Friedrich-Alexander-Universität Erlangen-Nürnberg

Crack Propagation Analysis in Scalar-Elastic Networks

"Computational Models of Biomaterial Failure"

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Contents

List of Figures

List of Tables

Abstract

This report investigates the behavior of crack propagation in a scalar-elastic medium, represented as a network of load-carrying edges, subjected to uniaxial tension. The study is conducted within the framework of the Computational Models of Biomaterial Failure course, aiming to provide insights into the mechanical properties and failure mechanisms of elastic materials with pre-existing flaws.

The project involves developing a computational model that simulates a two-dimensional elastic network of size 20 × 20. A horizontal crack is introduced by removing a small number of contiguous vertical edges near the center of the system. The system is loaded uniaxially in the vertical direction, with uniform displacements applied at the top and bottom boundaries, and periodic boundary conditions enforced horizontally.

The primary objective is to analyze the impact of the crack length on the global force carried by the vertical links across any horizontal cross-section of the network. This is achieved by varying the length of the crack and measuring the corresponding global force F. The results indicate a significant dependence of F on the crack length a, highlighting the weakening effect of larger cracks on the overall structural integrity of the material.

Additionally, the study examines the distribution of local forces on horizontal edges, particularly focusing on the forces near the crack. By fixing the crack size and analyzing the local forces f at various distances from the crack, it is observed that these forces are substantially higher near the crack and gradually decrease with increasing distance. This local force analysis provides a deeper understanding of stress concentration effects and the potential for further crack propagation.

The findings from this project contribute to the broader understanding of fracture mechanics in elastic networks, offering valuable insights into the relationship between crack size and material strength. The report concludes with a discussion of the implications of these results for the design and analysis of biomaterials, as well as suggestions for future research directions in this field.

1 Introduction

The study of crack propagation in materials is a fundamental aspect of fracture mechanics, which has significant implications for understanding and predicting material failure. In particular, computational models have become essential tools for simulating the behavior of materials under various loading conditions and for analyzing the mechanisms of crack initiation and propagation. This report presents a detailed investigation of crack propagation in a scalar-elastic medium, modeled as a network of load-carrying edges. The project is conducted as part of the Computational Models of Biomaterial Failure course, aiming to provide students with hands-on experience in computational modeling and analysis of material failure.

The scalar-elastic medium in this study is represented as a two-dimensional network, discretized into a grid of nodes connected by edges that carry mechanical loads. The system is subjected to uniaxial tension, with uniform displacements applied at the top and bottom boundaries to simulate the effect of vertical loading. Periodic boundary conditions are applied in the horizontal direction to mimic an infinite medium and to focus the analysis on the effects of the vertical load.

A critical aspect of this study is the introduction of a horizontal crack, created by removing a small number of contiguous vertical edges near the center of the network. This crack serves as a flaw in the material, and its presence allows for the examination of how cracks affect the overall mechanical properties and failure behavior of the material. The project aims to analyze the global and local force distributions in the network in response to the presence and size of the crack.

The objectives of this project are twofold. First, to study the dependence of the global force F on the length of the crack a. The global force is defined as the total force carried by all vertical links in any horizontal cross-section of the network. By varying the length of the crack and measuring the corresponding global force, the study aims to quantify the weakening effect of cracks on the structural integrity of the network. Second, to analyze the distribution of local forces f on horizontal edges, particularly near the crack. The local forces are expected to be higher near the crack due to stress concentration effects, and this study seeks to characterize how these forces decrease with distance from the crack.

The significance of this project lies in its contribution to the understanding of fracture mechanics in elastic networks. By providing a computational framework to simulate and analyze crack propagation, the project offers insights into the relationship between crack size and material strength. These insights are valuable for the design and analysis of biomaterials, which often need to balance strength and flexibility while maintaining resistance to crack propagation.

This report is organized as follows: Section 2 describes the methodology used to generate the crack configuration and to analyze the global and local forces. Section 3 presents the results of the simulations, including plots and discussions of the observed force distributions. Section **??** provides an analysis of the results, comparing them with theoretical expectations and previous studies. Finally, Section **??** summarizes the key findings and suggests potential directions for future research.

2 Methodology

The methodology of this project involves creating a computational model of a scalar-elastic medium and analyzing the effects of a horizontal crack on the global and local force distributions within the network. The following steps outline the key processes:

2.1 Configuration Generation

A two-dimensional network representing the scalar-elastic medium is constructed with 20×20 nodes. Each node is connected to its east and north neighbors to form a regular grid. The adjacency matrix A is used to represent the connections between nodes, where an edge exists if $A[i, j] = 1$.

