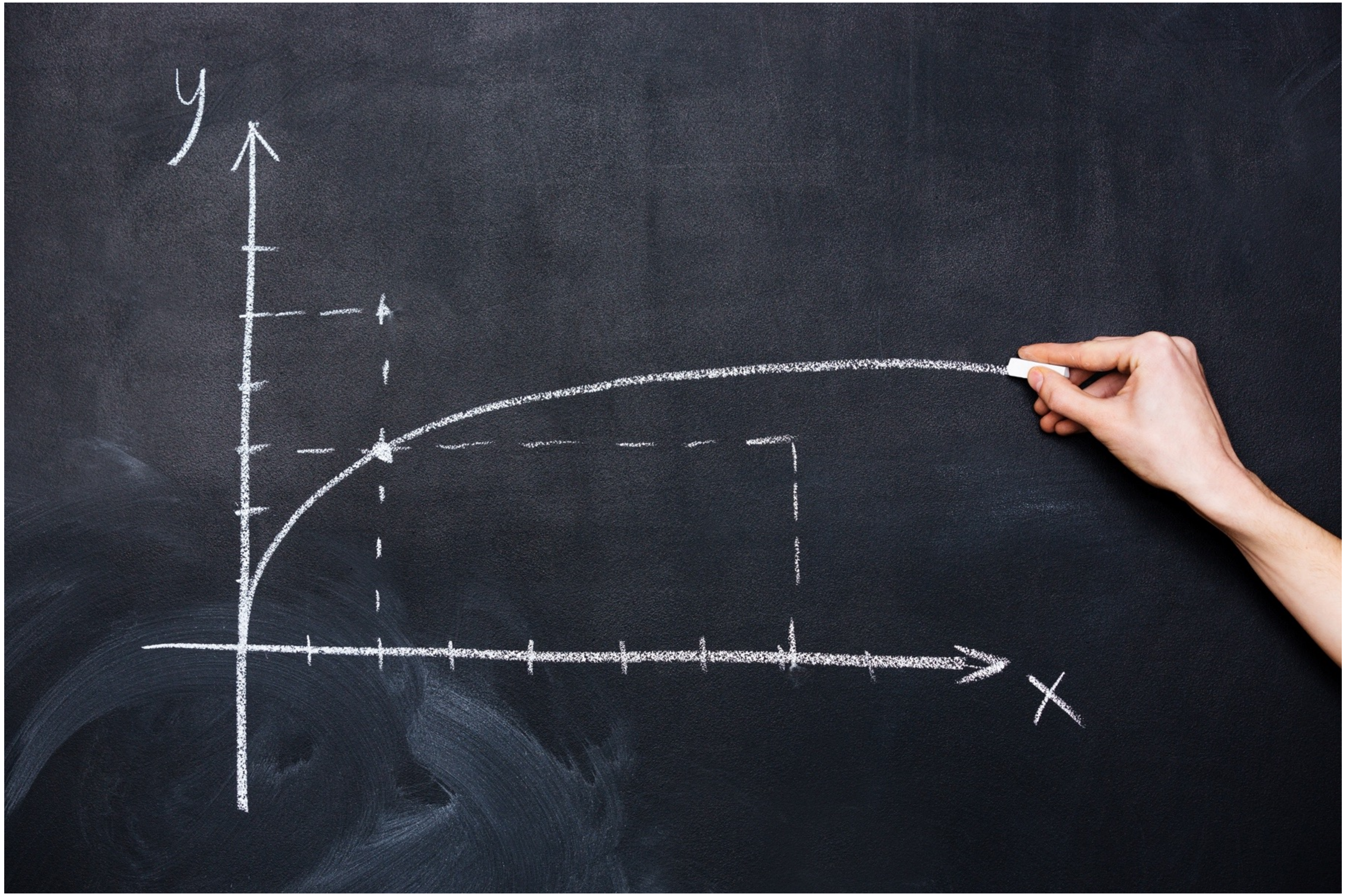
Continuous Cost Models



Cost-Driven Design Optimization

Cost Modeling to optimize costs often consists of two types:

Cost Modeling

Discrete
Cost Modeling

The minimum cost is selected from several different (discrete) options.

Continuous
Cost Modeling

Cost is a continuous function of a design parameter or cost driver.

Continuous Cost Models

First some theory:

General Approach to Cost Minimization with Continuous Cost Models:

1. Identify the Primary Cost Driver, X

2. Develop a Cost Model such that Total Cost = f(X)

3. Calculate the minimum in Total Cost with respect to X

This involves more analytical work and may involve a bit of calculus.
But we will show some easy approaches to defining the minimum cost as well!

Let’s try an example…

You want to visit your friends to do some mountain biking.

They live 1200 miles away, and as need your mountainbike, you must drive to get there. You know you can do it in 2 long days.

Because you are very cost conscious, you want to know the optimum speed you should travel to minimize your total cost of getting there.

The road is mostly interstate highways, with speed limits of 85 mph.

How fast should you drive to minimize total costs?

Travel Costs to Visit Your Friends…

Your cost for trip is a combination of two things:

- The cost of fuel to travel the 1200 miles
- The cost of all the snacks you need along the way, and the cost of the hotel at the end of the first day.

The faster you drive, the lower your car’s fuel economy, and the more fuel you consume.

The slower you drive, the more snacks you need, and your cost of food increases.

Let’s build a cost model to find the optimum speed for your journey!

Travel Costs to Visit Your Friends…

The Cost of Fuel

- Your cost of fuel is determined by the number gallons you consume, N, multiplied by the price per gallon, P:

$$C_{Fuel} = N \; x \; P$$

- The number of gallons you consume is determined from the distance traveled, D (in miles), divided by your car's fuel economy, FE (in mpg, miles per gallon):

$$N = \frac{D}{FE}$$

- Putting that all together…

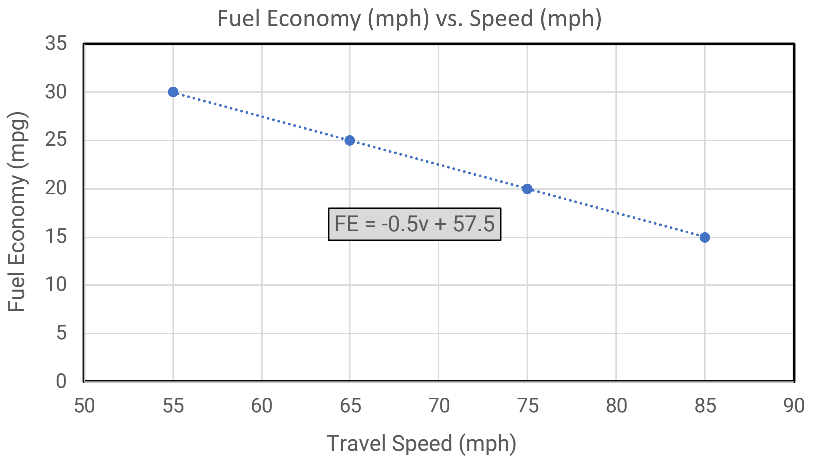
$$C_{Fuel} = \frac{D \; x \; P}{FE}$$

Travel Costs to Visit Your Friends…

The Cost of Fuel

- What you also know, is that the faster you drive, the lower your car's fuel economy. You have done some measurements and found the following:

Speed	FE (miles per gallon)
55	30
65	25
75	20
85	15



Between 55 and 85 mph, the relationship between your car's fuel economy and speed is:

$$FE = -0.5v + 57.5$$

where v = your speed (or more technically, your "velocity")

Travel Costs to Visit Your Friends…

The Cost of Fuel

- Combining all these terms:

$$C_{Fuel} = \frac{D\ x\ P}{-0.5v + 57.5}$$

- Now we have all that we need to calculate the cost of fuel for your trip.

- Let's plug in some values:

  $\mathrm{D} = 1200\ \mathrm{miles}$

  $\mathrm{P} = \$4.00\ \mathrm{per\ gallon}$

- Therefore:

$-miles/gallon

$$C_{Fuel} = \frac{4800}{-0.5v + 57.5}$$

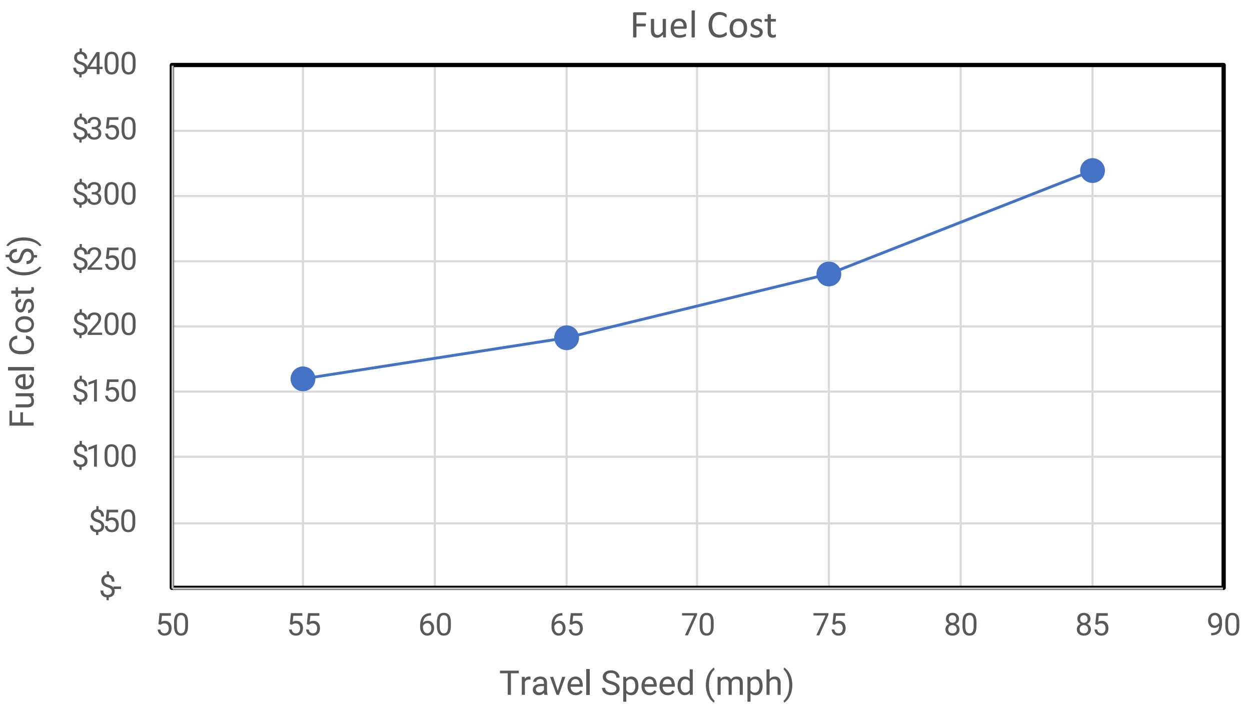
miles/gallon

Travel Costs to Visit Your Friends…

The faster you go, the higher your fuel costs.

The Cost of Fuel

$$C_{Fuel} = \frac{4800}{-0.5v + 57.5}$$



Travel Costs to Visit Your Friends…

The Cost of Food and Lodging

- Your cost of food and lodging, $C_{F\&L}$, is determined by the cost of all the snacks *you* consume, S, and the cost of the hotel for the night, H:

$$C_{F\&L} = S + H$$

- But the cost for your snacks depends on the number of hours you are on the road, which is related to how fast you are driving! If you spend \$10 for every hour of driving, and if the trip takes "N" hours, then your Snack Costs are:

$$S = \left(\frac{\$10}{hr}\right) x\ R(hrs)$$

- The # of hours driving, R, is just the distance, D (miles), divided by speed, v (mph):

$$R = \frac{D}{v}$$

Travel Costs to Visit Your Friends…

The Cost of Food and Lodging

- Putting all that together, we get:

$$C_{F\&L} = \frac{\$10\ x\ D}{v} + H$$

- Now, for your trip of 1200 miles (the D), and your hotel (the H), which you expect to cost $100, we can simplify this to:

$$C_{F\&L} = \frac{\$12000}{v} + \$100$$

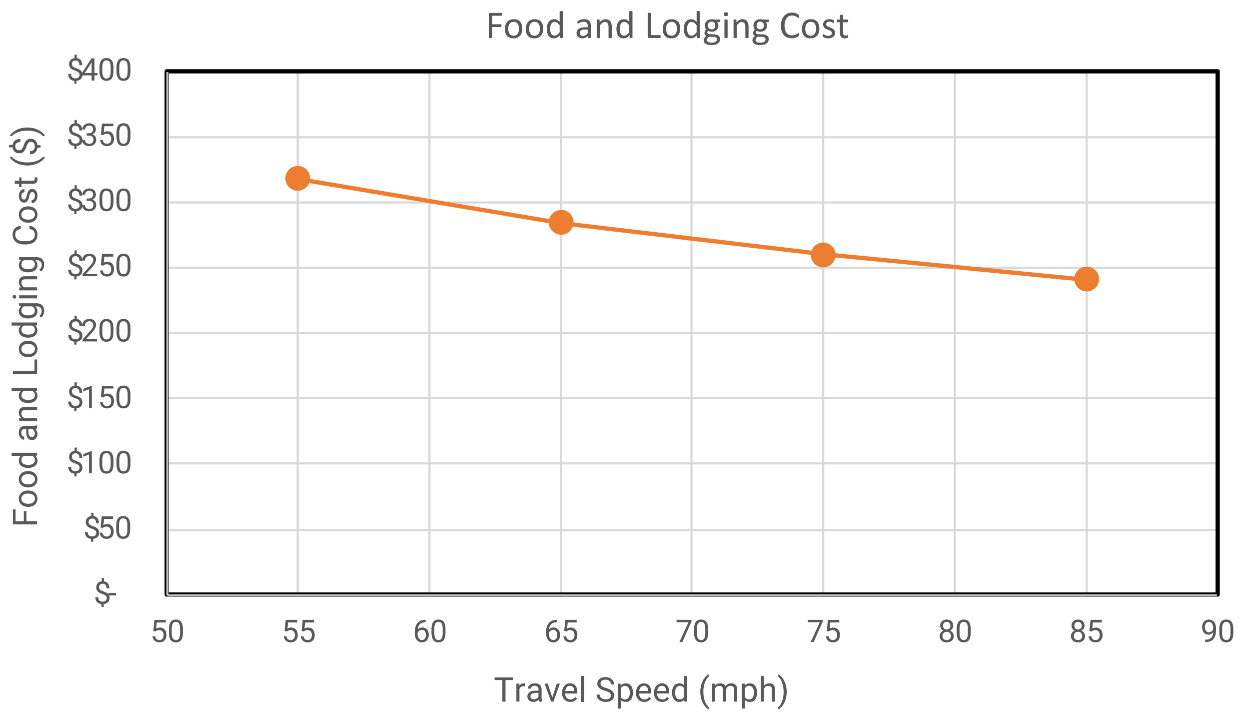
- Now we have everything for the Cost of Food and Lodging!

Travel Costs to Visit Your Friends…

The faster you go, the lower your food and fuel costs.

The Cost of Food and Lodging

$$C_{F\&L} = \frac{\$12000}{v} + \$100$$



Travel Costs to Visit Your Friends…

The Total Cost of your Trip

- The Total Cost of your Trip is just:

$$C_{Trip} = C_{Fuel} + C_{F\&L}$$

- when combining all the equations with our known values:

$$C_{Trip} = \frac{\$4800}{-0.5v + 57.5} + \frac{\$12000}{v} + \$100$$

You now a very nice continuous cost model that describes the total cost of your trip!

