Measures of Shape

Data Science for Quality Management: Describing Data Numerically

with **Wendy Martin**

Learning objectives:

Discriminate between skewness & kurtosis
Calculate the sample skewness & kurtosis

Measures of Shape

Measures of shape reflect the type of distribution sampled.

- Skewness is concerned with the symmetrical nature of the distribution, and
- Kurtosis is concerned with the peakedness of the distribution.

Skewness

- Skewness is the degree of departure from symmetry of a distribution

- Symbols
  Population (γ_3) and
  Sample (g_3)

Skewness

- Measures “lopsidedness”
- Symmetric distributions have zero skewness

Skewness: Calculations

- The most important group of measures of skewness and kurtosis use the third and fourth moments about the mean
- Moments about the mean are the average of the deviations from the mean raised to some power

Skewness: Calculations

- The r^{th} moment about the mean is:

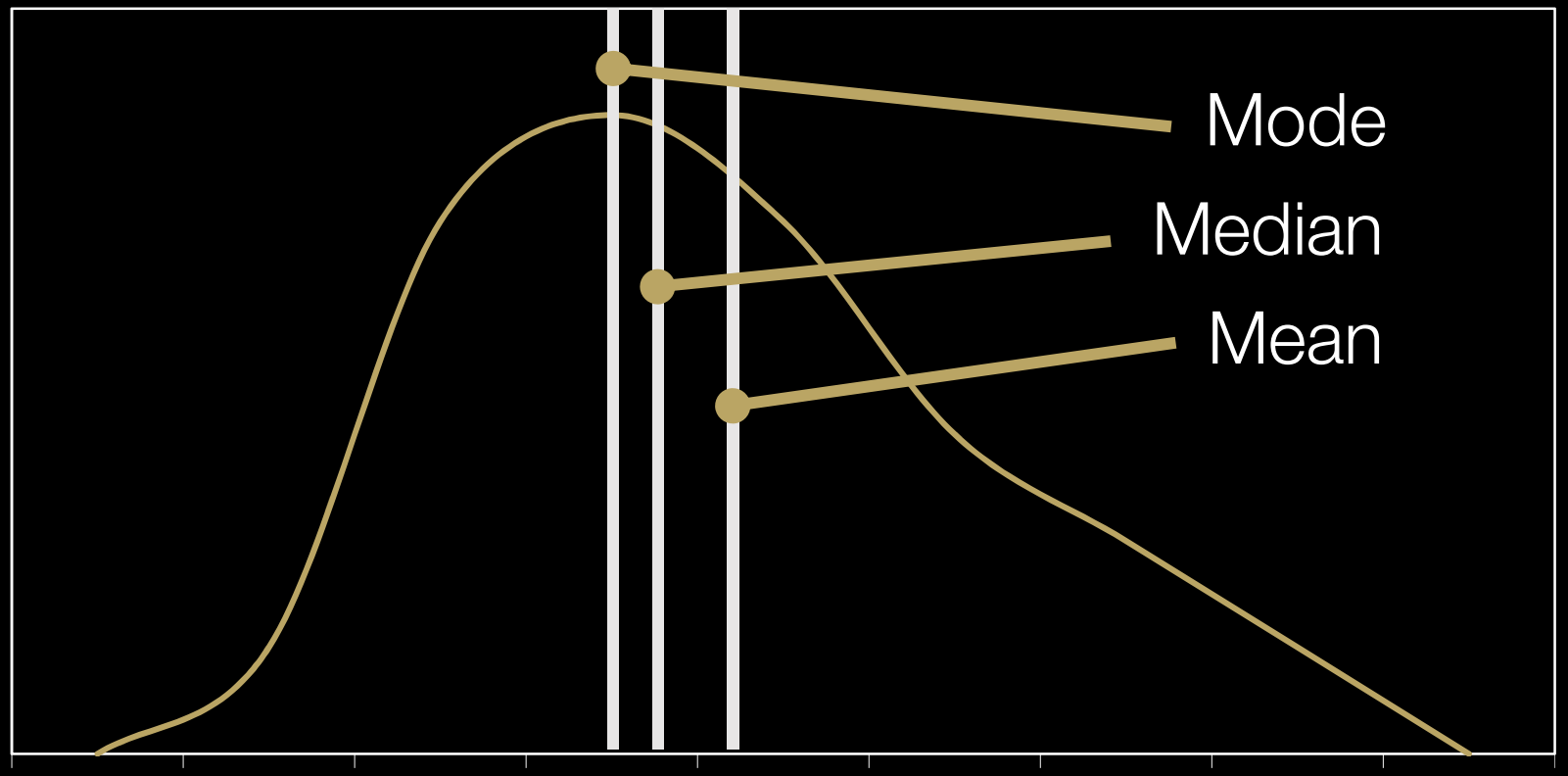
$$m_r = \frac{\sum (X - \bar{X})^r}{n}$$

