Some Practical Examples

Present Value given a Series of Annuities

Because of your promotions, you make $200,000 per year when you're 65. Now you want to withdraw $200,000 per year ("A") until you've, well, passed on.

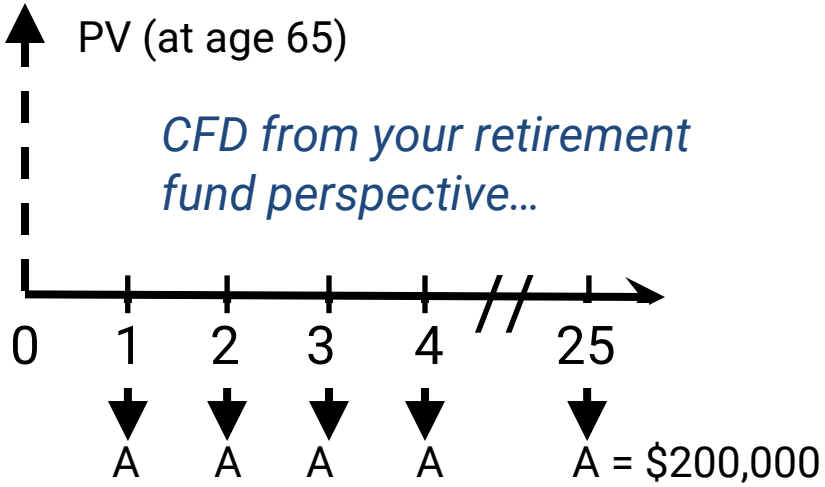
What does your retirement fund need to be in order to withdraw $200,000 per year until…

Assume you invest in a fund that returns 8%/yr, and you plan on living until 90 (N=25).

$$PV = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

PV = $200,000 (10.6748)

PV = $2,134,960



What happens in year 26?

Uniform Series Present Value

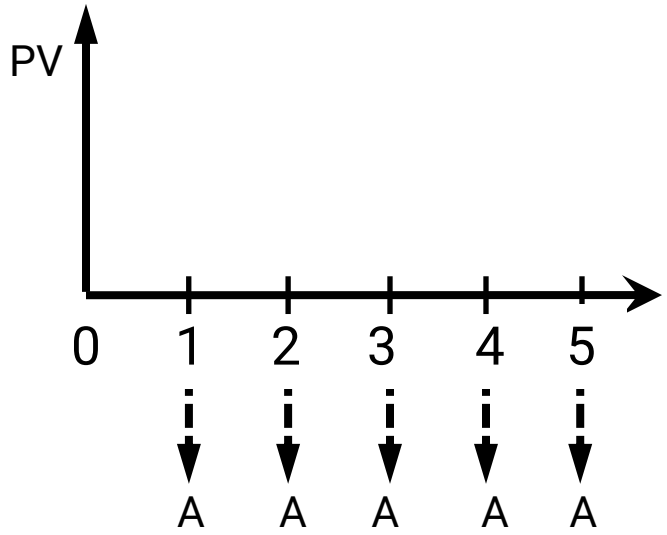
Ex. A = $200,000, interest rate = 8%, N = 25 years.

Use Excel's PV function once again…

	A	B	C	D	E	F
1	**TVM with Excel**					
2	**Present Value Calculations**					
3						
4	FV =	$0	*this is value of our investment at time = N*			
5	PMT =	-$200,000	*this is our A, at the end of each period*			
6	RATE =	8%	*this is our i*			
7	NPER =	25	*this is our N*			
8						
9	PV =	$2,134,955	*PV(rate, nper, pmt, fv, type)*			
10			*=PV(B6,B7,B5,B4)*			
11						

Loan Repayments

You decide to buy a new EV for $45,000. You pay $5,000 as a down-payment and borrow the rest, with the loan being a 5-year loan at a nominal interest of 6% compounded monthly. What is your monthly payment?



$$A = PV\left[\frac{i\,(1+i)^N}{(1+i)^N - 1}\right]$$

This is how you calculate monthly payments for a loan!

What are the terms for this equation?

PV = $40,000

N =(12 x 5) = 60

i =(6% / 12) = 0.5%

Loan Repayments

You decide to buy a new EV for $45,000. You pay $5,000 as a down-payment and borrow the rest. What is your monthly payment? The loan is a 5-year loan at a nominal interest rate of 6% compounded monthly.

	A	B	C	D	E
1	**Paying Off My Electric Vehicle**				
2					
3					
4	PV =	$40,000	*this is your initial loan value*		
5	FV =	$0	*your final value after paying off the loan*		
6	RATE =	0.50%	*this is the interest rate per period (month)*		
7	NPER =	60	*this is the number of payment periods*		
8					
9	PMT =	-$773.31	*PMT(rate, nper, pv, fv, type)*		
10			*=PMT(B6,B7,B4,B5)*		

Different ways of buying stuff…

You're thinking of buying a new EV that costs $28,650. You can do it one of three ways

Option 1: Pay cash that you have sitting around.

Option 2: Debt-financing: pay $3650 down, then finance the rest over 36 months at 1.9%, compounded monthly.

Option 3: Lease the car: pay something up-front, monthly payments, and turn it in at the end.

Electric Vehicle
$28,650

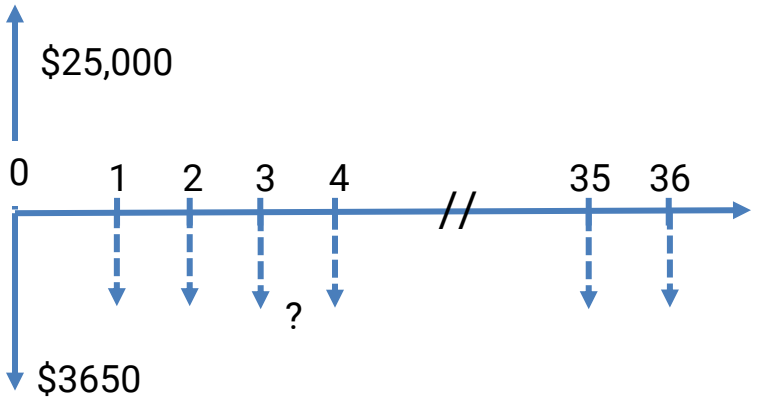
Which is the least costly option?

We'll consider the "lowest monthly payment" and "lowest overall cost".

Option 1: this is easy, you pay $28,650 and you're done.

Payment Options…

Option 2: Debt-financing: pay $3650 down, then finance the rest over 36 months at 1.9%, compounded monthly.



$$A = PV\left[\frac{i\,(1+i)^N}{(1+i)^N - 1}\right]$$

The interest rate per period: 1.9% / 12 = 0.1583% (0.001583)

The total # of periods (payments): 12 x 3 = 36

$$\text{A} = \$25{,}000\left[\frac{0.001583\,(1+0.001583)^{36}}{(1+0.001583)^{36} - 1}\right]$$

$$\text{A} = \$25{,}000\left[\frac{0.00168}{0.0586}\right]$$

$$\text{A} = \$25{,}000\,[.02860]$$

A = $714.97 per month

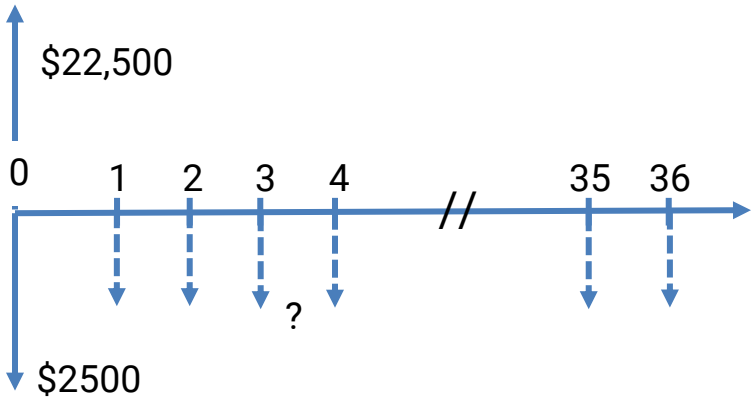
What did you pay in total?

Total Cost = $3650 + (36 x $716.72)

Total Cost = $29,452.05

Payment Options…

Option 2: Debt-financing: pay $3650 down, then finance the rest over 36 months at 1.9%, compounded monthly.



