Introduction to Probability
Part 2

Data Science for Quality Management: Probability and Probability Distributions

with **Wendy Martin**

Learning objectives:

Discriminate between marginal, joint and conditional probabilities

Discriminate between independent and dependent events

Learning objectives:

Calculate marginal, joint, and conditional probability under independent and dependent conditions.

Statistical Independence and Dependence

- Events which are statistically independent are those where the outcome of one event has no effect on the outcome of the second event

Statistical Independence and Dependence

- Events which effect subsequent events are termed dependent.

Independent Conditions – Marginal Probability

- P(A) Independent Event (e.g. coin toss)

Independent Conditions – Joint Probability

- The probability of two or more events occurring together (or in succession) is the product of their marginal probabilities
- $P(AB) = P(A) \times P(B)$, where

Independent Conditions – Joint Probability

- P(AB) = probability of events A and B occurring together or in succession; joint probability
- P(A) = marginal probability of (A)
- P(B) = marginal probability of (B)

Independent Conditions – Joint Probability Example 1

- The probability of a machine operator producing a defective part at any point in time is 0.05. What is the probability that three bad parts will be produced in succession?

Independent Conditions – Joint Probability Example 1

- P(ABC) = P(A) x P(B) x P(C)
- P(3 Defectives) = P(Def) x P(Def) x P(Def)
- P(3 Def) = 0.05 x 0.05 x 0.05
- P(3 Def) = 0.000125

Independent Conditions – Conditional Probability

- P(B|A) = Probability of event B occurring, given that A has occurred.
- P(B|A) = P(B)…because A and B are independent!

Dependent Conditions – Conditional Probability

- Note that this is equivalent to calculating the probability of the part being defective, given a sample space of B, after A has been drawn.

Dependent Conditions – Conditional Probability Example

- Assuming a randomly selected part is from Vendor A, what is the P that it is also defective?

Vendor	# Defective	# Not Defective	Total
Vendor A	15	85	100
Vendor B	10	55	65
Total	25	140	**165**

Dependent Conditions – Conditional Probability Example

Dependent Conditions – Conditional Probability Example

Vendor	# Defective	# Not Defective	Total
Vendor A	15	85	100
Vendor B	10	55	65
Total	25	140	**165**

- Note: This is the same as observing that given the 15 defectives out of 100 Vendor A parts, then

Dependent Conditions

Note also that the P(Defective and Vendor A) constitutes a joint probability under statistical dependence. Creating a table of joint P values for the sample space:

Event	P	Fraction
Vendor A and Defective	0.0909	$^{15}/_{165}$
Vendor A and Not Defective	0.5151	$^{85}/_{165}$
Vendor B and Defective	0.0606	$^{10}/_{165}$
Vendor B and Not Defective	0.3333	$^{55}/_{165}$

Dependent Conditions – Conditional Probability Example

- As a second example, assume that a non-defective part has been drawn. What is the P that it is from Vendor B?

Dependent Conditions – Conditional Probability Example

- Note that should a non-defective part have been selected, the P of it being a part from Vendor B is

Joint Probabilities Under Statistical Dependence

- The formula for joint probabilities under statistical dependence is a variation of the conditional probability formula

Joint Probabilities Under Statistical Dependence

$$P(B|A) = \frac{P(BA)}{P(A)} \Rightarrow P(BA) = P(B|A)\, x\, P(A)$$

Noting that $\Rightarrow$ $P(BA) = P(AB)$

And that P(BA) = P of events B and A happening together or in succession

Joint Probabilities Under Statistical Dependence Example

- As an example, we can check any of the joint probability calculations from the joint P table; for example,
- P(A and Def) = 0.0909 or ${}^{15}/_{165}$

Joint Probabilities Under Statistical Dependence Example

$$P(BA) = P(B|A)\ x\ P(A)$$

$$P(BA) = \frac{P(BA)}{P(A)}\ x\ P(A)$$

$$\frac{P(A\ and\ Def)}{P(Def)}\ x\ P(Def) = \frac{0.0909}{0.1515}\ x\ 0.1515 = 0.0909$$

Sources

The material used in the PowerPoint presentations associated with this course was drawn from a number of sources. Specifically, much of the content included was adopted or adapted from the following previously-published material:

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982
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