Skewness: Calculations

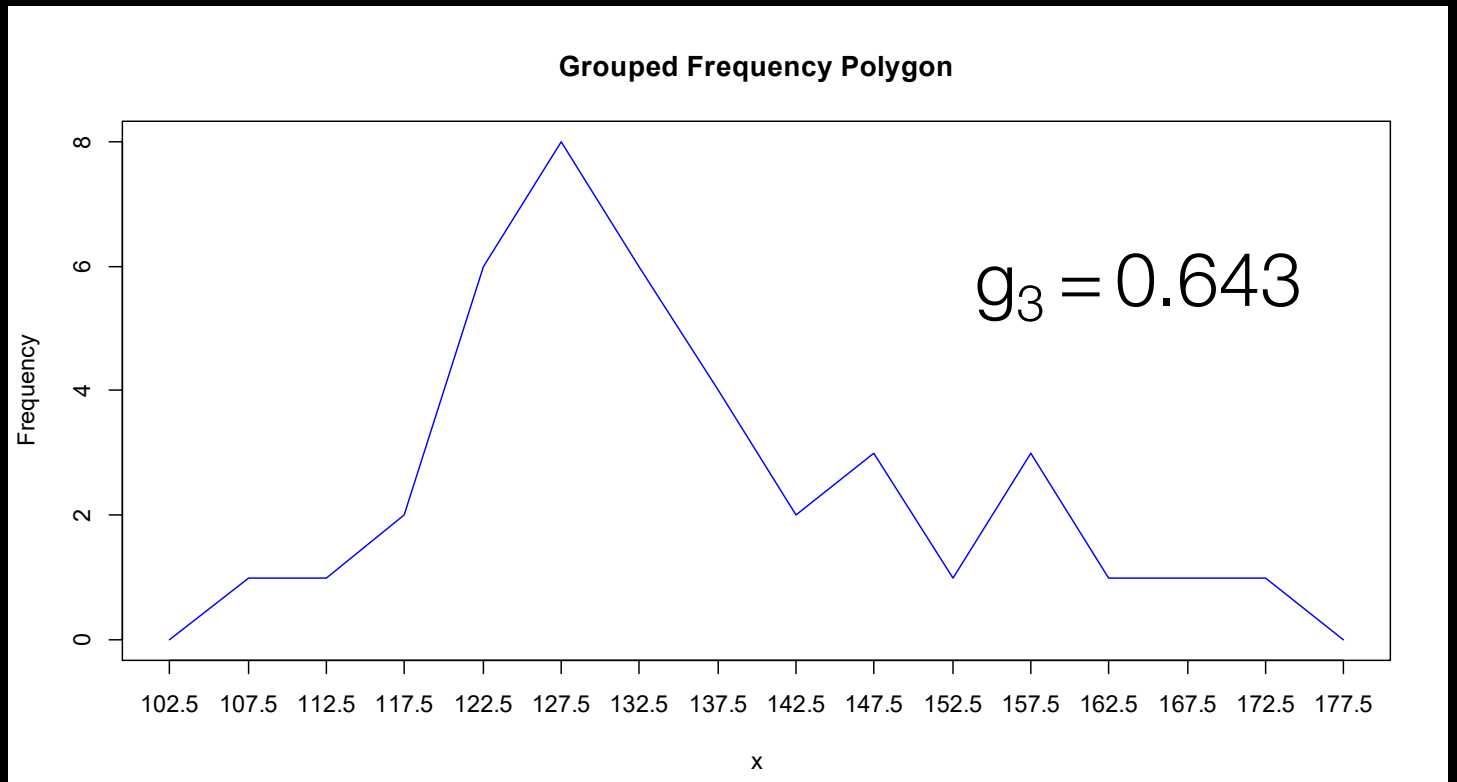
- A measure of skewness may then be calculated as follows
- The sign displays the direction of skewness

$$g_3 = \left[\frac{\sqrt{n(n-1)}}{n-2}\right] x \frac{m_3}{m_2^{3/2}}$$

Skewed Distributions



Skewed Distributions



How to Calculate in RStudio

- In R Studio:

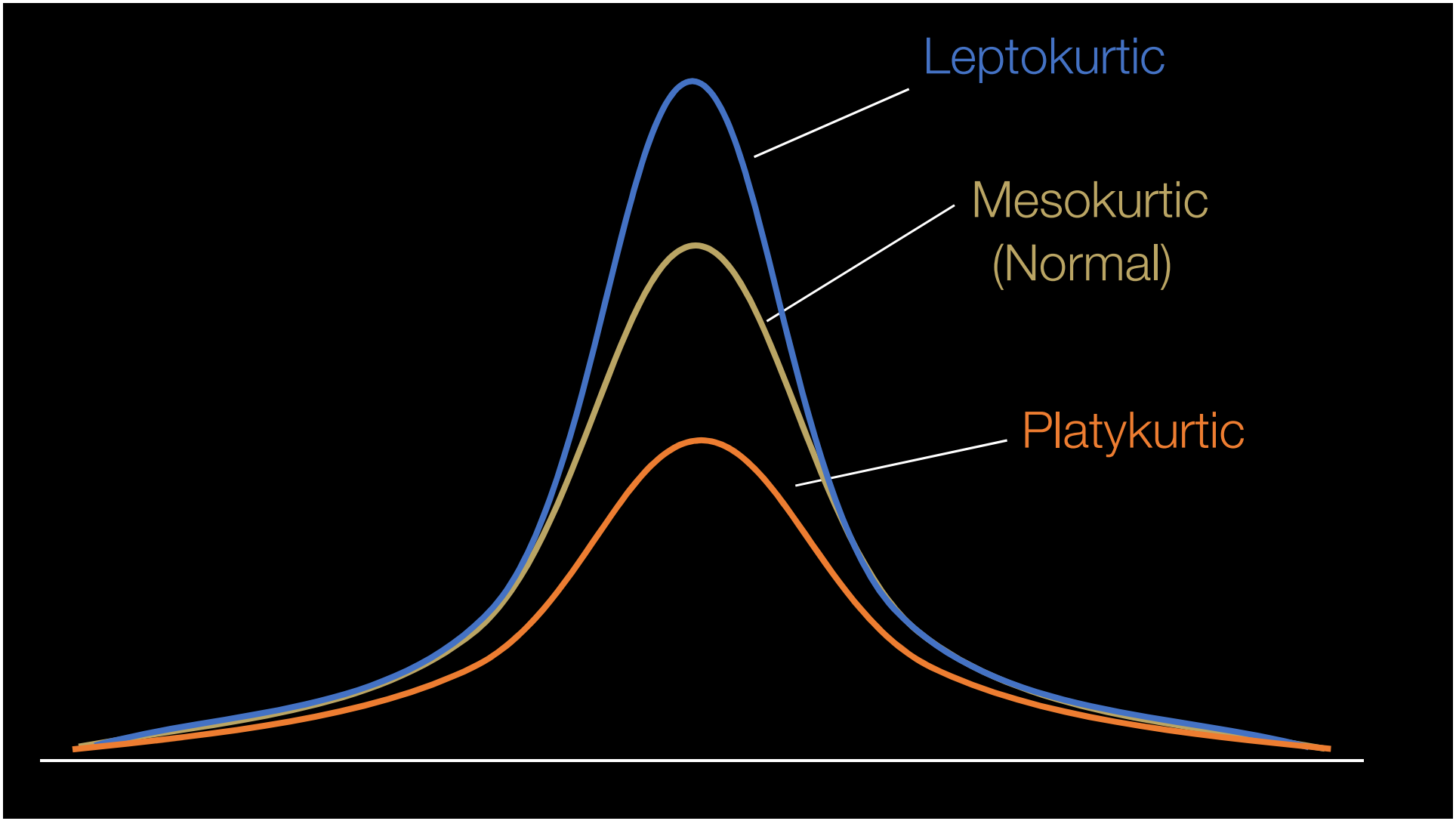
```
> skewness(castings$weight)
```

Kurtosis

- Kurtosis is the degree of peakedness of a distribution
- An intermediate distribution, with zero kurtosis, is known as a mesokurtic distribution

Kurtosis

- A symmetrical leptokurtic distribution has a higher peak and has heavier tails, and has positive kurtosis
- A symmetrical platykurtic distribution has a lower peak and lighter tails, and has negative kurtosis