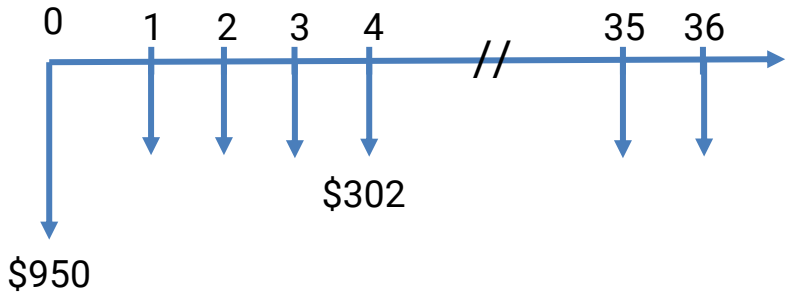
$$A = P\left[\frac{i\,(1+i)^N}{(1+i)^N - 1}\right]$$

	A	B	C	D	E	F
1	**Financing My Car**					
2	Loan Amt:	$25,000				
3	APR:	1.9%				
4						
5	PV =	$25,000	*this is your initial loan value*			
6	FV =	$0	*your final value after paying off the loan*			
7	RATE =	0.16%	*this is the interest rate per period (month)*			
8	NPER =	36	*this is the number of payment periods*			
9						
10	PMT =	-$714.97	*PMT(rate, nper, pv, fv, type)*			
11			*=PMT(B6,B7,B4,B5)*			
12						

Let's check to make sure we did that right!

Payment Options…

Option 3: You lease the car: $950 due at signing, monthly payments = $302.
Residual value = $21,000.



Ok, this isn't too complicated…

What did you pay in total?

Total Cost = $950 + (36 x $302)

Total Cost = $11,822

Seems like leasing is a much less expensive option.

But aren't we missing something?

Payment Options…

Option 3: You lease the car: $950 due at signing, monthly payments = $302.
Residual value = $21,000.

At the end of the lease, you have to:

1) turn the car in
2) buy it for the residual value of $21,000

You love your new EV, and decide to buy it with cash.

Now what did you pay in total?

Total Cost = $950 + (36 x $302) + $21,000

Total Cost = $32,822

Payment Options…

You're thinking of buying n EV that costs $28,650. You can do it one of three ways

Option 1: Pay cash that you have sitting around.

Option 2: Debt-financing: pay $3650 down, then finance the rest over 36 months at 1.9%, compounded monthly.

Option 3: Lease the car: pay something up-front, monthly payments, and turn it in at the end.

Electric Vehicle
$28,650

Option 1: Buying the car	= $28,650
Option 2: Financing the car	= $29,452
Option 3: Leasing the car	= $32,822

So paying cash is really the cheapest option, and leasing is the most expensive.

One last example…

You're thinking of buying a nice condo that costs $500,000.

You have two options: You need to pay 20% as a down-payment = $100,000

Option 1: Finance it for 30 years at 2.5% per year, compounded monthly

Option 2: Finance it for 15 years at 2.125% per year, compounded monthly

	A	B	C	D	E	F	G
1	**Financing My Condo**						
2	Loan Amt:	$400,000					
3							
4	Term:	30	years		15	years	
5	APR:	2.5%	per year		2.125%	per year	
6	PV =	$400,000			PV =	$400,000	
7	FV =	$0			FV =	$0	
8	RATE =	0.21%			RATE =	0.18%	
9	NPER =	360			NPER =	180	
10							
11	PMT =	-$1,580	*per month*		PMT =	-$2,597.12	
12							
13	Total Cost:	-$568,974			Total Cost:	-$467,482	
14	Principal Paid:	-$400,000			Principal Paid:	-$400,000	
15	Interest Paid:	-$168,974			Interest Paid:	-$67,482	
16							

Over the life of the loan, the 30-year loan will cost you almost $100,000 more than the 15-year loan.

That's the trade-off for paying $1000 less per month.

One last example…

You're thinking of buying a nice condo that costs $500,000.