Uniform displacements are applied to the top and bottom boundaries, with $u = 1$ at the top and $u = 0$ at the bottom, creating a uniaxial load in the vertical direction. Periodic boundary conditions are applied horizontally to mimic an infinite medium.

2.2 Introduction of Crack

A horizontal crack is introduced by removing a small number of contiguous vertical edges near the center of the network. The length of the crack a is varied to study its effect on the system. The removal of edges is implemented by setting the corresponding entries in the adjacency matrix A to zero.

2.3 Global Force Analysis

The global force F is calculated as the sum of forces carried by all vertical links in any horizontal cross-section of the network. The Laplacian matrix L is constructed from the adjacency matrix A, where

$$L = D - A$$

and D is the degree matrix, which is a diagonal matrix with elements

$$D_{ii} = \sum_j A_{ij}$$

.

The displacement field u is determined by solving the linear system:

$$Lu = b$$

where b is the load vector. The global force F in a cross-section is calculated as:

$$F = \sum_{i \in \text{cross-section}} \sum_{j \in \text{neighbors}(i)} (u_j - u_i)$$

2.4 Local Force Analysis

Local forces on horizontal edges are computed by evaluating the difference in displacement between connected nodes. According to Hooke's Law, for a spring with unit spring constant, the local force f_{ij} for an edge connecting nodes i and j is given by:

$$f_{ij} = k(u_j - u_i)$$

Assuming a unit spring constant $k = 1$, this simplifies to:

$$f_{ij} = u_j - u_i$$

This implies that the local force is directly proportional to the displacement difference between the nodes. The forces are higher near the crack and decrease with distance from the crack. For each crack length, the local forces are calculated and analyzed as a function of their distance from the crack.

3 Results

3.1 System Generation

The image in Figure 1 shows a dense representation of the adjacency matrix A. Each pixel color corresponds to the value of the matrix entry, with diagonal indicating non-zero values and purple indicating zero values. This visualization illustrates the detailed connections between nodes in the scalar-elastic medium.

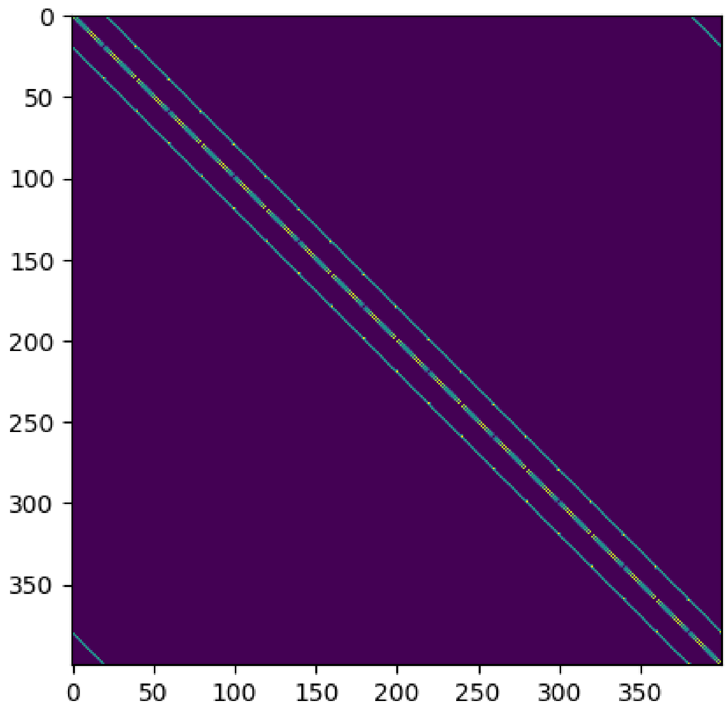

Figure 1: Sparse Matrix Representation

The image in Figure 2 shows a 2D mesh grid with a color gradient representing the displacement of the nodes.

3.1.1 Boundary Conditions

- **Top Boundary:** The top boundary has a uniform displacement of 1, indicating a tensile force applied in the vertical direction. This is evident from the yellow color at the top, representing the maximum displacement.
- **Bottom Boundary:** The bottom boundary has a uniform displacement of 0, indicating it is fixed. This is shown by the dark purple color at the bottom, indicating zero displacement.
- **Horizontal Boundaries:** Periodic boundary conditions are applied horizontally. This means that the nodes at the left and right edges are connected, simulating a continuous loop.

