Ideas:

- Backwards Selection and Forward Selection. Just a lot of that.

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In [ ]:
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Module 6: Peer Reviewed Assignment

Outline:

The objectives for this assignment:

1. Apply the processes of model selection with real datasets.
2. Understand why and how some problems are simpler to solve with some forms of model selection, and others are more difficult.
3. Be able to explain the balance between model power and simplicity.
4. Observe the difference between different model selection criterion.

General tips:

1. Read the questions carefully to understand what is being asked.
2. This work will be reviewed by another human, so make sure that you are clear and concise in what your explanations and answers.

In [10]:
```r
# This cell loads in the necesary packages
library(tidyverse)
library(leaps)
```

```
Registered S3 method overwritten by 'rvest':
  method            from
  read_xml.response xml2
── Attaching packages ─────────────────────────────────────── tidyve
rse 1.2.1 ──
✔ tibble  2.1.1       ✔ purrr   0.3.2
✔ tidyr   0.8.3       ✔ dplyr   0.8.0.1
✔ readr   1.3.1       ✔ stringr 1.4.0
✔ tibble  2.1.1       ✔ forcats 0.4.0
── Conflicts ────────────────────────────────────────── tidyverse_co
nflicts() ──
✖ tidyr::complete() masks RCurl::complete()
✖ dplyr::filter()   masks stats::filter()
✖ dplyr::lag()      masks stats::lag()
```

Problem 1: We Need Concrete Evidence!

Ralphie (https://en.wikipedia.org/wiki/Ralphie_the_Buffalo) is studying to become a civil engineer. That means she has to know everything about concrete, including what ingredients go in it and how they affect the concrete's properties. She's currently writting up a project about concrete flow, and has asked you to help her figure out which ingredients are the most important. Let's use our new model selection techniques to help Ralphie out!

Data Source: Yeh, I-Cheng, "Modeling slump flow of concrete using second-order regressions and artificial neural networks," Cement and Concrete Composites, Vol.29, No. 6, 474-480, 2007.

In [4]:

```
#concrete.data = read.table("Concrete.data", header=TRUE, sep=",")
library(RCurl) #a package that includes the function getURL(), whic
h allows for reading data from github.
library(ggplot2)
url = getURL(paste0("https://raw.githubusercontent.com/bzaharatos/-
Statistical-Modeling-for-Data-Science-Applications/master/Modern%20
Regression%20Analysis%20/Datasets/slump_test.data"))
concrete.data = read.csv(text = url, sep = ",")

concrete.data = concrete.data[, c(-1, -9, -11)]
names(concrete.data) = c("cement", "slag", "ash", "water", "sp", "c
ourse.agg", "fine.agg", "flow")

head(concrete.data)
```

```
Loading required package: bitops
Registered S3 methods overwritten by 'ggplot2':
  method         from
  [.quosures     rlang
  c.quosures     rlang
  print.quosures rlang
```

cement	slag	ash	water	sp	course.agg	fine.agg	flow
273	82	105	210	9	904	680	62.0
163	149	191	180	12	843	746	20.0
162	148	191	179	16	840	743	20.0
162	148	190	179	19	838	741	21.5
154	112	144	220	10	923	658	64.0
147	89	115	202	9	860	829	55.0

1. (a) Initial Inspections

Sometimes, the best way to start is to just jump in and mess around with the model. So let's do that. Create a linear model with `flow` as the response and all other columns as predictors.

Just by looking at the summary for your model, is there reason to believe that our model could be simpler?

In [5]:
```r
lmod <- lm(flow ~., data = concrete.data)
summary(lmod)
```

```
Call:
lm(formula = flow ~ ., data = concrete.data)

Residuals:
    Min      1Q  Median      3Q     Max 
-30.880 -10.428   1.815   9.601  22.953 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)  
(Intercept) -252.87467  350.06649  -0.722   0.4718  
cement         0.05364    0.11236   0.477   0.6342  
slag          -0.00569    0.15638  -0.036   0.9710  
ash            0.06115    0.11402   0.536   0.5930  
water          0.73180    0.35282   2.074   0.0408 *
sp             0.29833    0.66263   0.450   0.6536  
course.agg     0.07366    0.13510   0.545   0.5869  
fine.agg       0.09402    0.14191   0.663   0.5092  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 12.84 on 95 degrees of freedom
Multiple R-squared:  0.5022,	Adjusted R-squared:  0.4656 
F-statistic: 13.69 on 7 and 95 DF,  p-value: 3.915e-12
```

From looking at the summary, Many of the predictors are statistically insignificant, which suggests that a smaller model might be sufficient.

