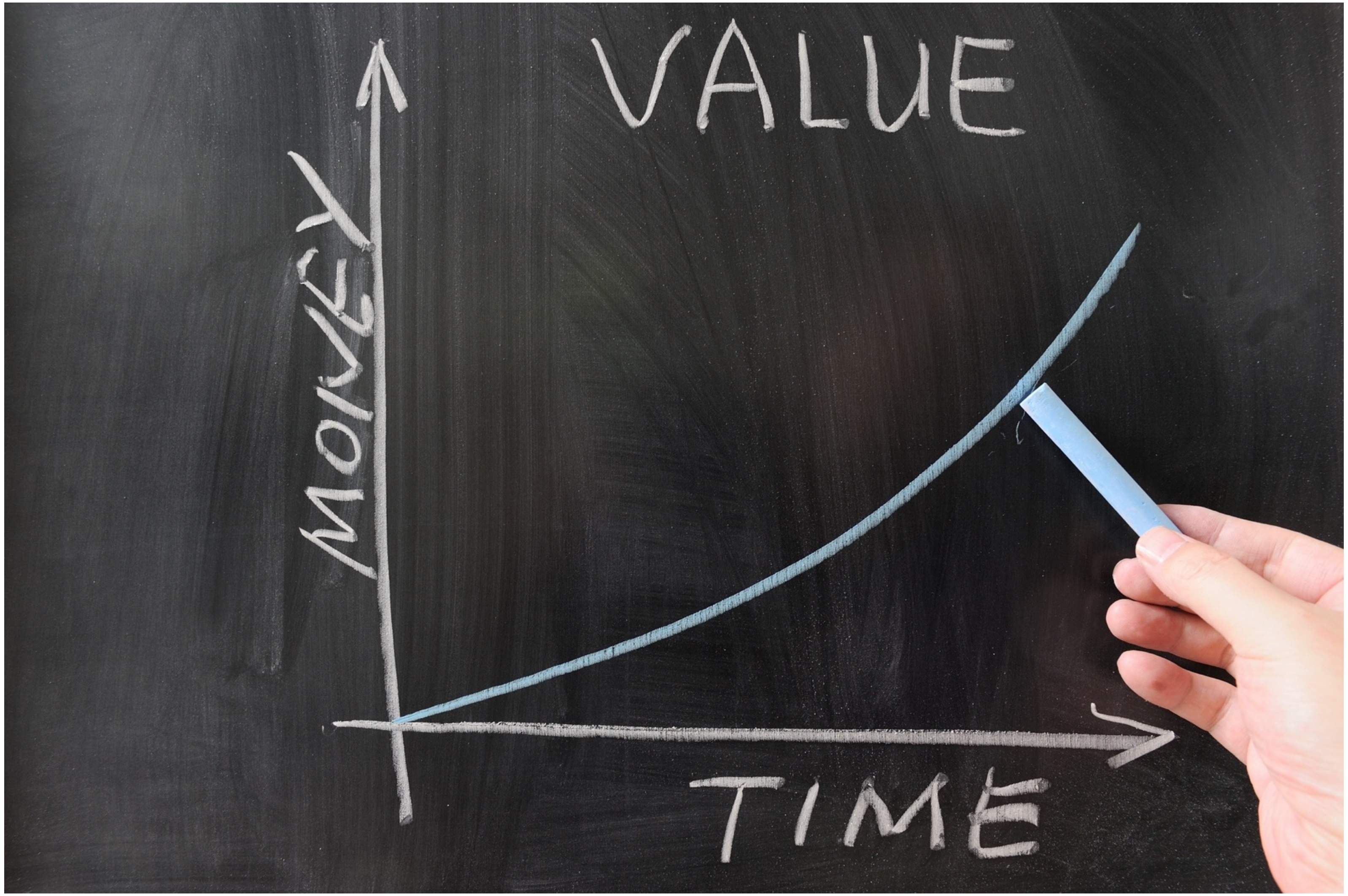
Time Value of Money – Part 2



Time Value of Money - Summary

Single Payment Cash Flows:

- Compound Amount: $FV = PV\,(1+i)^N$
- Present Value: $PV = FV/(1+i)^N$

Uniform Series Cash Flows:

