Simple and Compound Interest

The Concept of Interest…

The Basics

Money is such a valuable asset people are willing to pay to have it access to it.

If you are trustworthy, people – notably banks - are willing to loan you money in return for you paying them a little more than the loan.

Banks will pay *you* interest for the opportunity to use your money to provide loans to those people it!

Interest: the amount charged to "borrow" or "loan" money

Simple Interest

Simple Interest: the interest calculated based on the original investment, the interest rate and a period of time:

$$I = P \times i \times N$$

Where:

I = Interest

P = The Principal (the amount you invest at the beginning)

i = interest rate (stated in %, but always calculated as a decimal)

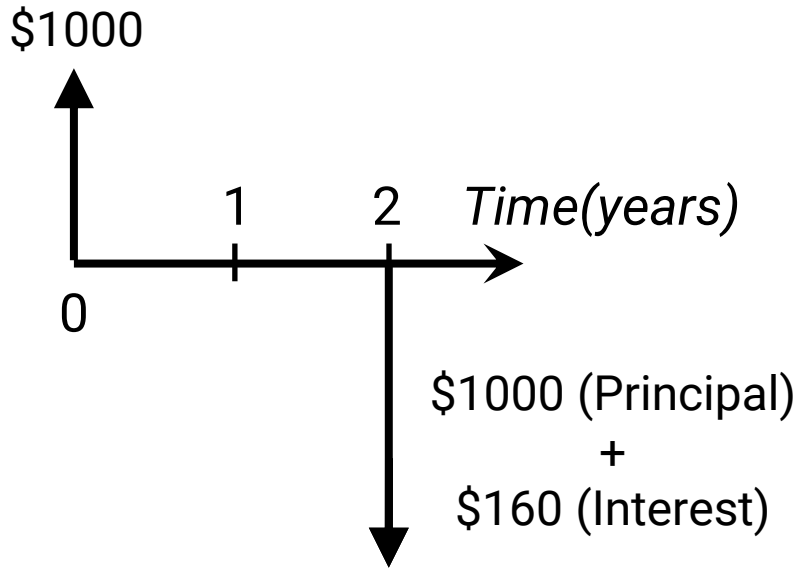
N = the # of time "*periods*" under consideration (usually in months or years)

Simple Interest – A Simple Example

Example: A friend loans you $1000 at 8% per year simple interest, payable in two years. What is the amount of interest you'll pay?

P = $1000 i = 8%/yr (or 0.08/yr) N = 2 years

Interest Owed = P x i x N = ($1000) x (0.08/yr) x (2yr) = $160



Future Value based on Simple Interest

Consider this situation:

You make an investment of $1000. It earns simple interest every year at a rate of 5%.

What is the future value of your investment, 5 years from now?

The future value of your investment is your initial investment plus the interest you earned over the 5-years:

$FV = P + I$

$FV = P + (P \times i \times N)$ → $FV = \$1000 + (\$1000 \times 0.05 \times 5)$

$FV = \$1000 + \$250$

$FV = \$1250$

Simple Interest – Your Turn…

Example: Your Grandmother loans you $10,000 to develop an app for tech-savvy senior citizens. She charges you 4% simple interest per year, everything payable in 3 years.

What is the amount of interest you'll pay? What is the future value of her investment?

$I = P \times i \times N$

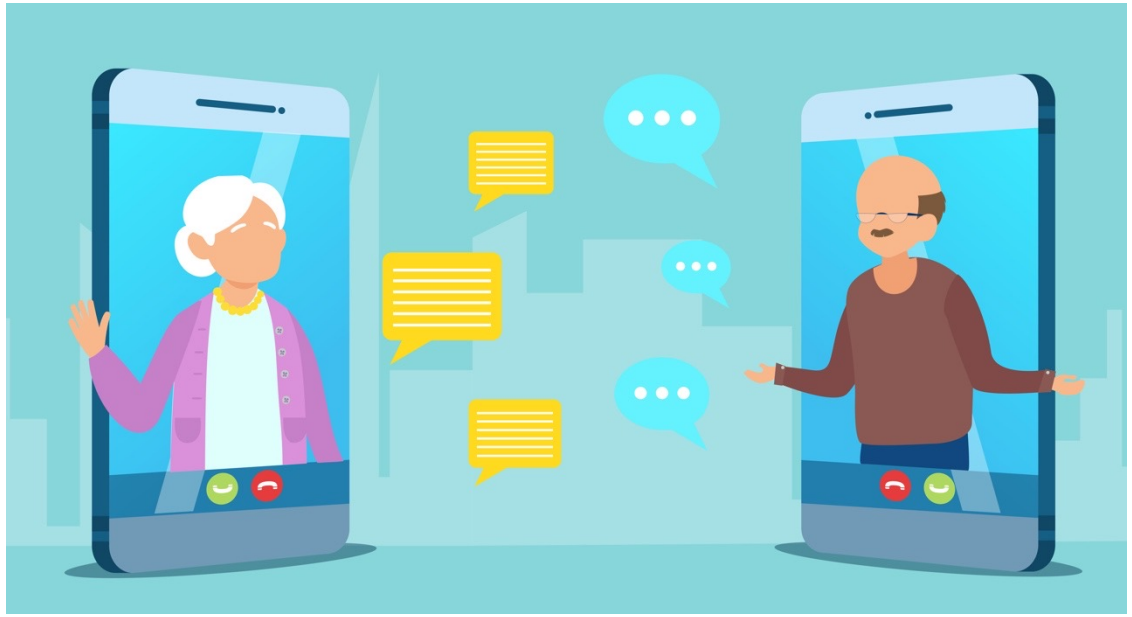
$I = \$10{,}000 \times 0.04 \times 3$

$I = \$1200$

$FV = P + I$

$FV = \$10{,}000 + \$1200$

$FV = \$11{,}200$



Compound Interest

Compound Interest: Interest added to the Principal at the end of each period, which then becomes the "new" Principal for the next period.

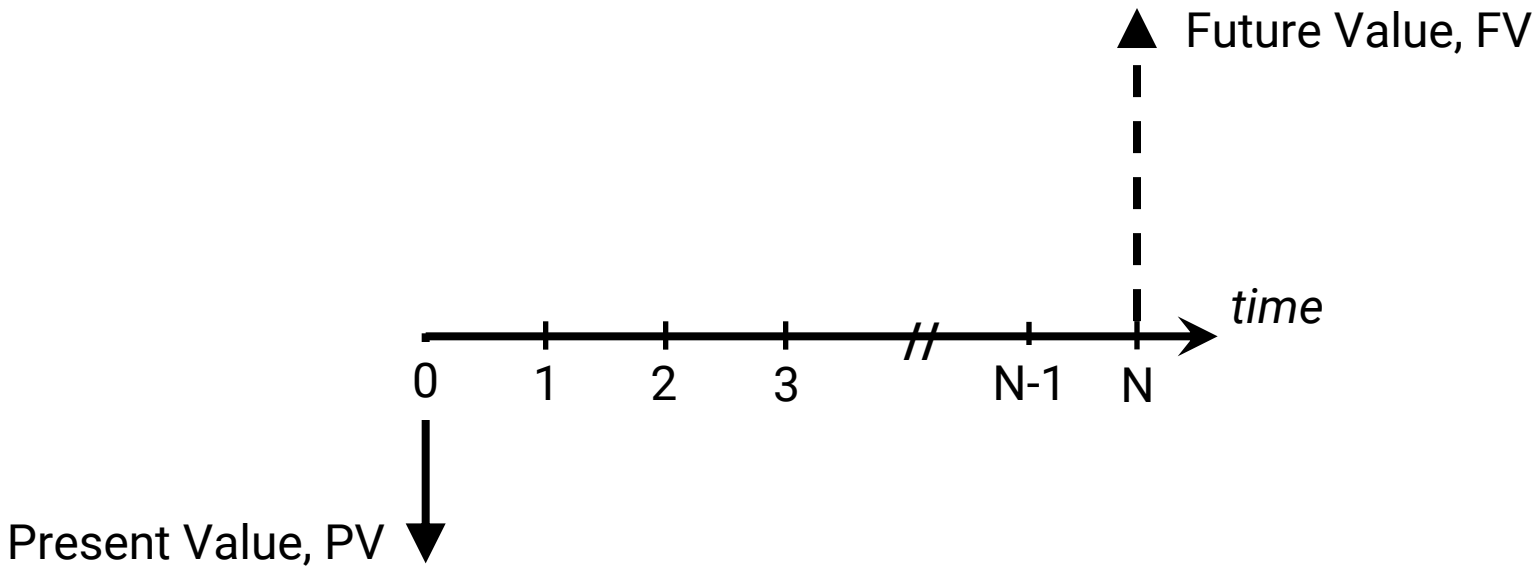
Ex. You put $1000 into an interest-bearing savings account that has a return (interest rate) of 5% per year. What is your investment worth in 3 years?

Year	Amount at Beginning of Year	Interest Earned at End of Year (EOY)	Total Amount at End of Year (EOY)
1	$1,000.00	$50.00	$1,050.00
2	$1,050.00	$52.50	$1,102.50
3	$1,102.50	$55.12	**$1,157.62**

Compound Interest: Interest is earned on top of interest!

The Future Value of an Investment…

What is the Future Value, “FV”, of an investment having an initial Present Value, “PV”, an interest rate, “i”, and “N” periods?



Let’s start by breaking the investment into one period (year) “pieces”.

The Future Value of an Investment…

Ex. You invest $1000 in an interest-bearing account that has a return (interest rate) of 5% per year. What is your investment worth in "N" years?

Year	Amount at Beginning of Year	Interest Earned at End of Year (EOY)	Total Amount at End of Year (EOY)
1	PV = $1,000.00	$50.00	$1,050.00
2	$1,050.00	$52.50	$1,102.50
3	$1,102.50	$55.12	**$1,157.62**
…	…	…	…
N	?	?	**FV_N**

Can we establish a formal relationship between PV, FV , N and i?

The Future Value of an Investment…

Ex. You invest $1000 in an interest-bearing account that has a return (interest rate) of 5% per year. What is your investment worth in "N" years?

Year	Amount at Beginning of Year	Interest Earned at End of Year (EOY)	Total Amount at End of Year (EOY)
1	**PV_1 = $1,000.00**	**I_1 = $50.00**	**FV_1 = $1,050.00**
2	$1,050.00	$52.50	$1,102.50
3	$1,102.50	$55.12	$1,157.62
…	…	…	…
N	PV_N	I_N	FV_N

$$FV_1 = PV_1 + I_1$$

$$FV_1 = PV_1 + (PV_1 * i)$$

$$FV_1 = PV_1 (1+i)$$

$$FV_1 = PV_1 (1+i)$$

The Future Value of an Investment…

Ex. You invest $1000 in an interest-bearing account that has a return (interest rate) of 5% per year. What is your investment worth in "N" years?

Year	Amount at Beginning of Year	Interest Earned at End of Year (EOY)	Total Amount at End of Year (EOY)
1	PV_1 = $1,000.00	I_1 = $50.00	FV_1 = $1,050.00
2	**PV_2 = $1,050.00**	**I_2 = $52.50**	**FV_2 = $1,102.50**
3	$1,102.50	$55.12	$1,157.62
…	…	…	…
N	PV_N	I_N	FV_N

$FV_1 = PV_1 (1+i)$

$FV_2 = PV_2 + I_2 = PV_2 + (PV_2 * i) = PV_2 (1+i)$

But $PV_2 = FV_1$

$FV_2 = [PV_1 (1+i)] (1+i)$

$FV_2 = PV_1 (1+i)^2$

$FV_2 = PV_1 (1+i)^2$

The Future Value of an Investment…

Ex. You invest $1000 in an interest-bearing account that has a return (interest rate) of 5% per year. What is your investment worth in "N" years?

Year	Amount at Beginning of Year	Interest Earned at End of Year (EOY)	Total Amount at End of Year (EOY)
1	PV_1 = $1,000.00	I_1 = $50.00	FV_1 = $1,050.00
2	PV_2 = $1,050.00	I_2 = $52.50	FV_2 = $1,102.50
3	**PV_3 = $1,102.50**	**I_3= $55.12**	**FV_3 = $1,157.62**
…	…	…	…
N	FV_{N-1}	I_N	FV_N

$FV_1 = PV\ (1+i)$

$FV_2 = PV\ (1+I)^2$

$FV_3 = PV_3 + I_3 = PV_3 + (PV_3*i) = PV_3\ (1+i)$

But $PV_3 = FV_2$

$FV_3 = PV_1\ (1+i)^2\ (1+i)$

$FV_3 = PV_1\ (1+i)^3$

$FV_3 = PV_1\ (1+i)^3$

The Future Value of an Investment…

Ex. You invest $1000 in an interest-bearing account that has a return (interest rate) of 5% per year. What is your investment worth in "N" years?

Year	Amount at Beginning of Year	Interest Earned at End of Year (EOY)	Total Amount at End of Year (EOY)	
1	PV = $1,000.00	I_1 = $50.00	FV_1 = $1,050.00	$FV_1 = PV\ (1+i)$
2	PV_2 = $1,050.00	I_2 = $52.50	FV_2 = $1,102.50	$FV_2 = PV\ (1+i)^2$
3	PV_3 = $1,102.50	I_3 = $55.12	FV_3 = $1,157.62	$FV_3 = PV\ (1+i)^3$
…	…	…	…	
N	**PV_N**	**I_N**	**FV_N**	**$FV_N = PV_1\ (1+i)^N$**

$$FV = PV\ (1+i)^N$$

The Future Value of an Investment…

How to Become Wealthy Without Trying Too Hard…

Example: Upon graduation, your grandparents gave you $25,000 to invest for retirement. If you invest in something that returns 6% per year, what is the value of your investment 40 years from now?

$$FV = PV\,(1+i)^N$$

where:

PV = $25,000

i = 6% per year

N = 40 years

$$FV = \$25{,}000 * (1+0.06)^{40}$$

FV = $257,143 !

Future Value of an Investment

How much more do you realize due to compounding interest compared to just simple interest?

Compound Interest Case

$$FV = PV\,(1+i)^N$$

where: PV = $25,000
i = 6% per year
N = 40 years

$$FV = \$25{,}000 \times (1+0.06)^{40}$$
$$FV = \$25{,}000 \times 10.2857$$

FV = $257,143 !

Simple Interest Case

$$FV = P + I = P + (P \times i \times N)$$

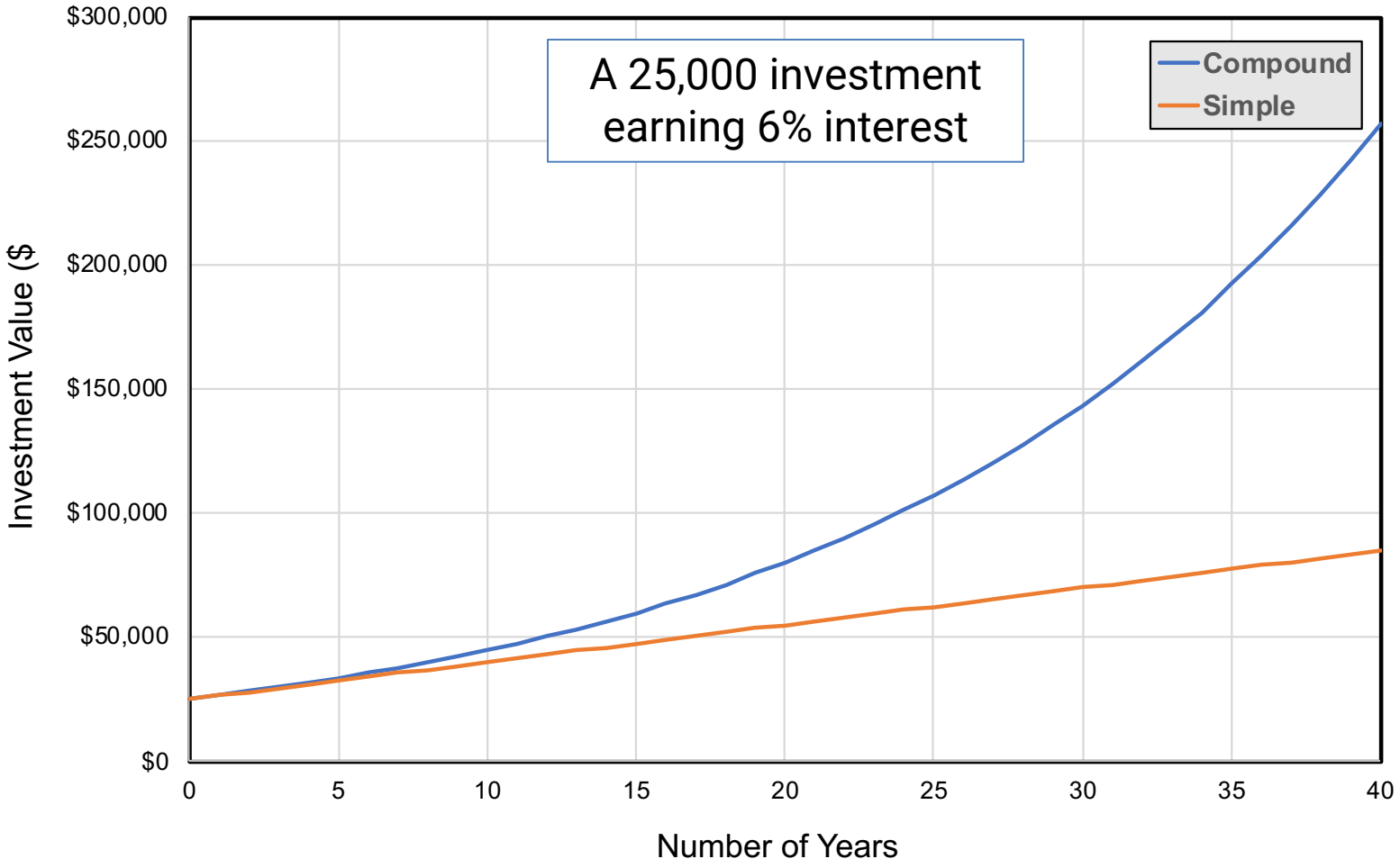
where: P = $25,000
i = 6% per year
N = 40 years

$$FV = \$25{,}000 + (\$25{,}000 \times 0.06 \times 40)$$
$$FV = \$25{,}000 + \$60{,}000$$

FV = $85,000 !

Future Value of an Investment

How much more do you realize due to compounding interest compared to just simple interest?



Main Takeaways…

- Simple Interest Case: the future value (FV) is determined by the principal (P), the interest rate (i) and the number of periods (N):

