signal is equal to the sum of all the frequency hops.
- Therefore the bandwidth of the transmitted FH/MFSK signal is very large of the order of few GHz.

FH / MFSK Receiver :

- The block diagram of an FH-MFSK receiver is as shown in Fig. 6.11.



(E-485) Fig. 6.11 : An FH-MFSK receiver

Operation of FH / MFSK receiver :

- The received signal is applied to a mixer. The other input to the mixer comes from a digital frequency synthesizer.
- This digital synthesizer is driven by a PN code generator which is synchronized with the PN code generator at the transmitter and generates the same code sequence.
- Therefore the frequency hops produced at the synthesizer output will be identical to those at the synthesizer output at the transmitter.
- At the output of the multiplier we get the input signals, their sum and difference (as far as frequency is concerned).
- Out of these frequency components, the difference frequency component is selected by the bandpass filter that follows the multiplier.
- This difference signal is the MFSK signal. Thus the mixer removes the frequency hopping.
- The MFSK signal at the mixer output is then applied to a non-coherent MFSK demodulator. At the output of the MFSK detector we obtain the digital modulating signal b (t).
- The non-coherent M-ary FSK detector can be implemented by using a bank of M, non coherent matched filters.
- Each matched filter is matched to one of the tones of the MFSK signal. The largest output out of the M available outputs of filters is selected to obtain the digital modulating signal.

Q. 18 Consider a slow hop spread spectrum system with binary FSK, two symbols per frequency hop, and a PN sequence generator with outputs with the binary message of 011011011000. The message is transmitted using the following PN sequence with k = 3 : {010, 110, 101, 100, 000, 101, 011, 001, 001, 111, 011, 001}, plot the output frequencies for the input message. **May 13**

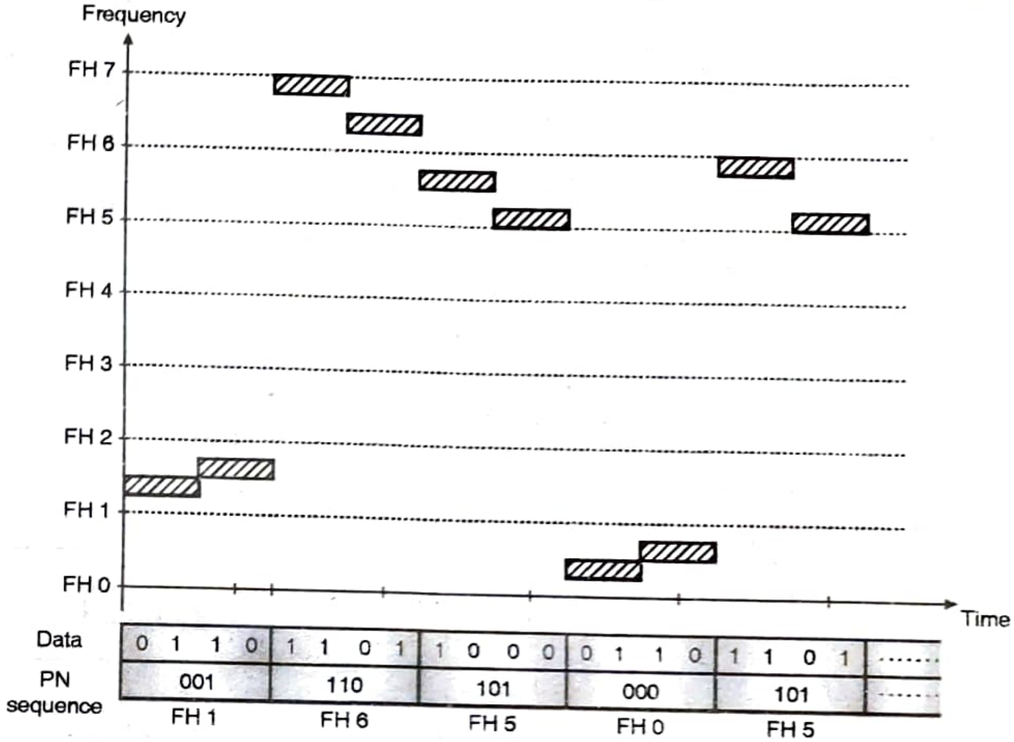
Ans. :

Given : Slow hop S.S. with BFSK, two symbols per frequency hop,

Data message : 011011011000, k = 3

- The plot of output frequencies versus data inputs is as shown in Fig. 6.12.
- The relation between data symbols, and PN sequence is as follows,

Data symbol	01 10	11 01	10 00	01 10	.....
PN sequence	001	110	101	000	.....



(E-1364) Fig. 6.12

Q. 19 Explain in brief : Fast frequency hopping.

May 15. Dec. 18. Dec. 19

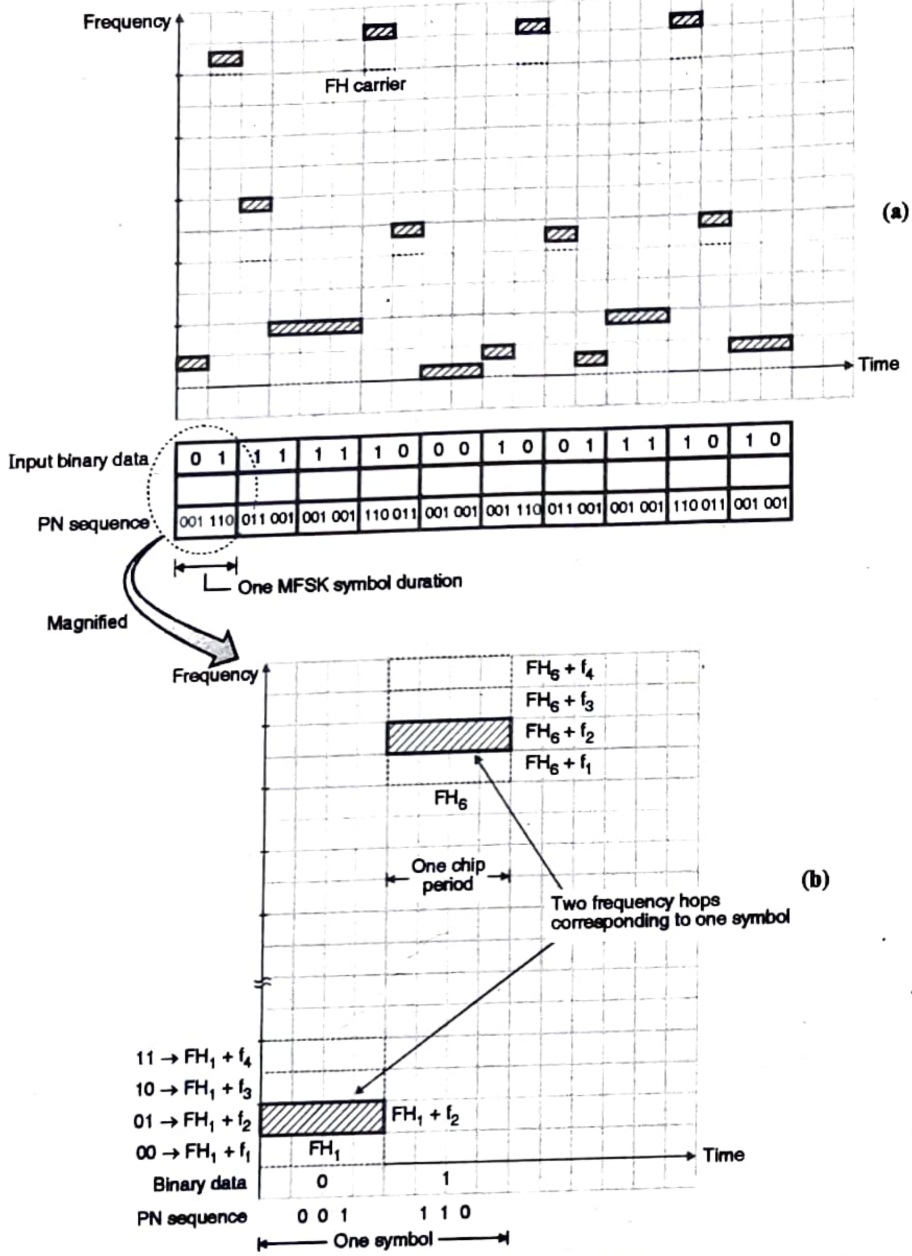
Ans. :

Fast frequency hopping :

- The fast FH/MFSK system is different than the slow FH/MFSK system.
- Because in the fast FH/MFSK system, there are multiple hops for each M-ary symbol. Hence each hop is a "chip".

$\therefore$ Chip rate R_c = Rate of hopping R_h

- The fast frequency hopping is used for defeating a smart jammer who tries to interfere the transmission.
- Before the jammer could understand the frequency band which is being used by the transmitter, the transmitted signal is hopped to a new carrier frequency.
- The principle of fast frequency hopping is illustrated in Fig. 6.13.
- The data sequence used for the fast hopping is same as the one used for the slow hopping.
- The number of bits per MFSK symbol = K = 2. Therefore the number of MFSK tones = $2^K = 4$.
- The length of PN segment per hop i.e. n = 3. Therefore the total number of frequency hops = $2^3 = 8$.
- The PN sequence decides the hopping frequency (shown by dotted lines in Fig. 6.13(a). Two successive input binary bits 0 1 form the first symbol.
- During this symbol duration the PN sequence (3 digit) has two distinct values viz 001 and 110.
- Therefore one symbol duration corresponds to two frequency hops.
- As shown in Fig. 6.13(b) the frequency of the MFSK modulator for symbol 01 is f_2 and the outputs of the synthesizer corresponding to 001 and 110 outputs of the PN sequence generator are say FH_1 and FH_6.



(E-487) **Fig. 6.13 : Waveforms of fast hopping system**

- Therefore the transmitted frequencies are ($FH_1 + f_2$) and ($FH_6 + f_2$).
- The operation for the first symbol 01 is summarized below.

Summary of operation in the first symbol duration :

Symbol : 01	
Frequency of MFSK modulator = f_2	
Output of PN code generator	**Frequency of synthesizer (hop)**
0 0 1	FH_1
1 1 0	FH_6
Transmitted frequencies : ($f_2 + FH_1$) and ($f_2 + FH_6$)	

Q. 20 Compare DSSS with FHSS system.

Dec. 15, May 16, May 18

Ans. :

Comparison of DS-SS and FHSS :

Table 6.2 : Comparison of DS-SS and FHSS

Sr. No.	Parameter	Direct sequence spread spectrum	Frequency hopping spread spectrum
1.	Definition	PN sequence of large bandwidth is multiplied with narrow band data signal.	Data bits are transmitted in different frequency slots which are changed by PN sequence.

Sr. No.	Parameter	Direct sequence spread spectrum	Frequency hopping spread spectrum
2.	Chip rate	It is fixed $R_c = \frac{1}{T_c}$	$R_c = \max(R_h, R_s)$
3.	Modulation technique	BPSK	M-ary FSK
4.	Processing gain	$PG = \frac{T_b}{T_c} = N$	$PG = 2^t$
5.	Effect of loading	Less	More
6.	Acquisition time	Long	Short
7.	Effect of distance	This system is distance relative.	Effect of distance is less.

Q. 21 Represent variation of the frequency of a fast hop spread spectrum system with binary FSK, having following parameters :

Number of bits per MFSK symbol K = 2

Number of MFSK tones $M = 2^K = 4$

Length of PN segment per hop K = 3

Total number of frequency hops $2^K = 8$

For the binary message of 01111110001001111010

Generate the PN sequence for the message to be transmitted. The period of the PN sequence is $2^4 - 1 = 15$ with initial shift register content of 1100. Dec. 13, May 15, May 16

Ans. :

Given : Fast hop spread spectrum with BFSK.

Number of bits/symbol, K = 2

Number of MFSK tones, $M = 2^K = 4$

Length of PN segment per hop, K = 3

Number of frequency hops, $2^K = 8$

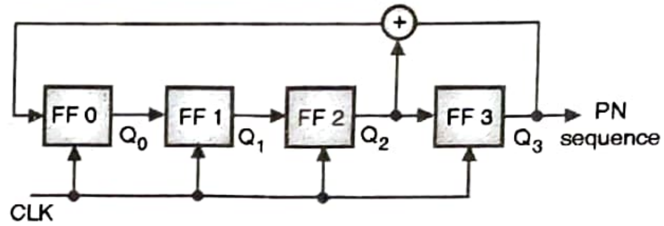
Binary message :

01 11 11 10 00 10 01 11 10 10

Step 1 : Generate the PN sequence :

- The circuit diagram of the PN sequence generator is as shown in Fig. 6.14.
- The initial state is given as,

$$Q_3\,Q_2\,Q_1\,Q_0 = 1\;1\;0\;0$$



(E-1375) Fig. 6.14 : PN sequence generator

- Table 6.3 summarizes the operation of PN generator.
- Thus the PN sequence generated is as follows :

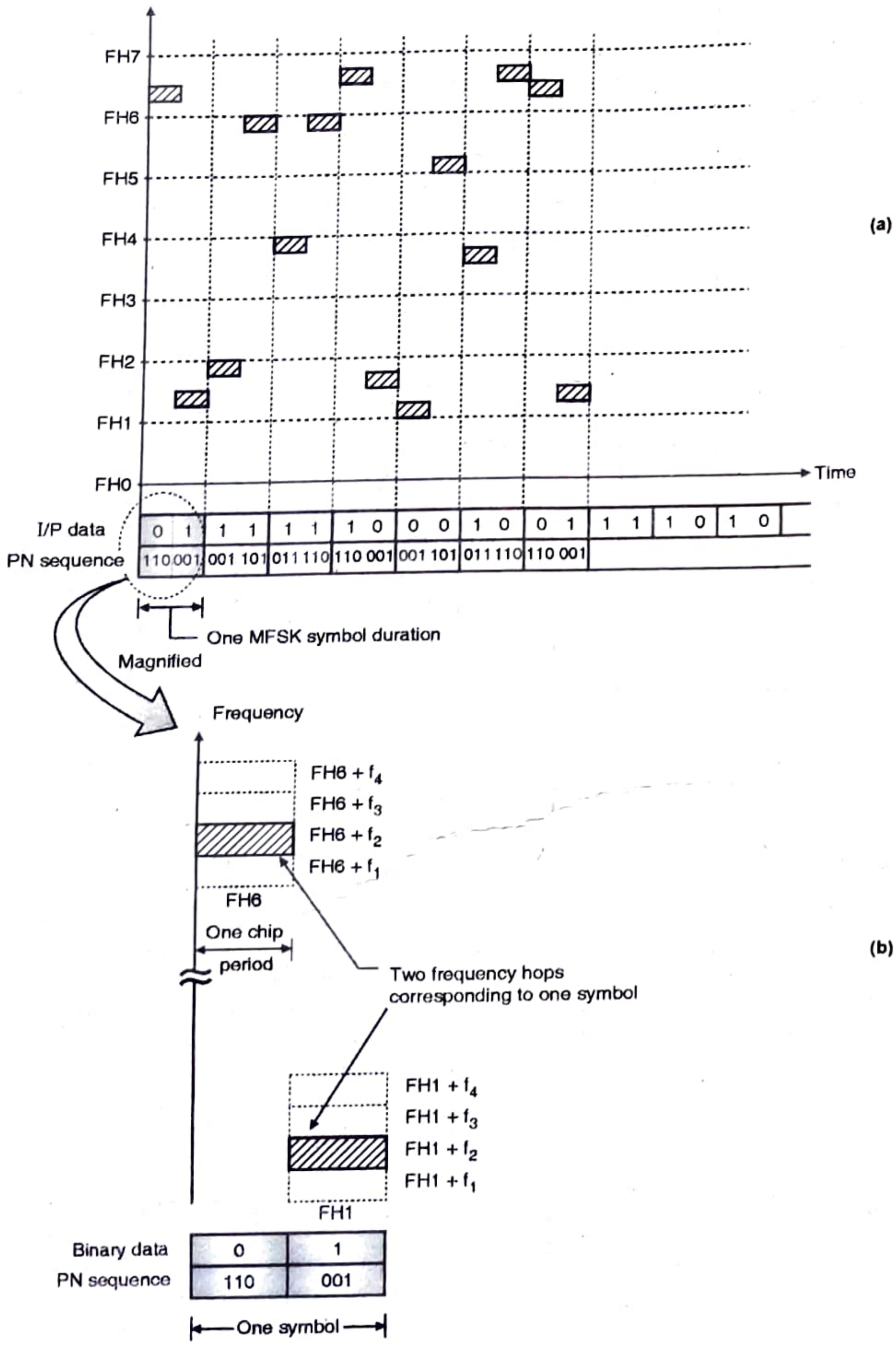
110 001 001 101 011 110 ....

(E-1376) Table 6.3 : PN sequence generator output

Clock pulse number	Shift register outputs Q_3	Q_2	Q_1	Q_0	EXOR output $Q_3 + Q_2$	PN sequence Q_5
0	1	1	0	0	0	1
1	1	0	0	0	1	1
2	0	0	0	1	0	0
3	0	0	1	0	0	0
4	0	1	0	0	1	0
5	1	0	0	1	1	1
6	0	0	1	1	0	0
7	0	1	1	0	1	0
8	1	1	0	1	0	1
9	1	0	1	0	1	1
10	0	1	0	1	1	0
11	1	0	1	1	1	1
12	0	1	1	1	1	0
13	1	1	1	1	0	1
14	1	1	1	0	0	1
15	1	1	0	0	0	1

Step 2 : Represent variation in output frequency :

- The principle of fast frequency hopping is illustrated in Fig. 6.15(a).
- The number of bits per MFSK symbol = K = 2.
- Therefore the number of MFSK tones = $2^K = 4$.
- The length of PN segment per hop i.e. n = 3.
- Therefore the total number of frequency hops = $2^3 = 8$.
- The PN sequence decides the hopping frequency (shown by dotted lines in Fig. 6.15(a).



(E-1377) **Fig. 6.15**

- Two successive input binary bits 0 1 form the first symbol.
- During this symbol duration the PN sequence (3 digit) has two distinct values viz 110 and 001.
- Therefore one symbol duration corresponds to two frequency hops.

- As shown in Fig. 6.15(b) the frequency of the MFSK modulator for symbol 01 is f_2 and the outputs of the synthesizer corresponding to 110 and 001 outputs of the PN sequence generator are say FH_6 and FH_1 respectively.

Q. 22 Write advantages and disadvantages of FHSS. Dec. 17, May 18, Dec. 18

Ans. :

Advantages of FH-SS System :

1. The synchronization is not greatly dependent on the distance.
2. The serial search system with FH-SS needs shorter time for acquisition.
3. The processing gain PG is higher than that of DS-SS system.

Disadvantages of FH-SS System :

1. The bandwidth of FH-SS system is too large (in GHz).
2. Complex and expensive digital frequency synthesizers are required to be used.

Q. 23 Represent variation of frequency of fast hop FHSS with binary FSK having following parameters :

1. **No. of bits per MFSK symbol K = 2.**
2. **No. of MFSK tones M = 2K = 4**
3. **Length of PN segment per hop = 3**
4. **Total number of frequency hops = 8. Generate PN sequence with initial shift register contents 1100.**

Represent variation of frequency for binary data 01111100. May 18

Ans. :

Given : Fast hop spread spectrum with BFSK.

Number of bits/symbol, $K = 2$

Number of MFSK tones, $M = 2^K = 4$

Length of PN segment per hop, $K = 3$

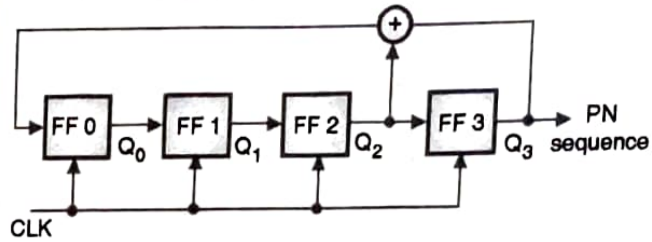
Number of frequency hops, $2^K = 8$

Binary message :

01 11 11 00

Step 1 : Generate the PN sequence :

- The circuit diagram of the PN sequence generator is as shown in Fig. 6.16.



(E-1375) **Fig. 6.16 : PN sequence generator**

- The initial state is given as,

$$Q_3\ Q_2\ Q_1\ Q_0 = 1\ 1\ 0\ 0$$

- Table 6.4 summarizes the operation of PN generator.

(E-1376) **Table 6.4 : PN sequence generator output**

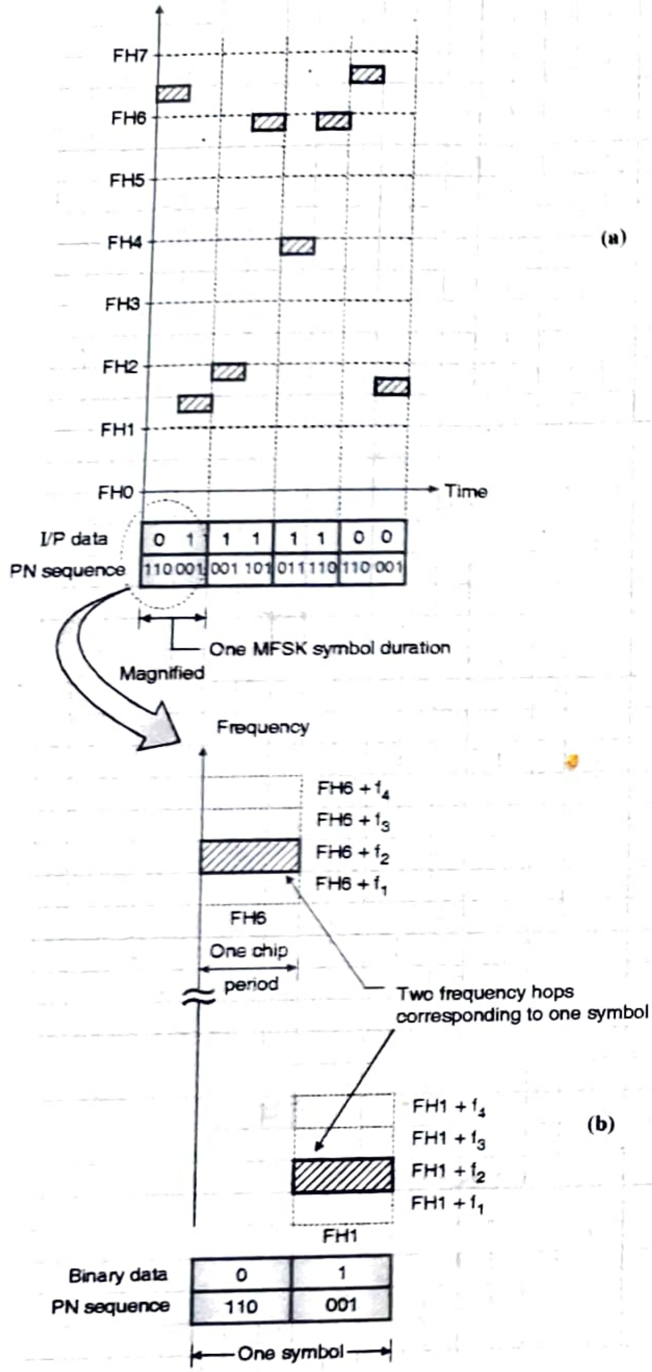
Clock pulse number	Shift register outputs Q_3	Q_2	Q_1	Q_0	EXOR output $Q_3 + Q_2$	PN sequence Q_5
0	1	1	0	0	0	1
1	1	0	0	0	1	1
2	0	0	0	1	0	0
3	0	0	1	0	0	0
4	0	1	0	0	1	0
5	1	0	0	1	1	1
6	0	0	1	1	0	0
7	0	1	1	0	1	0
8	1	1	0	1	0	1
9	1	0	1	0	1	1
10	0	1	0	1	1	0
11	1	0	1	1	1	1
12	0	1	1	1	1	0
13	1	1	1	1	0	1
14	1	1	1	0	0	1
15	1	1	0	0	0	1

- Thus the PN sequence generated is as follows :

110 001 001 101 011 110 ....

Step 2 : Represent variation in output frequency :

- The principle of fast frequency hopping is illustrated in Fig. 6.17(a) and (b).



(E-1991) Fig. 6.17

Chapter 7 : Information Theory

Q. 1 Define entropy and mutual information.

May 11, May 13, Dec. 14, April 18, May 18

Ans. :

Definition of entropy :

- The "Entropy" is defined as the average information per message.
- It is denoted by H and its units are bits/message.
- The entropy must be as high as possible in order to ensure maximum transfer of information.
- We will prove that the entropy depends only on the probabilities of the symbols that are being produced by the source.

Definition of mutual information :

- Mutual information is defined as the amount of information transferred when x_i is transmitted and y_j is received.
- It is defined as :

$$I(x_i ; y_j) = \log \frac{p(x_i / y_j)}{p(x_i)} \quad ...(1)$$

where $p(x_i / y_j)$ = The conditional probability of x_i provided y_j occurs.

$p(x_i)$ = Probability of x_i.

- Mutual information measures the amount of information transferred when x_i is transmitted and y_j is received.

Q. 1 A 3 bit PCM system generates 1000 samples/sec. If the quantized samples are produced by the system with probabilities $\left\{\frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}\right\}$. Then find the rate of information. If the samples are equiprobable, what will be rate of information ? **Dec. 15**

Ans. :

Given : 3 bit PCM system,

message rate r = 1000 samples/sec

Samples	Probabilities
S_1	1/4
S_2	1/4
S_3	1/8
S_4	1/8
S_5	1/16
S_6	1/16
S_7	1/16
S_8	1/16

1. **Source entropy :**

$$H = \sum_{m=1}^{4} P_k \log_2 (1/P_k)$$

$$= 2 \times [1/4 \log_2 (4)] + 2 \times [1/8 \log_2 (8)] + 4 \times [1/16 \log_2 (16)]$$

$$= 1 + 0.75 + 1$$

$$= 2.75 \text{ bits/message}$$

2. **Information rate (R) :**

$$R = r \times H = 1000 \times 2.75$$

$$= 2750 \text{ bits/sec}$$ **...Ans.**

3. **Entropy :**

- If the samples are equiprobable, then

$$S_1 = S_2 = S_3 = S_4 = S_5 = S_6 = S_7 = S_8 = \frac{1}{8}$$

$$\text{Then, Entropy} = 8 \times \frac{1}{8} \log_2 (8) = 3 \text{ bits}$$

$$\text{Information rate, } R = r \times H = 1000 \times 3$$

$$= 3000 \text{ bits/sec}$$ **...Ans.**

Q. 2 A source emits 1000 symbols per second from a range of 5 symbols, with probabilities $\left[\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}\right]$. Find entropy and information rate. **May 18, May 19**

Ans. :

Given : Symbol rate = 1000 per second, 5 symbols.

Symbol probabilities : $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}$

To find : H and R

1. **Entropy (H) :**

$$H = \frac{1}{2} \log_2 2 + \frac{1}{4} \log_2 4 + \frac{1}{8} \log_2 8 + \frac{2}{16} \log_2 16$$

$$\therefore H = 0.5 + 0.5 + \left(\frac{3}{8}\right) + 0.5$$

$$\therefore H = 1.875 \text{ bits / symbol}$$ **...Ans.**

2. **Information rate (R) :**

$$\therefore R = r \times H = 1000 \times 1.875$$

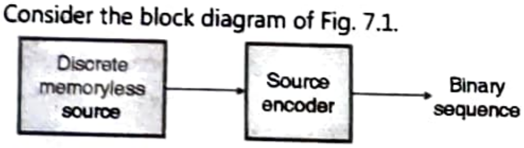
$$= 1875 \text{ bits / sec.}$$ **...Ans.**

Q. 3 Write a short note on : Shannon Source coding theorem. **Dec. 11, April 18, March 19**

Ans. :

Block diagram of source encoding :

- Consider the block diagram of Fig. 7.1.



(L-295) Fig. 7.1 : Source encoding

- The output of the DMS is being converted by the source encoder into a binary signal i.e. we get code words at the output of the encoder.

Statement of source coding theorem :

- Given a discrete memory less source of entropy H, the average code-word length L for any source encoding is bounded as,

$$L \geq H \quad ...(1)$$

Explanation :

- The source coding theorem states that the average code word length L should always be greater than or equal to the source entropy H.
- That means the value of L_{min} should be equal to H.

$$\therefore \quad L_{min} = H \qquad ...(2)$$

- Substituting this value of L_{min} into Equation $\eta = \frac{L_{min}}{L}$ we get,

$$\text{Code efficiency } \eta = \frac{H}{L} \qquad ...(3)$$

- The variable length coding is done in order to increase the efficiency of the source encoder.

Q. 4 Write the procedure for Shannon-Fano coding.

Dec. 08, Feb. 16, May 17

Ans. :

- Consider Table 7.1 which gives eight possible messages m_1, m_2, ...m_8 with their corresponding probabilities.

Table 7.1

Message	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8
Probability	1 / 2	1 / 8	1 / 8	1 / 16	1 / 16	1 / 16	1 / 32	1 / 32

- In order to obtain the words for each message follow the procedure given below :

Procedure to obtain Shannon - Fano codes :

Step 1 : List the source symbols (messages) in the order of decreasing probability.

Step 2 : Partition the set of symbols into two sets that are as close as possible to being equiprobable.

Step 3 : Assign 0 to each message in the upper set and 1 to each message in the lower set.

Step 4 : Continue this process, each time partitioning the sets with as nearly equal probabilities as possible until it is not possible to further partition the messages.

- By following this procedure we get the code for each message as shown in Table 7.2.