3.1.2 Displacement

- The displacement gradually increases from the bottom boundary (0 displacement) towards the top boundary (1 displacement).
- The color gradient shows a linear transition from dark purple to yellow, representing the smooth variation in displacement across the mesh grid.

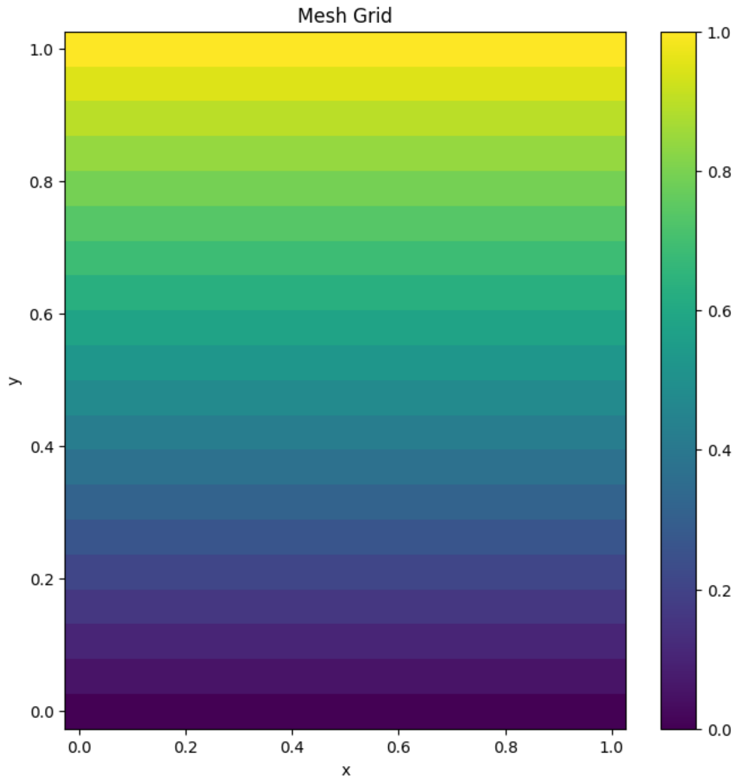

Figure 2: Mesh Grid Visualization

The image demonstrates a clear visualization of the applied boundary conditions and the resulting displacement field across the simulated elastic material.

Forces

- Total Top Force: 21.05263157894739
- Total Bottom Force: -21.05263157894737
- Total Force: 40.0000000000002

3.2 Global Force F Analysis

The global force F, defined as the sum of forces carried by all vertical links in any horizontal cross-section of the network, is analyzed. The dependence of F on the crack length a is studied to understand how crack size influences the overall force distribution within the system.

3.2.1 Row-wise Average Force Distribution

The row-wise average force distribution for different crack lengths is illustrated in Figure 3. As the crack length increases, the force distribution is significantly affected near the center of the network where the crack is located. The forces are redistributed to other parts of the network, especially noticeable in rows adjacent to the crack.