1. (b) Backwards Selection

Our model has 7 predictors. That is not too many, so we can use backwards selection to narrow them down to the most impactful.

Perform backwards selection on your model. You don't have to automate the backwards selection process.

Here, we see that several of the predictors have t-tests with high p-values (higher than the standard $\alpha = 0.05$). Instead of just removing all of those, let's perform backward selection, removing the predictor with the largest p-value greater than $\alpha_0 = 0.15$, `slag` in this case, and then refit the model. We can do this with the `update()` function. Remember that $\alpha_0 = 0.15$, sometimes called the "p-to-remove", is used as a cuttoff in backward selection, and if the goal is prediction, then 0.15 or 0.2 is thought to work well.

In [6]:
```r
# From the previous summary, the "worst" predictor is "slag", so we can remove it and check if the
# adjusted R^2, AIC, or BIC improve

# Starting R^2: 0.4656
# Starting AIC: 531.56005610193
# Starting BIC: 550.003159019538
lmod_1step <- update(lmod, ~ . - slag)
summary(lmod_1step)
# R^2 removing slag: 0.4711
# AIC removing slag: 529.561491684203
# BIC removing slag: 545.369865613581
# Adjusted R^2 has increased, AIC has decreased, and BIC has decreased, so this model is "better"
```

```
Call:
lm(formula = flow ~ cement + ash + water + sp + course.agg + 
    fine.agg, data = concrete.data)

Residuals:
    Min      1Q  Median      3Q     Max 
-30.843 -10.451   1.771   9.589  22.939 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -265.45032   55.46193  -4.786 6.16e-06 ***
cement         0.05766    0.02088   2.761 0.006899 ** 
ash            0.06524    0.01987   3.283 0.001434 ** 
water          0.74420    0.09117   8.163 1.28e-12 ***
sp             0.31366    0.50874   0.617 0.538997    
course.agg     0.07849    0.02447   3.207 0.001820 ** 
fine.agg       0.09909    0.02644   3.747 0.000305 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 12.78 on 96 degrees of freedom
Multiple R-squared:  0.5022,	Adjusted R-squared:  0.4711 
F-statistic: 16.14 on 6 and 96 DF,  p-value: 9.229e-13
```

In [7]:
```
# There is still a predictor with a p-value greater than 0.15, "s
p", so we can remove it
lmod_2step <- update(lmod_1step, ~ . - sp)
summary(lmod_2step)
# R^2 removing sp: 0.4745
# AIC removing sp: 527.968525227541
# BIC remocing sp: 541.142170168689
# Adjusted R^2 has increased, AIC has decreased, and BIC has decrea
sed, so this model is "better"

best.model <- lmod_2step
```

```
Call:
lm(formula = flow ~ cement + ash + water + course.agg + fine.agg,
    data = concrete.data)

Residuals:
    Min      1Q  Median      3Q     Max
-31.893 -10.125   1.773   9.559  23.914

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -249.50866   48.90884  -5.102 1.67e-06 ***
cement         0.05366    0.01979   2.712 0.007909 **
ash            0.06101    0.01859   3.281 0.001436 **
water          0.72313    0.08426   8.582 1.53e-13 ***
course.agg     0.07291    0.02266   3.217 0.001760 **
fine.agg       0.09554    0.02573   3.714 0.000341 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 12.74 on 97 degrees of freedom
Multiple R-squared:  0.5003,	Adjusted R-squared:  0.4745
F-statistic: 19.42 on 5 and 97 DF,  p-value: 2.36e-13
```

There are no more "bad" predictors, so we've achieved the "best" model, according to backward selection.

1. (c) Objection!

Stop right there! Think about what you just did. You just removed the "worst" features from your model. But we know that a model will become less powerful when we remove features so we should check that it's still just as powerful as the original model. Use a test to check whether the model at the end of backward selection is significantly different than the model with all the features.

Describe why we want to balance explanatory power with simplicity.

In [8]:
```
anova(best.model, lmod)
```

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
97	15733.53	NA	NA	NA	NA
95	15671.26	2	62.27123	0.1887457	0.8283068

The f-test being run by anova() is testing the following hypothesis:

$$H_0 : \beta_{slag} = \beta_{sp} = 0$$
$$H_1 : \beta_{slag} \neq \beta_{sp} \neq 0$$

From anova, we have a p-value of 0.8283068, which means we fail to reject the null hypothesis, which means the extra parameters in the full model are not statistically significant, and therefore we do not need them and the reduced model is better than the full model!

1. (d) Checking our Model

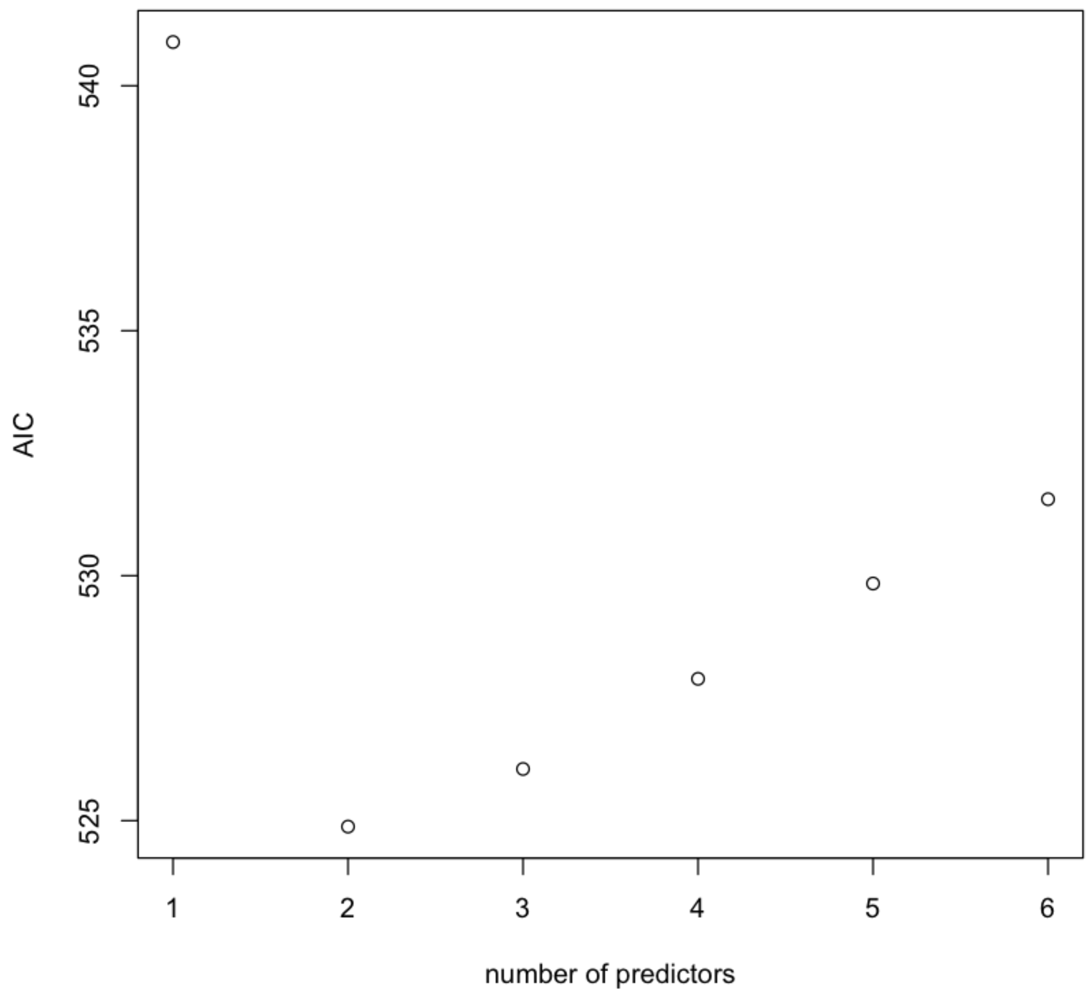
Ralphie is nervous about her project and wants to make sure our model is correct. She's found a function called `regsubsets()` in the leaps package which allows us to see which subsets of arguments produce the best combinations. Ralphie wrote up the code for you and the documentation for the function can be found [here (https://www.rdocumentation.org/packages/leaps/versions/2.1-1/topics/regsubsets)](https://www.rdocumentation.org/packages/leaps/versions/2.1-1/topics/regsubsets). For each of the subsets of features, calculate the AIC, BIC and adjusted R^2. Plot the results of each criterion, with the score on the y-axis and the number of features on the x-axis.