(E-2105) Table 7.2 : An example to demonstrate application of Shannon - Fano algorithm

Message	Probability	Col. I	Col. II	Col. III	Col. IV	Col. V	Code word	No. of bits / code word
m_1	1/2	0 Partition	Stop	–	–	–	0	1
m_2	1/8	1	0	0 Partition	Stop	–	100	3
m_3	1/8	1	0 Partition	1	Stop	–	101	3
m_4	1/16	1	1	0	0 Partition	Stop	1100	4
m_5	1/16	1	1	0 Partition	1	Stop	1101	4
m_6	1/16	1	1	1	0 Partition	Stop	1110	4
m_7	1/32	1	1	1	1	0 Partition	11110	5
m_8	1/32	1	1	1	1	1	11111	5

- Consider column I in the Table 7.2. We divide the total 8 messages into two partitions or groups, such that the sum of probabilities of each group is the same.
- Thus in column I we have m_1 in one group and all the other messages in the other group.
- This is because the probability of m_1 is 1/2 and the sum of probabilities of m_2 to m_8 is also 1/2.
- Then assign a bit 0 to the message in one partition i.e. m_1 and assign bit 1 to all the messages (m_2 to m_8) in the other partition as shown in Table 7.2.
- This process is continued, until each message finds itself alone in a partition.
- So in column II, we stop for message m_1 and partition the remaining messages m_2 to m_8 into two groups (m_2, m_3 one group and remaining in second group).
- The codes generated for different messages are as listed in the last column.
- Observe that the message m_1 which has the highest probability has only one bit in its code.

- Whereas as the probability decreases the number of bits in the encoded word will increase e.g. m_8 has probability of 1/32 and the code word has 5 bits to represent it.

Average code word length (L) :

- The average code word length L is given by,

$$L = \sum_{k=1}^{M} p_k \times \text{(length of message } m_k \text{ in bits)} \quad \text{... (1)}$$

Where p_k = Probability of k^{th} message

m_k = k^{th} message

$$L = \left(\frac{1}{2} \times 1\right) + [(1/8 \times 3) \times 2]$$
$$+ [(1/16 \times 4) \times 3] + [(1/32 \times 5) \times 2]$$

$$\therefore \quad L = 2.3125 \text{ bits/message} \quad \text{...(2)}$$

Average information per message (H) :

- The average information per message is given by,

$$H = \sum_{i=1}^{k} p(x_i) \cdot \log_2 [1/p(x_i)] \quad \text{... (3)}$$

Code efficiency (η) :

- The code efficiency (η) is defined as the ratio of the average information per message (H) and the average code word length (L).

$$\therefore \text{Code efficiency } (\eta) = \frac{\text{Average information per message}}{\text{Average code word length}}$$

$$\therefore \quad \eta = \frac{H}{L}$$

- There are no units for the code efficiency.
- The code efficiency can be expressed in percentage as,

$$\eta = \frac{H}{L} \times 100\,\% \quad \text{... (4)}$$

- The code efficiency should be as high as possible.

Code redundancy :

- Redundancy involves transmission of extra bits alongwith the data bits.
- These bits actually do not contain any information, but they ensure the detection and correction of errors introduced during the transmission.
- Redundancy of a code is denoted by γ and its value should be as small as possible to improve the efficiency of a code.

$$\gamma = 1 - \eta$$

Q. 5 A zero memory source emits six messages (N, I, R, K, A, T) with probabilities (0.30, 0.10, 0.02, 0.15, 0.40, 0.03) respectively. Given that 'A' is coded as '0'. Find :

1. **Entropy of source.**
2. **Determine Shannon Fano code and tabulate them.**
3. **What is the original symbol sequence of the Shannon Fano coded signal (11001111011111110100).**

May 11, Dec. 13, April 18

Ans. :

1. Entropy of the source (H) :

$$H = \sum_{k=1}^{6} p_k \log_2 (1/p_k)$$

$$= 0.4 \log_2 (1/0.4) + 0.3 \times \log_2 (1/0.3) + 0.15 \log_2 (1/0.15)$$
$$+ 0.1 \log_2 (1/0.1) + 0.03 \log_2 (1/0.03) + 0.02 \log_2 (1/0.02)$$
$$= 0.5288 + 0.5210 + 0.4105 + 0.3322 + 0.1518 + 0.1129$$

$$\therefore \ H = 2.057 \text{ bits/message} \qquad \text{...Ans.}$$

2. Shannon Fano code :

Table 7.3

Message	Probability	Step 1	Step 2	Step 3	Step 4	Step 5	Code
A	0.4	0 Partition	Stop				0
N	0.3	1	0 Partition	Stop			10
K	0.15	1	1	0 Partition	Stop		110
I	0.1	1	1	1	0 Partition	Stop	1110
T	0.03	1	1	1	1	0 Partition	11110
R	0.02	1	1	1	1	1	11111

3. Original Symbol Sequence : (E-1493)

$$\underbrace{110}_{K}\ \underbrace{0}_{A}\ \underbrace{11110}_{T}\ \underbrace{11111}_{R}\ |\ \underbrace{110}_{I}\ \underbrace{10}_{N}\ \underbrace{0}_{A}$$

Q. 6 Obtain the coding efficiency of a Shannon Fano for a zero memory source that emits eight messages with respective probabilities as given below. Use three letters for encoding such as – 1, 0, 1.

P = [0.3, 0.12, 0.12, 0.12, 0.12, 0.08, 0.07, 0.07]

X = $[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]$ **May 17**

Ans. :

Step 1 : Obtain the codes using Shannon Fano :

Table 7.4

Message	Probability	Step 1	Step 2	Step 3	Code	No. of bits per code
X_1	0.3	– 1	Stop		– 1	1
X_2	0.12	0	– 1	Stop	0, – 1	2
X_3	0.12	0	0	Stop	00	2
X_4	0.12	0	1	Stop	01	2
X_5	0.12	1	– 1	Stop	0, – 1	2
X_6	0.08	1	0	Stop	10	2
X_7	0.07	1	1	– 1	11– 1	3
X_8	0.07	1	1	0	110	3

Step 2 : Average codeword length L :

$$L = \sum_{j=1}^{8} n_j\, p(x_j)$$

$$= (1 \times 0.3) + (4 \times 0.12 \times 2) + (1 \times 0.08 \times 2) + (2 \times 0.07 \times 3)$$

$$= 1.84 \text{ bits / message}$$

Step 3 : Source entropy H(X) :

$$H(X) = \sum_{j=1}^{8} p(x_j) \log_2 \frac{1}{p(x_j)}$$

$$= 0.3 \times \log_2 (1/0.3) + 4 \times 0.12 \log_2 (1/0.12) + 0.08 \log_2 (1/0.08) + 2 \times 0.07 \log_2 (1/0.07)$$

$$= 0.5210 + 1.4683 + 0.2915 + 0.5371$$

$$= 2.82 \text{ bits / message}$$

Step 4 : Coding efficiency :

$$\%\eta = \frac{H(X)}{L \times \log M} \times 100$$

– But M = 3, because we are using three levels for coding.

$$\therefore \ \%\eta = \frac{2.82}{1.84 \times \log_2 3} \times 100 = 96.69\%$$

Q. 7 A zero memory source emits seven symbols with probabilities (0.2, 0.15, 0.02, 0.1, 0.4, 0.08, 0.05). Compute coding efficiency, when above symbols are encoded by Shannon-Fano source coding technique. March 19

Ans. :

– The Shannon-Fano codes are as shown in Table 7.5.

(E-2067) Table 7.5

Symbol	Probability	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Code	Code length
S_1	0.4	0 Partition	Stop here					0	1
S_2	0.2	1	0 Partition	Stop here				10	2
S_3	0.15	1	1	0 Partition	Stop here			110	3
S_4	0.1	1	1	1	0 Partition	Stop here		1110	4
S_5	0.08	1	1	1	1	0 Partition	Stop here	11110	5
S_6	0.05	1	1	1	1	1	0 Partition	111110	6
S_7	0.02	1	1	1	1	1	1	111111	6

0.60
0.60

0.35
0.40

Average code word length (L) :

$$L = (0.4 \times 1) + (0.2 \times 2) + (0.15 \times 3) + (0.1 \times 4) + (0.08 \times 5) + (0.05 \times 6) + (0.02 \times 6)$$

$$\therefore L = 2.47 \text{ bits}$$

Source entropy (H) :

$$H = -0.4 \log_2 0.4 - 0.2 \log_2 0.2 - 0.15 \log_2 0.15 - 0.1 \log_2 0.1 - 0.08 \log_2 0.08 - 0.05 \log_2 0.05 - 0.02 \log_2 0.02$$

$$H = 0.5287 + 0.4643 + 0.4105 + 0.3321 + 0.2915 + 0.2160 + 0.1128$$

$$\therefore H = 2.356 \text{ bits / message}$$

Code efficiency :

$$\% \eta = \frac{H}{L} \times 100 = \frac{2.356}{2.47} \times 100$$

$$\therefore \% \eta = 95.38 \%$$ ...Ans.

Q. 8 Explain Huffman algorithm with suitable example. **Dec. 07, Dec. 08, Dec. 13**

Ans. :

Huffman algorithm :

- The Huffman code is a source code. It is a prefix code as well.
- Here word length of the code word approaches the fundamental limit set by the entropy of discrete memory less source.
- This code is "optimum" as it provides the smallest average code word length for a given discrete memory less source.
- It encodes each message transmitted by a DMS with different value of number of bits based on their probabilities.

Huffman encoding algorithm :

- The Huffman encoding algorithm is as follows :
  1. The source symbols (messages) are arranged in the order of decreasing probability. The two source symbols having the lowest probability are assigned with binary digits 0 and 1.
  2. These two source symbols (messages) are then "combined" into a new source symbol (message) with probability equal to the sum of the two original probabilities. The probability of the new symbol is placed in the list of symbols as per its value.
  3. This procedure is repeated until we are left with only two source symbols (messages) for which a 0 and a 1 are assigned.
  4. The code of each original source symbol is obtained by working backward and tracing the sequence of 0s and 1s assigned to that symbol.
- The Huffman coding can be shown in the form of an algorithm as follows :

1. List source symbols (messages) in the order of decreasing probability.

↓

2. The two source symbols of lowest probability are assigned numbers 0 and 1.

↓

3. These two source symbols are combined into a new message.

↓

4. The probability of this new message is equal to the sum of probabilities of the two original symbols.

↓

5. The probability of this new message is placed in the list according to its value.

↓

6. Repeat this procedure until we are lift with only two source symbols, for which a 0 and a 1 are assigned.

(E-104)

Q. 9 A source emits letters from an alphabet A = {a_1, a_2, a_3, a_4, a_5} with probabilities P (a_1) = 0.15, P (a_2) = 0.04, P (a_3) = 0.26, P (a_4) = 0.05, P (a_5) = 0.50.

1. **Calculate the entropy of the source.**
2. **Find a Huffman code for the source.**
3. **Find the average length of the code.**
4. **Redundancy.**
5. **Variance.** **Dec. 14**

Ans. :

Step 1 : Entropy of the source (H) :

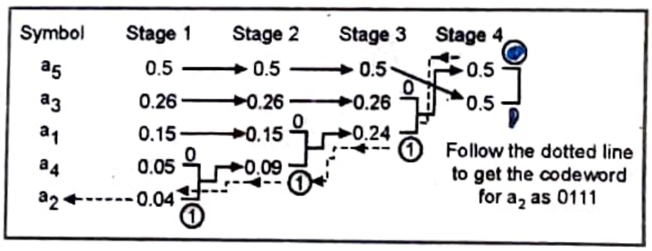
$$H = \sum_{k=1}^{5} - p_k \log_2 p_k$$

$$\therefore H = -0.15 \log_2 0.15 - 0.04 \log_2 0.04 - 0.26 \log_2 0.26 - 0.05 \log_2 0.05 - 0.5 \log_2 0.5$$

$$\therefore H = 0.4105 + 0.1858 + 0.5052 + 0.2161 + 0.5$$

$$= 1.8176 \text{ bits / message}$$ ...Ans.

Step 2 : Huffman code for the source :



(E-1688) **Fig. 7.2**

- Similarly we can obtain the codewords for the remaining symbols as shown in Table 7.6.

Table 7.6 : Codewords

Symbol	a_1	a_2	a_3	a_4	a_5
Probability	0.15	0.04	0.26	0.05	0.5
Codeword	010	0111	00	0110	1
Codeword length	3	4	2	4	1

Step 3 : Average length of the code (L) :

$$L = \sum_{k=1}^{5} p_k \times \text{(length of the message in bits)}$$

$$= (0.15 \times 3) + (0.04 \times 4) + (0.26 \times 2) + (0.05 \times 4) + (0.5 \times 1)$$

$$\therefore L = 1.83 \text{ bits / symbol} \quad \text{...Ans.}$$

Step 4 : Efficiency of the code (η) :

$$\% \eta = \frac{H}{L} \times 100 = \frac{1.8176}{1.83} \times 100$$

$$\therefore \% \eta = 99.32\% \quad \text{...Ans.}$$

Step 5 : Redundancy :

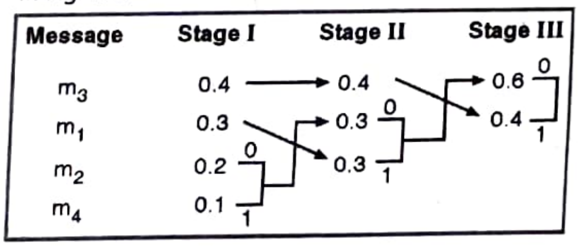
$$\text{Redundancy } R = 1 - \eta = 1 - 0.9932$$

$$= 6.8 \times 10^{-3} \text{ or } 0.68\ \% \quad \text{...Ans.}$$

Q. 10 Design a Huffman code for a source generating 4 different types of messages with probabilities 0.3, 0.2, 0.4, 0.1. Find the coding efficiency. **Dec. 15, Dec. 19**

Ans. :

– The Huffman code for the source alphabet is as shown in Fig. 7.3.



(G-1917) Fig. 7.3

– The code word for messages are given in Table 7.7.

Table 7.7

Message	Probability	Code word	Code word length
m_3	0.4	1	1
m_1	0.3	01	2
m_2	0.2	000	3
m_4	0.1	001	3

To compute the code efficiency :

1. The average code length = L

$$= \sum_{m=1}^{4} P_k \times \text{(Length of symbol in bits)}$$

From Table P. 7.7,

$$L = (0.4 \times 1) + (0.3 \times 2) + (0.2 \times 3) + (0.1 \times 3)$$

$$= 1.9 \text{ bits/symbol}$$

2. The average information per message,

$$H = \sum_{m=1}^{4} P(m_i) \log_2 [1/P(m_i)]$$

$$H = [0.4 \log_2 (2.5)] + [0.3 \log_2 (3.33)] + [0.2 \log_2 (5)] + [0.1 \log_2 (10)]$$

$$= 0.528 + 0.520 + 0.464 + 0.332$$

$$\therefore H = 1.844 \text{ bits/message}$$

3. Code efficiency $= \eta = \frac{H}{L} \times 100 = \frac{1.844}{1.9} \times 100$

$$\therefore \eta = 97.05\ \%$$

Redundancy :

$$R = 1 - \eta = 1 - 0.9705 = 0.0295 \quad \text{...Ans.}$$

Q. 11 Obtain the coding efficiency of a Shannon Fano and Huffman code for a zero memory source that emits six messages (G, N, H, A, E, S) with probabilities of {0.19, 0.15, 0.02, 0.16, 0.4, 0.08} respectively. **Dec. 16**

Ans. :

Part I : Shannon Fano coding :

1. The Shannon Fano code is constructed as follows (Table 7.8).

Table 7.8 : Shannon Fano code

Message	Probability	Step 1	Step 2	Step 3	Step 4	Code
E	0.4	0	0 Partition			00
G	0.19	0 Partition	1			01
A	0.16	1	0 Partition			10
N	0.15	1	1	0 Partition		110
S	0.08	1	1	1	0 Partition	1110
H	0.02	1	1	1	1	1111

2. **Average information per message (H) :**

$$H = \sum_{i=1}^{6} P(x_i) \log_2 [1/P(x_i)]$$

$$= 0.4 \log_2 (1/0.4) + 0.19 \log_2 (1/0.19) + 0.16 \log_2 (1/0.16) + 0.15 \log_2 (1/0.15) + 0.08 \log_2 (1/0.08) + 0.02 \log_2 (1/0.02)$$

$$= 0.5288 + 0.4552 + 0.423 + 0.4105 + 0.2915 + 0.1129$$

$$\therefore H = 2.22 \text{ bits/message} \quad ...(1)$$

3. **Average codeword length (L) :**

$$L = \sum_{k=1}^{6} P_k \times (\text{length of } m_k \text{ in bits})$$

$$= (0.4 \times 2) + (0.19 \times 2) + (0.16 \times 2) + (0.15 \times 3) + (0.08 \times 4) + (0.02 \times 4)$$

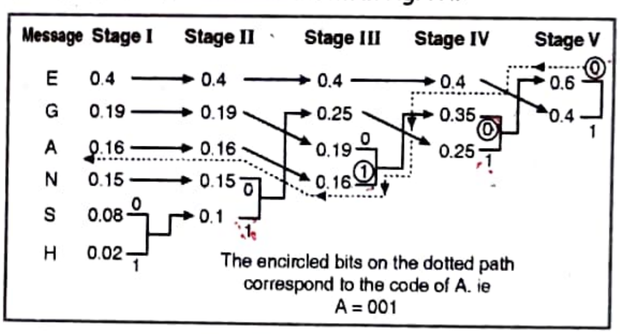
$$\therefore L = 2.35 \text{ bits/ message} \quad ...(2)$$

4. **Code efficiency (η) :**

$$\eta = \frac{H}{L} \times 100 = \frac{2.22}{2.35} \times 100 = 94.47\ \% \quad \textbf{...Ans.}$$

Part II : Huffman's code :

1. The Huffman code is as shown in Fig. 7.4.



(E-1649) Fig. 7.4

- Follow the path indicated by the dotted line to obtain the codeword for message A as 001.
- Similarly we can obtain the codewords for remaining message.
- These are as listed in Table 7.9.

Table 7.9

Message	E	G	A	N	S	H
Codeword	1	000	001	010	0110	0111

2. **Average information per message (H) :**

$$H = \sum P(x_i) \times \log_2 [(1/P(x_i))]$$

$\therefore$ As calculated in part I,

$$H = 2.22 \text{ bits/ message} \quad ...(3)$$

3. **Average code length (L) :**

$$L = \sum_{k=1}^{6} P_k \times (\text{length of } m_k \text{ in bits})$$

$$\therefore L = (0.4 \times 1) + (0.19 \times 3) + (0.16 \times 3) + (0.15 \times 3) + (0.08 \times 4) + (0.02 \times 4)$$

$$= 2.3 \text{ bits/message} \quad ...(4)$$

4. **Code efficiency (η) :**

$$\eta = \frac{H}{L} \times 100 = \frac{2.22}{2.3} = 96.52\ \% \quad \textbf{...Ans.}$$

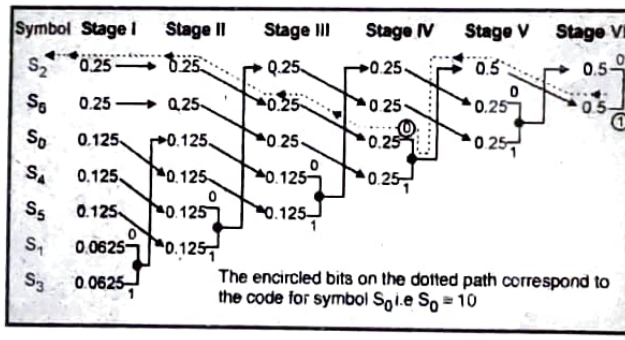
Q. 12 A DMS channel has following symbols and their probabilities. Apply Huffman coding technique to generate a code with minimum variance. Calculate code efficiency.

S_0	S_1	S_2	S_3	S_4	S_5	S_6
0.125	0.0625	0.25	0.0625	0.125	0.125	0.25

Dec. 18

Ans. :

- The Huffman code for the source alphabets is as shown in Fig. 7.5.



(E-2012) Fig. 7.5 : Huffman code

- Follow the path indicated by the dotted line to obtain the code word for symbol S_2 as 10.
- Similarly we can obtain the code words for the remaining symbols.
- These are as listed in Table 7.10.

Table 7.10

Symbol	Probability	Code word	Code word length
S_2	0.25	10	2 bit
S_6	0.25	11	2 bit
S_0	0.125	001	3 bit

Symbol	Probability	Code word	Code word length
S_4	0.125	010	3 bit
S_5	0.125	011	3 bit
S_1	0.0625	0000	4 bit
S_3	0.0625	0001	4 bit

To compute the efficiency :

1. The average code length = L

$$= \sum_{k=0}^{6} P_k \times \text{(length of symbol in bits)}$$

– From Table P. 7.10 ,

$$L = (0.25 \times 2) + (0.25 \times 2) + (0.125 \times 3) \times 3 + (0.0625 \times 4) \times 2$$

$$\therefore \quad L = 2.625 \text{ bits/symbol}$$

2. The average information per message = H

$$= \sum_{i=0}^{6} p(x_i) \log_2 [1 / p(x_i)]$$

$$\therefore H = [0.25 \log_2 (4)] \times 2 + [0.125 \log_2 (8)] \times 3 + [0.0625 \log_2 (16)] \times 2$$

$$= [0.25 \times 2 \times 2] + [0.125 \times 3 \times 3] + [0.0625 \times 4 \times 2]$$

$\therefore$ H = 2.625 bits/message.

3. Code efficiency $\eta = \frac{H}{L} \times 100 = \frac{2.625}{2.625} \times 100$

$$\therefore \eta = 100\%$$

Note : As the average information per symbol (H) is equal to the average code length (L), the code efficiency is 100%.

Q. 13 Encode following symbols using Huffman source coding technique and calculate coding efficiency.

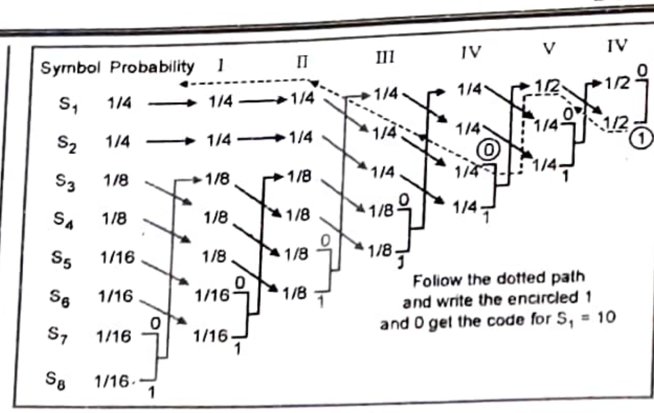
$$\left[\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \frac{1}{16}, \frac{1}{4}, \frac{1}{16}, \frac{1}{8}\right]$$

March 19

Ans. :

Step 1 : Huffman coding :

– The coding is as shown in Fig. 7.6.



(E-2068) Fig. 7.6

– The codewords for all messages are listed in Table 7.11.

Table 7.11

Message	1/4	1/4	1/8	1/8	1/16	1/16	1/16	1/16
Code	10	11	010	011	0000	0001	0011	0011
length	2	2	3	3	4	4	4	4

Step 2 : Source entropy (H) :

$$H = \sum_{i=1}^{8} P(x_i) \log_2 [1 / P(x_i)]$$

$$= \frac{1}{4} \log_2 4 + \frac{1}{4} \log_2 4 + 2\left[\frac{1}{8} \log_2 8\right] + 4\left[\frac{1}{16} \log_2 16\right]$$

$$H = 0.5 + 0.5 + (2 \times 0.375) + (4 \times 0.25)$$

$$= 2.75 \text{ bits / symbol}$$

Step 3 : Average code length (L) :

$$L = \sum_{k=1}^{8} P_k \times \text{length of symbol in bits}$$

$$= 2\left(\frac{1}{4} \times 2\right) + 2\left(\frac{1}{8} \times 3\right) + 4\left(\frac{1}{16} \times 4\right) = 1 + 0.75 + 1$$

$$= 2.75 \text{ bits / symbol}$$

Step 4 : Coding efficiency (η) :

$$\eta = \frac{H}{L} = \frac{2.75}{2.75} = 1 \text{ or } 100\%$$

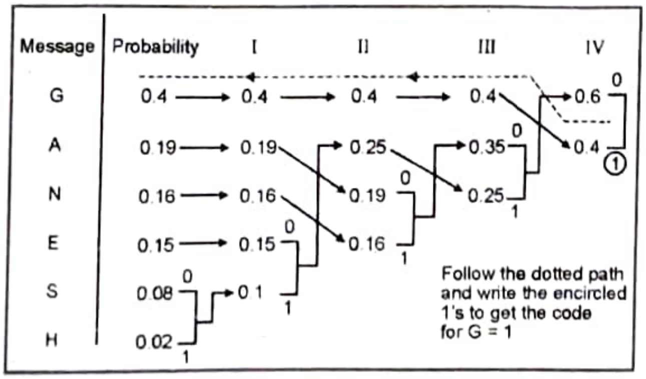
...Ans.

Q. 14 Apply Huffman coding for the symbols [A E H N G S] generated by a DMS with probabilities [0.19 0.15 0.02 0.16 0.4 0.08]. Also calculate coding efficiency.

May 19

Ans. :

– The Huffman coding is as shown in Fig. 7.7.



(E-2078) **Fig. 7.7 : Huffman coding**

– Similarly we can obtain the codes for all other messages. They are listed in Table 7.12.

(E-2082) **Table 7.12**

Message	A	E	H	N	G	S
Probability	0.19	0.15	0.02	0.16	0.4	0.08
Codeword	000	010	0111	001	1	0110
Length	3	3	4	3	1	4

1. Average codeword length (L) :

$$L = \sum_{k=1}^{6} P_k \text{. length of } k^{th} \text{ codeword}$$

$$= (0.19 \times 3) + (0.15 \times 3) + (0.02 \times 4) + (0.16 \times 3) + (0.4 \times 1) + (0.08 \times 4)$$

$$\therefore L = 2.3 \text{ bits / message}$$

2. Source entropy (H) :

$$H = \sum_{k=1}^{6} P_k \log_2 (1/P_k)$$

$$H = 0.19 \log_2 (1/0.19) + 0.15 \log_2 (1/0.15) + 0.02 \log_2 (1/0.02) + 0.16 \log_2 (1/0.16) + 0.4 \log_2 (1/0.4) + 0.08 \log_2 (1/0.08) = 2.2209 \text{ bits / symbol}$$

3. Coding efficiency : (η) :

$$\% \eta = \frac{H}{L} \times 100 = \frac{2.2209}{2.3} \times 100$$

$$= 96.56 \%$$ ...Ans.

Q. 15 Prove the properties of mutual information.

Dec. 07, Dec. 16, March 19

Ans. :

Properties of mutual information :

– The mutual information has the following properties :

Property 1 :

– The mutual information of a channel is symmetric. That means,

$$I(X;Y) = I(Y;X) \quad ...(1)$$

– where the mutual information $I(X;Y)$ is a measure of uncertainty about the channel input i.e. X and this uncertainty is resolved by making observations on the output side of the channel i.e. Y.

– And the mutual information $I(Y;X)$ is a measure of uncertainty on the channel output side i.e. Y which is resolved by sending the channel input.

Q. 16 A discrete source transmits messages x_1, x_2, x_3 with probabilities $p(x_1) = 0.3$, $p(x_2) = 0.25$, $p(x_3) = 0.45$. The source is connected to the channel whose conditional probability matrix is

$$P(Y/X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0.3 & 0.7 \end{bmatrix} \end{array}$$

Calculate all the entropies and mutual information with this channel.

May 01, May 18

Ans. :

Step 1 : Obtain the joint probability matrix P (X, Y) :

– The given matrix P (Y/X) is the conditional probability matrix.

– We can obtain the joint probability matrix P (X, Y) as :

$$P(X, Y) = P[Y/X] \cdot P(X)$$

$$\therefore P(X, Y) = \begin{bmatrix} 0.9 \times 0.3 & 0.1 \times 0.3 & 0 \\ 0 & 0.8 \times 0.25 & 0.2 \times 0.25 \\ 0 & 0.3 \times 0.45 & 0.7 \times 0.45 \end{bmatrix}$$

$$\therefore P(X, Y) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.27 & 0.03 & 0 \\ 0 & 0.2 & 0.05 \\ 0 & 0.135 & 0.315 \end{bmatrix} \end{array} \quad ...(1)$$

Step 2 : Obtain the probabilities p (y_1), p (y_2) and p (y_3) :

– The probabilities p (y_1), p (y_2) and p (y_3) can be obtained by adding the column entries of P (X , Y) matrix of Equation (1).

$$\therefore \quad p(y_1) = 0.27 + 0 + 0 = 0.27$$

$$p(y_2) = 0.03 + 0.2 + 0.135 = 0.365$$

$$p(y_3) = 0 + 0.05 + 0.315 = 0.365$$

Step 3 : Conditional probability matrix P (X/Y) :

– The conditional probability matrix P (X/Y) can be obtained by dividing the columns of the joint probability matrix P (X , Y) of Equation (1) by p (y_1), p (y_2) and p (y_3) respectively.

$$\therefore \quad P(X/Y) = \begin{bmatrix} 0.27/0.27 & 0.03/0.365 & 0/0.365 \\ 0/0.27 & 0.2/0.365 & 0.05/0.365 \\ 0/0.27 & 0.135/0.365 & 0.315/0.365 \end{bmatrix}$$

$$\therefore \quad P(X/Y) = \begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 1 & 0.082 & 0 \\ 0 & 0.5479 & 0.1369 \\ 0 & 0.3698 & 0.863 \end{bmatrix} \end{array}$$

Step 4 : The marginal entropies H (X) and H (Y) :

$$H(X) = \sum_{i=1}^{3} p(x_i) \log_2 [1/p(x_i)]$$

$$= p(x_1) \log_2 [1/p(x_1)] + p(x_2) \log_2 [1/p(x_2)] + p(x_3) \log_2 [1/p(x_3)]$$

– Substituting the values of p (x_1), p (x_2) and p (x_3) we get,

$$H(X) = 0.3 \log_2 (1/0.3) + 0.25 \log_2 (1/0.25) + 0.45 \log_2 (1/0.45)$$

$$= [(0.3 \times 1.7369) + (0.25 \times 2) + (0.45 \times 1.152)]$$

$$\therefore \quad H(X) = [0.521 + 0.5 + 0.5184]$$

$$= 1.5394 \text{ bits/message}$$ ...Ans.

$$\text{Similarly } H(Y) = p(y_1) \log_2 [1/y_1] + p(y_2) \log_2 [1/y_2] + p(y_3) \log_2 [1/y_3]$$

$$= 0.27 \log_2 [1/0.27] + 0.365 \times 2 \times \log_2 [1/0.365]$$

$$H(Y) = 0.51 + 1.0614$$

$$= 1.5714 \text{ bits/message}$$ ...Ans.