You have two options: You need to pay 20% as a down-payment = $100,000

Option 1: Finance it for 30 years at 5% per year, compounded monthly

Option 2: Finance it for 15 years at 4% per year, compounded monthly

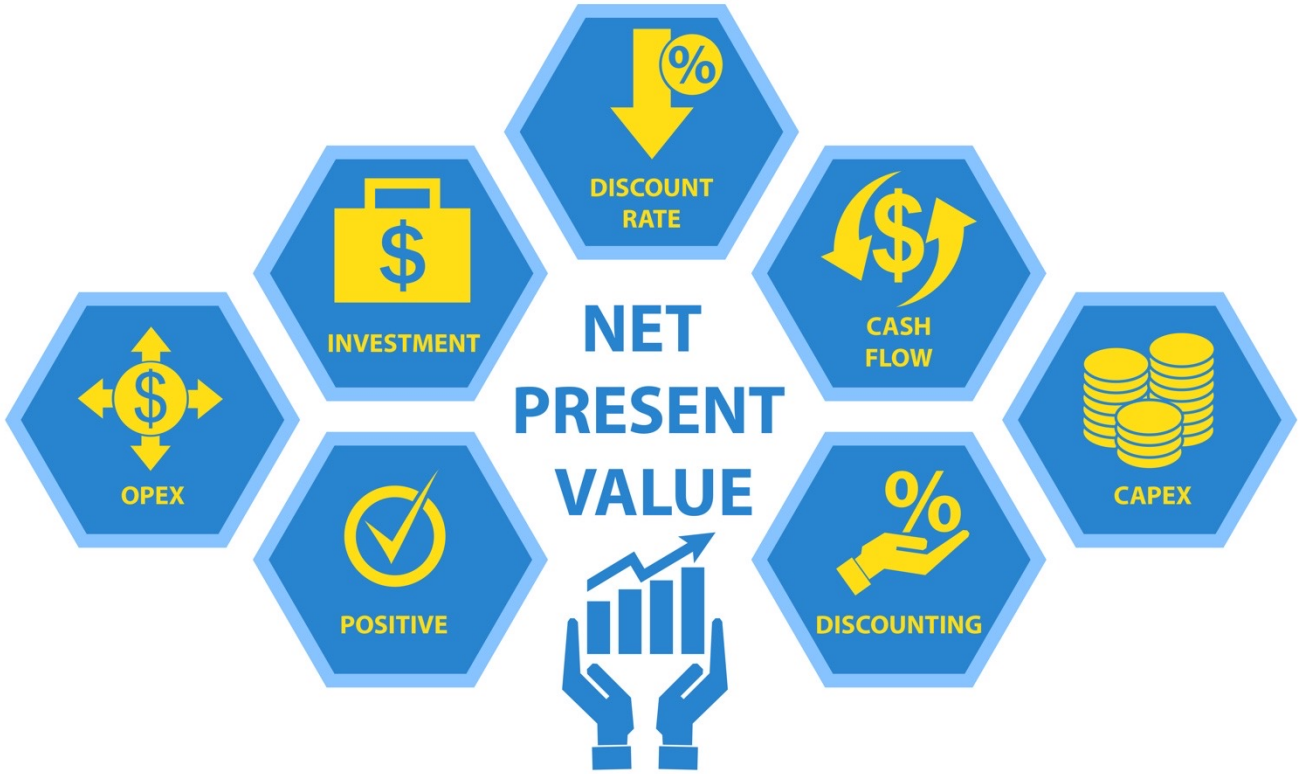
	A	B	C	D	E	F	G
1	**Financing My Condo**						
2	Loan Amt:	$400,000					
3							
4	Term:	30	years		15	years	
5	APR:	5.0%	per year		4.000%	per year	
6	PV =	$400,000			PV =	$400,000	
7	FV =	$0			FV =	$0	
8	RATE =	0.42%			RATE =	0.33%	
9	NPER =	360			NPER =	180	
10							
11	PMT =	-$2,147	*per month*		PMT =	-$2,958.75	
12							
13	Total Cost:	-$773,023			Total Cost:	-$532,575	
14	Principal Paid:	-$400,000			Principal Paid:	-$400,000	
15	Interest Paid:	-$373,023			Interest Paid:	-$132,575	
16							

Over the life of the loan, the 30-year loan will cost you almost $250,000 more than the 15-year loan.

Key Point Here: the cheapest monthly solution is rarely the cheapest solution overall!

Next Time…

Project Valuation Techniques



Credits & References

Slide 1: Accounting report and financial statement on desk by Leonid, Adobe Stock.

Slide 2: Grim Reaper Image Source, https://www.publicdomainpictures.net/pictures/310000/nahled/sensemann.png (accessed May 9, 2022).

Slide 6 and 11: Electric Car in Charging Station Isolated by nerthuz, Adobe Stock (299554135.jpeg).

Slide 15: Concept of NPV - Net Present Value by Elnur, Adobe Stock (402998660.jpeg).