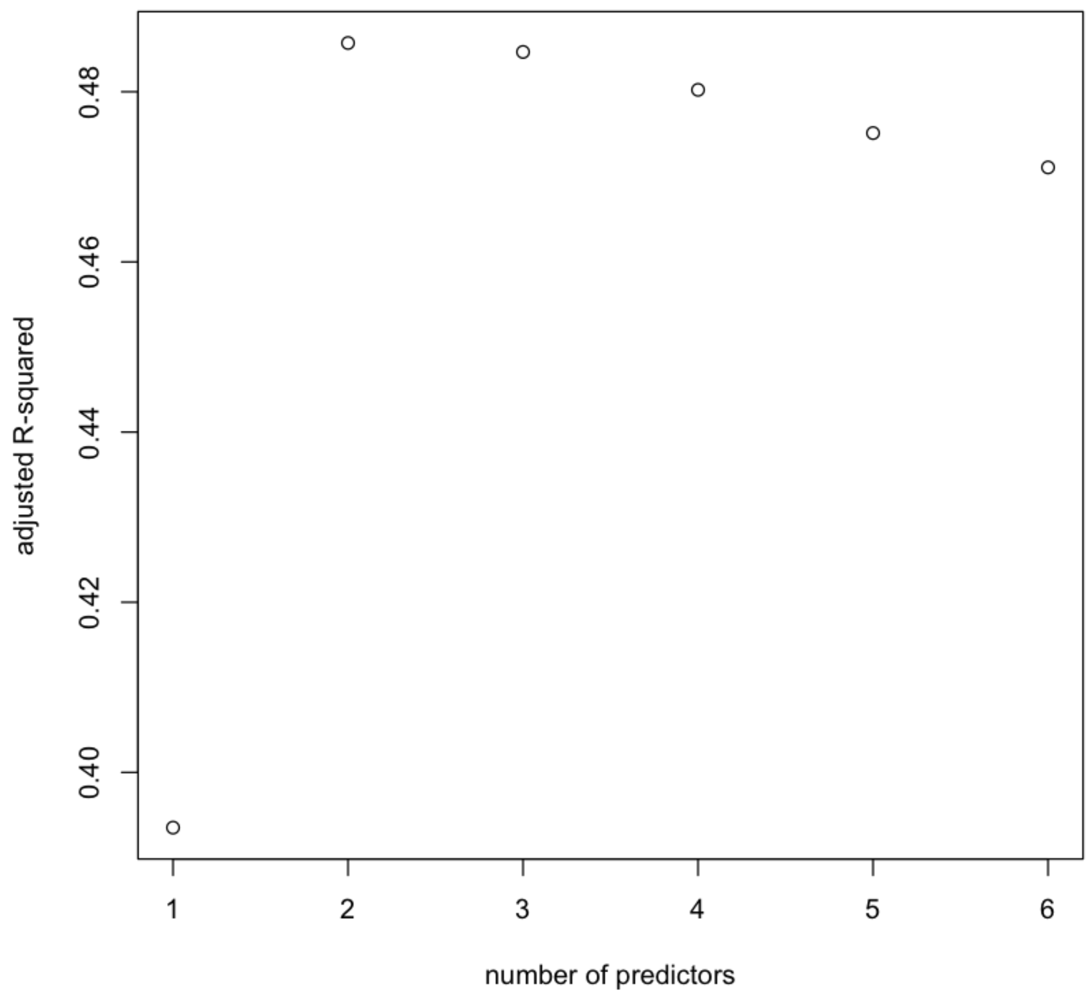
Do all of the criterion agree on how many features make the best model? Explain why the criterion will or will not always agree on the best model.

Hint: It may help to look at the attributes stored within the regsubsets summary using `names(rs)`.

