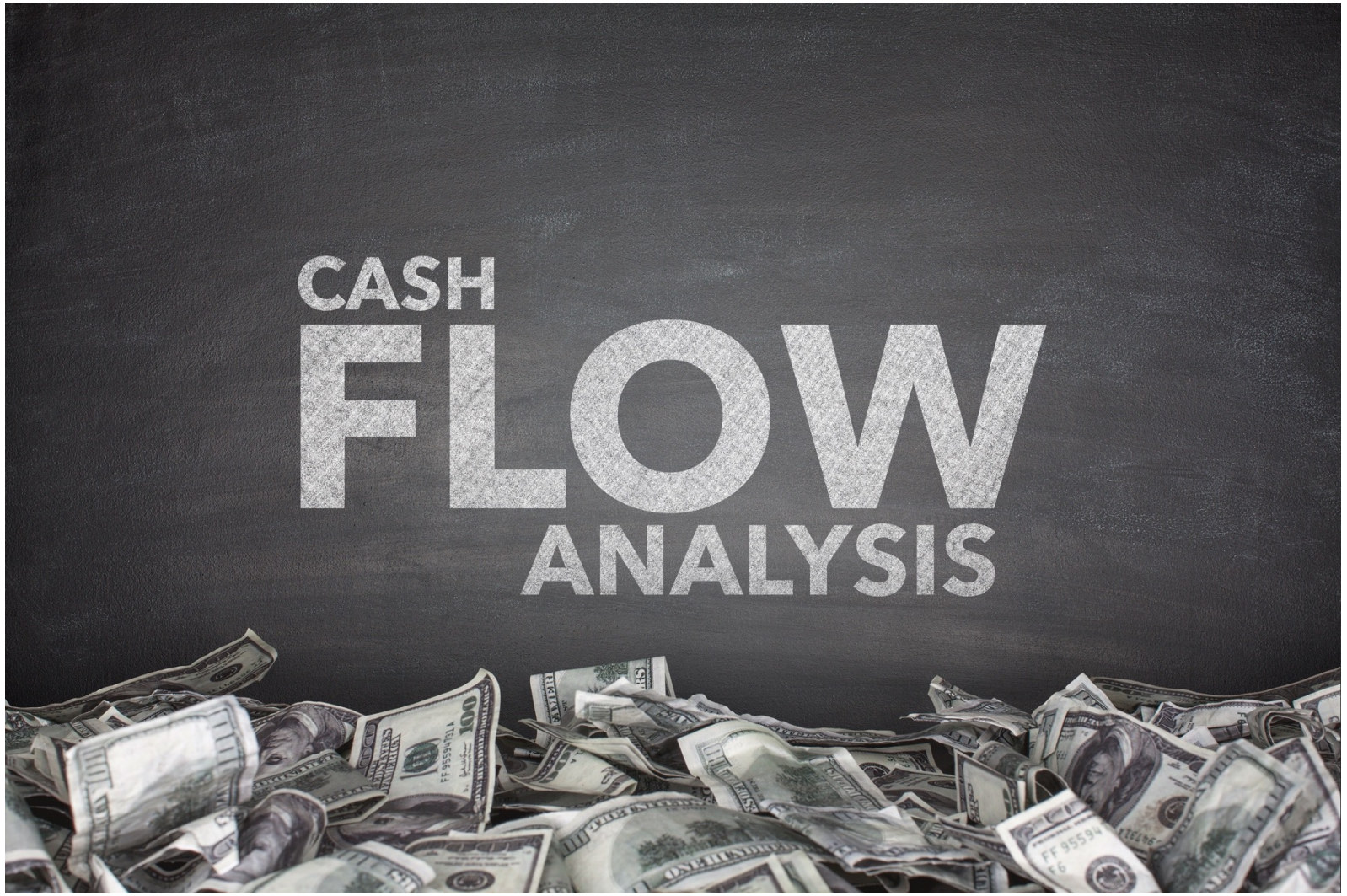
Multiple Payment Cash Flows



A More Realistic Situation…

As we have seen:

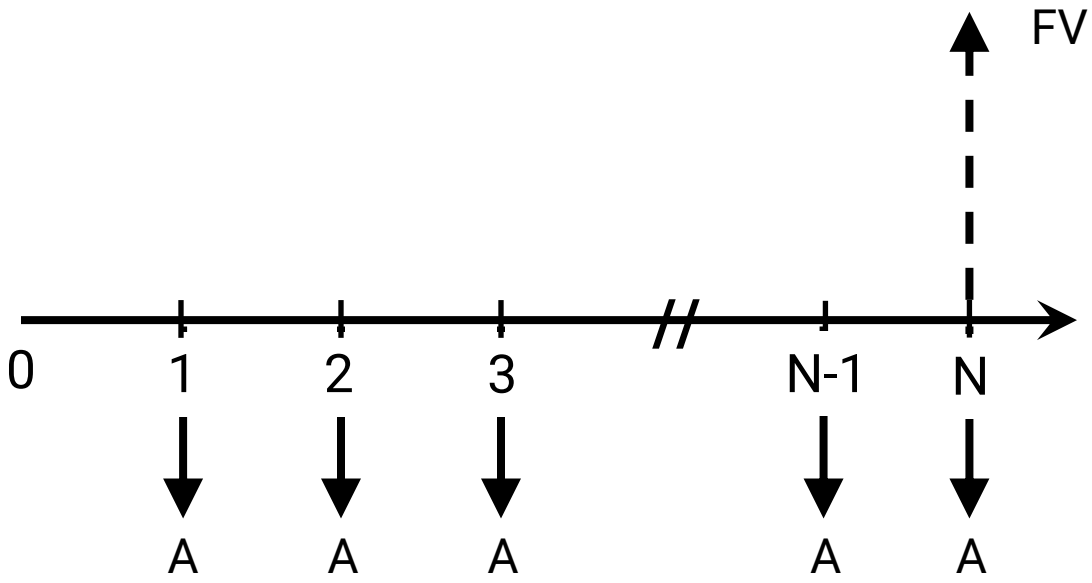
A single payment today = a long time to achieve our retirement goals.

More commonly, we contribute more frequently (usually monthly, and sometimes yearly).

What is the impact of investing more frequently on our retirement strategy?

What Happens When There is More than 1 Payment?

Example. You contribute an amount, A, to your retirement plan every month.



A uniform series of payments are called “Annuities”, with a value “A”.

Usually, annuities are paid at the ***end of the period (“Ordinary Annuity”)***.

In some cases, annuities come at the beginning of the period (“Annuities Due”).

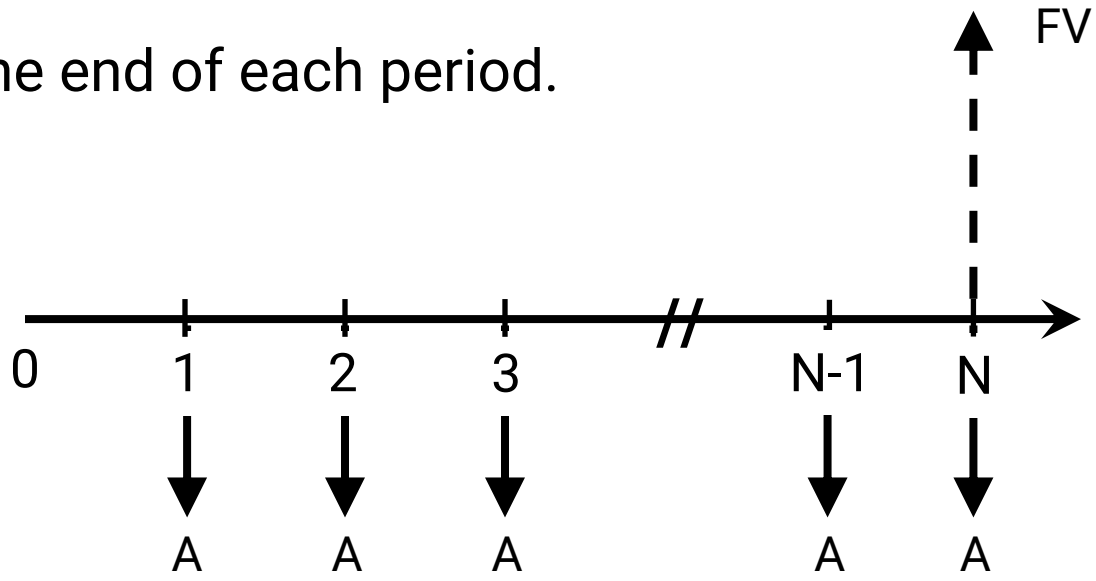
Calculating Future Value Given A, i, and N

See derivation in course resources…

The payments are the same every period.

There is no payment at time = 0.

All payments come at the end of each period.



Note: there is no initial payment at time = 0!

$$FV = A\left[\frac{(1+i)^N - 1}{i}\right]$$

Retirement based on Annual Deposits

Example: At the end of every year, you deposit $1,000 into an account that earns 6% interest per year. What is the future value of your investment after 40 years?

Let's start with:

$$FV = A\left[\frac{(1+i)^N - 1}{i}\right]$$

The answer is:

a) $44,793 b) $60,000 c) $77,385 d) $154,762 e) $82,759

When should I start investing?

You want to invest in your retirement, which you are hoping to do by age 62. Consider these two scenarios:

Scenario 1: You graduate and get a nice job. You invest $6000 at the end of each year in your retirement plan that returns 8% per year.

What is the value of your investment after 40 years, when you turn 62?

$$FV = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$1,554,340 !

Scenario 2: You graduate and get a nice job. But you have a lot to buy (car, condo), and a lot of student loans to pay off. Reluctantly, you decide you can't start investing until you're financially ready, at about age 32.

Ten years later, at age 32, you happily start investing $6000 at the end of each year in your retirement plan that returns 8% per year.

What is the value of your investment after 30 years (when you're 62)?

$679,700 !

When should I start investing?

You want to invest in your retirement, which you are hoping to do by age 62. Consider these two scenarios:

Scenario 1: You graduate and get a nice job. You invest $6000 at th… your retirement plan tl…

What is the value of y… years, when you turn 6…

$$FV = A\left[\frac{(1+i)^{\quad}-1}{i}\right]$$

$1,554,340 !

You lose more than half the value of your investment by waiting those 10 years!

Scenario 2: You graduate and get a nice job. …buy (car, condo), and a lot …y off. Reluctantly, you …investing until you're …bout age 32.

… 32, you happily start …e end of each year in your retirement plan that returns 8% per year.

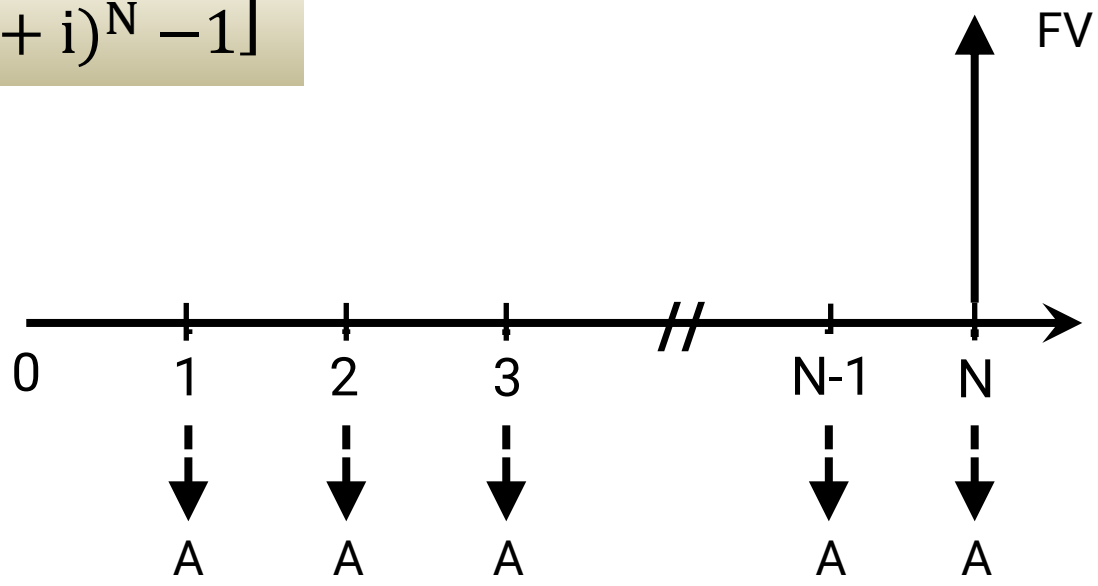
What is the value of your investment after 30 years (when you're 62)?

$679,700 !

What is the Annuity, given FV, i, and N

If you know the Future Value of your investment, what is the required annuity payment, A, to get there?

$$A = FV\left[\frac{i}{(1+i)^N - 1}\right]$$



An Example…

You're thinking 40 years from now is a long time. 30 years from seems ok. If your investment has an 8% return, how much should you invest each year to reach $1M?

$$A = FV\left[\frac{i}{(1+i)^N - 1}\right]$$

$$A = \$1{,}000{,}000\left[\frac{0.08}{(1+0.08)^{30} - 1}\right]$$

$$A = \$1{,}000{,}000 \text{ x } (0.00883)$$

A = $8,830 per year

Future Value & Annuities

$$FV = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$A = FV\left[\frac{i}{(1+i)^N - 1}\right]$$

These are referred to as “Uniform Series Cash Flows”

$$\left[\frac{(1+i)^N - 1}{i}\right]$$

“Uniform Series Compound Amount Factor”

$$\left[\frac{i}{(1+i)^N - 1}\right]$$

“Uniform Series Sinking Fund Factor”

Main Takeaways…

Most of the time, we invest periodically, such as at the end of the month with our company's retirement plan.

These uniform, series of payments are known as Annuities; payments that come at the end of the period are known as Ordinary Annuities.

The future value of an investment from an ordinary annuity is given by:

$$FV = A\left[\frac{(1+i)^N - 1}{i}\right]$$

Given the future value of an investment, the required annuity is obtained from:

$$A = FV\left[\frac{i}{(1+i)^N - 1}\right]$$

Next Time…

Spreadsheets to the Rescue!

Credits & References

Slide 1: Cash flow analysis on blackboard by Tuomas Kujansuu, Adobe Stock (72683988.jpeg).

Slide 2: Young man looking at an older himself in the mirror by Tommaso Lizzul, Adobe Stock (71278066.jpeg)

Slide 10: One person is answering question about his work. Is it reasonable, by Richelle, Adobe Stock (354552706.jpeg).

Slide 12: Accounting or Financial Management Software Program on Laptop Screen in Office Desk, by Menara Grafis, Adobe Stock (432160164.jpeg).