Step 5 : The conditional entropy H (X / Y) :

$$H(X/Y) = -\sum_{i=1}^{3} \sum_{j=1}^{3} p(x_i, y_j) \log_2 p(x_i/y_j)$$

$\therefore$ H (X/Y)

$$= -p(x_1, y_1) \log_2 p(x_1/y_1) - p(x_1, y_2) \log_2 p(x_1/y_2) - p(x_1, y_3) \log_2 p(x_1/y_3) - p(x_2, y_1) \log_2 p(x_2/y_1) - p(x_2, y_2) \log_2 p(x_2/y_2) - p(x_2, y_3) \log_2 p(x_2/y_3) - p(x_3, y_1) \log_2 p(x_3/y_1) - p(x_3, y_2) \log_2 p(x_3/y_2) - p(x_3, y_3) \log_2 p(x_3/y_3)$$

– Refer the joint and conditional matrices given in Fig. 7.8.

P(X / Y)

	y_1	y_2	y_3
x_1	0.27	0.03	0
x_2	0	0.2	0.05
x_3	0	0.135	0.315

P(X / Y)

	y_1	y_2	y_3
x_1	1	0.0821	0
x_2	0	0.5479	0.1369
x_3	0	0.3698	0.863

(E-1700) **Fig. 7.8**

– Substituting various values from these two matrices we get,

$$H(X/Y) = -0.27 \log_2 1 - 0.03 \log_2 (0.0821) - 0 - 0 - 0.2 \log_2 (0.5479) - 0.05 \log_2 (0.1369) - 0 - 0.135 \log_2 (0.3698) - 0.315 \log_2 (0.863) \quad ...(2)$$

$$= 0 + 0.108 + 0.1736 + 0.1434 + 0.1937 + 0.0669$$

$$\therefore \quad H(X/Y) = 0.6856 \text{ bits / message}$$ ...Ans.

Step 6 : The joint entropy H (X , Y) :

– The joint entropy H (X , Y) is given by,

$$H(X, Y) = -\sum_{i=1}^{3} \sum_{j=1}^{3} p(x_i, y_j) \cdot \log_2 p(x_i, y_j)$$

$\therefore$ H (X, Y)

$$= -[p(x_1, y_1) \log_2 p(x_1, y_1) + p(x_1, y_2) \log_2 p(x_1, y_2) + p(x_1, y_3) \log_2 p(x_1, y_3) + p(x_2, y_1) \log_2 p(x_2, y_1) + p(x_2, y_2) \log_2 p(x_2, y_2) + p(x_2, y_3) \log_2 p(x_2, y_3) + p(x_3, y_1) \log_2 p(x_3, y_1) + p(x_3, y_2) \log_2 p(x_3, y_2) + p(x_3, y_3) \log_2 p(x_3, y_3)]$$

– Referring to the joint matrix we get,

$$H(X, Y) = -[0.27 \log_2 0.27 + 0.03 \log_2 0.03 + 0 + 0 + 0.2 \log 0.2 + 0.05 \log 0.05 + 0 + 0.135 \log_2 0.135 + 0.315 \log_2 0.315]$$

$$= [0.51 + 0.1517 + 0.4643 + 0.216 + 0.39 + 0.5249]$$

$$\therefore \quad H(X, Y) = 2.2569 \text{ bits/message}$$ ...Ans.

Step 7 : The mutual information :

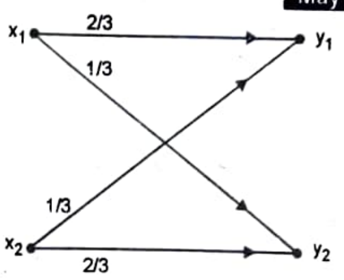
– Mutual information, is given by,

$$I[X, Y] = H(X) - H(X/Y) = 1.5394 - 0.6856$$

$$= 0.8538 \text{ bits.}$$ ...Ans.

Q. 17 A discrete source transmits messages X_1 and X_2 with probabilities 3/4 and 1/4. The source is connected to the channel given below. Calculate all entropies and mutual information.

May 07, Dec. 16



(E-1190) Fig. 7.9

Ans. :

Given : $p(x_1) = 3/4,\ p(x_2) = 1/4$

$\therefore\ p(X) = [\,3/4, 1/4\,]$

Step 1 : Find P (Y /X) :

- The conditional probability matrix for the given channel is as follows :

$$P(Y/X) = \begin{bmatrix} P(Y_1/X_1) & P(Y_2/X_1) \\ P(Y_1/X_2) & P(Y_2/X_2) \end{bmatrix}$$

$$= \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \quad \text{... Referring the given Fig. 7.9.}$$

Step 2 : Find P (X,Y) :

- P (X, Y) is a joint probability matrix. We can obtain it as follows :

$$p(x_1, y_1) = p(x_1) \cdot p(y_1/x_1) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$

$$p(x_1, y_2) = p(x_1) \cdot p(y_2/x_1) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$$

$$p(x_2, y_2) = p(x_2)\, p(y_1/x_2) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

$$p(x_2, y_2) = p(x_2)\, p(y_2/x_2) = \frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$$

$$\therefore\ P(X, Y) = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 1/2 & 1/4 \\ 1/12 & 1/6 \end{bmatrix} \end{array}$$

Step 3 : Find p (y_1), p (y_2) and P (Y) :

$$P(y_1) = p(x_1, y_1) + p(x_2, y_1) = 1/2 + 1/12 = 7/12$$

$$P(y_2) = p(x_1, y_2) + p(x_2, y_2) = 1/4 + 1/6 = 5/12$$

$$\therefore\ P(Y) = [7/12, 5/12]$$

Step 4 : Find all entropies :

$$H(X) = p(x_1) \log_2 [\,1/p(x_1)\,] + p(x_2) \log_2 [1/p(x_2)\,]$$

$$= 3/4 \log_2 (4/3) + 1/4 \log_2 (4)$$

$$= 0.81 \text{ bits/message} \quad \textbf{...Ans.}$$

$$H(Y) = p(y_1) \log_2 [\,1/p(y_1)] + p(y_2) \log_2 [\,1/p(y_2)]$$

$$= \frac{7}{12} \log_2 (12/7) + \frac{5}{12} \log_2 (12/5)$$

$$= 0.98 \text{ bits/ message} \quad \textbf{...Ans.}$$

$$H(XY) = \sum_{j=1}^{m} \sum_{k=1}^{n} p(x_i, y_j) \log_2 \frac{1}{p(x_i, y_j)}$$

$$= \frac{1}{2} \log_2 2 + \frac{1}{4} \log_2 4 + \frac{1}{12} \log_2 12 + \frac{1}{6} \log_2 6$$

$$= 1.73 \text{ bits/message} \quad \textbf{...Ans.}$$

$$H(X/Y) = H(XY) - H(Y)$$

$$= 1.73 - 0.98 = 0.75 \text{ bits/ message} \quad \textbf{...Ans.}$$

$$H(Y/X) = H(XY) - H(X)$$

$$= 1.73 - 0.81 = 0.92 \text{ bits / message} \quad \textbf{...Ans.}$$

Step 5 : Mutual information :

$$I(X, Y) = H(X) - H(X/Y) = 0.81 - 0.75$$

$$= 0.06 \text{ bits/ message} \quad \textbf{...Ans.}$$

Q. 18 Prove that for a finite variance σ^2, the Gaussian random variable has the largest differential entropy attainable by any random variable.

Show that entropy is given by $\frac{1}{2} \log_2 [2\pi e \sigma^2]$

May 08, May 16

Ans. :

- Consider an arbitrary pair of random variable X and Y.
- Let their PDFs are denoted by $f_Y(x)$ and $f_X(x)$ respectively.

$$\sum_{k=0}^{k-1} p_k \log_2 \left[\frac{q_k}{p_k}\right] \le 0 \qquad \text{for } q_k = p_k \text{ for all k} \quad \text{... (1)}$$

- We can apply this fundamental inequality to the given situation to write,

$$\int_{-\infty}^{\infty} f_Y(x) \log_2 \left[\frac{f_X(x)}{f_Y(x)}\right] dx \le 0 \quad \text{... (2)}$$

OR

$$-\int_{-\infty}^{\infty} f_Y(x) \log_2 f_Y(x)\, dx \le -\int_{-\infty}^{\infty} f_Y(x) \log_2 f_X(x)\, dx \quad \text{.... (3)}$$

- But LHS of Equation (3) represents the differential entropy of random variable Y.

$$\therefore h(Y) \le -\int_{-\infty}^{\infty} f_Y(x) \log_2 f_X(x)\, dx$$

- Now imagine that random variables X and Y are described as follows :
  1. X and Y have the same mean m and same variable σ^2
  2. R. V. X. has a Gaussian distribution.

$$\therefore f_x(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-m)^2/2\sigma^2} \quad ...(5)$$

- Substitute Equation (5) into (4) and change the base of logarithm from 2 to e = 2.7183 to get,

$$\therefore h(Y) \le -\log_2 e \int_{-\infty}^{\infty} f_Y(x)\left[\frac{-(x-m)^2}{2\sigma^2} - \log_e(\sqrt{2\pi\sigma})\right] dx \quad (6)$$

- Now let us use the following properties of R. V. Y.

$$\int_{-\infty}^{\infty} f_Y(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} (x-m)^2 f_Y(x)\, dx = \sigma^2$$

- Hence Equation (6) gets simplified as follows :

$$h(Y) \le \frac{1}{2}\log_2(2\pi e\sigma^2) \quad ...(7)$$

- The RHS of Equation (7) represents the differential entropy of the Gaussian R. V. X.

$$\therefore h(X) = \frac{1}{2}\log_2(2\pi e\sigma^2) \quad ...(8)$$

- So from Equations (7) and (8) we can write,

$$h(Y) \le h(X) \begin{cases} X = \text{Gaussian Random Variable} \\ Y = \text{Another Random Variable} \end{cases} \quad ...(9)$$

and the equality holds only if X = Y.

- Equation (9) tells us that for a finite variance σ^2 the Gaussian random variable X has the largest differential entropy than any other random variable Y.
- Consider the RHS of Equation (6). It is the differential entropy h(x)

$$\text{RHS} = h(X)$$

$$= -\log_2 e \int_{-\infty}^{\infty} f_Y(x)\, dx \left[\frac{-(x-m)^2}{2\sigma^2} - \log_e(\sqrt{2\pi\sigma})\right] dx$$

$$= -\log_2 e \left\{\int_{-\infty}^{\infty} \frac{-(x-m)^2}{2\sigma^2} f_Y(x)\, dx - \int_{-\infty}^{\infty} \log_e \sqrt{2\pi\sigma}\; f_Y(x)\, dx\right\}$$

$$= -\log_2 e\left\{-\frac{1}{2\sigma^2}\times\sigma^2 - \log_e\sqrt{2\pi\sigma}\times 1\right\} \quad ...(4)$$

$$= \frac{1}{2}\log_2 e + \log_2 e \log_e \sqrt{2\pi\sigma}$$

$$\therefore h(X) = \frac{1}{2}\log_2(2\pi e\,\sigma^2) \quad \text{...Ans.}$$

Q. 19 A discrete source emits messages x_1 and x_2 with probabilities 3/4 and 1/4 with binary symmetric channels. Find H(X), H(Y), H(X,Y), H(X/Y), H(Y/X), I (X ; Y) if probability p = 1/3 draw channel diagram. **May 12, Dec. 19**

Ans. :

Given : p (x_1) = 3/4, p (x_2) = 1/4, BSC.

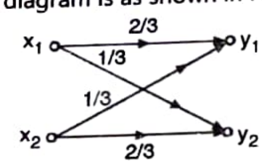
p = 1/3 ∴ (1 – p) = 2/3

To find : H(X), H(Y), H(X,Y), H(X/Y), I(X;Y)

To draw the channel diagram.

Step 1 : Draw the channel diagram :

- The channel diagram is as shown in Fig. 7.10.



(E-1698) Fig. 7.10 : Channel diagram

Step 2 : Find H(X) :

- Source entropy H(X) is given by,

$$H(X) = -p(x_1)\log_2 p(x_1) - p(x_2)\log_2 p(x_2)$$

$$= -\frac{3}{4}\log_2(0.75) - \frac{1}{4}\log_2(0.25)$$

$$= 0.3113 + 0.5 = 0.8113 \text{ bits/message} \quad \text{...Ans.}$$

Step 3 : Find H(Y) and H(X, Y) :

- From the channel diagram, we can obtain the conditional probability matrix as follows :

$$P(Y/X) = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \end{array}$$

Also $P(X) = [x_1 \quad x_2] = [3/4 \quad 1/4]$

- We can obtain the joint probability matrix P (X,Y) with the help of P (X) and P (Y/X) as follows :

$$P(x_1, y_1) = P(x_1)\cdot P(y_1/x_1) = 3/4 \times 2/3 = 1/2$$

$$P(x_1, y_2) = P(x_1)\cdot P(y_2/x_1) = 3/4 \times 1/3 = 1/4$$

$$P(x_2, y_1) = P(x_2)\cdot P(y_1/x_2) = 1/4 \times 1/3 = 1/12$$

$P(x_2, y_2) = P(x_2) \cdot P(y_2/x_2) = 1/4 \times 2/3 = 1/6$

$$P(X, Y) = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 1/2 & 1/4 \\ 1/12 & 1/6 \end{bmatrix} \end{array}$$

– From this matrix, we can obtain P (Y) as follows :

$P(y_1) = P(x_1, y_1) + P(x_2, y_1) = 1/2 + 1/12 = 7/12$

– Similarly,

$P(y_2) = P(x_1, y_2) + P(x_2, y_2) = 1/4 + 1/6 = 5/12$

$\therefore P(Y) = [7/12 \quad 5/12]$

$\therefore H(Y) = -P(y_1) \log_2 P(y_1) - P(y_2) \log_2 P(y_2)$

$= -7/12 \log_2 (7/12) - 5/12 \log_2 (5/12)$

$\therefore H(Y) = 0.4536 + 0.5263 = 0.98$ bits/message ...Ans.

– Now let us obtain H (XY)

$$H(XY) = -\sum_{j=1}^{m} \sum_{k=1}^{n} P(x_j y_k) \log_2 P(x_j y_k)$$

– From the P (X,Y) matrix we get,

$H(XY) = -1/2 \log_2 (1/2) - 1/4 \log_2 (1/4)$

$- 1/12 \log_2 (1/12) - 1/6 \log_2 (1/6)$

$\therefore H(XY) = 1.73$ bits / message ...Ans.

Step 4 : Find H (X/Y), H (Y/X) and I (X ; Y) :

$H(X/Y) = H(XY) - H(Y) = 1.73 - 0.98$

$= 0.75$ bits / message ...Ans.

$H(Y/X) = H(XY) - H(X) = 1.73 - 0.8113$

$= 0.9187$ bits/message ...Ans.

$I(X;Y) = H(X) - H(X/Y) = 0.8113 - 0.75$

$= 0.0613$ bits/message ...Ans.

Q. 20 Find H (x), H (y), H (x, y) and I (x, y) if the joint probabilities of communication system are given as :

$$P(x, y) = \begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}$$

Feb. 16

Ans. :

– The given channel matrix is,

$$P(x, y) = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix} \end{array}$$

Step 1 : Obtain the individual probabilities :

$P(x_1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(x_2) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(y_1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(y_2) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Step 2 : Find H(X) and H(Y) :

$$H(X) = P(x_1) \log_2 \left(\frac{1}{P(x_1)}\right) + P(x_2) \log_2 \left(\frac{1}{P(x_2)}\right)$$

$H(X) = \frac{1}{2} \log_2 2 + \frac{1}{2} \log_2 2 = 1$ ...Ans.

$$H(Y) = P(y_1) \log_2 \left[\frac{1}{P(y_1)}\right] + P(y_2) \log_2 \left[\frac{1}{P(y_2)}\right]$$

$= \frac{1}{2} \log_2 2 + \frac{1}{2} \log_2 2 = 1$ ...Ans.

Step 3 : Find H (X, Y) :

H (X, Y)

$$= P(x_1, y_1) \log_2 \left[\frac{1}{P(x_1, y_1)}\right] + P(x_1, y_2) \log_2 \left[\frac{1}{P(x_1, y_2)}\right]$$

$$+ P(x_2, y_1) \log_2 \left[\frac{1}{P(x_2, y_1)}\right] + P(x_2, y_2) \log_2 \left[\frac{1}{P(x_2, y_2)}\right]$$

$= 4 \times \frac{1}{4} \log_2 4$

$\therefore H(X, Y) = 4 \times \frac{1}{4} \times 2 \log_2 2 = 2$ bits ...Ans.

Step 4 : Find I (X, Y) :

$I(X, Y) = H(X) - H(X/Y)$

$H(X/Y) = H(X, Y) - H(Y) = 2 - 1 = 1$ bit.

$\therefore I(X, Y) = 1 - 1 = 0$ bit/message ...Ans.

Q. 21 Find mutual information for the channel matrix given below. **Dec. 13, May 17**

$$P(X, Y) = \begin{bmatrix} 0.3 & 0.05 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0.15 & 0.05 \\ 0 & 0.05 & 0.15 \end{bmatrix}$$

OR

The joint probability matrix representing transmitter and receiver is given below. Find all entropies and mutual information of the communication system.

$$P(X, Y) = \begin{bmatrix} 0.3 & 0.05 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0.15 & 0.05 \\ 0 & 0.05 & 0.15 \end{bmatrix}$$

Ans. :

– The given matrix is a joint probability matrix.

Step 1 : Find P(X) and P(Y) :

Find P(X) :

– The sum of row elements will give us the matrix P(X).

$$\therefore P(X) = [\,0.35 \;\; 0.25 \;\; \underbrace{0.20}_{\text{Sum of row - 3 elements}} \;\; \underbrace{0.20}_{\text{Sum of row - 4 elements}}\,]$$

(E-1701)

Find P(Y) :

– The sum of column elements will give us matrix P(Y).

$$\therefore P(Y) = [\,0.3 \;\; 0.5 \;\; \underbrace{0.2}_{\text{Sum of column - 3 elements}}\,]$$

(E-1702)

Step 2 : Find H(X), H(Y) and H (X, Y) :

$$H(X) = \sum_{j=1}^{m} p(x_j) \log \frac{1}{p(x_j)}$$

– From the matrix P(x) we get,

$$H(X) = 0.35 \log \frac{1}{0.35} + 0.25 \log \frac{1}{0.25} + 0.2 \log \frac{1}{0.2} + 0.2 \log \frac{1}{0.2}$$

$$= 1.96 \text{ bits / message}$$

– From the matrix P(Y) we get,

$$H(Y) = \sum_{k=1}^{n} p(y_k) \log \frac{1}{p(y_k)}$$

$$= 0.3 \log \frac{1}{0.3} + 0.05 \log \frac{1}{0.05} + 0.2 \log \frac{1}{0.2}$$

$$= 1.49 \text{ bits / message}$$

$$H(X,Y) = \sum_{j=1}^{m} \sum_{k=1}^{n} p(x_j\, y_k) \log \frac{1}{(x_j\, y_k)}$$

– From the given matrix p (x, y) we get,

$$H(X,Y) = 0.3 \log \frac{1}{0.3} + 0.05 \log \frac{1}{0.05} + 0.25 \log \frac{1}{0.25} + 0.15 \log \frac{1}{0.15} + 0.05 \log \frac{1}{0.05} + 0.05 \log \frac{1}{0.05} + 0.15 \log \frac{1}{0.15}$$

$$\therefore H(X,Y) = 2.49 \text{ bits / message}$$

Step 3 : Find the mutual information I (X, Y) :

$$H(X/Y) = H(XY) - H(Y)$$

$$= 2.49 - 1.49 = 1 \text{ bit / message}$$

$$\therefore I(X;Y) = H(X) - H(X/Y)$$

$$= 1.96 - 1 = 0.96 \text{ bits / message} \qquad \text{...Ans.}$$

Q. 22 A DMC having channel transition matrix as, $\begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$ emits equiprobable messages X_1 and X_2. Draw channel diagram and find H(X), H(Y), H(X, Y), H(X / Y), I(X, Y), Comment on type of channel. **April 18**

Ans. :

Step 1 : Draw the channel diagram :

– The given channel matrix is as follows :

$$p(x, y) = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

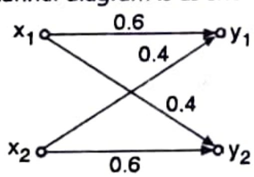
$$\therefore p(y_1 / x_1) = 0.6$$

$$p(y_2 / x_2) = 0.4$$

$$p(y_2 / x_1) = 0.4$$

$$p(y_2 / x_2) = 0.6$$

– Hence the channel diagram is as shown in Fig. 7.11.



(G-2474) Fig. 7.11 : Channel diagram

Step 2 : Obtain H(X) and H(Y) :

Find H(X) :

$$H(X) = -p(x_1) \log_2 p(x_1) - p(x_2) \log_2 p(x_2)$$

$$= -\frac{1}{2} \log_2 (1/2) - \frac{1}{2} \log_2 (1/2)$$

$$\therefore H(X) = 1 \text{ bit/message} \qquad \text{...Ans.}$$

Find p (y_1) and p (y_2) :

– Let us find p (y_1) and p (y_2). Refer Fig. P. 7.11 to write,

$$p(y_1) = 0.6\, p(x_1) + 0.4\, p(x_2)$$

$$= (0.6 \times 0.5) + (0.4 \times 0.5) = 0.5$$

$$p(y_2) = 0.4\, p(x_1) + 0.6\, p(x_2)$$

$$= (0.4 \times 0.5) + (0.6 \times 0.5) = 0.5$$

Find H (Y) :

$$H(Y) = -p(y_1)\log_2 p(y_1) - p(y_2)\log_2 p(y_2)$$

$$\therefore H(Y) = -0.5\log_2 0.5 - 0.5\log_2 0.5$$

$$= 1 \text{ bit / message}$$ ...Ans.

Step 3 : Calculate H (X, Y) :

$$H(X,Y) = -p(x_1,y_1)\log_2 p(x_1,y_1) - p(x_1,y_2)\log_2 p(x_1,y_2) - p(x_2,y_1)\log_2 p(x_2,y_1) - p(x_2,y_2)\log_2 p(x_2,y_2)$$

$$\therefore H(X,Y) = -0.6\log_2 0.6 - 0.4\log_2 0.4 - 0.4\log_2 0.4 - 0.6\log_2 0.6$$

$$= 2(0.442) + (0.529 \times 2)$$

$$= 1.942 \text{ bits / message}$$

Step 4 : Find H (X / Y) :

$$H(X/Y) = H(X,Y) - H(Y) = 1.942 - 1$$

$$\therefore H(X/Y) = 0.942 \text{ bits/message}$$ ...Ans.

Step 5 : Find I (X, Y) :

$$I(X,Y) = H(X) - H(X/Y)$$

$$= 1 - 0.942$$

$$= 0.058$$ ...Ans.

Q. 23 Define channel capacity. **Dec. 18**

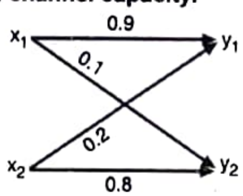
Ans. :

Definition :

- The channel capacity is denoted by "C" and in simple terms defined as the maximum possible bit rate a channel can support without allowing introduction of any errors.
- The unit of channel capacity is bits/sec.

Q. 24 Consider the given binary channel :

1. **Construct the channel matrix.**
2. **Find out the value of P (y) ; if the source is equiprobable.**
3. **Calculate all entropies, mutual information and channel capacity.** **Dec. 14**



(G-1741) **Fig. 7.12**

Ans. :

Step 1 : Construct the channel matrix :

- The channel matrix for the given channel is given by,

$$P(Y/X) = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$ ...Ans.

Step 2 : Value of P(Y) :

- The sources are equiprobable.

$$\therefore P(x_1) = P(x_2) = 0.5$$

$$\therefore P(X) = [0.5 \quad 0.5]$$

- We know that $p(x_j, y_k) = p(x_j) \cdot p(y_k/x_j)$
- Using this expression we get,

$$P(x_1, y_1) = P(x_1) \cdot P(y_1/x_1) = 0.5 \times 0.9 = 0.45$$

$$P(x_1\, y_2) = P(x_1) \cdot P(y_2/x_1) = 0.5 \times 0.1 = 0.05$$

$$P(x_2\, y_1) = P(x_2)\, P(y_1/x_2) = 0.5 \times 0.2 = 0.1$$

$$P(x_2\, y_2) = P(x_2)\, P(y_2/x_2) = 0.5 \times 0.8 = 0.4$$

- Hence joint probability matrix P (X, Y) is as follows :

$$P(X,Y) = \begin{bmatrix} 0.45 & 0.05 \\ 0.1 & 0.4 \end{bmatrix}$$

$P(y_1) = 0.45 + 0.1 = 0.55$ ...Adding elements of column 1

$P(y_2) = 0.05 + 0.4 = 0.45$ ...Adding elements of column 2

$\therefore P(Y) = [0.55 \quad 0.45]$ ...Ans.

- We can also find P(Y) using the following alternative method.

$$P(Y) = [P(X)]\,[P(Y/X)]$$

$$= [0.5 \quad 0.5]\begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

$$= [0.55 \quad 0.45]$$ ...Ans.

Step 3 : Calculate all the entropies :

1. $$H(X) = p(x_1)\log_2(1/p(x_1)) + p(x_2)\log_2(1/p(x_2))$$

$$= 0.5\log_2 1/0.5 + 0.5\log_2(1/0.5)$$

$$= 1 \text{ bit/message}$$ ...Ans.

2. $$H(Y) = p(y_1)\log_2[1/p(y_1)] + p(y_2)\log_2[1/p(y_2)]$$

$$= 0.55\log_2[1/0.55] + 0.45\log_2[1/0.45]$$

$$= 0.4744 + 0.5184$$

$$= 0.9928 \text{ bits/message}$$ ...Ans.

3. $$H(X,Y) = \sum_{i=1}^{m}\sum_{j=1}^{n} p(x_i\, y_j)\log_2 \frac{1}{p(x_i\, y_j)}$$

- Referring to the P (X,Y) matrix we get,

$$H(X,Y) = -0.45 \log_2 (0.45) - 0.05 \log_2 (0.05) - 0.1 \log_2 (0.1) - 0.4 \log_2 (0.4)$$
$$= 0.5184 + 0.2161 + 0.3323 + 0.5288$$
$$= 1.5955 \text{ bits / message} \quad \text{...Ans.}$$

4. $H(X/Y) = H(XY) - H(Y)$
$$= 1.5955 - 0.9928 = 0.6027 \text{ bits/message} \quad \text{...Ans.}$$

5. $H(Y/X) = H(XY) - H(X)$
$$= 1.5955 - 1 = 0.5955 \text{ bits/message} \quad \text{...Ans.}$$

Step 4 : Calculate the mutual information :

$$I(X,Y) = H(X) + H(Y) - H(XY)$$
$$= 1 + 0.9928 - 1.5955$$
$$= 0.3973 \text{ bits/message} \quad \text{...Ans.}$$

Step 5 : Channel capacity :

– From Fig. 7.12 we get,

$P_{11} = 0.9, P_{12} = 0.1, P_{21} = 0.2, P_{22} = 0.8$

$$\therefore \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 0.9 \log_2 0.9 + 0.1 \log_2 0.1 \\ 0.2 \log_2 0.2 + 0.8 \log_2 0.8 \end{bmatrix}$$
$$= \begin{bmatrix} -0.4690 \\ -0.7219 \end{bmatrix}$$

$$\therefore 0.9 Q_1 + 0.1 Q_2 = -0.4690$$
$$\therefore Q_2 = \frac{(-0.4690 - 0.9 Q_1)}{0.1}$$
$$= -4.69 - 9 Q_1 \quad \text{...(1)}$$

$$\text{Also } 0.2 Q_1 + 0.8 Q_2 = -0.7219 \quad \text{...(2)}$$

– Substituting Equation (2) into (1) we get,

$$0.2 Q_1 + 0.8 (-4.69 - 9 Q_1) = -0.7219$$
$$\therefore 0.2 Q_1 - 3.752 - 7.2 Q_1 = -0.7219$$
$$\therefore Q_1 = \frac{3.03}{-7} = -0.4328 \quad \text{...(3)}$$

– Substituting Equation (3) into Equation (1) we get,

$$Q_2 = -4.69 - 9 \times -0.4328$$
$$= -0.7942 \quad \text{...(4)}$$

– Channel capacity is given by,

$$C = \log_2 (2^{Q1} + 2^{Q2})$$
$$\therefore C = \log_2 (2^{-0.4328} + 2^{-0.7942})$$
$$\therefore C = \log_2 (0.7408 + 0.5767)$$
$$= \log_2 (1.3175)$$
$$\therefore C = 0.3978 \text{ bits/message} \quad \text{...Ans.}$$

Q. 25 Explain channel coding theorem.

Dec. 11, April 18, May 18, Dec. 18

Ans. :

Statement :

– The statement of the Shannon's theorem is : Given that a source of M equally likely messages with M >> 1, which is generating information at a rate R. Given that a channel of capacity "C" exists.

– Then if,

$$R \leq C$$

– Then there exists a "coding" technique such that the output of the source may be transmitted over the channel with a probability of error in the received message which may be made arbitrarily small.

Meaning :

1. The theorem talks about the rate of transmission of information (R) over a communication channel.
2. The channel capacity "C" is a rate of transmission in bits/sec. According to the theorem if $R \leq C$, then it is possible to use a coding technique and make an error free transmission even in the presence of noise.