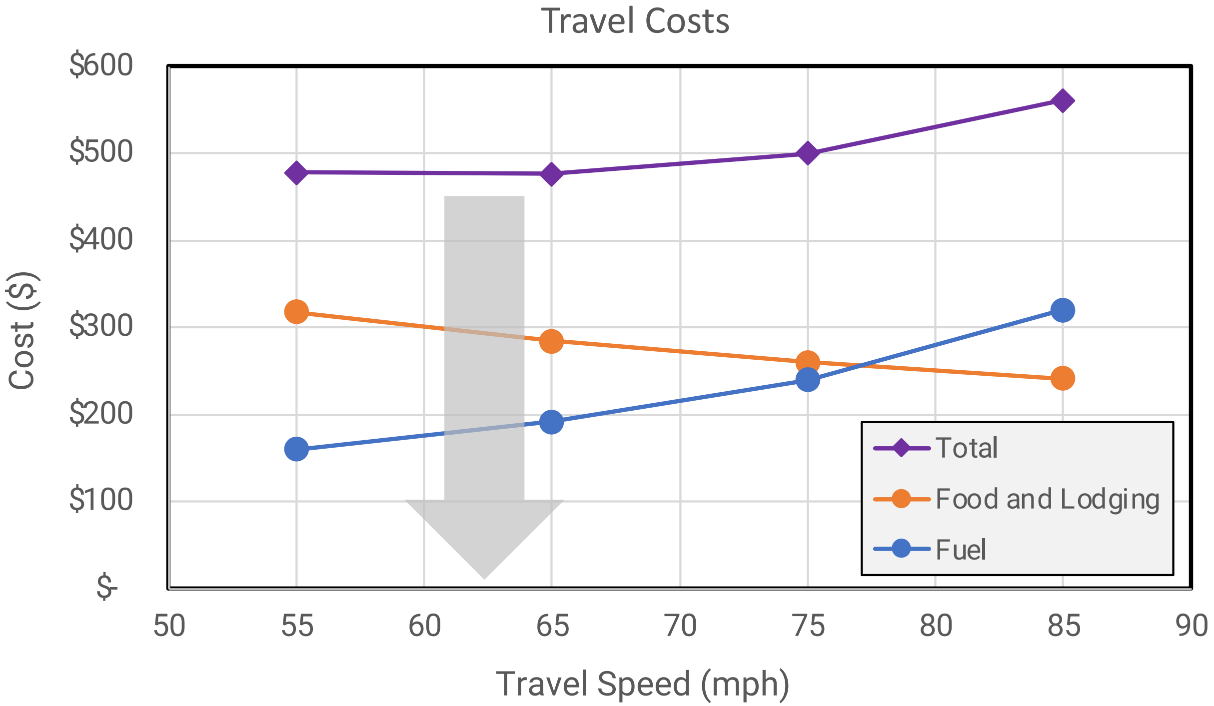
Time to see what the answer is!

Travel Costs to Visit Your Friends…

Your optimum speed is between 60-65 mph!

The Total Cost of Your Trip

$$C_{Trip} = \frac{\$4800}{-0.5v + 57.5} + \frac{\$12000}{v} + \$100$$



The Analytical Approach to This…

Mathematically, we could have solved for the minimum in cost using our knowledge of calculus and determining the minimum in our cost function:

$$C_{Trip} = \frac{\$4800}{-0.5v + 57.5} + \frac{\$12000}{v} + \$100$$

- The way to do that is by first taking the derivative of C_{Trip} relative to velocity and setting it equal to zero:

$$\frac{dC_{Trip}}{dv} = 0$$

- Then solve for ”v” to obtain the speed where C_{Trip} is a minimum.

In many cases, the analytical approach can get complex, forcing you to resort to numerical or graphical techniques, like we did in this example!

Main Takeaways…

- Continuous cost models are used to determine the minimum in cost from a wide range of potential conditions.

- In Continuous Cost Models:
  - One first defines the primary cost driver
  - A Cost Model is established applicable to the relevant range of conditions
  - The minimum in cost can be determined analytically by $dC/dx = 0$, and solving for x.
  - But often the model's complexity requires a numerical or graphical solution

While they may be complex at times, continuous cost models are routinely used to determine the minimum in cost relative to the primary cost driver.

Next Time…

Cost Estimation Techniques

Credits & References

Slide 1: Hand drawing graph of mathematical function parabola on blackboard by Drobot Dean, Adobe Stock (106199736.jpeg).

Slide 17: One arrow hitting center target, horizontal banner by Olivier Le Moal, Adobe Stock (36961324.jpeg).