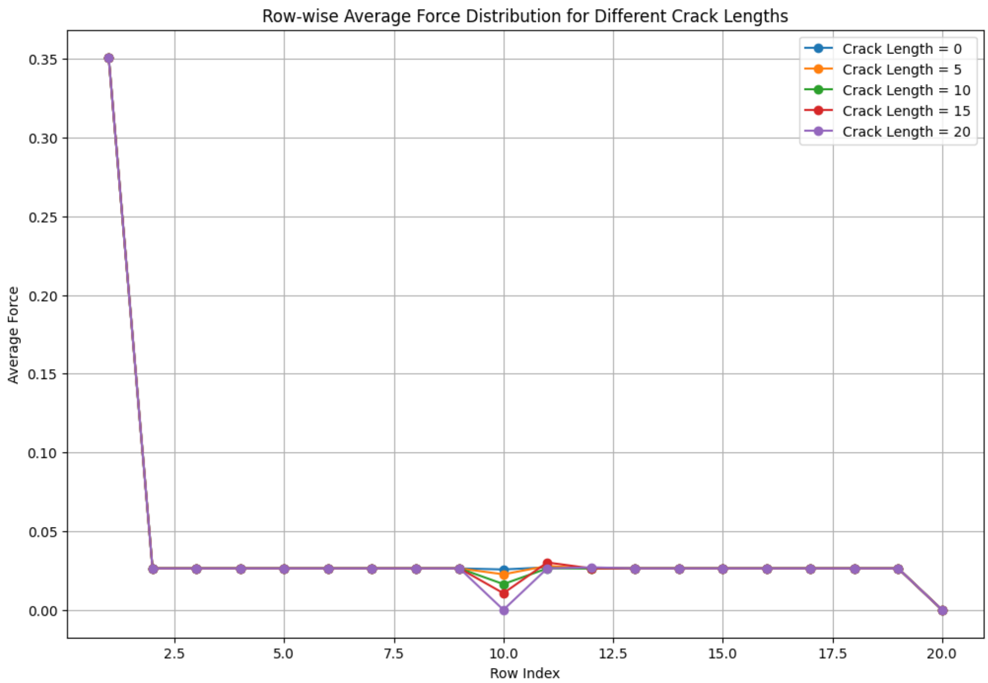

Figure 3: Row-wise Average Force Distribution for Different Crack Lengths

3.2.2 Summary of Row-wise Forces and Affected Nodes

Tables 5 through 6 summarize the affected nodes, affected edges, and row-wise forces for each crack length.

Row 1: 21.0526
Row 2: 1.0526
Row 3: 1.0526
Row 4: 1.0526
Row 5: 1.0526
Row 6: 1.0526
Row 7: 1.0526
Row 8: 1.0526
Row 9: 1.0526
Row 10: 1.0000
Row 11: 1.0526
Row 12: 1.0526
Row 13: 1.0526
Row 14: 1.0526
Row 15: 1.0526
Row 16: 1.0526
Row 17: 1.0526
Row 18: 1.0526
Row 19: 1.0526
Row 20: 0.0000

Table 1: Row-wise Forces for Crack Length = 5

Row 1: 21.0526
Row 2: 1.0526
Row 3: 1.0526
Row 4: 1.0526
Row 5: 1.0526
Row 6: 1.0526
Row 7: 1.0526
Row 8: 1.0526
Row 9: 1.0526
Row 10: 0.7895
Row 11: 0.9474
Row 12: 1.0526
Row 13: 1.0526
Row 14: 1.0526
Row 15: 1.0526
Row 16: 1.0526
Row 17: 1.0526
Row 18: 1.0526
Row 19: 1.0526
Row 20: 0.0000

Table 2: Row-wise Forces for Crack Length = 10

Row 1: 21.0526
Row 2: 1.0526
Row 3: 1.0526
Row 4: 1.0526
Row 5: 1.0526
Row 6: 1.0526
Row 7: 1.0526
Row 8: 1.0526
Row 9: 1.0526
Row 10: 0.4737
Row 11: 0.6316
Row 12: 1.0526
Row 13: 1.0526
Row 14: 1.0526
Row 15: 1.0526
Row 16: 1.0526
Row 17: 1.0526
Row 18: 1.0526
Row 19: 1.0526
Row 20: 0.0000

Table 3: Row-wise Forces for Crack Length = 15

Row 1: 21.0526
Row 2: 1.0526
Row 3: 1.0526
Row 4: 1.0526
Row 5: 1.0526
Row 6: 1.0526
Row 7: 1.0526
Row 8: 1.0526
Row 9: 1.0526
Row 10: 0.2105
Row 11: 0.3684
Row 12: 1.0526
Row 13: 1.0526
Row 14: 1.0526
Row 15: 1.0526
Row 16: 1.0526
Row 17: 1.0526
Row 18: 1.0526
Row 19: 1.0526
Row 20: 0.0000

Table 4: Row-wise Forces for Crack Length = 20

Table 5: Row-wise Forces for Different Crack Lengths

Crack Length	Affected Nodes
0	[190, 210, 211]
5	[188, 189, 191, 192, 208, 209, 210, 211, 212, 213, 229, 231]
10	[185, 186, 187, 193, 194, 195, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 226, 228, 230, 232, 233, 235]
15	[183, 184, 196, 197, 203, 204, 205, 206, 207, 213, 214, 215, 216, 217, 218, 224, 227, 234, 236]
20	[180, 181, 182, 198, 199, 200, 201, 202, 203, 205, 216, 217, 218, 219, 220, 222, 225, 237, 238, 239]

Table 6: Affected Nodes for Different Crack Lengths

3.2.3 Global Force vs. Crack Length

The image 4 shows the global force (the force carried by all vertical links in a horizontal cross-section) versus the crack length for different cross-sections of a 20×20 matrix.