– There is a negative statement associated with the Shannon's theorem. It's statement is as follows :

Negative statement of Shannon's theorem :

– Given a source of equally likely messages with M >> 1, which is generating an information at a rate R, and if

$$R > C$$

– then the probability of error is close to unity for every possible set of M transmitted signals.

Meaning :

1. If the information rate R exceeds a specific value "C", then the error probability will increase towards unity as M increases.
2. When R > C, complexity of coding is increased which results in an increase in the probability of error.

Critical rate :

– Let the source entropy by H, the source produces symbols one per T_s seconds.

– Let the channel capacity be C and be used every T_c seconds.

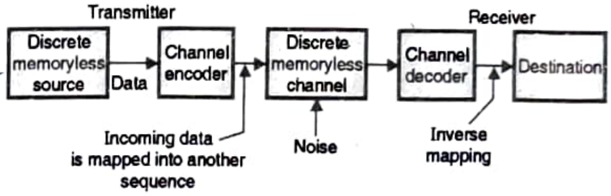
Channel coding :

– Due to the presence of noise in a communication channel, the errors are introduced in the digital signals.

– If the channel is too noisy the error probability is very high.

remaining

- A probability of error equal to 10^{-6} or lower is required for many applications.
- It is possible to raise the level of performance by using the **channel coding.**
- The goal of designing a channel coding scheme is to increase the resistance of the digital communication system to the channel noise.
- Channel coding is a process of mapping the incoming data sequence into another sequence applied to the channel and inverse mapping the sequence at the output of the channel into an output data sequence in such a way that the effect of noise is minimized.
- This process is illustrated in Fig. 7.13.



(E-149) **Fig. 7.13 : Digital communication system showing channel coding**

- The process of mapping takes place at the transmitter and it is performed by the channel encoder.
- The inverse mapping takes place at the receiver by the channel decoder as shown in Fig. 7.13.
- Then according to channel coding theorem we can write,

$$R \leq C$$

$$H \times r \leq C / T_c$$

$$\text{Where } r = \text{Symbol rate} = 1 / T_s$$

$$\therefore \quad H / T_s \leq C / T_c \qquad ...(1)$$

- The ratio C / T_c is called as critical rate and if $H \times r = C / T_c$ then the system is said to be signaling at the critical rate.

Importance of channel coding theorem :

- The channel coding theorem is the most important theorem of information theory because it specifies the channel capacity C as the fundamental limit on the rate at which the transmission of reliable error free message can take place over a discrete memoryless channel.

Limitations of channel coding theorem :

1. This theorem does not tell us how to construct a code so as to transmit without errors on a noisy channel.
2. This theorem does not have an accurate value of the probability of symbol error after decoding the channel output. But it tells us that the probability of symbol error tends to zero with increase in the code length.

Q. 26 State and explain Shannon's information capacity theorem.

Imp

Dec. 11, May 12, May 18, May 19

Ans. :

Statement of the theorem :

- The channel capacity of a continuous channel of bandwidth B Hz having an additive white Gaussian noise of power spectral density $N_0/2$ over a limited bandwidth of B is given by,

$$C = B \log_2\left(1 + \frac{S}{N_0 B}\right) \text{ bits per second} \qquad ...(1)$$

- The channel capacity of a white, band limited Gaussian channel is given by,

$$C = B \log_2\left(1 + \frac{S}{N}\right) \text{ bits/sec}$$

where, B = Channel bandwidth

S = Average transmitted signal power

N = Noise power within the channel bandwidth

- We know that,

$$\text{power } S = \int_{-B}^{B} \text{power spectral density.}$$

- Using this equation we can find the noise power "N" as follows,

$$\text{Noise power } N = \int_{-B}^{B} \frac{N_0}{2}\, df$$

where $(N_0 / 2)$ is the power spectral density of the white Gaussian noise.

$$\therefore \quad N = N_0 B \qquad ...(2)$$

- Substituting this value of N we get,

$$\therefore \quad C = B \log_2\left[1 + \frac{S}{N_0 B}\right] \qquad ...(3)$$

Observations :

- The major observations from Equation (3) are as follows :

1. This theorem provides a relation among three important system parameters namely channel bandwidth B, average transmitted power (S) and the noise power spectral density at the channel output.
2. Channel capacity C depends directly on the bandwidth B, but it depends on the ratio $(S / N_0 B)$ in the logarithmic manner.
3. Therefore it is easier to increase the information capacity C of a channel by increasing bandwidth B rather than by increasing the signal power S if we assume the noise variance to be constant.
4. This theorem implies that if the average transmitted power S and channel bandwidth B are constant, we can transmit information at the rate of C bits per second with a low probability of error by using a suitable encoding system.
5. It is not possible to transmit at a rate higher than C bits per seconds by any encoding system without a definite probability of error.

- Hence this theorem defines the fundamental limit on the rate of error free transmission for a power limited, band limited Gaussian channel.
- This limit can be approached if and only if the statistical properties of the transmitted signal approximate those of white Gaussian noise.

> **Note :** It is always easier to increase the information capacity of a communication channel by increasing the bandwidth instead of increasing the transmitted power.

Q. 27 A channel has a bandwidth of 5 kHz and a signal to noise power ratio 63. Determine the bandwidth needed if the S/N power ratio is reduced to 31. What will be the signal power required if the channel bandwidth is reduced to 3 kHz ? Dec. 11, May 19

Ans. :

1. To determine the channel capacity :

- It is given that B = 5 kHz and $\frac{S}{N} = 63$. Hence using the Shannon Hartley theorem the channel capacity is given by,

$$C = B \log_2 \left[1 + \frac{S}{N}\right] = 5 \times 10^3 \log_2 [1 + 63]$$

$$\therefore \quad C = 30 \times 10^3 \text{ bits/sec} \qquad ...(1)$$

2. To determine the new bandwidth :

- The new value of $\frac{S}{N} = 31$. Assuming the channel capacity "C" to be constant we can write,

$$30 \times 10^3 = B \log_2 [1 + 31]$$

$$\therefore \quad B = \frac{30 \times 10^3}{5} = 6 \text{ kHz} \qquad ...(2)$$

3. To determine the new signal power :

- Given that the new bandwidth is 3 kHz. We know that noise power $N = N_0 B$.
- Let the noise power corresponding to a bandwidth of 6 kHz be $N_1 = 6 N_0$ and the noise power corresponding to the new bandwidth of 3 kHz be $N_2 = 3 N_0$.

$$\therefore \quad \frac{N_1}{N_2} = \frac{6 N_0}{3 N_0} = 2 \qquad ...(3)$$

The old signal to noise ratio $= \frac{S_1}{N_1} = 31$

$$\therefore \quad S_1 = 31 N_1 \qquad ...(4)$$

- The new signal to noise ratio $= \frac{S_2}{N_2}$. We do not know its value, hence let us find it out.

$$30 \times 10^3 = 3 \times 10^3 \log_2 \left(1 + \frac{S_2}{N_2}\right)$$

$$\therefore \quad \frac{S_2}{N_2} = 1023 \qquad ...(5)$$

$$\therefore \quad S_2 = 1023 N_2$$

But from Equation (3), $N_2 = \frac{N_1}{2}$, substituting we get,

$$\therefore \quad S_2 = 1023 \frac{N_1}{2} \qquad ...(6)$$

- Dividing Equation (6) by Equation (4) we get,

$$\frac{S_2}{S_1} = \frac{1023 N_1}{2 \times 31 N_1} = 16.5$$

$$\therefore \quad S_2 = 16.5 S_1 \qquad \textbf{...Ans.}$$

- Thus if the bandwidth is reduced by 50% then the signal power must be increased 16.5 times i.e. 1650% to get the same capacity.

Q. 28 The channel capacity is given by,

$C = B \log_2\left[1 + \frac{S}{N}\right]$ In the presence of white Gaussian noise, with a constant signal power the channel capacity reaches its upper limit with increase in the bandwidth B. Prove that this upper limit of C is given by, $C_\infty = 1.44 \frac{S}{N_0}$. **Feb. 16**

Ans. :

Consider the equation $C = B \log_2\left[1 + \frac{S}{N}\right]$

- As the noise present is white Gaussian noise, the noise power N can be expressed as,

$$N = N_0 B \quad ...(1)$$

- This is because the power spectral density of white Gaussian noise is $(N_0/2)$. Substitute this value of N into the equation for C to get,

$$C = B \log_2\left[1 + \frac{S}{N_0 B}\right] \quad ...(2)$$

- Rearranging the Equation (2) as follows :

$$C = \frac{S}{N_0} \times \frac{N_0}{S} \cdot B \log_2\left[1 + \frac{S}{N_0 B}\right]$$

$$= \frac{S}{N_0} \cdot \log_2\left[1 + \frac{S}{N_0 B}\right]^{N_0 B/S}$$

$$C = \frac{S}{N_0} \log_2\left[1 + \frac{S}{N_0 B}\right]^{1/(S/N_0 B)} \quad ...(3)$$

- Now as the bandwidth B approaches ∞, the channel capacity will approach its upper limit denoted by "C_∞".

$$\therefore \quad C_\infty = \lim_{B \to \infty} C$$

$$= \lim_{B \to \infty} \frac{S}{N_0} \log_2\left[1 + \frac{S}{N_0 B}\right]^{1/(S/N_0 B)} \quad ...(4)$$

- Let us substitute $x = \frac{S}{N_0 B}$ in the Equation (4). Also as $B \to \infty$, $x \to 0$

$$\therefore \quad C_\infty = \lim_{x \to 0} \frac{S}{N_0} \log_2 [1 + x]^{1/x}$$

$$\therefore C_\infty = \frac{S}{N_0} \lim_{x \to 0} \log_2 [1 + x]^{1/x} \quad ...(5)$$

- Let us use the standard relation stating that

$$\lim_{x \to 0} (1 + x)^{1/x} = e$$

$$\text{Therefore,} \quad C_\infty = \frac{S}{N_0} \log_2 e = \frac{S}{N_0} \frac{\log_{10} e}{\log_{10} 2}$$

$$\therefore \quad C_\infty = 1.44 \frac{S}{N_0}$$

- This is the value of the upper limit of channel capacity when the bandwidth approaches infinity.

Q. 29 In a facsimile transmission of a picture, there are about [2.25×10^6] picture elements per frame. For good reproduction, twelve brightness levels are necessary. Assuming all these levels to be equiprobable, calculate the channel bandwidth required to transmit one picture in every three minutes for a single to noise power ratio of 30 dB. If SNR requirement increases to 40 dB, calculate the new bandwidth. Explain the trade-off between bandwidth and SNR, by comparing the two results. **Dec. 14**

Ans. :

Given :

Number of picture elements per frame = 2.25×10^6

Number of brightness levels = 12 = M

All the twelve brightness levels are equiprobable.

Number of pictures per minute = 1/3

SNR_1 = 30 dB $\quad SNR_2$ = 40 dB

1. Calculate the information rate :

- The number of picture elements per frame is 2.25×10^6 and these elements can be of any brightness out of the possible 12 brightness levels.
- The information rate (R) = Number of messages/sec. × Average information per message.

$$R = r \times H \quad ...(1)$$

$$\text{Where} \quad r = \frac{2.25 \times 10^6}{3 \text{ minutes}} = \frac{2.25 \times 10^6}{180 \text{ sec}}$$

$$= 12500 \text{ elements/sec.} \quad ...(2)$$

$$\text{and} \quad H = \log_2 M = \log_2 12$$

...as all brightness levels are equiprobable. ...(3)

$$\therefore \quad R = 12{,}500 \times \log_2 12$$

$$\therefore \quad R = 44.812 \text{ k bits/sec.} \quad ...(4)$$

2. Calculate the bandwidth B :

– The Shannon's capacity theorem states that,

$$R \leq C \text{ where } C = B \log_2\left[1 + \frac{S}{N}\right] \quad ...(5)$$

Substitute $\frac{S}{N}$ = 30 dB = 1000 we get,

$$\therefore 44.812 \times 10^3 \leq B \log_2 [1 + 1000]$$

$$\therefore B \geq \frac{44.812 \times 10^3}{9.96}$$

$$\therefore B \geq 4.4959 \text{ kHz.}$$ **...Ans.**

3. BW for S/N = 40 dB :

– For signal to noise ratio of 40 dB or 10,000 let us calculate new value of bandwidth.

$$\therefore 44.812 \times 10^3 \leq B \log_2 [1 + 10000]$$

$$\therefore B \geq \frac{44.812 \times 10^3}{13.287}$$

$$\therefore B \geq 3.372 \text{ kHz.}$$ **...Ans.**

Trade off between bandwidth and SNR : As the signal to noise ratio is increased from 30 dB to 40 dB, the bandwidth will have to be decreased.

Q. 30 A voice grade telephone channel has a bandwidth of 3400 Hz. If the signal-to-noise ratio on the channel is 30 dB, determine the capacity of channel. If the above channel is to be used to transmit 48 kbps of data, determine the minimum SNR required for the channel. **May 11, April 18**

Ans. :

Part I : Capacity C of the channel

Given : B = 3400 Hz, S/N = 30 dB

$$P/N = 30 \text{ dB} = 10 \log_{10} [S/N]$$

$$[P/N] = \text{Antilog } 3 = 1000$$

$$\text{Capacity } C = B \log_2\left[1 + \frac{S}{N}\right]$$

$$\therefore C = 3400 \log_2 [1 + 1000]$$

$$\therefore C = 33.89 \text{ kbps}$$ **...Ans.**

Part II : New SNR

Given : C = 48 kbps

$$\therefore 48 \times 10^3 = 3400 \log_2\left[1 + \frac{S}{N}\right]$$

$$\therefore 17776 = 1 + S/N$$

$$\therefore S/N = 17775$$ **...Ans.**

$$\text{or } [P/N] \text{ dB} = 10 \log_{10} [17775] = 42.5 \text{ dB}$$ **...Ans.**

– Thus in order to transmit the data at 48 kbps the minimum SNR should be 42.5 dB.

Q. 31 An ideal communication system with average power limitation and White Gaussian noise has a bandwidth of 1 MHz and S/N ratio of 10.

1. **Determine the channel capacity**
2. **If S/N ratio drop to 5, what bandwidth is required for the same channel capacity ?**
3. **If bandwidth is decreased to 0.5 MHz, what S/N ratio is required to maintain the same channel capacity ?** **Dec. 08, March 19**

Ans. :

Given : $B = 1 \times 10^6$ Hz, S/N = 10

Channel capacity :

$$C = B \log_2\left[1 + \frac{S}{N}\right]$$

$$C = 1 \times 10^6 \log_2 [1 + 10] = 1 \times 10^6 \frac{\log_{10} 11}{\log_{10} 2}$$

$$= 1 \times 10^6 \times 3.3219$$

$$= 3.3219 \text{ Mbps}$$ **...Ans.**

New bandwidth :

Given : New S/N = 5

$$\therefore 3.3219 \times 10^6 = B \log_2 [1 + 5]$$

$$\therefore 3.3219 \times 10^6 = B \frac{\log_{10} 6}{\log_{10} 2}$$

$$B = \frac{3.3219 \times 10^6}{2.5849}$$

$$= 1.285 \text{ MHz}$$ **...Ans.**

New SNR :

Given : B = 0.5 MHz

$$\therefore 3.3219 = 0.5 \log_2\left[1 + \frac{S}{N}\right]$$

$$\therefore 1 + \frac{S}{N} = 2^{6.6438}$$

$$= 99.99 \approx = 100$$

$$\therefore \frac{S}{N} = 100 - 1$$

$$= 99 \text{ or } 19.96 \text{ dB}$$ **...Ans.**

Q. 32 Find capacity of a channel having bandwidth 1 MHz and signal to noise ratio of 10 dB.

May 16

Ans. :

Given : B = 1 MHz, S/N = 10 dB

1. **S/N ratio :**

$$[S/N]_{dB} = 10 \log_{10} (S/N)$$

$$\therefore 10 = 10 \log_{10}(S/N)$$

$$\therefore (S/N)_{ratio} = 10$$

2. **Channel capacity :**

$$C = B \log_2 \left(1 + \frac{S}{N}\right) = 1 \times 10^6 \log_2 (1 + 10)$$

$$= 1 \times 10^6 \frac{\log_{10} 11}{\log_{10} 2}$$

$$\therefore C = 3.46 \text{ Mbps}$$ ...Ans.

Q. 33 Explain Shannon's third theorem (Information capacity theorem or Shannon Hartley theorem) and prove that when B → ∞, then the channel capacity $C = \frac{S}{N_0} \log_2 = 1.44 \frac{S}{N_0}$.

Dec. 11, May 12, May 13, May 14, Dec. 14, Dec. 16

Ans. :

- The information capacity theorem is applicable to the Gaussian channel which is a power limited as well as band limited channel.
- Consider an ideal system, with R_b as the rate of data transmission and a channel capacity C.
- The relation between R_b and C of an ideal system is as follows :

$$R_b = C \qquad ...(1)$$

- Let E_b denote the energy transmitted per bit duration.
- Then the average transmitted power is given by,

$$S = \frac{E_b}{T_b} = E_b \times R_b = E_b \times C \qquad ...(2)$$

- Therefore the expression for the channel capacity of an ideal system is given by,

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

- Substituting $S = E_bC$ we get,

$$C = B \log_2\left(1 + \frac{E_bC}{N}\right) = B \log_2\left(1 + \frac{E_bC}{N_0B}\right) \qquad ...(3)$$

$$\therefore \frac{C}{B} = \log_2\left[1 + \frac{E_b}{N_0} \times \frac{C}{B}\right] \qquad ...(4)$$

$$\therefore 1 + \frac{E_b}{N_0} \times \frac{C}{B} = 2^{C/B}$$

$$\therefore \frac{E_b}{N_0} = \frac{B}{C} (2^{C/B - 1}) \qquad ...(5)$$

- The (E_b/N_0) should be sufficiently large so as to ensure an error free communication.
- If the value of (E_b/N_0) reduces below a particular value called as the **limiting value,** then the error free communication becomes impossible.

$$\text{Let } \frac{E_b}{N_0} \times \frac{C}{B} = x$$

Substitute this value into Equation (4) we get,

$$\frac{C}{B} = x \log_2 (1 + x)^{1/x} \qquad ...(6)$$

- Also

$$\frac{C}{B} = \frac{E_b}{N_0} \times \frac{C}{B} \log_2 (1 + x)^{1/x} \qquad ...(7)$$

$$\therefore 1 = \frac{E_b}{N_0} \log_2 (1 + x)^{1/x} \qquad ...(8)$$

- As $\frac{C}{B}$ tends to zero, x also tends to zero. Then Equation (8) gets modified as follows :

$$1 = \frac{E_b}{N_0} \lim_{x \to 0} \log_2 (1 + x)^{1/x}$$

$$\therefore 1 = \frac{E_b}{N_0} \log_2 e$$

$$\therefore \frac{E_b}{N_0} = \frac{1}{\log_2 e} = 0.693$$

$$\therefore \left[\frac{E_b}{N_0}\right]_{dB} = 10 \log 0.693 = -1.6 \text{ dB} \qquad ...(9)$$

- With increase in bandwidth, the following term,

$$\frac{E_b}{N_0} = \frac{2^{C/B} - 1}{(C/B)} \text{ will decrease}$$

- As bandwidth B tends to ∞ (B →∞)

$$\left.\frac{E_b}{N_0}\right|_{B\to\infty} = \log_e 2 = 0.693$$

$$\therefore \lim_{B\to\infty}\left(\frac{E_b}{N_0}\right)_{\text{in dB}} = -1.6 \quad ...(10)$$

- This is called as **Shannon's limit.** We can obtain the corresponding channel capacity by substituting $\lim B \to \infty$ in the following expression :

$$C = B \log_2\left[1 + \frac{S}{N_0 B}\right]$$

$$C_\infty = 1.44 \frac{S}{N_0} \quad ...(11)$$

- The ratio (E_b/N_0) is called as signal energy per bit, to noise power spectral density.
- This ratio has been expressed in terms of the bandwidth efficiency (C/B) in Equation (5).

unit - IV

Chapter 8 : Linear Block Codes

Q. 1 Define and give example : Code rate **Dec. 19**

Ans. :

Definition :

- The code rate is defined as the ratio of the number of message bits (k) to the total number of bits (n) in a code word.

$$\therefore \quad \text{Code rate } (r) = \frac{k}{n} \quad ...(1)$$

Example :

- The code rate of a (7,4) code is r = 4/7.

Q. 2 Define and give example Hamming weight **March 19, Dec. 19**

Ans. :

Definition :

- The Hamming weight of a code word x is defined as the number of non-zero elements in the code word.
- Hamming weight of a code vector (code word) is the distance between that code word and an all zero code vector. (a code having all elements equal to zero).
- For example, the Hamming weight of the following code word is 3, as it contains 3 non-zero elements.

Code word = 1 0 0 1 0 0 1

Q. 3 Define and give example : Hamming distance. **May 17, March 19, Dec. 19**

Ans. :

Definition :

- Consider two code vectors (or code words) having the same number of elements.
- The "Hamming distance" or simply distance between the two code words is defined as the number of locations in which their respective elements differ.

Example :

- For example consider the two code words given below :

Codeword - 1 :	1	1	1	1	0	1	0	0
		↕		↕			↕	↕
Codeword - 2 :	0	1	0	1	1	1	1	0

(E-1705)

- Note that the bits 2, 4, 7 and 8 are different from each other. Hence Hamming distance is 4.

Q. 4 Define and give example : Minimum hamming distance. **Dec. 19**

Ans. :

Definition :

- The minimum distance "d_{min}" of a linear block code is defined as the smallest Hamming distance between any pair of code vectors in the code.
- Therefore the minimum distance is same as the smallest Hamming weight of difference between any pair of code vectors.
- It can be proved that the minimum distance of a linear block code is the smallest Hamming weight of the non-zero code vectors in the code.

Q. 5 What are parity bits ? **Feb. 16, Dec. 18**

Ans. :

Definition of parity bit :

- A parity bit or a check bit is a bit added to a string of binary bits to ensure that the total number of 1-bit in the string including the parity bit is either even or odd.

Q. 6 Explain two dimensional parity check.

Dec. 05, May 06, May 16

Ans. :

Two Dimensional Parity Check :

- When a large number of binary words are being transmitted or received in succession, the resulting collection of bits is considered as a **block of data,** with rows and columns as shown in Fig. 8.1.

Characters		C	O	M	P	U	T	E	R	
7 bit ASCII codes (Message bits)	b_1	1	1	1	0	1	0	1	0	1
	b_2	1	1	0	0	0	0	0	1	1
	b_3	0	1	1	0	1	1	1	0	1
	b_4	0	1	1	0	0	0	0	0	0
	b_5	0	0	0	1	1	1	0	1	0
	b_6	0	0	0	0	0	0	0	0	0
	b_7	1	1	1	1	1	1	1	1	0
VRC bits (even parity) →		1	1	0	0	0	1	1	1	1

These bits will make the parity of each column even

These bits will make the parity of each row even

← LRC bits (even parity)

(L-315) **Fig. 8.1 : Vertical and longitudinal parity check bits**

LRC and VRC Bits :

- The parity bits are produced for each row and column of such block of data.
- The two sets of parity bits so generated are known as :
  1. Longitudinal Redundancy Check **(LRC)** bits
  2. Vertical Redundancy Check **(VRC)** bits.
- The LRC bits indicate the parity of rows and VRC bits indicate the parity of columns as shown in Fig. 8.1.

The Vertical Redundancy Check (VRC) Bits :

- As shown in Fig. 8.1 the VRC bits are parity bits associated with the ASCII code of each character.
- Each VRC bit will make the parity of its corresponding column "an even parity".
- For example consider column 1 corresponding to character "C".
- The ASCII code for the character C is as follows:

$$C = b_7 \ldots\ldots b_1 = 1\,0\,0\,0\,0\,1\,1$$

- Thus it has three 1's. Therefore, the 8^{th} bit which is a VRC bit is made "1" in order to make the parity even.
- Similarly the other VRC bits are found as shown in Fig. 8.1.

Character	C	
b_1	1	
b_2	1	← Column - 1 of the data block
b_3	0	
b_4	0	
b_5	0	
b_6	0	
b_7	1	
VRC bit →	1	← VRC bit = 1 to make the parity of first column even

(G-1944)

The Longitudinal Redundancy Check (LRC) Bits :

- The LRC bits are parity bits associated with the rows of the data block of Fig. 8.1.
- Each LRC bit will make the parity of the corresponding row, an even parity.
- For example, consider row 1 of Fig. 8.1 as follows.

Row 1 :	b_1	1	1	1	0	1	0	1	0	1	← LRC bit to make parity even

(G-1945)

- The contents of first row are (1 1 1 0 1 0 1 0), which has five 1's.
- Hence, in order to make the parity of the first row even, we select the LRC bit = 1.

How to locate the error ?

- Even a single error in any bit will result in an incorrect "LRC" in one of the rows and an incorrect VRC in one of the columns.
- The bit which is common to the row and column is the bit in error.
- However there is still a limitation on the block parity code, which is that, multiple errors in rows and columns can be only detected but they cannot be corrected.
- This is because, it is not possible to locate the bits which are in error.

Q. 7 Explain linearity property of linear block code with examples.

April 18

Ans. :

Linearity Property :

- A block code is said to be **linear** if the sum of any two code words produces a valid code word.

Example :

- Check if the following code is linear or not.

  Code X = {0000, 0101, 1010, 1111}

- Add any two code words, as follows :

$$\begin{array}{r} 0\,0\,0\,0 \\ +\ 0\,1\,0\,1 \\ \hline \underbrace{0\,1\,0\,1}_{\text{Valid codeword}} \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\ 1\,0\,1\,0 \\ \hline \underbrace{1\,1\,1\,1}_{\text{Valid codeword}} \end{array}$$

$$\begin{array}{r} 0\,1\,0\,1 \\ +\ 1\,1\,1\,1 \\ \hline \underbrace{1\,0\,1\,0}_{\text{Valid codeword}} \end{array} \qquad \begin{array}{r} 1\,0\,1\,0 \\ +\ 1\,1\,1\,1 \\ \hline \underbrace{0\,1\,0\,1}_{\text{Valid codeword}} \end{array}$$

(G-2475) **Fig. 8.2 : Addition of code words**

- Fig. 8.2 shows that the given code is a **linear block code.**

Q. 8 Parity matrix of (7,4) LBC is as follows :

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Find the code words for the message :

i) 0 1 0 1 ii) 1 0 1 0 **March 19**

Ans. :

Given : Linear block code, n = 7, k = 4, n-k = 3

Step 1 : Find the code word for message 0101 :

- We know that, parity bits B = M × P

$$\therefore \quad [b_0, b_1, b_2] = [m_0\, m_1\, m_2\, m_3] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$\therefore$ For $m_0\, m_1\, m_2\, m_3$ = 0101

$$[b_0, b_1, b_2] = [0101] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$b_0 = 0 \oplus 1 \oplus 0 \oplus 0 = 1$$

$$b_1 = 0 \oplus 1 \oplus 0 \oplus 1 = 0$$

$$b_2 = 0 \oplus 1 \oplus 0 \oplus 1 = 0$$

$$\therefore \quad [b_0, b_1, b_2] = [100]$$

$\therefore$ Code word for 0101 = 0101100 **...Ans.**

Step 2 : Find the code word for message 1010 :

$$[b_0, b_1, b_2] = [1010] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad b_0 = 1 \oplus 0 \oplus 1 \oplus 0 = 0$$

$$b_1 = 0 \oplus 0 \oplus 1 \oplus 0 = 1$$

$$b_2 = 1 \oplus 0 \oplus 0 \oplus 0 = 1$$

$$\therefore \quad [b_0, b_1, b_2] = [011]$$

$\therefore$ Code word for 1010 = 1010011 **...Ans.**

Q. 9 For a (4, 2) linear block code, the generator matrix is given as :

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Find all code words that can be generated. Comment on error correction capability of the code. **Feb. 16**

Ans. :

Given : n = 4, k = 2

$$G = [I \mid P]$$

$$\therefore \quad P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Parity bits $[b_0, b_1] = [m_0\, m_1]\, [P]$

Code word for $m_0\, m_1$ = 00 :

$$[b_0, b_1] = [0, 0] \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = [0, 0]$$

$$\therefore \quad \text{Codeword} = \underbrace{\boxed{0\,0}}_{\text{Message}}\ \underbrace{\boxed{0\,0}}_{\text{Parity}}$$ **...Ans.**

(E-1706)

- Similarly we can find the code words for the other messages. Table 8.1 enlists all the code words.

Table 8.1 : Code words

Message	Codeword	Hamming Distance
0 0	0 0 0 0	0
0 1	0 1 0 1	2 ← d_{min}
1 0	1 0 1 1	3
1 1	1 1 1 0	3

Step 1 : Error detecting and correcting capability :

- The minimum Hamming distance from Table 8.1 is,

$$d_{min} = 2$$

- $\therefore$ Error detecting capability is,

$$S \le (d_{min} - 1)$$

$$\therefore \quad S \le 2 - 1$$

$$\therefore \quad S \le 1$$ **...Ans.**

- Thus at the most one error can be detected.
- Number of correctable errors,

$$t \le \frac{d_{min} - 1}{2}$$

$$\therefore \quad t \le \frac{2 - 1}{2}$$

$$\therefore t \leq \frac{1}{2}$$ ...Ans.

– Thus no error can be corrected.

Q. 10 For a systematic (6, 3) LBC, the parity matrix is given by,

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

1. **Find all possible code vectors.**
2. **Find error detecting and correction capabilities.** **May 18**

Ans. :

Given : A LBC, n = 6, k = 3, (n – k) = 3.

To find : 1. All code words

2. Error detecting and correcting capability.

Part I : Find all code words :

– We know that,

Parity bits B = M × P

$$\therefore [b_0, b_1, b_2] = [m_0, m_1, m_2] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

– For $m_0\, m_1\, m_2 = 001$

$$[b_0, b_1, b_2] = [0, 0, 1] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = 110$$

– Similarly we can obtain the parity check bits and code words for all the messages as given in Table 8.2.

(G-2472) Table 8.2

Message bits m_0 m_1 m_2	Parity bits b_0 b_1 b_2	Codewords	Hamming distance
0 0 0	0 0 0	0 0 0 0 0 0	0
0 0 1	1 1 0	0 0 1 1 1 0	3
0 1 0	0 1 1	0 1 0 0 1 1	3 $\}d_{min}$
0 1 1	1 0 1	0 1 1 1 0 1	4
1 0 0	1 0 1	1 0 0 1 0 1	3
1 0 1	0 1 1	1 0 1 0 1 1	4
1 1 0	1 1 0	1 1 0 1 1 0	4
1 1 1	0 0 0	1 1 1 0 0 0	3

Part II : Error detecting and correcting capability :

– From Table 8.2, $d_{min}=3$

Error detecting capability :

$$S \leq (d_{min} - 1)$$

$$\therefore S \leq 2$$

– Hence at the most two errors can be detected.

Error correcting capability :

– Number of correctable errors

$$t \leq \frac{d_{min} - 1}{2}$$

$$\therefore t \leq \frac{3-1}{2}$$

$$\therefore t \leq 1$$

– Hence at the most one error can be corrected.

Q. 11 Find the parity check matrix for decoding linear block code if generator matrix is given as :

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Feb. 16

Ans. :

Step 1 : Separate the P matrix from G :

– The systematic form of generator matrix is given by,

$$G = [I_k \mid P]$$

– The given code is (6, 3) block code.

$$\therefore n = 6, k = 3.$$

– So identity matrix will be 3 × 3 matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Step 2 : Obtain the parity check matrix :

– Parity check matrix is of size (n – k) × n i.e. here it will be 3 × 6 matrix.

$$H = [P^T \mid I]$$

$$\text{But } P_T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } I_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \text{...Ans.}$$

Q. 12 Consider a (7, 4) linear block code whose generator matrix is given by :

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

1. **Obtain the code word for the message 0101.**
2. **Find the parity check matrix.**

Dec. 08, Dec. 11

Ans. :

Step 1 : Obtain the P matrix : (E-532)

Given generator matrix
$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$
$$\underbrace{}_{I_{4 \times 4}} \quad \underbrace{}_{P_{4 \times 3}}$$

– Therefore the P matrix is given by,

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{4 \times 3} \quad \text{...(1)}$$

Step 2 : To obtain the parity (check) bits :

– For the message [0 1 0 1], the parity bits can be obtained by using the expression.

$$B = MP$$

$$\therefore \quad [b_0\ b_1\ b_2]_{1 \times 3} = [0\,1\,0\,1] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{4 \times 3} = [1\ 0\ 0]$$

– Therefore the complete code word, for the message word 0 1 0 1 is (E-533)

Complete code word =

0	1	0	1	1	0	0

Message bits (first four) | Parity bits (last three)

– Similarly we can obtain the code words for the remaining message words.

– All the message vectors, the corresponding parity bits and code words are given in Table 8.3.

– The code weights are also given in the Table 8.3.

Table 8.3 : Code vectors for all the message vectors

Sr. No.	Message vector, M				Parity bits, B			Code words, X						
	m_3	m_2	m_1	m_0	b_2	b_1	b_0	X_6	X_5	X_4	X_3	X_2	X_1	X_0
1.	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.	0	0	0	1	1	0	1	0	0	0	1	1	0	1
3.	0	0	1	0	0	1	1	0	0	1	0	0	1	1
4.	0	0	1	1	0	1	0	0	0	1	1	0	1	0
5.	0	1	0	0	0	1	1	0	1	0	0	0	1	1
6.	0	1	0	1	1	1	0	0	1	0	1	1	1	0
7.	0	1	1	0	1	0	0	0	1	1	0	1	0	0
8.	0	1	1	1	0	0	1	0	1	1	1	0	0	1
9.	1	0	0	0	1	1	0	1	0	0	0	1	1	0
10.	1	0	0	1	0	1	1	1	0	0	1	0	1	1
11.	1	0	1	0	0	0	1	1	0	1	0	0	0	1
12.	1	0	1	1	1	0	0	1	0	1	1	1	0	0
13.	1	1	0	0	1	0	1	1	1	0	0	1	0	1
14.	1	1	0	1	0	0	0	1	1	0	1	0	0	0
15.	1	1	1	0	0	1	0	1	1	1	0	0	1	0
16.	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Step 3 : To obtain the parity check matrix :

– The parity check matrix [H] is a 3 × 7 matrix,

$$H = [P^T : I_{n-k}] = [P^T : I_3]$$

– The transpose matrix P^T is given by :

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}_{3 \times 4}$$

$$\therefore \quad H = [P^T : I_{3 \times 3}]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 0 & 1 \end{bmatrix}_{3 \times 7} \quad \text{...Ans.}$$

– This is the required parity check matrix.

Q. 13 The parity check matrix of a particular (7, 4) linear block code is given by,

$$[H] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

1. **Find the generator matrix (G).**
2. **List all the code vectors.**
3. **What is the minimum distance between the code vectors ?**
4. **How many errors can be detected ? How many can be corrected ?** **Dec. 15**

Ans. :

Given : $n = 7, k = 4, (n - k) = 3$

Step 1 : Find the generator matrix G :

Given...
$$H = \left[\begin{array}{cccc:ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$
$$\underbrace{\qquad P^T \qquad}\underbrace{\quad I_3 \quad}$$

(E-1707)

– The generator matrix is a (k × n) i.e. (4 × 7) matrix as follows :

$$G = [I_4 : P]$$ (E-1708)

$$\therefore \quad G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$
$$\underbrace{\qquad I_4 \qquad}\underbrace{\quad P \quad}$$

...Ans.

Step 2 : Find all the code words :

– The parity bits for the message [0 0 0 1] is obtained as follows :

$$B = M \times P$$

$$\therefore [b_0\, b_1\, b_2] = [0\,0\,0\,1] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= [0\,1\,1]$$ ...Ans.

$\therefore$ Codeword = 0001011

– For message [0 0 1 0] the parity bits are,

$$\therefore [b_0\, b_1\, b_2] = [0\,0\,1\,0] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= [1\,0\,1]$$ ...Ans.

$\therefore$ Codeword = 0010101 ...Ans.

– Similarly we can obtain the remaining code words.

– All the code words are listed in Table 8.4. The Hamming distance is equal to the number of 1s in a code word.

Table 8.4

Message	Codeword	Hamming Distance
0 0 0 0	0 0 0 0 0 0 0	0
0 0 0 1	0 0 0 1 0 1 1	3
0 0 1 0	0 0 1 0 1 0 1	3
0 0 1 1	0 0 1 1 1 0 1	4
0 1 0 0	0 1 0 0 1 0 1	3
0 1 0 1	0 1 0 1 1 0 1	4
0 1 1 0	0 1 1 0 0 1 1	4
0 1 1 1	0 1 1 1 0 0 0	3
1 0 0 0	1 0 0 0 1 1 1	4
1 0 0 1	1 0 0 1 1 0 0	3
1 0 1 0	1 0 1 0 0 1 0	3
1 0 1 1	1 0 1 1 0 0 1	4
1 1 0 0	1 1 0 0 0 0 1	3
1 1 0 1	1 1 0 1 0 1 0	4
1 1 1 0	1 1 1 0 1 0 0	4
1 1 1 1	1 1 1 1 1 1 1	7

Step 3 : Find d_{min} :

– From Table 8.4, $d_{min} = 3$.

Step 4 : Error detection and correction capability :

– Number of detectable errors is,

$$S \le (d_{min} - 1)$$

$$\therefore S \le 2$$ ...Ans.

– Thus at the most two errors can be detected.

– Number of correctable errors will be,

$$t \le \frac{d_{min} - 1}{2}$$

$$\therefore t \le 1$$ ...Ans.

– Thus at the most one error can be corrected.

Q. 14 What is a standard array decoding ? Explain with suitable example. May 16

Ans. :

– We can construct a standard array by using the coset leaders.

– All the valid code words starting with the all zero code word are written in the first row.

– Then we need to write down all the syndromes in the syndrome column (as shown in Table 8.5) and the coset leaders which are actually the error patterns in the coset leader column.

– The remaining column of n-tuples can be written by adding coset leader and the code word in the first row.

- These n-tuples actually correspond to the received code word with the corresponding syndrome written in the syndrome column.

(E-2115) Table 8.5 : Standard array for X = {000, 111}

Syndrome	Coset leader	n- Tuples	
0 0 (no errors)	0 0 0	1 1 1	Code vectors
1 1	1 0 0	0 1 1	Single errors
1 0	0 1 0	1 0 1	
0 1	0 0 1	1 1 0	

Q. 15 Explain the syndrome decoding operation for (n,k) block code with the help of diagram. **March 19**

Ans. :

- The procedure for syndrome decoding is as given below :
  1. From the received codeword Y, compute the corresponding syndrome $S = Y H^T$.
  2. Identify the error pattern (or coset leader) corresponding to syndrome S computed in the first step and call it as E.
  3. Obtain the transmitted codeword X as follows :

$$X = Y \oplus E$$

Q. 16 Obtain code words for (6, 3) LBC which has generator matrix of G = [100101; 010011; 001110]. Find all possible code words. Obtain corrected code word, if received code word is r = [001110] **Dec. 18**

Ans. :

Part I : To obtain all the code words :

- Given generator matrix is,

$$G = \Bigg[\underbrace{\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}}_{I_{3\times3}}\ \ \underbrace{\begin{matrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 0\end{matrix}}_{P_{3\times3}}\Bigg]$$

(E-2015)

- This is a (6, 3) LBC hence k = 3 (message bits)

1. Code word for 001 :

Parity bits $[\,b_0, b_1, b_2\,] = [m_0, m_1, m_2\,]\,[\,P\,]$

$$\therefore\ b_0, b_1, b_2 = [0\,0\,1]\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 0\end{bmatrix} = [1\,1\,0]$$

- ∴ Code word for 001 is,

$$\text{Codeword} = \underbrace{\boxed{0\ 0\ 1}}_{\text{Message}}\underbrace{\boxed{1\ 1\ 0}}_{\text{Parity}}$$

(E-2016)

2. Other code words :

- Similarly we can obtain the code words for all other messages.
- They are listed in Table 8.6.

Table 8.6

Message word	Parity bits	Code word
0 0 0	0 0 0	0 0 0 0 0 0
0 0 1	1 1 0	0 0 1 1 1 0
0 1 0	0 1 1	0 1 0 0 1 1
0 1 1	1 0 1	0 1 1 1 0 1
1 0 0	1 0 1	1 0 0 1 0 1
1 0 1	0 1 1	1 0 1 0 1 1
1 1 0	1 1 0	1 1 0 1 1 0
1 1 1	0 0 0	1 1 1 0 0 0

Part II : Obtain the correct code word :

1. Obtain the parity check matrix :

$$H = [\,I_{n-k} : P^T\,]$$

$$\therefore\ H = \Bigg[\underbrace{\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}}_{I_{3\times3}}\ \vdots\ \underbrace{\begin{matrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 0\end{matrix}}_{P^T}\Bigg]$$

(E-2017)

2. Find the syndrome :

Received code word r = [0 0 1 1 1 0]

Syndrome S = $r.H^T$

(E-2018)

$$\therefore\ S = [0\,0\,1\,1\,1\,0]\underbrace{\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 0\end{bmatrix}}_{H^T}$$

Syndrome is same as the fifth row of H^T matrix.

$$\therefore\ S = [0\,1\,1]$$

3. **Obtain the correct code word :**

– Since the syndrome matches with the fifth row of H^T matrix, the fifth bit in the received code word is wrong.

$\therefore$ Correct codeword = 0 0 1 1 0 0

Corrected bit (E-2020)

Q. 17 For a systematic (7,4) LBC, the parity matrix is given by [110 ; 011 ; 111 ; 101]

1. **Construct generator matrix.**
2. **Find code vectors for messages 1100, 0011.**
3. **If the received code vector is R = 0111101, find the corrected code word.** May 19

Ans. :

Step 1 : Generator matrix (G) :

$$G = [I|P] = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$\longleftarrow I_k \longrightarrow \longleftarrow P \longrightarrow$

(E-2073) **Fig. 8.2(a)**

Step 2 : Code vectors for the messages 1100, 0011 :

– We know that, B = MP

1. $$[b_0, b_1, b_2] = [1100]\begin{bmatrix}110\\011\\111\\101\end{bmatrix} = 101$$

$\therefore$ Code word = 1100101 ...**Ans.**

2. $$[b_0, b_1, b_2] = [0011]\begin{bmatrix}110\\011\\111\\101\end{bmatrix} = 010$$

$\therefore$ Code word = 0011010 ...**Ans.**

Step 3 : Find the parity check matrix :

– The parity check matrix $H = [P^T : I_{n-K}]$

$$\therefore H = \left[\begin{array}{cccc:ccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\longleftarrow P_T \longrightarrow \longleftarrow I_3 \longrightarrow$

(E-2074) **Fig. 8.2(b)**

$$\therefore H^T = \begin{bmatrix}110\\011\\111\\101\\100\\010\\001\end{bmatrix}$$

Step 4 : Find the corrected codeword :

Given R = 0111101

$\therefore$ Syndrome S = RH^T

$$= [0\,1\,1\,1\,1\,0\,1]\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\1 & 1 & 1\\1 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = 100$$

Syndrome "S" corresponds to 5^{th} row of H^T

(E-2075) **Fig. 8.2(c)**

– This syndrome is equal to the fifth row of H^T.

– Hence the error exists in the 5^{th} bit of the received code word R.

$\therefore$ R = | 0 | 1 | 1 | 1 | 1 | 0 | 1 |

error

(E-2076) **Fig. 8.2(d)**

$\therefore$ Correct transmitted code word is,

| 0 | 1 | 1 | 1 | 0 | 0 | 1 | ...**Ans.**

Q. 18 For (6,3) systematic linear code, the three parity digits are given by $c_4 = m_1 \oplus m_2$, $c_5 = m_1 \oplus m_2 \oplus m_3$ and $c_6 = m_1 \oplus m_3$.

1. **Determine generator matrix.**
2. **Comment on error detection & correction ability of code.**
3. **If received sequence is 101101 determine message word.** Dec. 19

Ans. :

Step 1 : The parity matrix P and generator matrix G :

– The relation between the check (parity) bits, message bits and the parity matrix P is given by :

$$[C_4\ C_5\ C_6]_{1\times 3} = [m_1, m_2, m_3]_{1\times 3}\,[P]_{3\times 3} \quad ...(1)$$

$$\therefore [C_4\ C_5\ C_6] = [m_1\ m_2\ m_3]\begin{bmatrix}P_{11} & P_{12} & P_{13}\\P_{21} & P_{22} & P_{23}\\P_{31} & P_{32} & P_{33}\end{bmatrix} \quad ...(2)$$

$$\therefore \left.\begin{aligned} C_4 &= P_{11}\, m_1 \oplus P_{21}\, m_2 \oplus P_{31}\, m_3 \\ C_5 &= P_{12}\, m_1 \oplus P_{22}\, m_2 \oplus P_{32}\, m_3 \\ C_6 &= P_{13}\, m_1 \oplus P_{23}\, m_2 \oplus P_{33}\, m_3 \end{aligned}\right\} \quad ...(3)$$

- Comparing Equation (3) with the given equations for C_4, C_5, C_6 we get,

$$P_{11} = 1 \quad P_{12} = 1 \quad P_{13} = 1$$

$$P_{21} = 1 \quad P_{22} = 1 \quad P_{23} = 0$$

$$P_{31} = 1 \quad P_{32} = 0 \quad P_{33} = 1$$

- Hence the parity matrix is as shown below :

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{3\times 3}$$

- This is the required parity matrix. The generator matrix is given by :

$$G = [I_k : P] = [I_3 : P_{3\times 3}]$$

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 1 & 1 & 0 \\ 0 & 1 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & : & 1 & 0 & 1 \end{bmatrix} \quad ...\textbf{Ans.}$$

Step 2 : Obtain the code words :

- It has been given that,

$$C_4 = m_1 \oplus m_2 \quad C_5 = m_1 \oplus m_2 \oplus m_3$$

$$C_6 = m_1 \oplus m_3$$

- Using these equations we can obtain the check bits for various combinations of the bits d_1, d_2, and d_3.
- After that the corresponding code words are obtained as shown in Table 8.7.

For $m_1\, m_2\, m_3 = 001$

$$C_4 = m_1 \oplus m_2 = 0 \oplus 0 = 0$$

$$C_5 = m_1 \oplus m_2 \oplus m_3 = 0 \oplus 0 \oplus 1 = 1$$

$$C_6 = m_1 \oplus m_3 = 0 \oplus 1 = 1$$

$\therefore C_4\, C_5\, C_6 =$ 011 and the code word is given by :

Code word for $m_1\, m_2\, m_3 = 001$

$$\underbrace{\boxed{0}\,\boxed{0}\,\boxed{1}}_{\text{Data bits}}\;\underbrace{\boxed{0}\,\boxed{1}\,\boxed{1}}_{\text{Check bits}}$$

(E-1745(a))

- Similarly the other code words are obtained. They are listed in Table 8.7.

Table 8.7 : Code words

Sr. No.	Message vector $m_1\, m_2\, m_3$	Check bits $C_4\, C_5\, C_6$	Code vectors or code words $m_1\, m_2\, m_3$	$C_4\, C_5\, C_6$	Code weight W (X)
1.	0 0 0	0 0 0	0 0 0	0 0 0	0
2.	0 0 1	0 1 1	0 0 1	0 1 1	3
3.	0 1 0	1 1 0	0 1 0	1 1 0	3
4.	0 1 1	1 0 1	0 1 1	1 0 1	4
5.	1 0 0	1 1 1	1 0 0	1 1 1	4
6.	1 0 1	1 0 0	1 0 1	1 0 0	3
7.	1 1 0	0 0 1	1 1 0	0 0 1	3
8.	1 1 1	0 1 0	1 1 1	0 1 0	4

Step 3 : Error correcting capacity :

- The error correcting capacity depends on the minimum distance d_{min}.

From Table 8.7, $d_{min} = 3$.

$\therefore$ Number of errors detectable is $d_{min} \ge s + 1$

$$\therefore \quad 3 \ge s + 1 \qquad \therefore \quad s \le 2$$

- So at the most two errors can be detected.

$$\text{and} \quad d_{min} \ge 2t + 1$$

$$\therefore \quad 3 \ge 2t + 1 \qquad \therefore \quad t \le 1$$

- Thus at the most one error can be corrected.

Step 4 : Find the corrected code word :

$$H^T = \begin{bmatrix} P \\ \cdots \\ I_{n-k} \end{bmatrix}_{n\times(n-k)}$$

- Substituting the P matrix and the identity matrix we get,

$$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- The first given code word is 101101.

Given : $Y_1 = [\,1\,0\,1\,1\,0\,1\,]$

- The syndrome for this code word is given by,

$$S_1 = Y_1 H^T$$

$$S_1 = [101101]\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1\,1\,0]$$

- As the syndrome matches with first row of H^T matrix.
- The first bit in the code word is in error.

$$Y = \boxed{1}\boxed{0}\boxed{1}\boxed{1}\boxed{0}\boxed{1}$$

Erroneous bit (G-2602(a))

- Hence the corrected code word is as follows :

$$X = \boxed{0}\boxed{0}\boxed{1}\boxed{1}\boxed{0}\boxed{1}$$

(G-2602(b))

Q. 19 For a systematic LBC, the parity check bits are

$$C_1 = M_1 \oplus M_2 \oplus M_3,$$
$$C_2 = M_2 \oplus M_3 \oplus M_4$$
$$C_3 = M_1 \oplus M_2 \oplus M_4$$

Find :

1. **Generator matrix.**
2. **Parity check matrix.**
3. **Error detecting and correcting capabilities.**
4. **Corrected code word for received code word [1101001].** April 18

Ans. :

Step 1 : Obtain the generator matrix (G) :

1. **Obtain parity matrix P :**

- The relation between the parity check bits, message bits and parity matrix P is as follows :

$$[C_1, C_2, C_3]_{1\times 3} = [M_1, M_2, M_3, M_4]\begin{bmatrix} P_1 & P_2 & P_3 \\ P_4 & P_5 & P_6 \\ P_7 & P_8 & P_9 \\ P_{10} & P_{11} & P_{12} \end{bmatrix}$$

$$\therefore C_1 = P_1 M_1 \oplus P_4 M_2 \oplus P_7 M_3 + P_{10} M_4 \quad ...(1)$$
$$C_2 = P_2 M_1 + P_5 M_2 \oplus P_8 M_3 + P_{11} M_4 \quad ...(2)$$
$$C_3 = P_3 M_1 + P_6 M_2 \oplus P_9 M_3 + P_{12} M_4 \quad ...(3)$$

- Comparing Equations (1), (2) and 3 with the given equations of C_1, C_2 and C_3 we get,

$P_1 = 1$ $P_2 = 0$ $P_3 = 1$

$P_4 = 1$ $P_5 = 1$ $P_6 = 1$

$P_7 = 1$ $P_8 = 1$ $P_9 = 0$

$P_{10} = 0$ $P_{11} = 1$ $P_{12} = 1$

- Hence the parity matrix is given by,

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

2. **Obtain the generator matrix :**

- The generator matrix is given by :

$$G = [I_k : P]$$

- But k = Number of message bits = 4

$$\therefore G = \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$ ...Ans..

- This is a (7, 4) LBC.

Step 2 : Obtain the parity check matrix (H) :

$$H = [P^T : I]$$

$$\text{But } P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\therefore H = \begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{\qquad\qquad}_{P^T} \quad \underbrace{\qquad\quad}_{I_{n-k}}$$

(G-2476) ...Ans.

Step 3 : Error detecting and correcting capabilities :

1. **Obtain d_{min} :**

- For obtaining d_{min}, add elements of each column of the parity check matrix H as follows (Mod - 2 addition).

$$\begin{array}{c} \text{Column :} \; 1\;2\;3\;4\;5\;6\;7 \end{array}$$

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$1 \oplus 0 \oplus 1 = 0$

$1 \oplus 1 \oplus 1 = 1$

$1 \oplus 1 \oplus 0 = 0$

$0 \oplus 1 \oplus 1 = 0$

$1 \oplus 0 \oplus 0 = 1$

$0 \oplus 1 \oplus 0 = 1$

$0 \oplus 0 \oplus 1 = 1$

(G-2477)

- Note that columns 1, 3 and 4 add up to zero vector.

$$\therefore d_{min} = 3$$

2. **Error detecting capability :**

$$S + 1 \le d_{min}$$

$$\therefore S \le d_{min} - 1 \qquad \therefore S \le 2$$

- So the code can at the most detect 2 errors.

3. **Error correcting capability :**

– We know that,

$$d_{min} \geq 2t + 1$$

$$\therefore \; 3 \geq 2t + 1$$

$$\therefore \; t \leq 1$$

– Hence this code can correct at the most one error.

Step 4 : Find the corrected code word :

Given : Y = [1 1 0 1 0 0 1]

$$\text{Syndrome } S = YH^T = [1\ 1\ 0\ 1\ 0\ 0\ 1]\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Syndrome 100 matches with the fourth row of H^T

(G-2478)

$$\therefore \; S = [1\ 0\ 0]$$

– As the syndrome matches with fourth row of H^T matrix, the fourth bit in the received code word is in error.

Y = | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

Erroneous bit **(G-2479)**

– Hence the corrected code word is as follows :

(G-2480) X = | 1 | 1 | 0 | 0 | 0 | 0 | 1 | **...Ans.**

Q. 20 The parity check matrix of a (7, 4) Hamming code is given as follows :

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

1. **Find generator matrix.**
2. **Find out all possible code words.**
3. **Determine error correcting capability.**

Dec. 13, May 17

Ans. :

Step 1 : Find the generator matrix G :

Given : n = 7, k = 4, (n – k) = 3 **(E-1732)**

$$H = \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \text{.....Given}$$

$$\longleftarrow P^T \longrightarrow \longleftarrow I_3 \longrightarrow$$

– The generator matrix will be a k × n i.e. 4 × 7 matrix as follows : **(E-1733)**

$$\therefore \; G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$\longleftarrow I_4 \longrightarrow \longleftarrow P \longrightarrow$$

...Ans.

Step 2 : Find all the code words :

– The parity bits for the first message [0 0 0 0] is obtained as follows :

$$B = M \times P$$

$$[b_0\ b_1\ b_2] = [0\ 0\ 0\ 0]\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = [0\ 0\ 0]$$

– Hence the corresponding code word = 0 0 0 0 0 0 0 **...Ans.**

– For the second message 0 0 0 1

$$[b_0\ b_1\ b_2] = [0\ 0\ 0\ 1]\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = [0\ 1\ 1]$$

– Hence the second codeword = 0 0 0 1 0 1 1 **...Ans.**

– Similarly we can find all the remaining code words.

– They are listed in Table 8.8.

Table 8.8

Message	Codeword	Hamming distance
0 0 0 0	0 0 0 0 0 0 0	0
0 0 0 1	0 0 0 1 0 1 1	3
0 0 1 0	0 0 1 0 1 1 0	3
0 0 1 1	0 0 1 1 1 0 1	4
0 1 0 0	0 1 0 0 1 1 1	4
0 1 0 1	0 1 0 1 1 0 0	3
0 1 1 0	0 1 1 0 0 1 0	3
0 1 1 1	0 1 1 1 0 1 0	4
1 0 0 0	1 0 0 0 1 0 1	3
1 0 0 1	1 0 0 1 1 1 0	4
1 0 1 0	1 0 1 0 0 1 1	4
1 0 1 1	1 0 1 1 0 0 0	3
1 1 0 0	1 1 0 0 0 1 0	3
1 1 0 1	1 1 0 1 0 0 1	3
1 1 1 0	1 1 1 0 1 0 0	4
1 1 1 1	1 1 1 1 1 1 1	7

Step 3 : Error correcting capability :

- From Table 8.8 the minimum Hamming distance $d_{min} = 3$.

$\therefore$ Error correcting capability $t_c = \dfrac{d_{min} - 1}{2} = \dfrac{3-1}{2} = 1$

- It can at the most correct 1 bit error.

Chapter 9 : Cyclic Codes

Q. 1 What are cyclic codes ? How are the cyclic codes represented ? What is requirement of generator polynomial for cyclic codes ? May 16

Ans. :

Cyclic codes :

- A code is called as the cyclic code if :
  1. It is a linear code and
  2. If it satisfies the cyclic property.

Polynomial Representation of Cyclic Codes :

- The cyclic property suggests that it is possible to treat the elements of code word of length "n" as the coefficients of a polynomial of a degree (n– 1).
- Thus the code word : $[x_0, x_1, x_2 \ldots\ldots, x_{n-1}]$
- can be expressed in the form of a code word polynomial as :

$$[X(D) = x_0 + x_1 D + x_2 D^2 + \ldots\ldots + x_{n-1} D^{n-1}] \quad \ldots(1)$$

- where D is an arbitrary real variable.
- If the given code word x is (1 1 0 1) then it can be expressed in the polynomial form as follows :

X = | 1 | 1 | 0 | 1 | Given code word

$(1 \times D^3) + (1 \times D^2) + (0 \times D^1) + (1 \times D^0)$

(E-536)

$\therefore$ $X(D) = D^3 + D^2 + 1 \leftarrow$ Corresponding code word polynomial

Q. 2 Explain the term : Generator polynomial. Dec. 14, Dec. 15, May 16, April 18

Ans. :

Generator polynomial :

- In the block codes we have defined the generator polynomial and have seen its role in generating the code words.
- In case of cyclic codes also, the generator polynomial can be defined and it will be very useful in the generation of code words.
- Let G(D) denote the generator polynomial of order (n – k) and let M(D) denote the message polynomial of order k.
- Generating polynomial of degree (n – k) given by,

$$G(D) = 1 + g_1 D + g_2 D^2 + \ldots\ldots + g_{n-k-1} D^{n-k-1} + D^{n-k} \quad \ldots(1)$$

- The generator polynomial can be expressed in the summation form as :

$$G(D) = 1 + \sum_{i=1}^{n-k-1} g_i D^i + D^{n-k} \quad \ldots(2)$$

- The other important point about the generator polynomial is that the degree of generator polynomial is equal to the number of parity bits (check bits) in the code word.
- The message polynomial of degree k is as follows :

$$M(D) = 1 + m_1 D + m_2 D^2 + \ldots + D^k \quad \ldots(3)$$

- Then the code word polynomial is given by,

$$X(D) = M(D) \cdot G(D) \quad \ldots(4)$$

- The code words obtained by using this method are called as **Non-systematic code words** and its degree is equal to "n".

Q. 3 For a (7,4) cyclic code, with generator polynomial $g(x) = x^3 + x^2 + 1$, what will be code words for following message words :
1. 1011
2. 1110

Dec. 15, Feb. 16

Ans. :

Part I : To obtain the non-systematic code vectors :

1. The number of message bits = k = 4.

   The number of parity bits = (n – k) = Degree of generator polynomial = 3.

   The total number of bits per code word n = 4 + 3 = 7.

2. The message polynomial for the message vector 1011 is,

$$M(x) = m_0 + m_1 x + m_2 x^2 + m_3 x^3 \quad \ldots(1)$$

- Substituting the values of $m_0, ..., m_3$ we get,

$$M(x) = 1 + x^2 + x^3 \quad ...(2)$$

3. The generator polynomial is given by,

$$g(x) = 1 + x^2 + x^3$$

4. The non-systematic cyclic code word polynomial is given by :

$$\begin{aligned} X(x) &= M(x) \cdot g(x) = (1 + x^2 + x^3)(1 + x^2 + x^3) \\ &= 1 + x^2 + x^3 + x^2 + x^4 + x^5 + x^3 + x^5 + x^6 \\ &= 1 + x^2(1+1) + x^3(1+1) + x^4 + x^5 + x^6 \end{aligned}$$

But $1 + 1 = 0$ ... Modulo-2 addition.

$$X(x) = 1 + x^4 + x^6$$

- Note that the degree of the code word polynomial is 6 i.e. $(n-1)$. The code word is given by,

$$X = (1\,0\,0\,0\,1\,0\,1)$$

- This is the nonsystematic code word for the message 1011.

Part II : To obtain the systematic code vector

Step 1 : Multiply M (x) by x^{n-k} :

$$x^{n-k} M(x) = x^3(x^3 + x^2 + 1) = x^6 + x^5 + x^3$$

Step 2 : Divide x^{n-k} M (x) by the generator polynomial :

(E-1756)

$$\begin{array}{r|l} & x^3 \leftarrow Q(x) \\ x^3 + x^2 + 1 & x^6 + x^5 + x^3 \\ & x^6 + x^5 + x^3 \\ \hline & 0 \quad 0 \quad 0 \leftarrow R(x) \end{array}$$

Step 3 : Obtain the codeword :

- Codeword polynomial X (x) is obtained as follows :

$$\begin{aligned} X(x) &= [x^{n-k} M(x)] \oplus R(x) \\ &= [x^6 + x^5 + x^3] \oplus [0 + 0 + 0] \\ &= x^6 + x^5 + 0x^4 + x^3 + 0x^2 + 0x + 0 \end{aligned}$$

- Hence the codeword is given by,

$$\therefore \; X = \underbrace{\boxed{1\,1\,0\,1}}_{\text{Message bits}}\underbrace{\boxed{0\,0\,0}}_{\text{Parity bits}}$$

(E-1757)

Q. 4 For cyclic code with generator polynomial $g(x) = x^3 + x^2 + 1$, obtain the code word for [1010]

Dec. 18, March 19

Ans. :

Code word for [1010] :

Given : $g(x) = x^3 + x^2 + 1$, $n = 7$, $k = 4$.

1. **Multiply M (x) by x^{n-k} :**

$M(x) = x^3 + x$ and $x^{n-k} = x^3$

$\therefore x^{n-k} M(x) = x^3(x^3 + x) = x^6 + x^4$

2. **Divide x^{n-k} M (x) by g (x) :**

(E-2021)

$$\begin{array}{r|l} & x^3 + x^2 + 1 \\ x^3 + x^2 + 1 & x^6 + x^4 \\ & x^6 + x^5 + x^3 \\ \hline & x^5 + x^4 + x^3 \\ & x^5 + x^4 + x^2 \\ \hline & x^3 + x^2 \\ & x^3 + x^2 + 1 \\ \hline & 1 \; : \text{Remainder } R(x) \end{array}$$

3. **Obtain the code word :**

- The code word polynomial is,

$$X(x) = [x^{n-k} M(x)] \oplus R(x) = x^6 + x^4 + 1$$

Hence the code word is :

$$X = \underbrace{\boxed{1\;0\;1\;0}}_{\text{Message bits}}\underbrace{\boxed{0\;0\;1}}_{\text{Parity bits}}$$

...Ans.

(E-2019)

Q. 5 For systematic (7, 4) cyclic code, find out the generator matrix and parity check matrix.

Given : $G(D) = D^3 + D + 1$ **May 18**

Ans. :

To obtain the generator matrix [G] :

- The i^{th} row of the generator matrix is given by,

$$D^{(n-i)} \oplus R_i(D) = Q_i(D)\, G(D) \quad \text{... where } i = 1, 2, \dots k \quad ...(1)$$

- It is given that the cyclic code is systematic (7, 4) code, $\therefore$ $n = 7$, $k = 4$ and $(n - k) = 3$.
- Substituting these values into the above expression, we get,

$$D^{(7-i)} \oplus R_i(D) = Q_i(D)\,(D^3 + D + 1) \; ... \; i = 1, 2, \dots 4.$$

- With $i = 1$, the above equation is given by,

$$D^6 \oplus R_i(D) = Q_i(D)\,(D^3 + D + 1) \quad ...(2)$$

- Let us obtain the value of $Q_i(D)$. The quotient $Q_i(D)$ can be obtained by dividing $D^{(n-i)}$ by $G(D)$.
- Therefore to obtain $Q_i(D)$, let us divide D^6 by $(D^3 + D + 1)$.

– The division takes place as follows :

$$
\begin{array}{rl}
 & D^3 + D + 1 \leftarrow \text{Quotient polynomial } Q_i(D) \\
D^3 + D + 1 \,\big) & D^6 \\
 & D^6 + D^4 + D^3 \\
\text{Mod - 2 additions} \rightarrow & \oplus \quad \oplus \quad \oplus \\ \hline
 & D^4 + D^3 \\
 & D^4 + 0D^3 + D^2 + D \\
\text{Mod - 2 additions} \rightarrow & \oplus \quad \oplus \quad \oplus \quad \oplus \\ \hline
 & D^3 + D^2 + D \\
 & D^3 + 0D^2 + D + 1 \\
\text{Mod - 2 additions} \rightarrow & \oplus \quad \oplus \quad \oplus \quad \oplus \\ \hline
 & D^2 + 0D + 1 \leftarrow \text{Remainder polynomial } R_i(D)
\end{array}
$$

(G-1984)

Here the quotient polynomial $Q_i(D) = D^3 + D + 1$

and the remainder polynomial $R_i(D) = D^2 + 0D + 1$

– Substituting these values into Equation (2) we get,

$$D^6 \oplus R_i(D) = (D^3 + D + 1)(D^3 + D + 1)$$

$$= D^6 + D^4 + D^3 + D^4 + D^2 + D + D^3 + D + 1$$

$$= D^6 + 0D^5 + (1 \oplus 1) D^4 + (1 \oplus 1) D^3 + D^2 + (1 \oplus 1) D + 1$$

$$= D^6 + 0D^5 + 0D^4 + 0D^3 + D^2 + 0D + 1$$

$\therefore$ 1st Row polynomial $\Rightarrow D^6 + 0D^5 + 0D^4 + 0D^3 + D^2 + 0D + 1$

$\therefore$ 1st Row elements $\Rightarrow$ 1 0 0 0 1 0 1

– Using the same procedure, we can obtain the polynomials for the other rows of the generator matrix as follows :

2nd Row polynomial $\Rightarrow D^5 + D^2 + D + 1$

3rd Row polynomial $\Rightarrow D^4 + D^2 + D$

4th Row polynomial $\Rightarrow D^3 + D + 1$

– These polynomials can be transformed into the generator matrix as follows :

– (E-1758)

$$
G = \begin{array}{r}
 \\ \text{Row 1} \rightarrow \\ \text{Row 2} \rightarrow \\ \text{Row 3} \rightarrow \\ \text{Row 4} \rightarrow
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} D^6 & D^5 & D^4 & D^3 & D^2 & D^1 & D^0 \end{array} \\
\left[\begin{array}{cccc:ccc}
1 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1
\end{array}\right]_{4 \times 7} \\
\longleftarrow I_{4 \times 4} \longrightarrow \longleftarrow P_{4 \times 3} \longrightarrow
\end{array}
$$

– This is the required generator matrix.

To obtain the parity check matrix [H] :

– The parity check matrix is given by :

$$H = [P^T : I_{3 \times 3}]$$

– The transpose matrix P^T is given by interchanging the rows and columns of the P matrix.

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}_{3 \times 4}$$

– Hence the parity check matrix is given by,

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 0 & 1 \end{bmatrix}_{3 \times 7}$$

– This is the required parity check matrix.

Note : This parity check matrix is of systematic type.

Q. 6 Construct a systematic (7, 4) cyclic code using the generator polynomial $G(x) = x^3 + x + 1$. What are the error correcting capabilities of this code ? Construct the decoding table and for the received code word 1 1 0 1 1 0 0, determine the transmitted data word. Dec. 13, Dec. 16

Ans. :

– From the given data it is clear that n = 7 and k = 4 for this code.

Step 1 : The generator matrix :

– Referring to Q. 5, the generator matrix for the given generator polynomial is :

$$G = \begin{bmatrix} 1\,0\,0\,0 : 1\,0\,1 \\ 0\,1\,0\,0 : 1\,1\,1 \\ 0\,0\,1\,0 : 1\,1\,0 \\ 0\,0\,0\,1 : 0\,1\,1 \end{bmatrix}$$ ...Ans.

Step 2 : Obtain code vectors (construction of code) :

– The generator matrix of step 1 can be used to calculate the code vectors, because

$$\boxed{X = MG}$$ ...(1)

Let M = [1010]

$$\therefore \quad X = [1\,0\,1\,0] \begin{bmatrix} 1\,0\,0\,0 : 1\,0\,1 \\ 0\,1\,0\,0 : 1\,1\,1 \\ 0\,0\,1\,0 : 1\,1\,0 \\ 0\,0\,0\,1 : 0\,1\,1 \end{bmatrix}$$

$$\therefore \quad X = \underbrace{1\,0\,1\,0}_{\text{Message bits}}\;\underbrace{0\,1\,1}_{\text{Parity bits}}$$

(E-1759)

– Similarly we can obtain the code words for other message vectors.

123

Table 9.1 : Code words

Message	Codeword	Hamming Distance
0000	0000 000	0
0001	0001 011	3
0010	0010 110	3
0011	0011 101	4
0100	0100 111	4
0101	0101 100	3
0110	0110 001	3
0111	0111 010	4
1000	1000 101	3
1001	1001 110	4
1010	1010 011	4
1011	1011 000	3
1100	1100 010	3
1101	1101 001	4
1110	1110 100	3
1111	1111 111	7

Step 3 : Error correcting capability :

- According to code words the minimum distance is $d_{min} = 3$.
- Therefore this code will be able to detect up to 2 errors and correct up to only 1 errors.

Step 4 : Obtain the transpose matrix H^T :

We know that $G = [I_k : P_{k \times (n-k)}]$

$$\text{But } G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

(E-554)

$$\longleftarrow I_{4 \times 4} \longrightarrow \longleftarrow P_{4 \times 3} \longrightarrow$$

$$\therefore \quad P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{4 \times 3} \quad \text{and} \quad P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}_{3 \times 4}$$

- The parity check matrix is given by :

$$H = [P^T : I_{n-k}]$$

$$\therefore \quad H = \left[\begin{array}{cccc:ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\therefore \quad H^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 5 : Prepare the decoding table :

- We can prepare the decoding table from the transpose of the parity check matrix H^T because each row of H^T represents a syndrome and a unique error pattern.
- Table 9.1(a) shows the error patterns and syndrome vectors.

Table 9.1(a) : Decoding table

Sr. No.		Syndrome vector			Error vector E with single error patterns						
1		0	0	0	0	0	0	0	0	0	0
2	First row of H^T	1	0	1	1	0	0	0	0	0	0
3	Second row of H^T	1	1	1	0	1	0	0	0	0	0
4	Third row of H^T	1	1	0	0	0	1	0	0	0	0
5	Fourth row of H^T	0	1	1	0	0	0	1	0	0	0
6	Fifth row of H^T	1	0	0	0	0	0	0	1	0	0
7	Sixth row of H^T	0	1	0	0	0	0	0	0	1	0
8	Seventh row of H^T	0	0	1	0	0	0	0	0	0	1

- This is the required decoding table.

Step 6 : Decode the input word :

The input word, $Y = 1101100$

- The received word can be expressed in the polynomial form as,

$$Y(x) = x^6 + x^5 + x^3 + x^2$$

- The syndrome vector is given by,

$$S(x) = \text{Remainder}\left[\frac{Y(x)}{G(x)}\right]$$

- So let us perform the division as follows :

$$Y(x) = x^6 + x^5 + 0x^4 + x^3 + x^2 + 0x + 0 \text{ and}$$

$$G(x) = x^3 + 0x^2 + x + 1$$

- The division takes place as follows :

$$
\begin{array}{r}
x^3 + x^2 + x + 1 \\
x^3 + x + 1 \,\big)\; x^6 + x^5 + x^3 + x^2 \\
x^6 + x^4 + x^3 \\
\hline
x^5 + x^4 + x^2 \\
x^5 + x^3 + x^2 \\
\hline
x^4 + x^3 \\
x^4 + x^2 + x \\
\hline
x^3 + x^2 + x \\
x^3 + x + 1 \\
\hline
x^2 + 0x + 1 \leftarrow \text{Remainder}
\end{array}
$$

(E-1760)

- The remainder polynomial is given by : $x^2 + 0x + 1$.

$$\therefore \quad S = [1\,0\,1]$$

- The nonzero syndrome indicates that there exists an error in the received code word.
- The decoding Table 9.1(a) shows that the error vector corresponding to the syndrome S = 1 0 1 is given by :

$$E = [1\,0\,0\,0\,0\,0\,0]$$

- Therefore the corrected code word is given by

$$X = Y \oplus E$$

$$= [1\,1\,0\,1\,1\,0\,0] \oplus [1\,0\,0\,0\,0\,0\,0]$$

$$\therefore \quad X = [0\,1\,0\,1\,1\,0\,0] \qquad \text{...Ans.}$$

Q. 7 Draw the encoder for a (7, 4) cyclic Hamming code generated by the generator polynomial, $G(D) = 1 + D + D^3$. May 08, April 18

Ans. :

- The generator polynomial is given by :

$$G(D) = D^3 + 0\,D^2 + D + 1 \qquad \text{...(1)}$$

- The generator polynomial of an (n, k) cyclic code is expressed as

$$G(D) = 1 + \sum_{i=1}^{n-k-1} g_i D^i + D^{n-k} \qquad \text{...(2)}$$

- For a (7, 4) cyclic Hamming code, n = 7 and k = 4

$$\therefore \quad G(D) = 1 + \sum_{i=1}^{7-4-1} g_i D^i + D^{7-4}$$

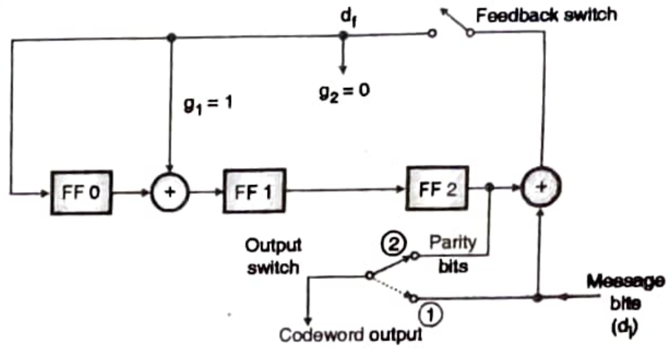
$$= 1 + g_1 D + g_2 D^2 + D^3$$

$$\therefore \quad G(D) = D^3 + g_2 D^2 + g_1 D + 1 \qquad \text{...(3)}$$

- Comparing Equations (1) and (3) we get

$$g_1 = 1 \text{ and } g_2 = 0 \qquad \text{...(4)}$$

- Therefore the encoder for a (7, 4) Hamming code is as shown in Fig. 9.1.



(E-1217) **Fig. 9.1 : Encoder for a cyclic hamming code**

Q. 8 Consider (7, 4) cyclic code with $g(x) = x^3 + x + 1$:

1. **Draw the hardware arrangement of cyclic encoder and verify the encoder by considering two different messages.**
2. **If received code vector is 1001101 and 11111110 find out transmitted (or corrected) code vectors. Draw the hardware arrangement of syndrome calculate (Cyclic decoder) and verify it.**

May [redacted]

Ans. :

1. Draw the encoder :

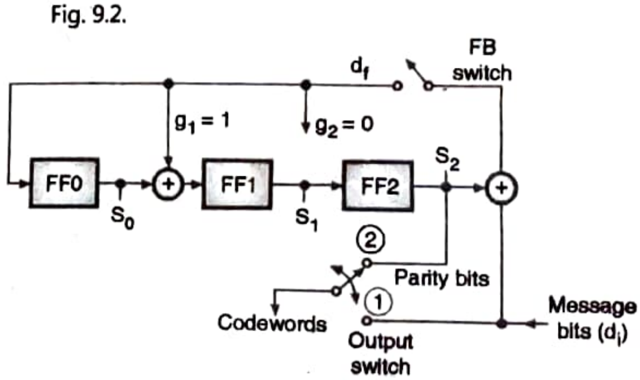
Given generator polynomial $g(x) = x^3 + x + 1$ ...(1)

Standard generator polynomial $g(x) = x^3 + g_2 x^2 + g_1 x + 1$ ...(2)

- Comparing Equations (1) and (2) we get,

$$g_2 = 0 \text{ and } g_1 = 1$$

- Therefore the required encoder is as shown in Fig. 9.2.



(E-1790) **Fig. 9.2 : Encoder**

1 0 0 1 1 0 0
1 0

2. Verification of encoder :

Let m = 0011

d_i	$d_f = d_i \oplus S_2$	$S_0 = d_f$	$S_1 = S_0 \oplus d_f$	$S_2 = S_1$
–	–	0	0	0
0	0	0	0	0
0	0	0	0	0
1	1	1	1	0
1	1	1	0	1

(E-2118)

Code word using encoder :

$\therefore$ Codeword X = $\boxed{0\ 0\ 1\ 1 \mid 1\ 0\ 1}$ (E-2116)

$$x^{n-k} M(x) = x^3 (x + 1) = x^4 + x^3$$

$$\begin{array}{r} x+1 \\ x^3+x+1 \;\big)\; x^4+x^3 \\ x^4+x^2+x \\ \hline x^3+x^2+x \\ x^3+x+1 \\ \hline x^2+1 \leftarrow \text{Remainder } R(x) \end{array}$$

(E-1791)

$$\therefore X(x) = [x^4 + x^3] \oplus [x^2 + 1]$$

$$= x^4 + x^3 + x^2 + 1$$

$\therefore$ Codeword X = $\boxed{0\ 0\ 1\ 1 \mid 1\ 0\ 1}$ ... Verified

(E-2117)

3. Find the transmitted code vectors :

1. Received code word Y_1 = 1 0 0 1 1 0 1

– Prepare the decoding table by finding out the syndrome for each error pattern.

Error pattern e_1 = 1 0 0 0 0 0 0

$\therefore e_1(x) = x^6$

$$\therefore \text{Syndrome polynomial } S(x) = \text{Rem.} \frac{e(x)}{g(x)}$$

$$\begin{array}{r} x^3+x+1 \\ x^3+x+1 \;\big)\; x^6 \\ x^6+x^4+x^3 \\ \hline x^4+x^3 \\ x^4+x^2+x \\ \hline x^3+x^2+x \\ x^3+x+1 \\ \hline \therefore S(x) = x^2+1 \end{array}$$

(E-1792)

$\therefore$ Syndrome S = [101]

– Similarly the other syndromes can be found. They have been listed in Table 9.2.

Table 9.2 : Decoding table

Error pattern							Syndrome		
1	0	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	0
0	0	1	0	0	0	0	1	1	0
0	0	0	1	0	0	0	0	1	1
0	0	0	0	1	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0
0	0	0	0	0	0	1	0	0	1

Given : Y_1 = 1 1 0 1 1 0 1

$$\therefore Y_1(x) = x^6 + x^5 + x^3 + x^2 + 1$$

$$\begin{array}{r} x^3+x^2+x+1 \\ x^3+x+1 \;\big)\; x^6+x^5+x^3+x^2+1 \\ x^6+x^4+x^3 \\ \hline x^5+x^4+x^2+1 \\ x^5+x^3+x^2 \\ \hline x^4+x^3+1 \\ x^4+x^2+x \\ \hline x^3+x^2+x+1 \\ x^3+x+1 \\ \hline x^2 \leftarrow S(x) \end{array}$$

(E-1793)

$\therefore$ Syndrome S = [100]

– In Table 9.2, the corresponding error pattern

e = 0 0 0 0 1 0 0

$\therefore X_1 = Y_1 \oplus e$ = (1 1 0 1 1 0 1) $\oplus$ (0 0 0 0 1 0 0)

$\therefore X_1$ = $\boxed{1\ 1\ 0\ 1\ 0\ 0\ 1}$ (E-2125)

Q. 9 Draw encoder for cyclic code having generator polynomial $g(x) = 1 + x^2 + x^3$. Generate code word for message [1011]. April 18

Ans. :

Given : $g(x) = 1 + x^2 + x^3$, Message = 1 0 1 1

To do : 1. Draw the encoder.

2. Obtain the code word.

Part I : Design the encoder :

– The generator polynomial is given by :

$$g(x) = x^3 + x^2 + 0x + 1 \quad ...(1)$$

- The generator polynomial of an (n, k) cyclic code is expressed as :

$$G(x) = 1 + \sum_{i=1}^{n-k-1} g_i\, x^i + x^{n-k} \quad ...(2)$$

- For a (7, 4) cyclic code, n = 7 and k = 4

$$\therefore \quad G(x) = 1 + \sum_{i=1}^{7-4-1} g_i\, x^i + x^{7-4}$$

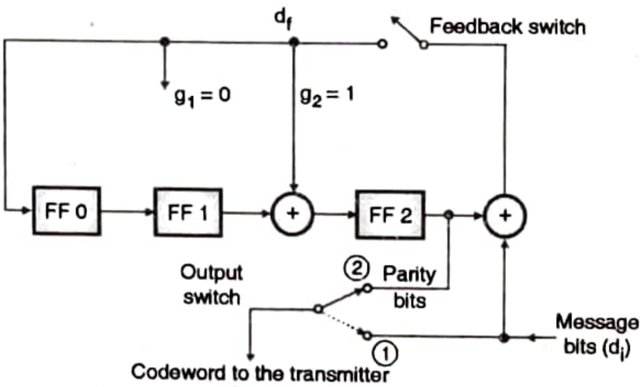
$$= 1 + g_1 x + g_2 x^2 + x^3$$

$$\therefore \quad G(x) = x^3 + g_2 x^2 + g_1 x + 1 \quad ...(3)$$

- Comparing Equations (2) and (4) we get,

$$g_1 = 0 \text{ and } g_2 = 1 \quad ...(4)$$

- Therefore the encoder for a (7, 4) cyclic code is as shown in Fig. 9.3.



(E-215(a)) **Fig. 9.3 : Encoder for a cyclic code**

Part II : Operation of encoder and verification :

- Initially the outputs of all flip flops is assumed to be equal to zero.
- Refer Table 9.3 to understand the generation of code words. The message is 1 0 1 1.

(G-2481) **Table 9.3 : Generation of codeword**

d_i	$d_f = d_i \oplus FF_2$	$FF_0 + d_f$	$FF_1 + FF_0$	$FF_2 = FF_1 \oplus d_f$
–	–	0	0	0
1	$1 \oplus 0 = 1$	1	0	1
0	$0 \oplus 1 = 1$	1	1	1
1	$1 \oplus 1 = 0$	0	1	1
1	$1 \oplus 1 = 0$	0	0	1

Parity bits

$\therefore$ Parity bits = 1 0 0

$\therefore$ Codeword = $\underbrace{1\ 0\ 1\ 1}_{\text{Message}}\ \underbrace{1\ 0\ 0}_{\text{Parity}}$ ...**Ans.**

Verification :

Given : $g(x) = x^3 + x^2 + 1$, n = 7, k = 4

1. **Multiply M(x) by x^{n-k} :**

$$\text{But} \quad M(x) = x^3 + x + 1 \text{ and } x^{n-4} = x^3$$

$$\therefore \quad x^{n-k} M(x) = x^3 (x^3 + x + 1) = x^6 + x^4 + x^3$$

2. **Divide $x^{n-k} M(x)$ by g(x) :**

(G-2482)

$$\begin{array}{r|l} & x^3 + x^2 \\ \hline x^3 + x^2 + 1 & x^6 + x^4 + x^3 \\ & x^6 + x^5 + x^3 \\ \hline & x^5 + x^4 \\ & x^5 + x^4 + x^2 \\ \hline & x^2 \end{array}$$

x^2 : Remainder R(x)

3. **Obtain the code word :**

- The code word polynomial is :

$$X(x) = [x^{n-k} M(x)] \oplus R(x)$$

$$\therefore \quad X(x) = x^6 + x^4 + x^3 + x^2$$

$$= x^6 + 0x^5 + x^4 + x^3 + x^2 + 0x + 0$$

- Hence the code word is,

$X = \underbrace{1\ 0\ 1\ 1}_{\text{Message bits}}\ \underbrace{1\ 0\ 0}_{\text{Parity bits}}$ .... hence verified.

(G-2483)

Q. 10 **Consider a (7, 4) cyclic code generated by $g(x) = 1 + x^2 + x^3$. Design an encoder using shift registers and using this, find out the code word for the message (1001). Suppose the received vector is r = (0010110), find the syndrome using syndrome circuit. Find out the generator matrix for the above cyclic code.**

Feb. 16, March 19, Dec. 19

Ans. :

Part 1 : To draw the encoder :

1. The received code word has 7 bits hence n = 7 and the degree of generator polynomial is 3 hence n – k = 3, so k = 4. Thus the given code is a (7, 4) cyclic code.
2. The generator polynomial of an (n, k) cyclic code is expressed as,

$$g(x) = 1 + \sum_{i=1}^{n-k-1} g_i x^i + x^{n-k}$$

$$\therefore \quad g(x) = 1 + \sum_{i=1}^{3-1} g_i x^i + x^3$$

$$\therefore \quad g(x) = 1 + g_1 x + g_2 x^2 + x^3 \quad \text{...(1)}$$

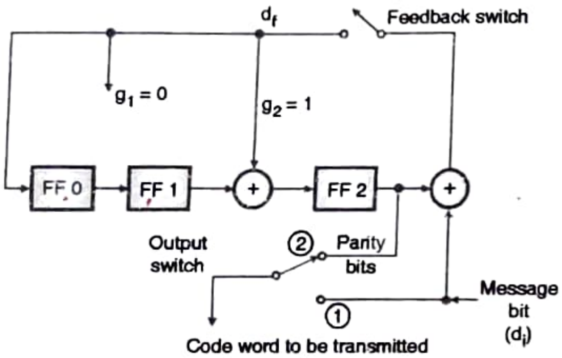
– The given generator polynomial is,

$$g(x) = 1 + 0x + x^2 + x^3 \quad \text{...(2)}$$

3. Comparing Equations (1) and (2) we get,

$$g_1 = 0, \; g_2 = 1$$

– Hence the encoder is as shown in Fig. 9.4(a).



(E-1218(a)) **Fig. 9.4(a) : Encoder**

Part 2 : To obtain the code word for message (1001) :

– Initially the output of all the flip-flops is assumed to be equal to zero.

– Refer Table 9.4(a) to understand the generation of code words.

(E-2119) **Table 9.4(a) : m = 1001**

d_i	$d_f = d_i \oplus FF2$	$FF0 = d_f$	$FF1 = FF0$	$FF2 = FF1 \oplus d_f$
–	–	0	0	0
1	1	1	0	1
0	1	1	1	1
0	1	1	1	0
1	1	1	1	0

$$\therefore \quad \text{Codeword } X = \underbrace{\boxed{1\;0\;0\;1}}_{\text{Message}}\underbrace{\boxed{0\;1\;1}}_{\text{Parity}} \quad \text{...Ans.}$$

(E-1842)

Verification :

Given : $M(D) = D^3 + 0\,D^2 + 0D + 1$

$$D^{n-k} = D^3$$

$$\therefore D^{n-k}\, M(D) = D^3 (D^3 + 0\,D^2 + 0\,D + 1) = D^6 + D^3$$

$$G(D) = D^3 + D^2 + 1$$

$$\therefore \quad X(D) = [D^{n-k} M(D)] \oplus R(D)$$

$$= [D^6 + 0\,D^5 + 0\,D^4 + D^3 + 0\,D^2 + 0\,D + 0] \oplus [D + 1]$$

$$= D^6 + 0\,D^5 + 0\,D^4 + D^3 + 0\,D^2 + D + 1$$

$$\therefore \quad \text{Codeword } X = \underbrace{\boxed{1\;0\;0\;1}}_{\text{Message}}\underbrace{\boxed{0\;1\;1}}_{\text{Parity}} \quad \text{...Ans.}$$

(E-1842)

Part 3 : Syndrome calculator :

– The given generator polynomial is,

$$g(x) = 1 + x^2 + x^3$$

$$g(x) = x^3 + x^2 + 0x + 1 \quad \text{...(3)}$$

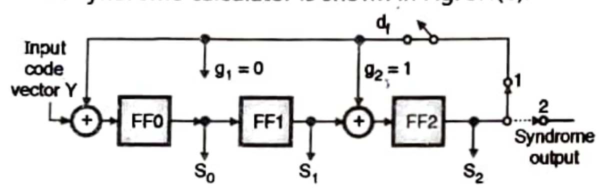
2. The general form of generator polynomial is,

$$g(x) = x^3 + g_2 x^2 + g_1 x + 1 \quad \text{...(4)}$$

– Comparing Equations (3) and (4) we get,

$$g_1 = 0, \; g_2 = 1$$

– The syndrome calculator is shown in Fig. 9.4(b).



(E-226(a)) **Fig. 9.4(b) : Syndrome calculator**

Part 4 : Calculation of syndrome :

– The output switch of Fig. 9.4(b) will be initially in position 1 until all the 7 bits of the received signal Y are shifted into the register.

– After that, the output switch is shifted to position 2. Clock pulses are then applied to the shift register to output the syndrome vector S.

– Table 9.4(b) explains the process of syndrome generation.

The received vector Y = 0010110

(E-1765) **Table 9.4(b) : Calculation of syndrome**

Shift	Input bit Y	d_f	Content of shift register		
			$S_0 = Y \oplus d_f$	$S_1 = S_0$	$S_2 = S_1 \oplus d_f$
–	–	–	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	1	0	1	0	0
4	0	0	0	1	1
5	1	1	0	0	1
6	1	1	0	0	1
7	0	1	1	0	1

Syndrome S = 101

$$
\begin{array}{r l}
 & D^3 + D^2 + D + 1 \\
D^3 + D^2 + 1 \,\big)\, & D^6 + D^3 \\
 & D^6 + D^5 + D^3 \\ \hline
 & D^5 \\
 & D^5 + D^4 + D^2 \\ \hline
 & D^4 + D^2 \\
 & D^4 + D^3 + D \\ \hline
 & D^3 + D^2 + D \\
 & D^3 + D^2 + 1 \\ \hline
 & D + 1 \leftarrow R(D)
\end{array}
$$

(E-1764)

$\therefore$ Syndrome (S) = 101

Imp

Q. 11 Suggest a suitable generator polynomial for a (7,4) systematic cyclic code and find code vector for the following data words :

1. 1010 2. 1111 3. 0001 4. 1000

Draw an encoder arrangement for the above code and explain its operation.