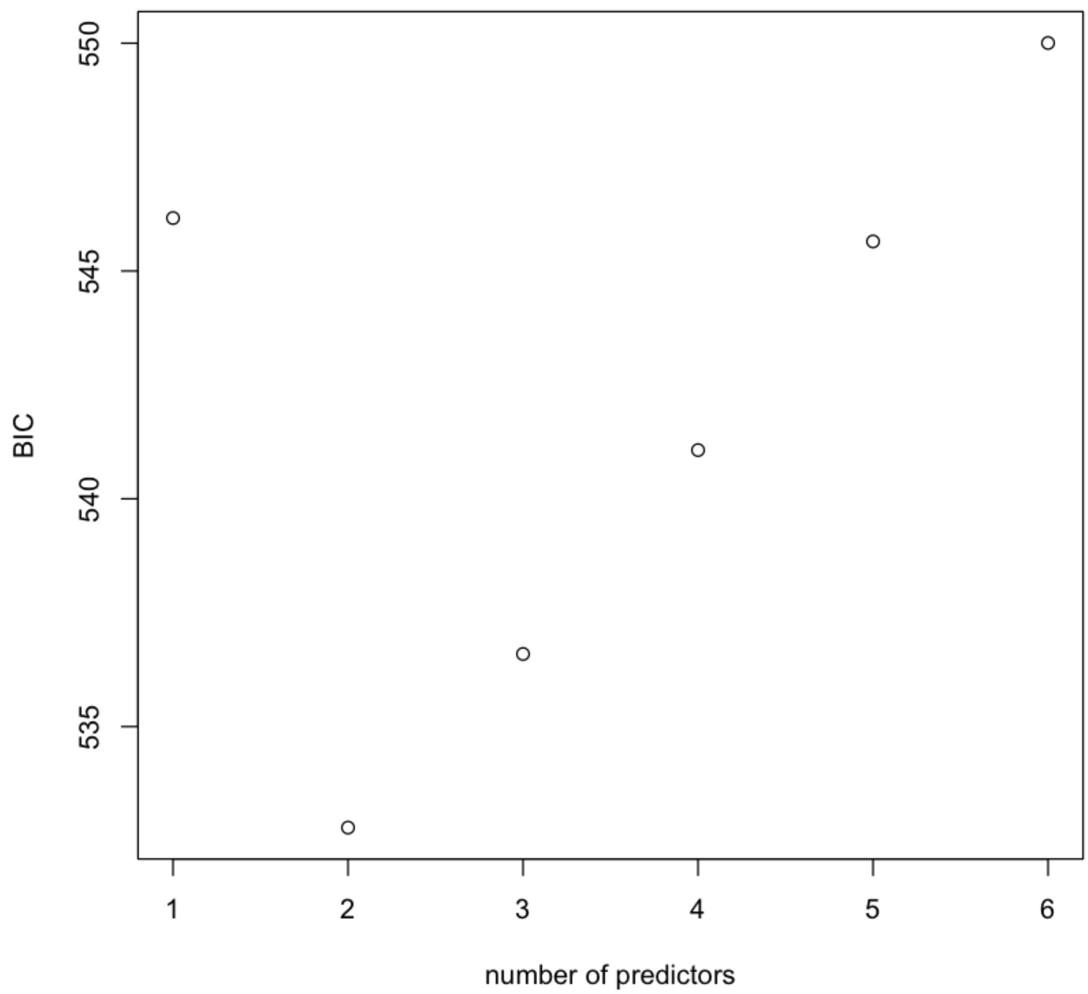
In [19]:
```
reg = regsubsets(flow ~ cement+slag+ash+water+sp+course.agg+fine.ag
g, data=concrete.data, nvmax=6)
rs = summary(reg)
rs$which

n = dim(concrete.data)[1]

AIC = 2*(2:7) + n*log(rs$rss/n);
plot(AIC ~ I(1:6), xlab = "number of predictors", ylab = "AIC")

plot(1:6, rs$adjr2, xlab = "number of predictors", ylab = "adjusted
R-squared")

BIC = log(n)*(2:7) + n*log(rs$rss/n)
plot(BIC ~ I(1:6), xlab = "number of predictors", ylab = "BIC")
```

(Intercept)	cement	slag	ash	water	sp	course.agg	fine.agg
TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE
TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE	FALSE
TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE	TRUE
TRUE	TRUE	TRUE	FALSE	TRUE	FALSE	FALSE	TRUE
TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	TRUE
TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE





0.40
0.42
0.44
0.46
0.48
adjusted R-squared
1
2
3
4
5
6
number of predictors



According to the plots of AIC, BIC, and adjusted R^2, the best model will have 2 predictors: `slag` and `water`.

In [ ]: