Module 5 Peer Review Assignment

Problem 1

Roll two six-sided fair dice. Let X denote the larger of the two values. Let Y denote the smaller of the two values.

a) Construct a table that gives the joint probability mass function for X and Y. (Note: "X is the larger value and Y is the smaller value in a two dice roll" means that for any two dice roll, X will be greater than or equal to Y).

	x=1	**x=2**	**x=3**	**x=4**	**x=5**	**x=6**
y=1	1/36	2/36	2/36	2/36	2/36	2/36
y=2	0/36	1/36	2/36	2/36	2/36	2/36
y=3	0/36	0/36	1/36	2/36	2/36	2/36
y=4	0/36	0/36	0/36	1/36	2/36	2/36
y=5	0/36	0/36	0/36	0/36	1/36	2/36
y=6	0/36	0/36	0/36	0/36	0/36	1/36

b) What is $P(X \geq 3, Y = 1)$?

$P(X \geq 3, Y = 1)$

In [3]:
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(2/36) + (2/36) +(2/36) +(2/36)
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0.2222222222222222

=> 2/9

c) What is $P(X \geq Y + 2)$?

(2/36) + (2/36) +(2/36) +(2/36) = (8/36)

(2/36) + (2/36) +(2/36) = 6/36

(2/36) + (2/36) = 4/36

(2/36)

(8/36) + (6/36) + (4/36) + (2/36) = (20/36)

In [2]:
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(8/36) + (6/36) + (4/36) + (2/36)
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0.5555555555555555

$$\begin{aligned} P(X \geq Y+2) &= P(X \geq 3, Y = 1) + P(X \geq 4, Y = 2) + P(X \geq 5, Y = 3) + P(X = (\\ &= 8/36 + 6/36 + 4/36 + 2/36 \\ &= 20/36 \end{aligned}$$

d) Are X and Y independent? Explain.

No, X and Y are dependent.

$$P(X=1)P(Y=1) = [(2/36) + (2/36) + (2/36) + (2/36) + (2/36) + (1/36)] = (11/36)$$
$$P(X=1, Y=1) = 1/36$$

Problem 2

Let (X, Y) be continuous random variables with joint PDF:

$$f(x,y) = \begin{cases} cxy^2 & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases}$$

Part a)

Solve for c. Show your work.

$$\begin{aligned} \int_0^1 \int_0^1 f(x,y)dxdy = 1 \\ \int_0^1 \int_0^1 cxy^2 dxdy \\ c\Big[(1/2)x^2\Big]_0^1 \Big[(1/3)y^3\Big]_0^1 \\ c\Big[(1/2)(1^2 - 0^2)\Big]\Big[(1/3)(1^3 - 0^3)\Big] \\ c(1/2)(1/3) \\ c(1/6) = 1 \\ c = 6 \end{aligned}$$

Part b)

Find the marginal distributions $f_X(x)$ and $f_Y(y)$. Show your work.

$$f_X(x) = \int_0^1 f(x, y)dy$$

$$\int_0^1 6xy^2 dy$$

$$6x\left[(1/3)y^3\right]_0^1$$

$$2x\left[1^3 - 0^3\right]$$

$$2x, \qquad 0 \le x \le 1$$

$$f_Y(y) = \int_0^1 6xy^2 dx$$

$$6y^2\left[(1/2)x^2\right]_0^1$$

$$3y^2\left[1^2 - 0^2\right]$$

$$3y^2, \qquad 0 \le y \le 1$$

Part c)

Solve for $E[X]$ and $E[Y]$. Show your work.

$$E[X] = \int_0^1 xf_X(x)dx$$

$$\int_0^1 x(2x)dx$$

$$2\left[(1/3)x^3\right]_0^1$$

$$(2/3)\left[1^3 - 0^3\right]$$

$$\frac{2}{3}\left[1 - 0\right]$$

$$= 2/3$$

$$E[Y] = \int_0^1 yf_Y(y)dy$$

$$\int_0^1 y(3y^2)dy$$

$$3\left[(1/4)y^4\right]_0^1$$

$$(3/4)\left[1^4 - 0^4\right]$$

$$(3/4)\left[1 - 0\right]$$

$$3/4$$

Part d)

Using the joint PDF, solve for $E[XY]$. Show your work.

$$E[XY] = \int_0^1 \int_0^1 xyf(x,y)dxdy$$

$$\int_0^1 \int_0^1 xy(6xy^2)dxdy$$

$$6\int_0^1 \int_0^1 x^2y^3dxdy$$

$$6\left[(1/3)x^3\right]_0^1\left[(1/4)y^4\right]_0^1$$

$$6(1/3)(1/4)\left[1^3 - 0^3\right]\left[1^4 - 0^4\right]$$

$$\frac{6}{12}[1-0][1-0]$$

$$\frac{1}{2}$$

Part e)

Are X and Y independent?

X and Y independent.

$E[XY] = 1/2$

$E[X]E[Y] = (2/3)(3/4) = (6/12) = (1/2)$

In [ ]: