APPM/MATH 4/5520

Final Exam Review Problems

- The final exam is on Thursday, December 15th in our normal classroom from 1:30am to 4:00pm. It is not cumulative.
- The exam will have 6 problems and you must choose and complete 5 out of 6 problems. There is no grad take-home part to this exam but grad students will have to do (get to do!) all 6 problems.
- There will be an optional review session on Wednesday, December 14th from 6:30 to 8pm in a room yet to be determined.
- Hang in there– you are almost done!

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1. Let $X_1, X_2, \ldots, X_n$ be a random sample from the Beta(a,b) distribution. Find a set of complete and sufficient statistics for estimating a and b.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the $\Gamma(3, \beta)$ distribution. Find the UMVUE (uniformly minimum variance unbiased estimator) of β. Give the variance of your estimator.

3. Let $X_1, X_2, \ldots X_n$ be a random sample from the Poisson distribution with parameter λ. Find the UMVUE (uniformly minimum variance unbiased estimator) of $\tau(\lambda) = \lambda^2$.

4. Let $X_1, X_2, \cdots, X_n$ be a random sample from a uniform distribution on $(0, \theta)$. We have already seen in class that the sample maximum $X_{(n)}$ is complete and sufficient for θ. Do not show this again.

   Find the UMVUE for $\tau(\theta) = \theta^p$ where $p > 0$.

5. Consider the distribution with pdf

$$f(x; \theta) = 1 - \theta^2(x - \frac{1}{2}), \qquad 0 < x < 1, \qquad -1 < \theta < 1.$$

   (a) Find the best test of size α for

$$H_0 : \theta = 0 \qquad \text{versus} \qquad H_1 : \theta = \theta_1,$$

   for some fixed $\theta_1 \neq 0$, based on a sample of size 1.

   (b) Find (if it exists) a UMP (uniformly most powerful) test of size α of

$$H_0 : \theta = 0 \qquad \text{versus} \qquad H_1 : \theta \neq 0$$

   based on a sample of size 1.

6. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the $N(0, \sigma^2)$ distribution. Derive the UMP test of size α for $H_0 : \sigma^2 = \sigma_0^2$ versus $H_1 : \sigma^2 > \sigma_0^2$.

7. Express the power function of your test from the previous problem in terms of the chi-squared distribution.

8. Let $X_1, X_2, \ldots, X_n$ be a random sample from the exponential distribution with rate θ. Consider the hypotheses $H_0 : \theta = \theta_0$ and $H_1 : \theta < \theta_0$.

   (a) Find a test of size α based on the minimium of the sample.

   (b) Find the UMP test of size α.

   (c) Find and compare the power functions of your two tests.

9. Let $X_1, X_2, \ldots, X_n$ be a random sample from the Poisson distribution with parameter λ. Use the Rao-Blackwell Theorem to find the UMVUE for $\tau(\lambda) = e^{-\lambda}$.

10. Consider a random sample of size n from the $unif(0, \theta)$ distribution. Find the UMP test of size α for $H_0 : \theta \geq \theta_0$ versus $H_1 : \theta < \theta_0$

11. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with pdf

$$f(x; \theta) = e^{\theta - x} I_{(\theta, \infty)}(x).$$

   (a) Find a complete and sufficient statistic for estimating θ.

   (b) Find the UMVUE for θ.

12. Consider a random sample $X_1, X_2, \ldots, X_n$ from a distribution with pdf $f(x; \theta) = \theta(1-x)^{\theta-1}$, $0 < x < 1$, $\theta > 0$.

    Give the form of the GLRT (generalized likelihood ratio test)for testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$?

13. Let $X_1, X_2, \ldots, X_n$ be a random sample from the $N(\mu, \sigma^2)$ distribution where σ^2 is known.

    Derive the GLRT of size α for testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$.

14. Suppose that $X \sim bin(n_1, p_1)$ and $Y \sim bin(n_2, p_2)$ and that we wish to test whether the proportions are equal:

$$H_0 : p_1 = p_2 = p \qquad \text{versus} \qquad H_1 : p_1 \neq p_2$$

    where p is unknown.

   (a) Give the GLR statistic.

   (b) Since your GLR does not simplify very well, give an approximate GLRT of size α based on large sample sizes.

15. Let $X_1, X_2, \ldots, X_n$ be a random sample from the uniform distribution on $(0, \theta]$. Find the exact (not asymptotic) distribution of $-2 \ln \lambda(\vec{X})$ where $\lambda(\vec{X})$ is the GLR for $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$. Based on this, find a GLRT of size α.

16. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the Poisson distribution with parameter λ. Find the posterior Bayes estimator of λ using the $\Gamma(\alpha, \beta)$ prior with α and β known.

17. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the $N(\mu, \sigma^2)$ distribution with σ^2 known. Find the posterior Bayes estimator of μ using a $N(0, 1)$ prior.