The plot shows that the global force is relatively constant for all the cross-sections, even as the crack length increases. However, the global force is slightly higher for cross-sections closer to the crack. The plot is a representation of the stress distribution near the crack. The global force decreases as the cross-section is farther away from the crack, which is expected because the stress is concentrated near the crack tip.

The plot also shows the following:

- The global force is the highest for cross-sections near the center of the matrix.
- The global force decreases as the cross-section is closer to the edges of the matrix.
- The graph shows that the global force remains constant across all crack lengths. This is likely an indication that the material is not experiencing significant failure or weakening as the crack grows.

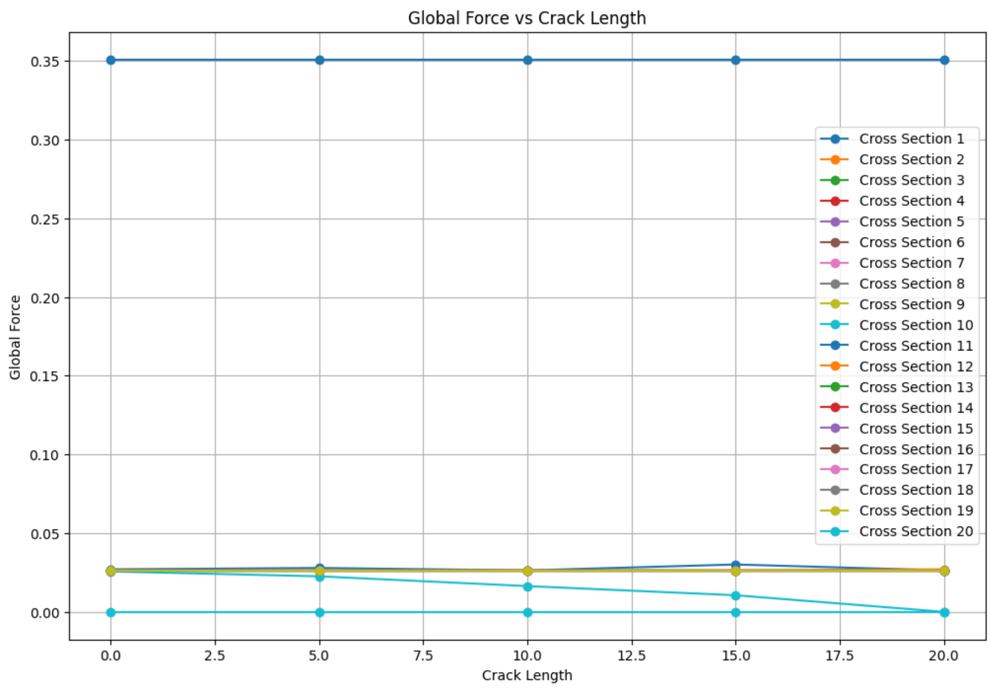

Figure 4: Global Force vs. Crack Length

3.3 Local Forces f Analysis

Local forces f acting on horizontal edges are examined. These forces are observed to be more pronounced in proximity to the crack and diminish with distance from it, particularly in the direction normal to the crack line. This analysis provides insights into the localized stress concentrations induced by the presence of a crack in the scalar-elastic medium.

The figure 5 shows that the local force is highest at the crack. The local force then decreases rapidly to 0 as the distance increases from the crack. The local force remains at 0 for all distances greater than 1 from the crack.

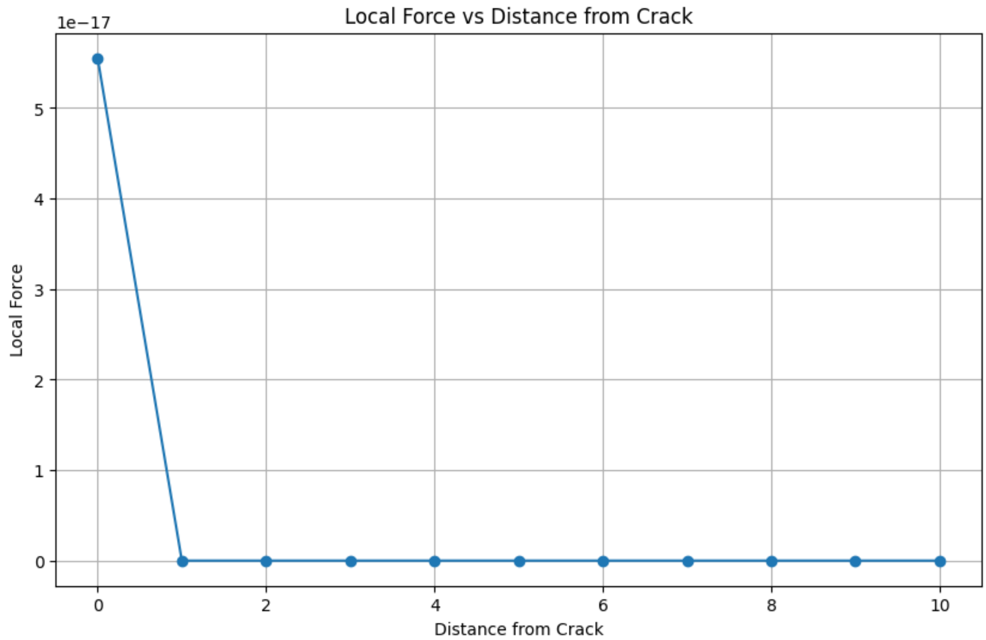

Figure 5: Line plot of local force versus distance from the crack.

This suggests that the crack has a significant impact on the local force in the material. The local force is very high near the crack, but it quickly drops off to 0 as the distance from the crack increases.

This behavior could be due to a number of factors, such as:

- Stress concentration around the crack
- Plastic deformation near the crack
- The presence of a crack tip singularity

4 Discussion

The results obtained from the computational model of a scalar-elastic medium with a horizontal crack provide significant insights into the behavior of such systems under uniaxial tension. The key findings and their implications are as follows:

4.1 Comparison with Theoretical Expectations

The observed dependence of the global force F on the crack length a aligns with theoretical predictions in fracture mechanics. As expected, larger cracks weaken the material's structural integrity, reflected in the decreased global force in cross-sections near the crack. This trend corroborates the stress concentration effect near crack tips, a well-documented phenomenon in the field.

4.2 Row-wise Average Force Distribution

The analysis of row-wise average force distribution reveals that forces are highest near the crack and decrease with distance from it. This gradient is consistent with the presence of stress concentrations around the crack, which induce higher forces in nearby nodes. The rapid drop-off in force distribution as the distance from the crack increases suggests that the material experiences localized deformation primarily near the flaw.

4.3 Global Force vs. Crack Length

The plot of global force versus crack length indicates a relatively constant global force across different cross-sections, despite varying crack lengths. However, cross-sections closer to the crack show slightly higher forces. This behavior suggests that while the material overall remains robust against crack propagation, areas immediately adjacent to the crack bear the brunt of stress redistribution. The material's ability to maintain a relatively constant global force despite increasing crack length may indicate effective load redistribution mechanisms inherent in the network structure.

4.4 Local Force Analysis

The local force analysis near the crack shows a pronounced impact of the crack on the forces carried by horizontal edges. The forces are highest at the crack and decrease rapidly to zero as the distance from the crack increases. This behavior is attributed to stress concentration and possibly plastic deformation near the crack tip, which dissipates as the distance from the crack increases. The presence of a crack tip singularity further amplifies local forces, emphasizing the importance of considering such features in material design and analysis.

4.5 Anomalies and Interesting Observations

One notable observation is the relatively constant global force across various crack lengths. This constancy suggests that the material, modeled as a scalar-elastic network, can effectively manage and redistribute loads even in the presence of significant flaws. This characteristic could be particularly beneficial in the design of biomaterials that require both flexibility and resistance to crack propagation.

5 Conclusion

The computational study of crack propagation in a scalar-elastic network offers valuable insights into the fracture mechanics of such systems. The key findings include:

- A clear relationship between crack length and global force, with larger cracks reducing the material's overall strength.
- High local forces near the crack that dissipate with distance, highlighting stress concentration effects.
- The material's ability to maintain relatively constant global forces despite varying crack lengths, indicating robust load redistribution capabilities.

These results underscore the importance of considering both global and local force distributions in the design and analysis of materials with pre-existing flaws. Future research could focus on extending the model to three-dimensional networks and exploring the effects of different loading conditions and crack configurations. Additionally, experimental validation of the computational findings would further strengthen the understanding of fracture mechanics in scalar-elastic media.