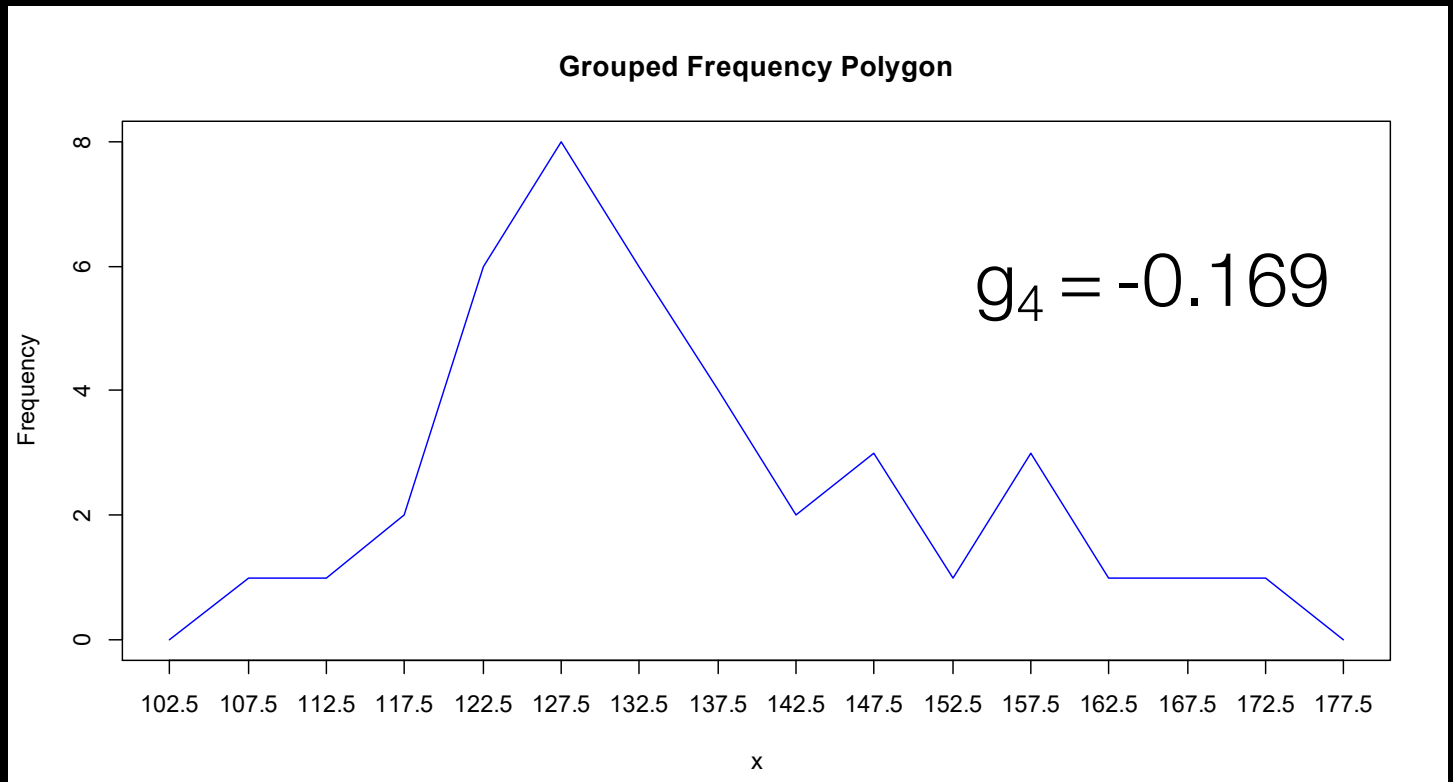

Leptokurtic
Mesokurtic
(Normal)
Platykurtic

Kurtosis: Calculations

- Symbols
  Population (γ_4) and
  Sample (g_4)

$$g_4 = \left[\frac{(n-1)(n+1)}{(n-2)(n-3)}\right] x \frac{m_4}{m_2^2} - 3\left[\frac{(n-1)^2}{(n-2)(n-3)}\right]$$

Skewed Distributions



Sources

The material used in the PowerPoint presentations associated with this course was drawn from a number of sources. Specifically, much of the content included was adopted or adapted from the following previously-published material:

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982
- Luftig, J. Advanced Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1984.
- Luftig, J. A Quality Improvement Strategy for Critical Product and Process Characteristics. Luftig & Associates, Inc. Farmington Hills, MI, 1991
- Luftig, J. Guidelines for Reporting the Capability of Critical Product Characteristics. Anheuser-Busch Companies, St. Louis, MO. 1994
- Spooner-Jordan, V. Understanding Variation. Luftig & Warren International, Southfield, MI 1996
- Luftig, J. and Petrovich, M. Quality with Confidence in Manufacturing. SPSS, Inc. Chicago, IL 1997
- Littlejohn, R., Ouellette, S., & Petrovich, M. Black Belt Business Improvement Specialist Training, Luftig & Warren International, 2000
- Ouellette, S. Six Sigma Champion Training, ROI Alliance, LLC & Luftig & Warren, International, Southfield, MI 2005