- Compound Amount: $FV = A\left[\dfrac{(1+i)^N - 1}{i}\right]$
- Sinking Fund: $A = FV\left[\dfrac{i}{(1+i)^N - 1}\right]$

- Using Excel, you can quickly calculate:
  - FV: using the FV function
  - PV: using the PV function
  - i: using the RATE function
  - N: using the NPER function
  - A: using the PMT function

Present Value given a Series of Annuities

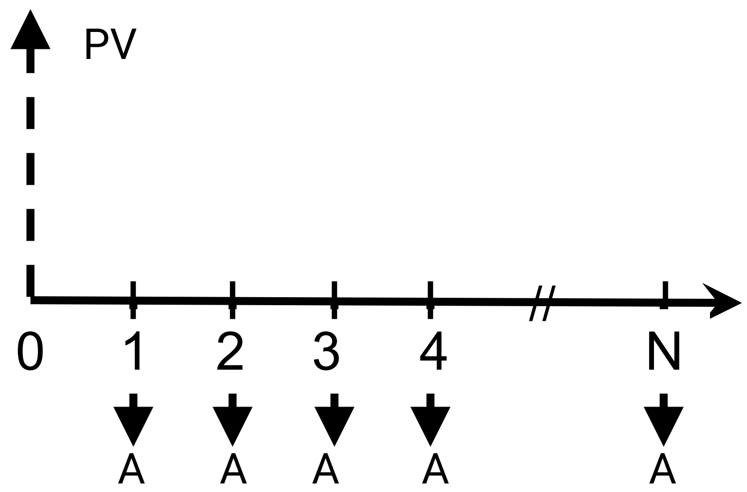
What is PV given A and some rate of return and length of time?

$$PV = FV/(1+i)^N$$

$$FV = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$PV = A\underbrace{\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]}_{\text{uniform series, present value factor}}$$

uniform series,
present value factor



The Present Value is determined by "discounting" each cash flow back to time=0.

Equal Series Present Value

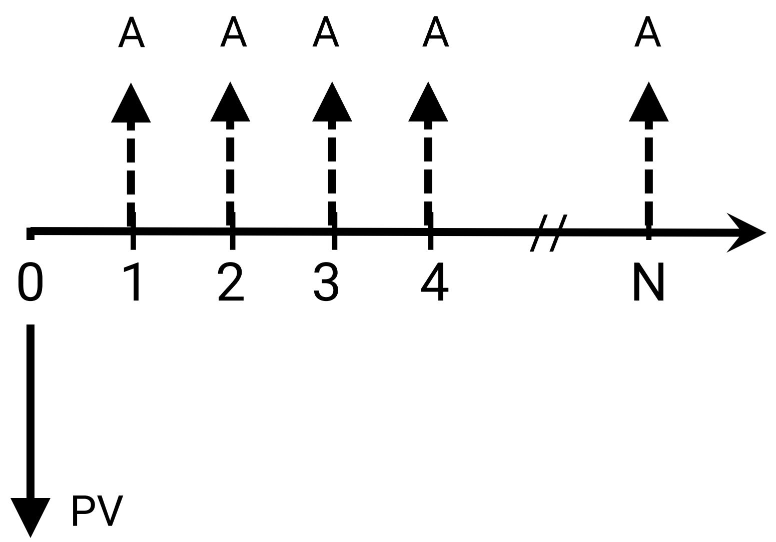
Ex. A = $200,000, interest rate = 8%, N = 25 years. Use Excel's PV function once again…

	A	B	C	D	E	F
1	**TVM with Excel**					
2	**Present Value Calculations**					
3						
4	FV =	$0	*this is value of our investment at time = N*			
5	PMT =	-$200,000	*this is our A, at the end of each period*			
6	RATE =	8%	*this is our i*			
7	NPER =	25	*this is our N*			
8						
9	PV =	$2,134,955	*PV(rate, nper, pmt, fv, type)*			
10			*=PV(B6,B7,B5,B4)*			
11						

Finding the Annuity given PV, i, and N

What is A given PV and some rate of return and length of time?

$$A = PV \underbrace{\left[\frac{i\,(1+i)^N}{(1+i)^N - 1}\right]}_{\text{uniform series, capital recovery factor}}$$



Capital Recovery: what profits are necessary to "recover" the cost of our capital expenditure? (aka, how much do we need to make to break even?)