Construct the decoding table for all single bit error patterns and determine the data vectors transmitted for the following received vector :

1. 1101101 2. 0101000

May 06, Dec. 08, May 11, April 18

Ans. :

Given : $n = 7, \quad k = 4$

Generator polynomial :

- Order of the generator polynomial is $(n - k) = 7 - 4 = 3$
- Moreover, to find the generator polynomial, we have to factorize $(x^7 + 1)$

$$x^7 + 1 = (x + 1)(x^3 + x + 1)(x^3 + x^2 + 1) \quad ...(A)$$

- The order of G (x) i.e. generator polynomial should be 3.
- We can select either second or third term on RHS of Equation (A).

$$\text{Let } G(x) = x^3 + x + 1$$ ...Ans.

Systematic code words :

Let the message $m_1 = 1010$

$\therefore$ Message polynomial $M(x) = x^3 + x$

Step 1 : Multiply M (x) by x^{n-k} :

$$x^{n-k} M(x) = x^3 (x^3 + x) = x^6 + x^4$$

Step 2 : Divide x^{n-k} M (x) by G (x) :

$$
\begin{array}{r l}
 & x^3 \\
x^3 + x + 1 \,\big)\, & x^6 + x^4 \\
 & x^6 + x^4 + x \\ \hline
 & x \leftarrow \text{Remainder } R(x)
\end{array}
$$

(E-1767)

Step 3 : Obtain the code word :

- Code word polynomial

$$X(x) = [x^{n-k} M(x)] \oplus R(x)$$

$$\therefore X(x) = [x^6 + x^4] \oplus [x]$$

$$\therefore X(x) = x^6 + 0x^5 + x^4 + 0x^3 + 0x^2 + x + 0$$

$\therefore$ Codeword X = $\boxed{1\ 0\ 1\ 0}\boxed{0\ 1\ 0}$ (E-1768)

Message bits | Parity bits

...Ans.

Code words for other data words :

1. $m_2 = 1111$

$$M(x_2) = x^3 + x^2 + x + 1$$

$$x^{n-k} M(x_2) = x^3 (x^3 + x^2 + x + 1)$$

$$= x^6 + x^5 + x^4 + x^3$$

$$
\begin{array}{r l}
 & x^3 + x^2 + 1 \\
x^3 + x + 1 \,\big)\, & x^6 + x^5 + x^4 + x^3 \\
 & x^6 + x^4 + x^3 \\ \hline
 & x^5 \\
 & x^5 + x^3 + x^2 \\ \hline
 & x^3 + x^2 \\
 & x^3 + x + 1 \\ \hline
 & x^2 + x + 1 \leftarrow \text{Remainder}
\end{array}
$$

(E-1769)

$$X(x) = [x^{n-k} M(x_2)] \oplus R(x)$$

$$= [x^6 + x^5 + x^4 + x^3] \oplus [x^2 + x + 1]$$

$$= x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

$\therefore$ Codeword X = $\boxed{1\ 1\ 1\ 1}\boxed{1\ 1\ 1}$

Message | Parity

(E-1770)

2. $m_3 = 0001 \quad \therefore M(x_3) = 1$

$$\therefore X(x) = M(x_3) \cdot g(x)$$

$$= 1 \cdot (x^3 + x + 1)$$

$$= x^3 + x + 1$$

$\therefore$ Codeword X = $\boxed{0\ 0\ 0\ 1}\boxed{0\ 1\ 1}$ (E-2126)

4. $m_4 = 1000 \quad \therefore M(x_4) = x^3$

$$x^{n-k} M(x_4) = x^3 \cdot x^3 = x^6$$

$$
\begin{array}{r}
x^3+x+1 \\
x^3+x+1 \overline{) \; x^6 \qquad\qquad} \\
x^6+x^4+x^3 \\
\hline
x^4+x^3 \\
x^4+x^2+x \\
\hline
x^3+x^2+x \\
x^3+x+1 \\
\hline
x^2+1
\end{array}
$$

(E-1771)

$$\therefore \quad X(x) = x^6 + x^2 + 1$$

$\therefore$ Codeword X = $\boxed{1\ 0\ 0\ 0\ 1\ 0\ 1}$ (E-2127)

Encoder arrangement :

- The generator polynomial is given by :

$$G(x) = x^3 + 0\,x^2 + x + 1 \qquad ...(1)$$

- The generator polynomial of an (n, k) cyclic code is expressed as,

$$G(x) = 1 + \sum_{i=1}^{n-k-1} g_i\, x^i + x^{n-k} \qquad ...(2)$$

- For a (7, 4) cyclic Hamming code, n = 7 and k = 4

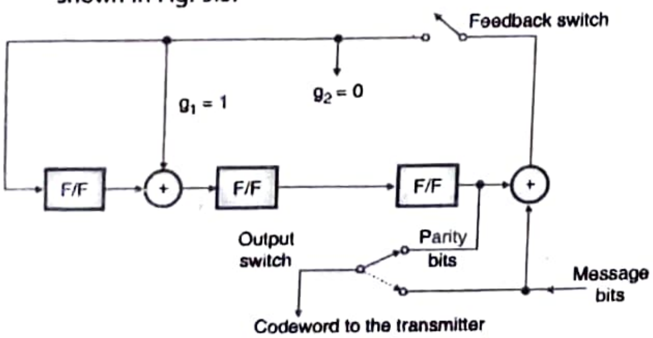
$$\therefore \quad G(x) = 1 + \sum_{i=1}^{7-4-1} g_i\, x^i + x^{7-4} = 1 + g_1\, x + g_2\, x^2 + x^3$$

$$\therefore \quad G(x) = x^3 + g_2\, x^2 + g_1\, x + 1 \qquad ...(3)$$

- Comparing Equations (1) and (3) we get,

$$g_1 = 1 \quad \text{and} \quad g_2 = 0 \qquad ...(4)$$

- Therefore the encoder for a (7, 4) Hamming code is as shown in Fig. 9.5.



(E-1507) **Fig. 9.5 : Encoder for 7, 4 cyclic code**

Decoding table and data vectors :

- The syndrome polynomial will have an order of (n – k – 1) = (7 – 4 – 1) = 2 and there will be 7 possible nonzero syndromes.

- Let us find the syndrome for each error pattern "e".

Suppose e = 1000000 $\therefore$ e (x) = x^6

$$\therefore \quad S(x) = \text{Remainder of } \frac{e(x)}{g(x)}$$

$$
\begin{array}{r}
x^3+x+1 \\
x^3+x+1 \overline{) \; x^6 \qquad\qquad} \\
x^6+x^4+x^3 \\
\hline
x^4+x^3 \\
x^4+x^2+x \\
\hline
x^3+x^2+x \\
x^3+x+1 \\
\hline
x^2+1 \leftarrow S(x)
\end{array}
$$

(E-1772)

$$\therefore \quad S(x) = x^2 + 1 \qquad \therefore \quad S = (1\ 0\ 1)$$

- Similarly we can obtain the syndromes for the remaining error patterns.
- They are as shown in Table 9.5.

Table 9.5 : Decoding table

Error pattern (e)	Syndrome (S)
1 0 0 0 0 0 0	1 0 1
0 1 0 0 0 0 0	0 1 0
0 0 1 0 0 0 0	1 1 0
0 0 0 1 0 0 0	0 1 1
0 0 0 0 1 0 0	1 0 0
0 0 0 0 0 1 0	0 1 0
0 0 0 0 0 0 1	0 0 1

To determine the data vectors :

1. Given Y = 1101101 :

$$\therefore \quad Y(x) = x^6 + x^5 + x^3 + x^2 + 1$$

$$
\begin{array}{r}
x^3+x^2+x+1 \\
x^3+x+1 \overline{) \; x^6+x^5+x^3+x^2+1} \\
x^6+x^4+x^3 \\
\hline
x^5+x^4+x^2+1 \\
x^5+x^3+x^2 \\
\hline
x^4+x^3+1 \\
x^4+x^2+x \\
\hline
x^3+x^2+x+1 \\
x^3+x+1 \\
\hline
x^2 \leftarrow S(x)
\end{array}
$$

(E-1773)

$$\therefore \quad \text{Syndrome } S = [100]$$

- This corresponds to the error pattern 0000100 in Table 9.5.

$$\therefore X = Y \oplus e$$

$$= (1101101) \oplus (0000100)$$

$$\therefore X = \boxed{1\ 1\ 0\ 1\ 0\ 0\ 1}$$

(E-2128)

2. Given Y = 0101000 :

$$\therefore Y(x) = x^5 + x^3$$

(E-1774)

$$\begin{array}{r} x^2 \\ x^3+x+1 \overline{\smash{)}\, x^5+x^3} \\ x^5+x^3+x^2 \\ \hline x^2 \leftarrow S(x) \end{array}$$

$$\therefore S(x) = x^2 \qquad \therefore S = 100$$

$$\therefore e = 0000100$$

$$\therefore X = Y \oplus e = (0101000) \oplus (0000100)$$

$$\therefore X = \boxed{0\ 1\ 0\ 1\ 1\ 0\ 0}$$

(E-2129)

Q. 12 For a (5, 1) cyclic code, the generator polynomial used is $g(x) = x^4 + x^3 + x^2 + x + 1$. Draw the encoder and decoder circuit for the cyclic code. May 16

Ans. :

1. Draw encoder :

- General form of generator polynomial is as follows :

$$g(x) = x^4 + g_3x^3 + g_2x^2 + g_1x + 1 \qquad ...(1)$$

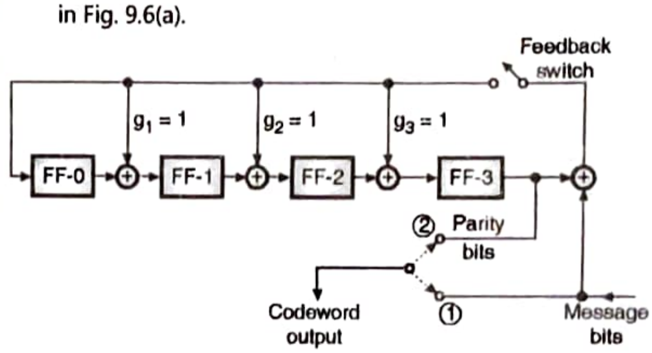
- The given generator polynomial is as follows :

$$g(x) = x^4 + x^3 + x^2 + x + 1 \qquad ...(2)$$

- Comparing Equations (1) and (2) we get,

$$g_1 = 1, g_2 = 1 \text{ and } g_3 = 1$$

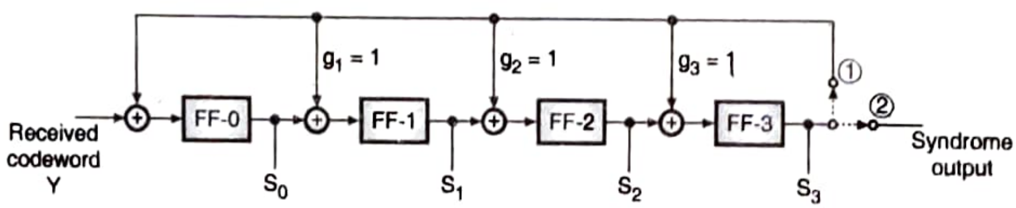
- Therefore, the encoder for (5, 1) cyclic code is as shown in Fig. 9.6(a).



(G-1910) **Fig. 9.6(a) : Encoder for (5, 1) cyclic code**

2. Draw decoder :

- The decoder is same as the syndrome calculator which is as shown in Fig. 9.6(b).



(G-1911) **Fig. 9.6(b) : Decoder for (5, 1) cyclic code**

Q. 13 Draw syndrome calculator for (7, 4) cyclic decoder and obtain syndrome for received code word [1001001]. Dec. 18

Ans. :

Assumption :

- Let the generator polynomial for the given cyclic code be,

$$g(x) = 1 + x^2 + x^3$$

1. Draw the syndrome calculator :

- The given generator polynomial is,

$$g(x) = 1 + x^2 + x^3$$

$$g(x) = x^3 + x^2 + 0x + 1 \qquad ...(1)$$

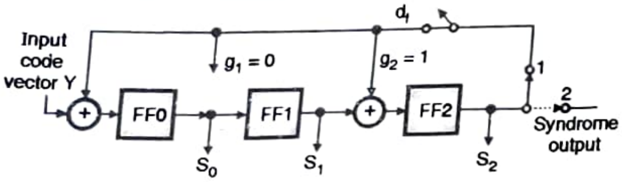
- The general form of generator polynomial is,

$$g(x) = x^3 + g_2x^2 + g_1x + 1 \qquad ...(2)$$

- Comparing Equations (1) and (2) we get,

$$g_1 = 0 \quad \text{and} \quad g_2 = 1$$

- The syndrome calculator is shown in Fig. 9.7.



(E-226(a)) **Fig. 9.7 : Syndrome calculator**

2. **Calculation of syndrome :**

- The output switch of Fig. 9.7 will be initially in position 1 until all the 7 bits of the received signal Y are shifted into the register.
- After that, the output switch is shifted to position 2. Clock pulses are then applied to the shift register to output the syndrome vector S.
- Table 9.6 explains the process of syndrome generation.

The received vector. Y = 1001001 .... Given

(E-2013) **Table 9.6 : Calculation of syndrome**

Shift	Input bit Y	d_f	Content of shift register		
			$S_0 = Y \oplus df$	$S_1 = S_0$	$S_2 = S_1 \oplus df$
–	–	–	0	0	0
1	1	0	1	1	1
2	0	1	1	1	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	0	0	0
6	0	0	0	0	0
7	1	0	1	1	1

Syndrome S = 111

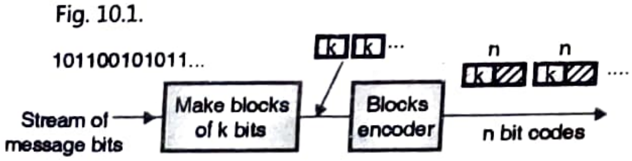
$\therefore$ Syndrome (S) = 111 ...**Ans.**

Chapter 10 : Convolutional Codes

Q. 1 "Convolutional coding can be alternative to block coding when block length is large". Justify. May 16

Ans. :

- For the block codes and cyclic codes there is always a one to one correspondence between the message word and the codeword.
- This type of encoding is useful for the applications having high data rates.
- In such applications, the incoming stream of message bits is divided to form blocks, these blocks are then encoded and the codeword are transmitted as shown in Fig. 10.1.



(E-1226) **Fig. 10.1 : Encoding in block codes**

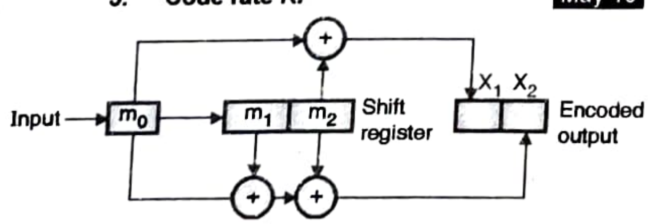
- The block length (n) is required to be large because larger block lengths reduce overheads and practically it is observed that the codes with larger block lengths work better.
- But the block lengths should not be too large as well, because if the block length is too large then the receiver has to wait for a very long time to allow all the bits in a codeword to be received.

Disadvantages of block codes :

- The block codes have the following disadvantages :
  1. A storage is required to be used at the transmitter to store the message block.
  2. The decoding process at the receiver cannot start unless the entire codeword is received.
- Due to the above two drawbacks, a delay gets introduced. Therefore block codes are not suitable for the real time systems which are sensitive to delays.

Q. 2 For the encoder of Fig. 10.2, calculate the following if the information frame length k = 1:

1. **Information frame length k**
2. **Codeword frame length n**
3. **Block length**
4. **Constraint length and**
5. **Code rate R.** May 13



(E-1229(a)) **Fig. 10.2 : Convolution encoder**

Ans. :

- From Fig. 10.2 it is clear that the information frame length k = 1.

- The encoded output is 2 bit long. So the codeword frame length n = 2. Length of the memory i.e. m = 2

Constraint length $= (m + 1)\,k = (2 + 1) \times 1 = 3$

$$\text{Code rate } R = \frac{k}{n} = \frac{1}{2}$$

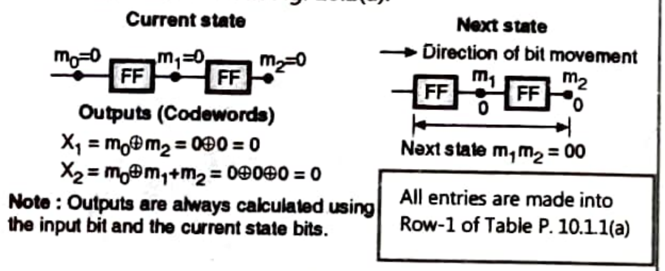
- Therefore the specifications of this encoder are either (2, 1, 2) or (1/2, 3).

Encoder operation :

- Consider the encoder shown in Fig. 10.2. Let the initial state be $m_1\, m_2 = 0\,0$

First input bit $m_0 = 0$:

- As the first input bit $m_0 = 0$ is entered the encoder status is as shown in Fig. 10.2(a).



(E-1800) **Fig. 10.2(a)**

- The current state $m_1\, m_2 = 00$ and the codewords are given by,

$$X_1 = m_0 \oplus m_2 = 0 \oplus 0 = 0$$

$$X_2 = m_0 \oplus m_1 \oplus m_2 = 0 \oplus 0 \oplus 0 = 0.$$

- The codewords are calculated from the input bit and the current state bits.
- The next state is obtained by shifting all the bits to right by one position.
- The original bit $m_2 = 0$ is flushed out and $m_0 = 0$ is taken in.

$\therefore$ Next state $m_1\, m_2 = 0\,0$

- The state $m_1\, m_2 = 00$ is defined as state "a".
- All these entries are made in the first row of Table 10.1(a).

Table 10.1(a) : Encoder operation

	Incoming bit m_0	Current state $m_1 m_2$	Next state $m_1 m_2$	Codeword $X_1 X_2$
Row - 1	0	0 0 (a)	0 0 (a)	0 0

Second input bit $m_0 = 1$:

- Next assume that a 1 input is applied. So $m_0 = 1$ and $m_1\, m_2 = 00$

$$\therefore X_1 = 1 \oplus 0 = 1$$

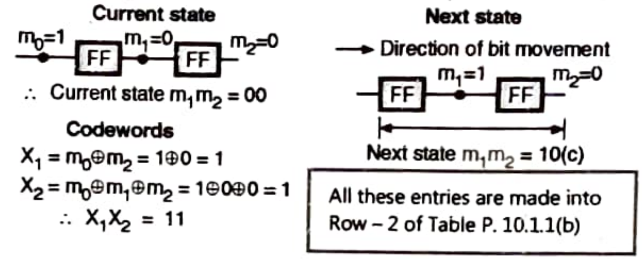
$$\text{and } X_2 = 1 \oplus 0 \oplus 0 = 1$$

$\therefore$ Codeword $X = X_1\, X_2 = 11.$

Again bit $m_2 = 0$ is discarded and $m_0 = 1$ is taken in by the shift register.

New state of shift register $= m_1\, m_2 = 10$

- Refer Fig. 10.2(b) to understand this operation.



(E-1801) **Fig. 10.2(b)**

- The operation will continue in this manner.
- Table 10.1 summarizes the encoder operation.

Table 10.1(b)

	Incoming bit m_0	Current state $m_1 m_2$	Next state $m_1 m_2$	Codeword $X_1 X_2$
Row - 1	0	0 0 (a)	0 0 (a)	0 0
Row - 2	1	0 0 (a)	1 0 (c)	1 1

- This table is also called as the **state table.**
- It can be seen from Table 10.1(c) that the input message sequence and the corresponding coded words is as follows :

Input bits : 01 01 01 01

Output bits : 00 11 11 00 01 10 10 01

(E-1982) **Table 10.1(c) : Encoder operation (State table)**

Input S_1	Present state S_2	S_3		Next state S_2	S_3	Output X_1	X_2
0	0	0	(a)	0	0 (a)	0	0
1	0	0		1	0 (c)	1	1
0	0	1	(b)	0	0 (a)	1	1
1	0	1		1	0 (c)	0	0
0	1	0	(c)	0	1 (b)	0	1
1	1	0		1	1 (d)	1	0
0	1	1	(d)	0	1 (b)	1	0
1	1	1		1	1 (d)	0	1

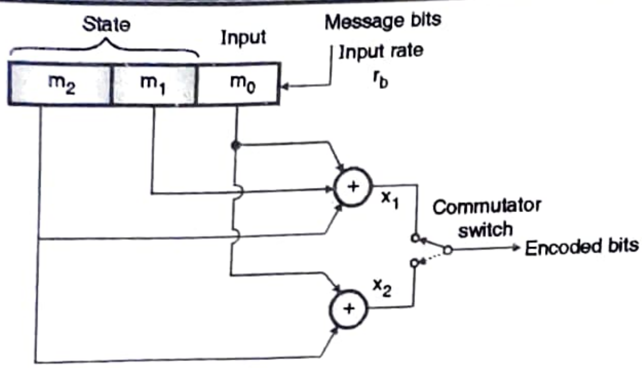
Q. 3 Define following terms related to convolutional code :

Imp

1. **Constraint length**
2. **Code rate** May 17, May 19, Dec. 19

Ans. :

- Fig. 10.3 shows the block diagram of a practical convolution encoder with n=2, k=1 and m = 2.



(E-229) **Fig. 10.3 : Convolution encoder with n = 2, k = 1 and m = 2**

Code Rate (R) :

- The code rate of the encoder of Fig. 10.3 is given by,

$$R = \frac{k}{n} \qquad ...(1)$$

Here k = Number of message bits = 1

n = Number of encoded bits per message bits = 2

$$\therefore \quad r = \frac{1}{2} \qquad ...(2)$$

Constraint length (k) :

- Each message bit influences a span of n (m + 1) successive output bits.
- The quantity n (m + 1) is called as the constraint length. It is measured in terms of encoded output bits.
- For the encoder of Fig. 10.3 the constraint length is 6 bits as n = 2 and m = 2, where m is the encoder's memory measured in terms of input message bits.
- Some authors define the constraint length in terms of message bits as the number of shifts over which a single message bit can influence the encoder output.
- For the encoder of Fig. 10.3 the number of shifts over which a single message bit influences the encoder output is 4.
- Therefore the constraint length is 3.

Q. 4 Explain with suitable example generator polynomial description of convolutional codes.

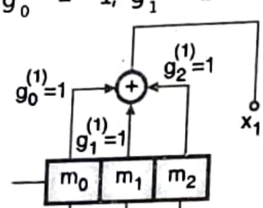
May 08, May 15, Dec. 15

Ans. :

Transform - Domain Approach (Polynomial Description of Convolution Codes :

- The convolution in time domain is transformed into the multiplication of Fourier transforms in the frequency domain.
- We can use this principle in the transform domain approach.
- In this process, the first step is to replace each path in the encoder by a polynomial in such a way that the coefficients of the polynomial are represented by the respective elements of the impulse response.
- For example, for the path corresponding to the top adder, it is given that,

$$g_0^{(1)} = 1,\ g_1^{(1)} = 1 \text{ and } g_2^{(1)} = 1$$



(E-1434)

Input-top adder-output path

- Therefore the input-top-adder-output path of the encoder can be expressed in terms of the polynomial as :

$$G^{(1)}(D) = g_0^{(1)} + g_1^{(1)} D + g_2^{(1)} D^2 \qquad ...(1)$$

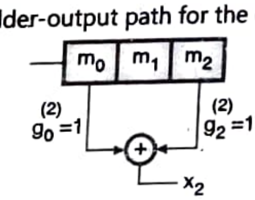
- Substituting the values we get,

$$G^{(1)}(D) = 1 + D + D^2 \qquad ...(2)$$

- The general expression is given by,

$$G^{(1)}(D) = g_0^{(1)} + g_1^{(1)} D + g_2^{(1)} D^2 + ...+ g_L^{(1)} D^2 ....+ g_L^{(1)} D_L \qquad ...(3)$$

- Similarly the polynomial corresponding to the input-bottom adder-output path for the encoder is given by,



(E-1435)

Input-bottom adder-output path

$$G^{(2)}(D) = g_0^{(2)} + g_1^{(2)} D + g_2^{(1)} D^2 \qquad ...(4)$$

Substituting $g_0^{(2)} = 1,\ g_1^{(2)} = 0$ and $g_2^{(2)} = 1$ we get,

$$G^{(2)}(D) = 1 + D^2 \qquad ...(5)$$

- The general form of polynomial is given by,

$$G^{(2)}(D) = g_0^{(2)} + g_1^{(2)} + g_2^{(2)} D^2 + ...+ g_L^{(2)} D^L$$

- The polynomials $G^{(1)}(D)$ and $G^{(2)}(D)$ of Equations (1) and (4) are called as the "generator polynomials" of the code.

- From the generator polynomials, we can obtain the codeword polynomials as follows : Codeword polynomial corresponding to top adder is given by,

$$x^{(1)}(D) = G^{(1)}(D) \cdot m(D) \quad ...(6)$$

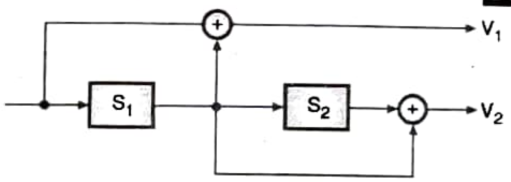
where m (D) = Message polynomial

- and the codeword polynomial corresponding to the bottom adder is given by,

$$x^{(2)}(D) = G^{(2)}(D) \cdot m(D) \quad ...(7)$$

- Once we get the two codeword polynomials, it is possible to obtain the final output sequences by simply using the individual coefficients.
- This is illustrated in the following example.

Q. 5 Draw the trellis diagram for the encoder shown in Fig. 10.4. **Dec. 15**

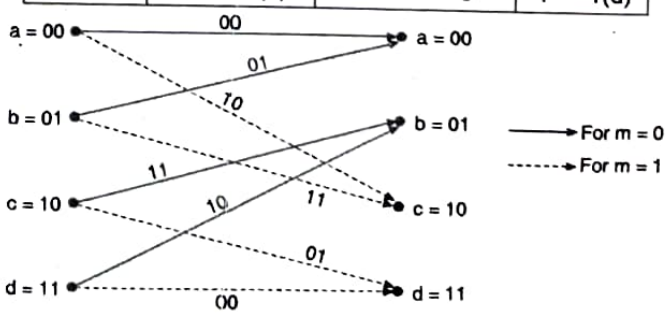


(G-1918) **Fig. 10.4**

Ans. :

Table 10.2 : Encoder operation

Incoming bit m	Present state S_1	Present state S_2	Output V_1 $m \oplus S_1$	Output V_2 $S_1 \oplus S_2$	Next state S_1	Next state S_2
Initially	0	0	0	0	–	–
0	0	0(a)	0	0	0	0(a)
1	0	0(a)	1	0	1	0(c)
0	0	1(b)	0	1	0	0(a)
1	0	1(b)	1	1	1	0(c)
0	1	0(c)	1	1	0	1(b)
1	1	0(c)	0	1	1	1(d)
0	1	1(d)	1	0	0	1(b)
1	1	1(d)	0	0	1	1(d)



(G-1919) **Fig. 10.4(a) : Trellis diagram for encoder**

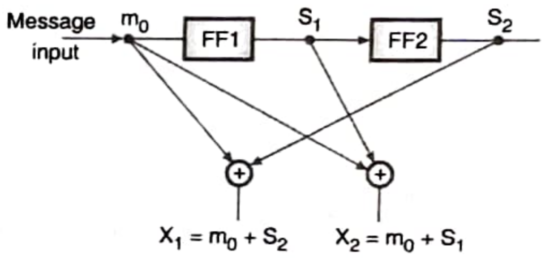
Q. 6 Draw the state diagram for convolutional encoder whose generators are given as :
g_{11} = [1 0 1] g_{12} = [1 1 0] **May 16**

Ans. :

Given : g_{11} = [1 0 1], g_{12} = [1 1 0]

Step 1 : Draw the encoder :

- As there are two generators, there will be two outputs and two mod-2 adders with each adder having at the most 3 inputs, there will be two flip flops.
- Hence the encoder is as shown in Fig. 10.5(a).



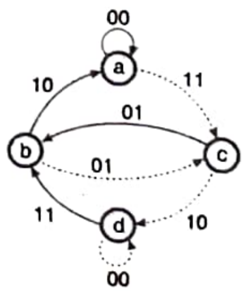
(G-1912) **Fig. 10.5(a) : Encoder**

Step 2 : Draw the state diagram :

Table 10.3

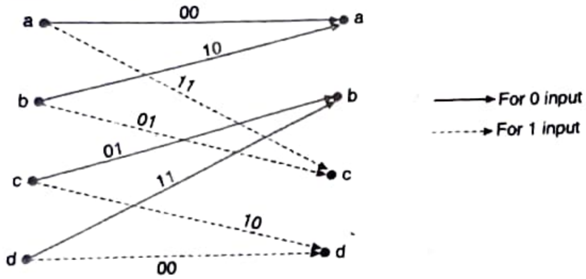
Incoming bit m_0	Current state S_1	Current state S_2	Next state S_1	Next state S_2	Codeword X_1	Codeword X_2
0	0	0 (a)	0	0 (a)	0	0
1	0	0 (a)	1	0 (c)	1	1
0	0	1 (b)	0	0 (a)	1	0
1	0	1 (b)	1	0 (c)	0	1
0	1	0 (c)	0	1 (b)	0	1
1	1	0 (c)	1	1 (d)	1	0
0	1	1 (d)	0	1 (b)	1	1
1	1	1 (d)	1	1 (d)	0	0

- Refer Table 10.3 and draw the state diagram as shown in Fig. 10.5(b).