$$FV = P + (P \times i \times N)$$

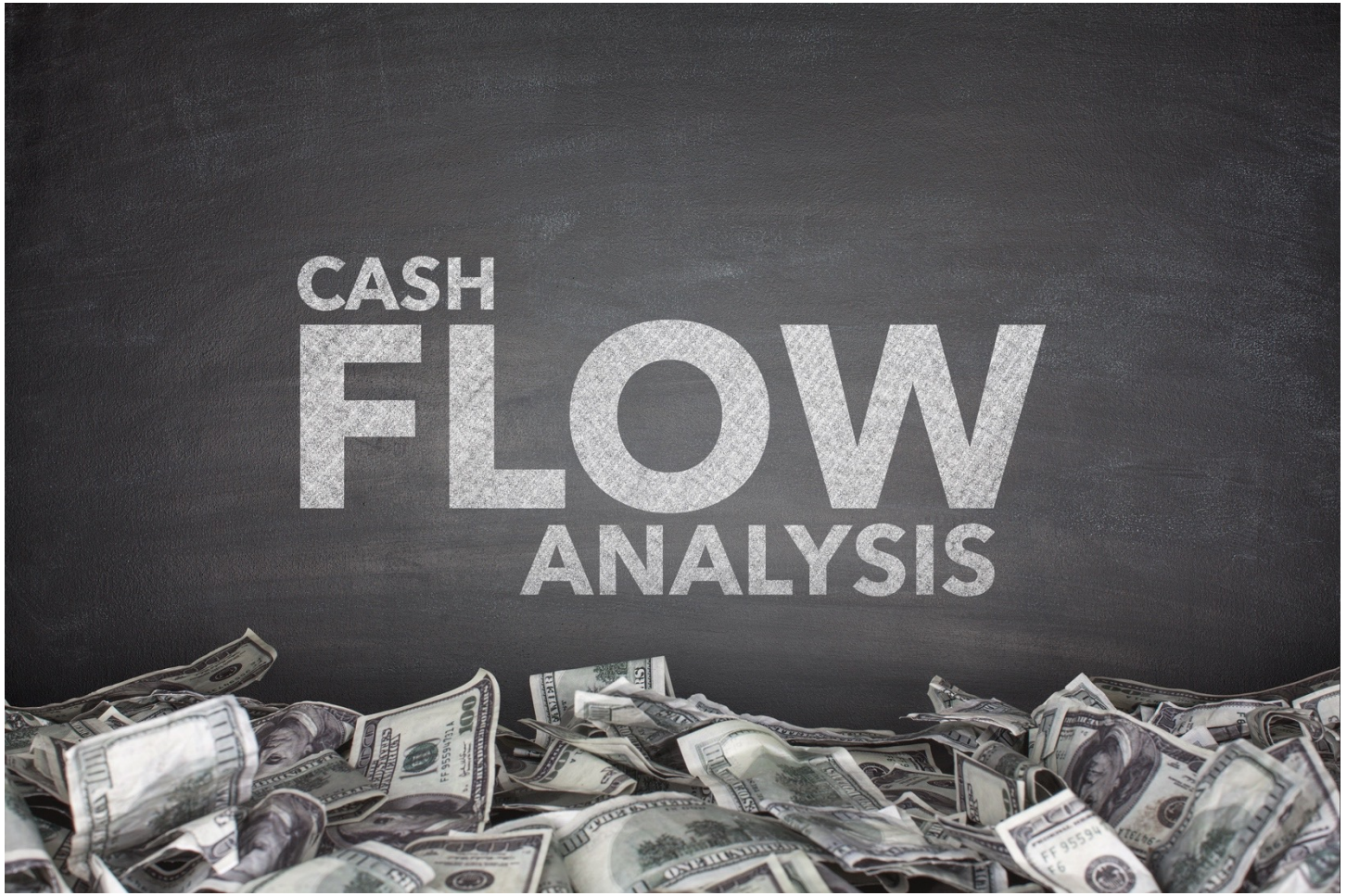
- Compound Interest Case: the future value (FV) increases over time because the investment collects interest, and the amount of interest is related to the interest obtained already.

$$FV = PV\ (1 + i)^N$$

- Compounding interest allows the initial investment to grow much faster than the case of simple interest.

Next Time…

Single Payment Cash Flows



Credits & References

Slide 1: Simple vs compound interest concept, flat tiny person vector illustration by VectorMine, Adobe Stock (321923382.jpeg).

Slide 6: Vector of a senior couple man and woman communication using smart phone video call by pathdoc, Adobe Stock (376488452.jpeg).

Slide 18: Cash flow analysis on blackboard by Tuomas Kujansuu, Adobe Stock (72683988.jpeg).