Perpetuities & Capitalized Costs

How to live off the interest without touching your principal, forever!

Perpetuities– when *N* is a long time…

Some annuities can go on forever, in this case, N → Infinity.

In essence, these annuities go on “in perpetuity”, and are called “Perpetuities”.

A **Perpetuity** is a series of annuity payments that go on forever…
The beauty of perpetuities is that they don’t impact the “principal”.

Ex. You deposit $100 (P) in a savings account earning 8% interest (i), or $8 per year (Pi).

EOY1: $100 + $8 = $108 You withdraw $8, leaving the remaining $100.
EOY2: $100 + $8 = $108 You withdraw $8, leaving the remaining $100.
….
EOY100: $100 + $8 = $108 You withdraw $8, leaving the remaining $100

This goes on forever…

In this case, the **Perpetuity = $8**.

The Principal remains the same: $100

Capitalized Costs – when *N* is a long time…

The **Capitalized Cost** is the amount of money that is set aside now, to yield perpetuity annuity payments well into the future without impacting the "principal".

Ex. You deposit \$100 (P) in an account earning 8% interest (i), or \$8 per year (Pi).

EOY1: \$100 + \$8 = \$108 You withdraw \$8, leaving the remaining \$100.
EOY2: \$100 + \$8 = \$108 You withdraw \$8, leaving the remaining \$100.
….
EOY100: \$100 + \$8 = \$108 You withdraw \$8, leaving the remaining \$100
…
This goes on forever…

In this case, the **Capitalized Cost = \$100**.

The Perpetuity = \$8

Capitalized Cost & Perpetuities

What is the PV necessary yield an <u>infinite</u> series of annuity payments, A?

$$PV = A\underbrace{\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]}_{=\frac{1}{i}}$$

When $N \rightarrow \infty$

Therefore: $$PV = \frac{A}{i}$$

$$\boxed{A = PV * i}$$

Where PV is called the "Capitalized Cost" necessary to generate an infinite series of payments, A (the perpetuity).

An Example of Capitalized Cost & Perpetuities

The Engineering Management Program wants to fund an Endowed Scholarship, which means annual student scholarships are paid from the interest from the endowment – forever.

If the program wants to award \$100,000 per year forever (the perpetuity), what is the required endowment (capitalized cost)? Assume 8% Interest.

Note that for $n \rightarrow \infty$ $A = PV * i$

Therefore: $PV = A / i$

$PV = \$100{,}000 / 0.08 = \$1{,}250{,}000$

The Endowment Fund (or Capitalized Cost) must be \$1.25M in order to generate \$100K in annual scholarships - forever.

Note that the endowment (\$1.25M), never goes down. It provides the same amount of scholarships each year - forever.

Another Example …

Being the great parent that you are, you've decided that you want to retire off the returns you make on your investments and leave the principal to your kids.

If you can live off $100,000 per year, what does the capitalized cost of your investment need to be, assuming that at retirement your rate of return is 4%.

$$PV = A / i$$

$$PV = \$100{,}000 / 0.04$$

$$PV = \$2{,}500{,}000$$

You must have $2.5M in your portfolio in order to generate $100K in annual income – forever – never touching your investment.