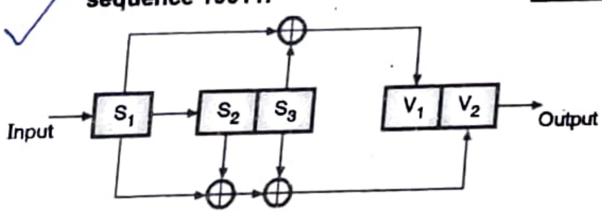
(G-1913) **Fig. 10.5(b) : State diagram**

Trellis diagram :



(G-1914) **Fig. 10.6 : Trellis diagram for encoder of Fig. 10.5(a)**

Q. 7 **For the convolution encoder shown in Fig. 10.7(a), sketch the state diagrams, code tree and trellis diagram. Find the output data sequence 10011.** **Dec. 16**



(E-1637) **Fig. 10.7(a)**

Ans. :

Outputs :

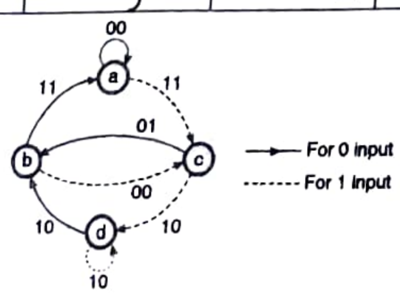
$$V_1 = S_1 \oplus S_3$$

$$V_2 = S_1 \oplus S_2 \oplus S_3$$

Step 1 : State table and state diagram

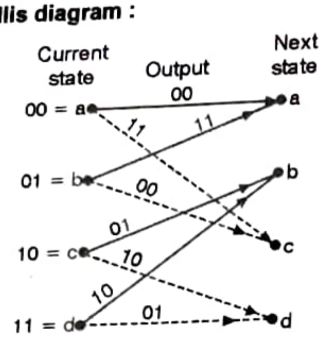
Table 10.4(a) : State table

Input S_1	Current state S_2	S_3		Next state S_2	S_3		Output V_1	V_2
0	0	0	(a)	0	0	(a)	0	0
1	0	0		1	0	(c)	1	1
0	0	1	(b)	0	0	(a)	1	1
1	0	1		1	0	(c)	0	0
0	1	0	(c)	0	1	(b)	0	1
1	1	0		1	1	(d)	1	0
0	1	1	(d)	0	1	(b)	1	0
1	1	1		1	1	(d)	0	1



(E-1831) **Fig. 10.7(b) : State diagram**

Step 2 : Trellis diagram :



(E-1832) **Fig. 10.7(c) : Trellis diagram**

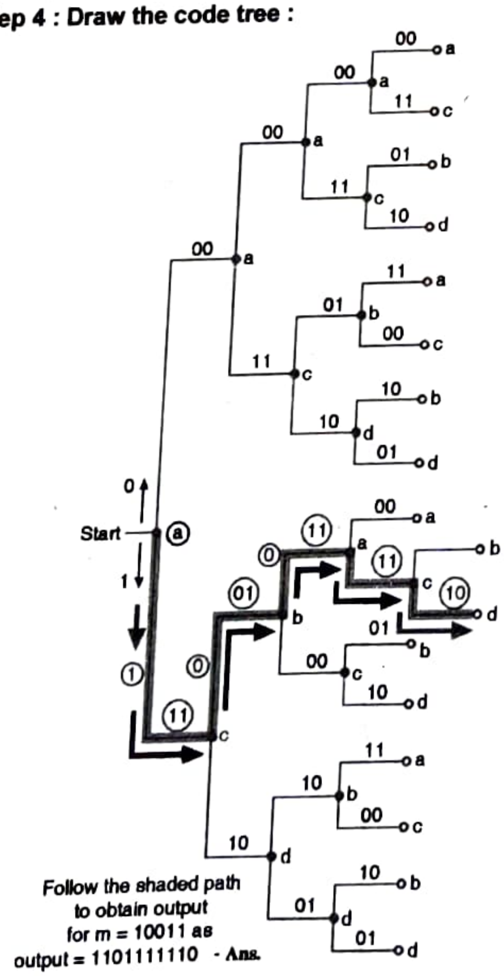
Step 3 : Find the output data sequence :

(E-2121) **Table 10.4(b)**

Input message	Present state	Next state	Output
1	a	c	1 1
0	c	b	0 1
0	b	a	1 1
1	a	c	1 1
1	c	d	1 0

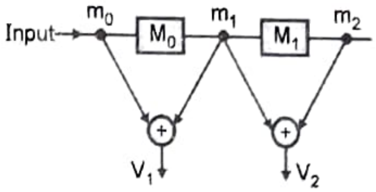
$\therefore$ Output sequence = 11 01 11 11 10 **...Ans.**

Step 4 : Draw the code tree :



(E-1833) **Fig. 10.7(d) : Code tree**

Q. 8 For the convolutional encoder shown in Fig. 10.9(a) show state table, state diagram and code tree. Find the codeword sequence for input message sequence 1011. **May 19**



(E-2083) **Fig. 10.9(a)**

Ans. :

Step 1 : State table :

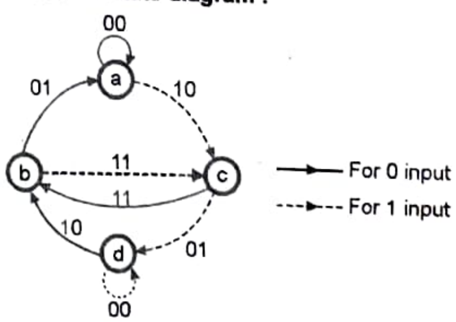
$$V_1 = m_0 + m_1,$$

$$V_2 = m_1 \oplus m_2.$$

Table 10.6(a) : Encoder operation

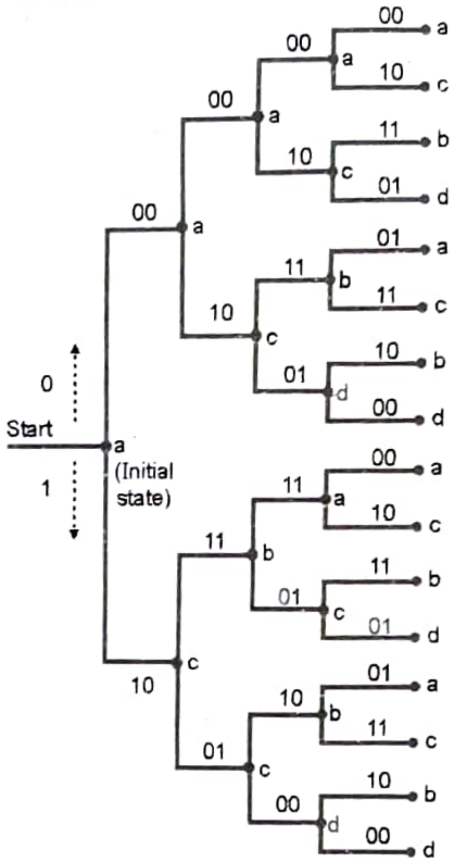
Input m_0	Current state $m_1\ m_2$	Next state $m_1\ m_2$	Output $V_1\ V_2$
Initially 0	0 0	-	0 0
0	0 0 (a)	0 0 (a)	0 0
1	0 0 (a)	1 0 (c)	1 0
0	0 1 (b)	0 0 (a)	0 1
1	0 1 (b)	1 0 (c)	1 1
0	1 0 (c)	0 1 (b)	1 1
1	1 0 (c)	1 1 (d)	0 1
0	1 1 (d)	0 1 (b)	1 0
1	1 1 (d)	1 1 (d)	0 0

Step 2 : Draw the state diagram :



(E-2079) **Fig. 10.9(b) : State diagram**

Step 3 : Draw the code tree :



(E-2080) **Fig. 10.9(c) : Codetree**

Step 4 : Find the codeword sequence :

(E-2081) **Table 10.6(b)**

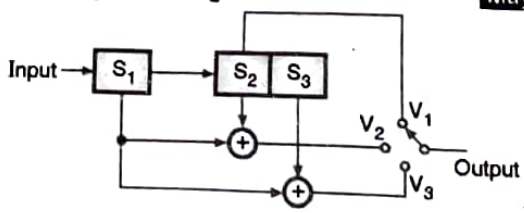
Input		1		0		1		1	
State	a		c		b		c		d
Output		10		11		11		01	

– Refer Table 10.6(b) to get output sequence.

Input sequence : 1 0 1 1

Output sequence : 1 0 1 1 1 1 0 1 **... Ans.**

Q. 9 For the convolutional encoder shown in Fig. 10.8(a), construct the code tree and trellis diagram. Find out the encoded sequence corresponding to the message sequence 10110 using trellis diagram. **May 17**



(E-1834) **Fig. 10.8(a) : Encoder**

Ans. :

Step 1 : Write the state table :

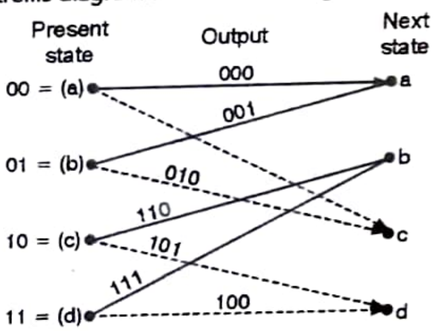
$$V_1 = S_2$$
$$V_2 = S_1 \oplus S_2$$
$$V_3 = S_1 \oplus S_3$$

Table 10.5(a) : State table

Input	Present state			Next state			Output		
S_1	S_2	S_3		S_2	S_3		V_1	V_2	V_3
0	0	0	(a)	0	0	(a)	0	0	0
1	0	0		1	0	(c)	0	1	1
0	0	1	(b)	0	0	(a)	0	0	1
1	0	1		1	0	(c)	0	1	0
0	1	0	(c)	0	1	(b)	1	1	0
1	1	0		1	1	(d)	1	0	1
0	1	1	(d)	0	1	(b)	1	1	1
1	1	1		1	1	(d)	1	0	0

Step 2 : Draw trellis diagram :

– The trellis diagram is as shown in Fig. 10.8(b).



(E-1835) **Fig. 10.8(b) : Trellis diagrm**

Step 3 : Obtain the encoded sequence :

– Refer Table 10.5(b) to obtain the encoded sequence.

Message sequence = 1 0 1 1 0

Table 10.5(b)

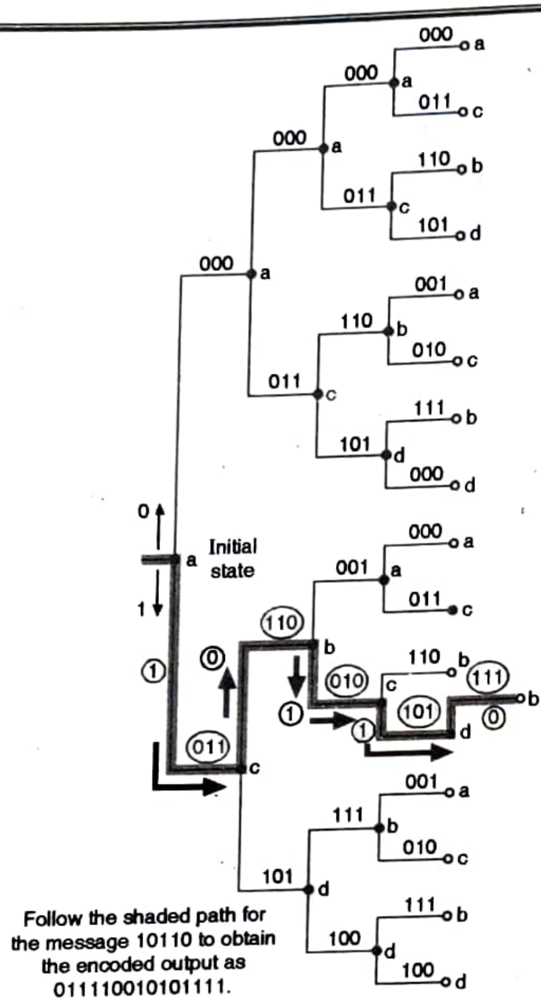
Input		1		0		1		1		0	
State	a		c		b		c		d		b
Output		011		110		010		101		111	

∴ Encoded sequence $V_1 V_2 V_3$ = 011 110 010 101 111

Step 4 : Draw the code tree :

– Refer Fig. 10.8(c) for the code tree. The encoded output obtained using the code tree is given by :

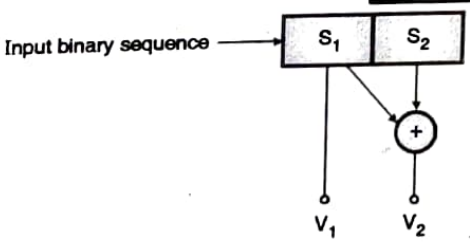
Encoded output = 011 110 010 101 111 **...Ans.**



(E-1836) **Fig. 10.8(c) : Code tree**

Q. 10 **Fig. 10.10(a) depicts a rate 1 / 2, constraint length N = 2, convolutional code encoder. Sketch the code tree for the same.**

Dec. 06, May 15



(E-253) **Fig. 10.10(a)**

Ans. :

– The constraint length k = 2 and its rate is 1/2. That means for a single input binary bit, two bits V_1 and V_2 are encoded at the output. **...Ans.**

– S_1 acts as input and S_2 acts as the state. For S_2, there are two possible values.

S_2 = 0 ...state "a"

S_2 = 1 ...state "b"

– Assume that S_1 and S_2 both are zero. From Fig. 10.10(a) we can write that,

$$\left.\begin{aligned} V_1 &= S_1 \\ \text{and} \quad V_2 &= S_1 \oplus S_2 \end{aligned}\right\} \quad \text{...(1)}$$

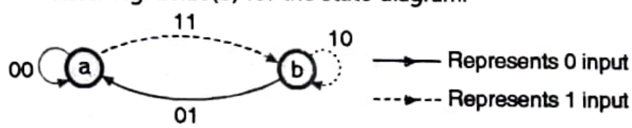
Step 1 : Prepare a state table :

Table 10.7 : State table

	Input bit S_1	Current state S_2	Next state S_2	Output $V_1\ V_2$
Initially	0	0	0	0 0
	0	0 (a)	0 (a)	0 0
	1	0 (a)	1 (b)	1 1
	0	1 (b)	0 (a)	0 1
	1	1 (b)	1 (b)	1 0

Step 2 : Draw the state diagram :

– Refer Fig. 10.10(b) for the state diagram.

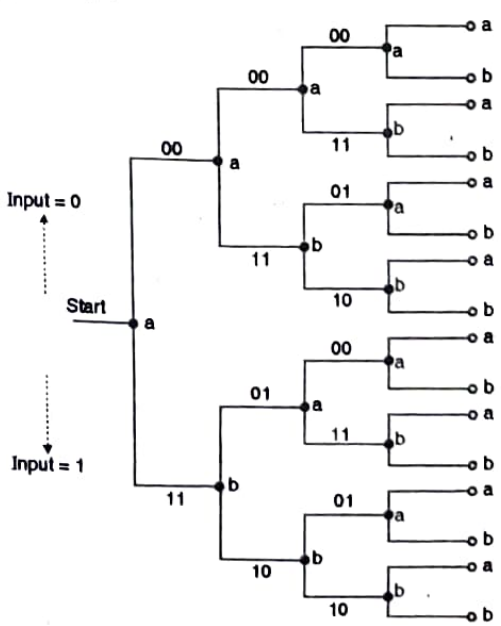


(E-1816) Fig. 10.10(b) : State diagram

Step 3 : Draw the code tree :

– The code tree is as shown in Fig. 10.10(c).

– It has been drawn by referring to the state diagram of Fig. 10.10(b).



(E-259) Fig. 10.10(c) : Code tree for the encoder of Fig. 10.10(a)

Q. 11 For systematic rate $\frac{1}{2}$ convolutional encoder with constraint length 2, parity bit is generated by mod–2 sum p = x + 1.

1. **Draw the encoder.**
2. **Draw state diagram, trellis diagram.**
3. **Find out the output for message (1 0 1).**

Dec. 19

Ans. :

– Refer Q. 10 for the encoder, state diagram and Trellis diagram.

Find the output of message 101 :

Table 10.6

Input		1		0		1
State	a		b		b	
Output		11		10		01

– Hence the output sequence is as follows :

Output = 11 10 01 **...Ans.**

Q. 12 Explain Viterbi's algorithm for decoding of convolutional codes.

Dec. 07, Dec. 08, Dec. 15, Dec. 18

Ans. :

Viterbi decoding :

– The ML (most likelihood) decoding is practically achieved via a minimum length decoder which is usually referred to as Viterbi algorithm (VA).

– The VA is therefore an optimal decoding technique for the memoryless channel.

– The Viterbi algorithm operates on the principle of maximum likelihood decoding and achieves optimum performance.

– The maximum likelihood decoder has to examine the entire received sequence Y and find a valid path which has the smallest Hamming distance from Y.

– But there are 2^N possible paths for a message sequence of N bits. These are a large number of paths.

– The Viterbi algorithm applies the maximum likelihood principle to limit the comparison of so many surviving paths, to make the maximum likelihood decoding practically feasible.

Q. 13 Explain Viterbi's algorithm for decoding of convolutional codes. **Dec. 15, May 19**

Ans. :

- When the decoder input is quantized into just two levels then the decoding is called hard decision decoding.
- The hard decision Viterbi decoding would search for a trellis path which has a minimum Hamming distance from r.
- Before we explain the Viterbi algorithm for the decoding of convolution codes, it is necessary to define certain important terms.

Metric :

- It is defined as the Hamming distance of each branch of each surviving path from the corresponding branch of Y (received signal).
- The metric is defined by assuming that 0's and 1's have the same transmission - error probability.

Surviving path :

- The surviving path is defined as the path of the decoded signal with minimum metric. Let the received signal be represented by Y.
- The Viterbi decoder assigns to each branch of each surviving path a metric.
- By summing the branch metrics we get the path metric.

Q. 14 What is sequential decoding of convolutional codes ? Explain in brief. What is its disadvantage ? **May 15, May 16, Dec. 18**

Ans. :

Sequential decoding :

- The principle of sequential decoding can be explained as follows :
- The sequential decoder generates hypotheses about the transmitted codeword sequence.
- It then computes a metric between these hypotheses and the received signal.
- The sequential decoder will go forward as long as the computed metric indicates that the choices made by the decoder are likely.
- But if the computed metric indicates that the choices made by decoder are unlikely, then the decoder will go backward and change the hypotheses.
- The process of changing the hypotheses continues by trial and error until the decoder finds a likely hypotheses.

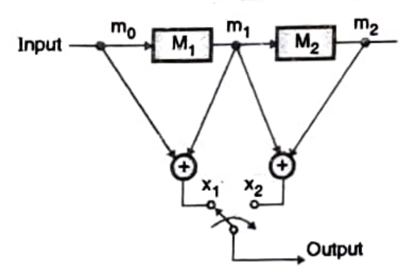
Implementation of sequential decoders :

- It is possible to implement a sequential decoder with hard or soft decisions.
- However practically the soft decisions are avoided because they increase the amount of storage required and the complexity of computation to a great extent.

Disadvantages :

1. It needs longer time.
2. It is not the optimum decoding method.
3. It is more complex.
4. It needs more storage.

Q. 15 For the convolutional encoder shown in Fig. 10.11(a), use Viterbi algorithm to decode the encoded sequence 10, 11, 11, 11, 01. **Dec. 12, Dec. 14, Dec. 18**



(E-279) **Fig. 10.11(a) : Convolutional encoder**

Ans. :

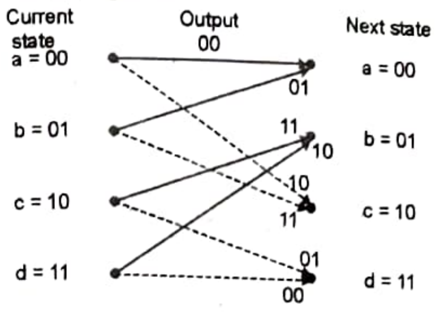
Step 1 : State table :

$$X_1 = m_0 + m_1, X_2 = m_1 \oplus m_2.$$

Table 10.8(a) : Encoder operation

Input m_0	Current state m_1 m_2	Next state m_1 m_2	Output x_1 x_2
Initially 0	0 0	-	0 0
0	0 0 (a)	0 0 (a)	0 0
1	0 0 (a)	1 0 (c)	1 0
0	0 1 (b)	0 0 (a)	0 1
1	0 1 (b)	1 0 (c)	1 1
0	1 0 (c)	0 1 (b)	1 1
1	1 0 (c)	1 1 (d)	0 1
0	1 1 (d)	0 1 (b)	1 0
1	1 1 (d)	1 1 (d)	0 0

Step 2 : Trellis diagram :

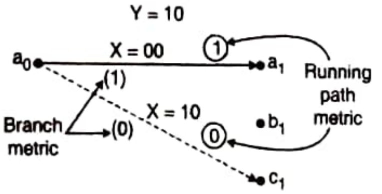


(E-280) **Fig. 10.11(b) : Trellis diagram**

Viterbi decoding :

Step 1 : Received signal Y = 10 (First block) :

- We will refer to the trellis diagram of Fig. 10.11(b).
- Consider the first block of received signal Y = 10. Now draw the trellis diagram for the first input bit as shown in Fig. 10.11(c) and write X = 00 or X = 10 and Y = 10 on the diagram.

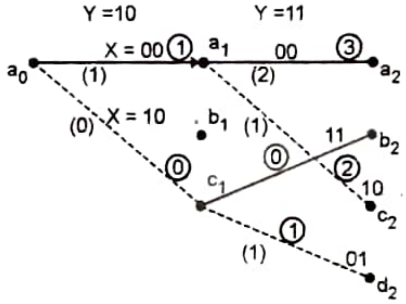


(E-281) **Fig. 10.11(c) : First step in Viterbi algorithm**

- Note that a_0 represents the current state while a_1, c_1 represent the next state.
- Now write the **Branch Metric** which are the numbers written in brackets in Fig. 10.11(c). These are obtained by taking difference between X and Y. So branch metric for branch $a_0\,a_1$ is (1) and that of branch $a_0\,c_1$ is (0).
- Then write the **Running Path Metrics** which are the encircled numbers in Fig. 10.11(c).
- These are obtained by summing the branch metric from a_0. Hence the running path metric of branch $a_0\,a_1 = 0 + 1 = 1$ and that of the branch $a_0\,c_1$ is 0.

Step 2 : Received signal Y = 11 (Second block) :

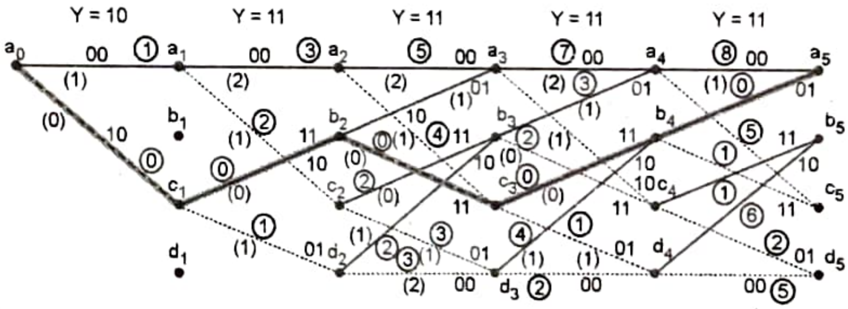
- Fig. 10.11(d) shows the second step in Viterbi algorithm corresponding to second block Y = 11 in the received signal.



(E-282) **Fig. 10.11(d) : Second step in Viterbi algorithm**

Step 3 : Complete the Viterbi diagram :

- We can proceed in the same manner to obtain the final Viterbi diagram as shown in Fig. 10.11(e).



(E-283) **Fig. 10.11(e) : Complete Viterbi diagram**

Step 4 : Maximum likelihood path :

- We have to choose a path which has the smallest value of running metric and the received signal should be decoded along this path.
- As shown in Fig. 10.11(e) there is one path having "0" running matrix. We choose this path

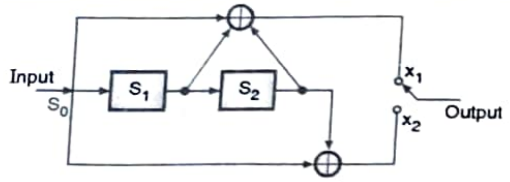
$$a_0 - c_1 - b_2 - c_3 - b_4 - a_5$$

- Which is shaded path in Fig. 10.11(e).
- If we travel along this path, then the decoded signal is,

  1 0 1 0 0

- A zero metric path indicates that there are no errors in the received signal.

Q. 16 A convolution encoder has code rate = 1/2 constraint length K = 3 as shown in Fig. 10.12. Draw the trellis diagram. By using Viterbi algorithm decode the sequence 010001000. Dec. 16



(E-1636(a)) **Fig. 10.12**

Ans. :

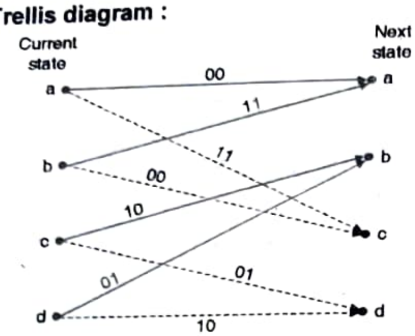
Step 1 : Encoder operation :

$$x_1 = S_0 \oplus S_1 \oplus S_2, \quad x_2 = S_0 \oplus S_2$$

Table 10.9 : Encoder operation

Input S_0	Current state S_1	S_2		Next state S_1	S_2		Output x_1	x_2
Initially 0	0	0		–			0	0
0	0	0	(a)	0	0	(a)	0	0
1	0	0	(a)	1	0	(c)	1	1
0	0	1	(b)	0	0	(a)	1	1
1	0	1	(b)	1	0	(c)	0	0
0	1	0	(c)	0	1	(b)	1	0
1	1	0	(c)	1	1	(d)	0	1
0	1	1	(d)	0	1	(b)	0	1
1	1	1	(d)	1	1	(d)	1	0

Step 2 : Trellis diagram :



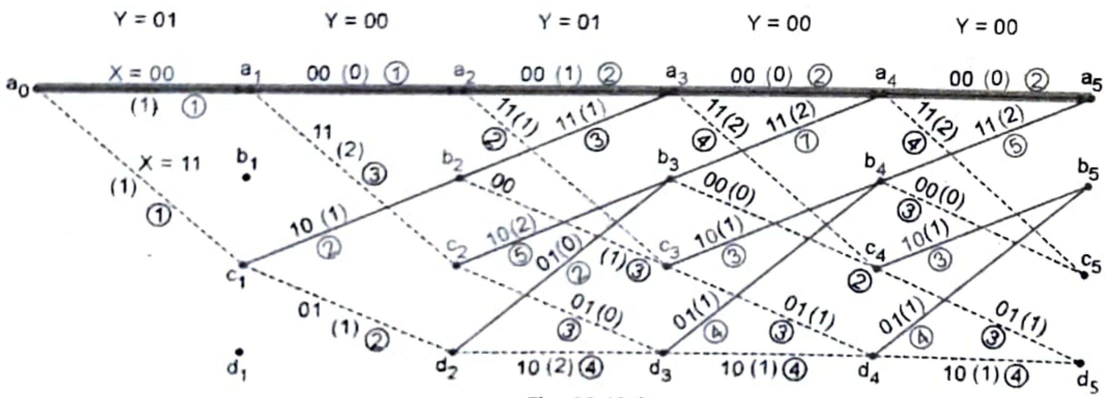
(E-1650) **Fig. 10.12(a) : Trellis diagram**

Step 3 : Viterbi algorithm : Refer Fig. 10.12(b).

Step 4 : Decoding :

- We have to choose the path having the smallest value of the running metric and decode the received signal along this path.
- As shown in Fig. 10.12(b) the path having smallest running metric i.e. 2 is ($a_0 - a_1 - a_2 - a_3 - a_4 - a_5$) which is shaded.
- By travelling along this path we can obtain the decoded sequence as,

$$m = 00000 \quad \text{...Ans.}$$



(E-1651) **Fig. 10.12(b)**

Q. 17 Define the following terms related to convolutional code with example : Free length and Coding gain. **May 18**

Ans. :

Free length :

- The error detection and correction capability of the block and cyclic codes is dependent on the minimum distance, d_{min} between the code vectors.
- But in case of a convolutional code the entire transmitted sequence must be considered as a single code vector.
- Therefore the free distance (d_{free}) is defined as the minimum distance between the code vectors.
- But the minimum distance between the code vectors is same as the minimum weight of the code vector. Hence the free distance is equal to the minimum weight of the code vector.

$\therefore$ Free distance d_{free} = Minimum distance

= Minimum weight of code vectors

- If X represents the transmitted signal then the free distance is given by,

$$d_{free} = [W(X)]_{min} \quad ...(1)$$

where $[W(X)]_{min}$ = Minimum weight of the code vector.

- The way minimum distance decides the capacity of the block or cyclic codes to detect and correct errors, the free distance will decide the error control capacity for the convolutional code.

Coding gain (A) :

- In the coding theory, coding gain (A) is defined as the signal to noise ratio (SNR) of an uncoded system and that of a coded system, required to reach the same bit error rate (BER).

$\therefore$ Coding gain = $(SNR)_{uncoded} - (SNR)_{coded}$

- The coding gain is used for comparing the different coding techniques.

$$\text{Coding gain (convolutional encoder)} = \frac{R\, d_{free}}{2} \quad ...(2)$$

Where R = Code rate and d_{free} = The free distance

- Thus the coding gain is directly proportional to the free distance d_{free}.

task

1. Huffman code
2. Shannon fano code

symbol - x_1 x_2 x_3 x_4

Huffman. probability - 0.4 0.3 0.2 0.1.

LIH.n.

Shannon fano = x_1 x_2 x_3 x_4 x_5 x_6.

0.4. 0.19. 0.16. 0.15 0.1.