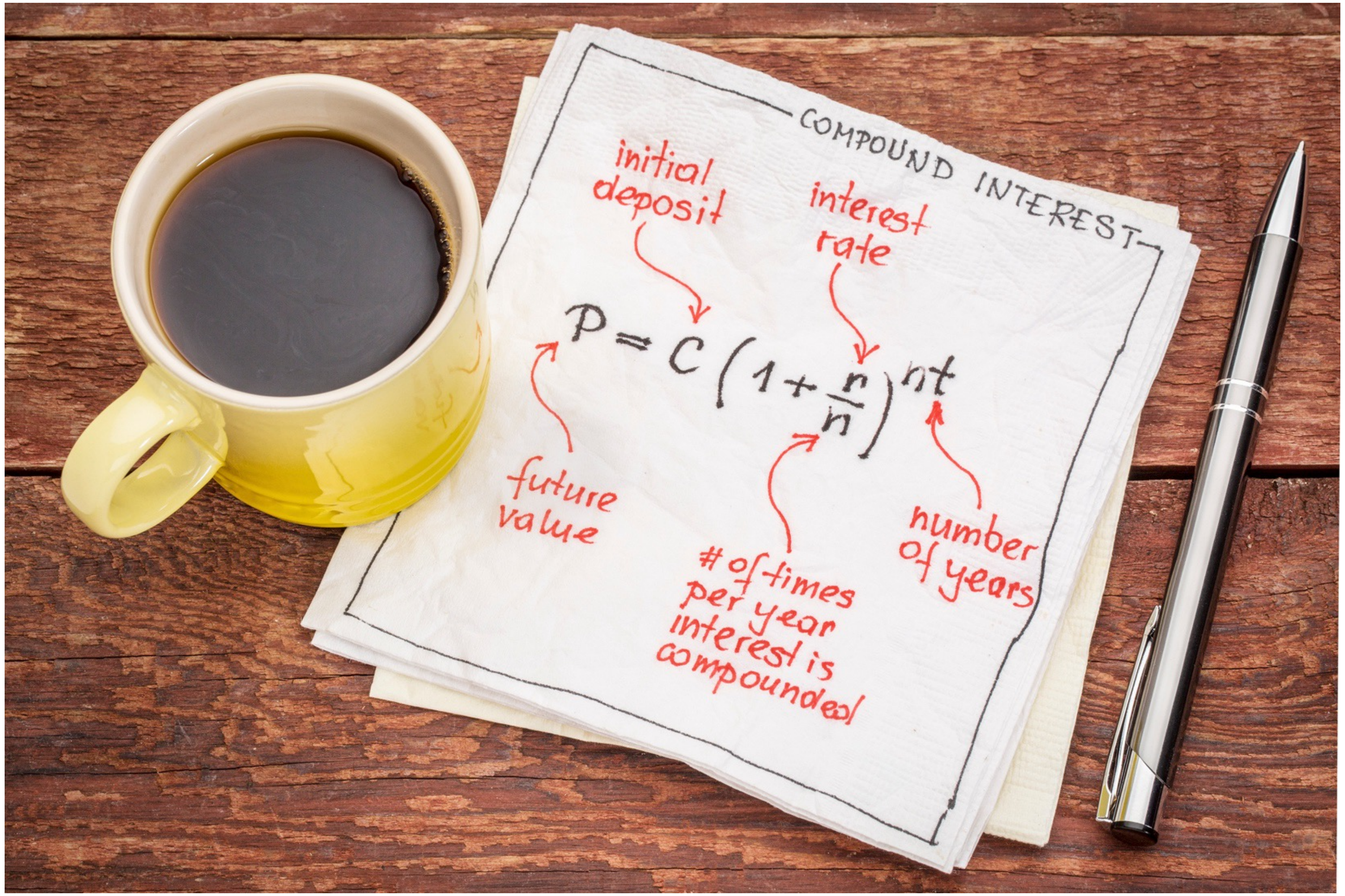
Summary…

- Uniform Series of Cash Flows:

		Excel Function
✓ Present Value:	$\mathrm{PV} = \mathrm{A}\left[\frac{(1+\mathrm{i})^{\mathrm{N}}-1}{\mathrm{i}\,(1+\mathrm{i})^{\mathrm{N}}}\right]$	PV
✓ Capital Recovery:	$\mathrm{A} = \mathrm{PV}\left[\frac{\mathrm{i}\,(1+\mathrm{i})^{\mathrm{N}}}{(1+\mathrm{i})^{\mathrm{N}}-1}\right]$	PMT
✓ Perpetual Annuities:	$\mathrm{A} = \mathrm{PV}\ \mathrm{x}\ \mathrm{i}$	PV x RATE
✓ Capitalized Costs:	$\mathrm{PV} = \frac{A}{i}$	PMT / RATE

Next Time…

Interest Rates & Compounding Periods



Credits & References

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