The Exponential Distribution

Data Science for Quality Management: Probability and Probability Distributions

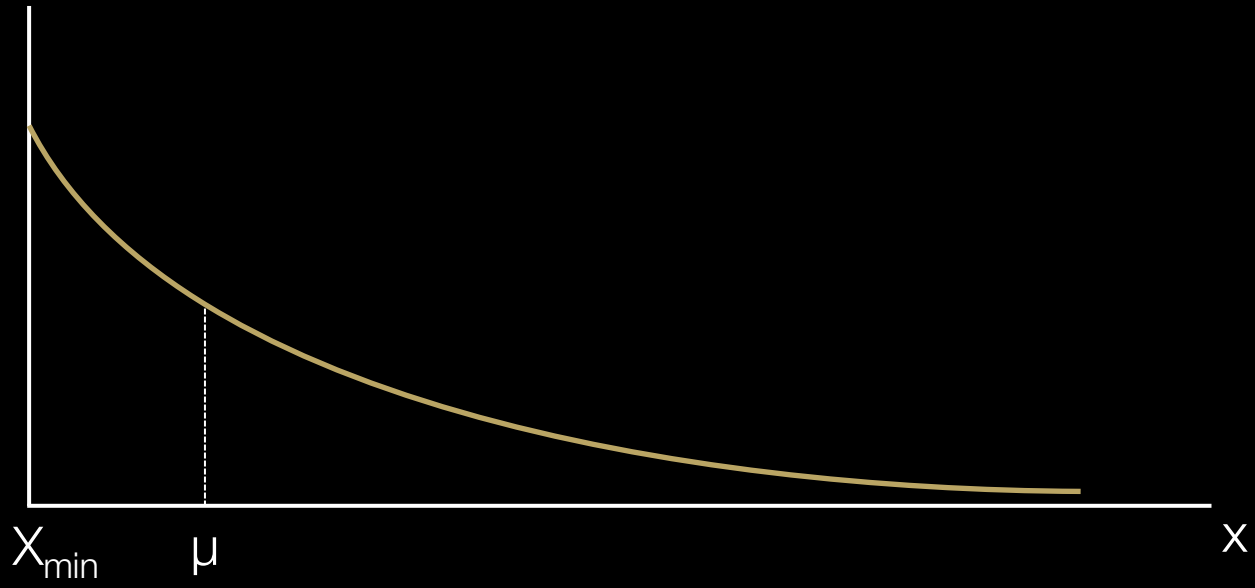
with **Wendy Martin**

Learning objectives:

Describe the Exponential probability distribution

Calculate probabilities using the Exponential distribution

The Exponential Distribution



The Exponential Distribution

- The exponential distribution occurs in a number of situations in the industrial environment.
- Time to failure often follows an exponential distribution.

The Exponential Distribution

- Measurement from a physical process that has a restraint, such as the location of a hole from a reference edge, where the reference edge is pressed against a fixture, may follow an exponential distribution.

The Exponential Distribution

- Roundness of shaft, measured by total indicator reading, may also follow this type of distribution.

The Exponential Distribution

- The exponential distribution is a continuous random variable probability distribution with the form:

$$y = \frac{1}{\mu - x_{min}} e^{\left[-\frac{x - x_{min}}{\mu - x_{min}}\right]}$$

The Exponential Distribution

- When x_{min} = 0, the equation reduces to:

$$y = \frac{1}{\mu} e^{\left[-\frac{x}{\mu}\right]}$$

The Exponential Distribution

- The normal distribution contains an area of 50% above and 50% below μ.
- With the exponential distribution, 36.8% of the area under the curve is above the average (μ) and 63.2% is below.

Applications / Observations

- Predictions based on an exponentially distributed process often only require the μ (and sometimes x_{min}) of the process.

Applications / Observations

- For prediction purposes, finding the area under the curve beyond the time period of concern is generally the point of interest.

- These prediction often relate to reliability issues or time between failure analyses.

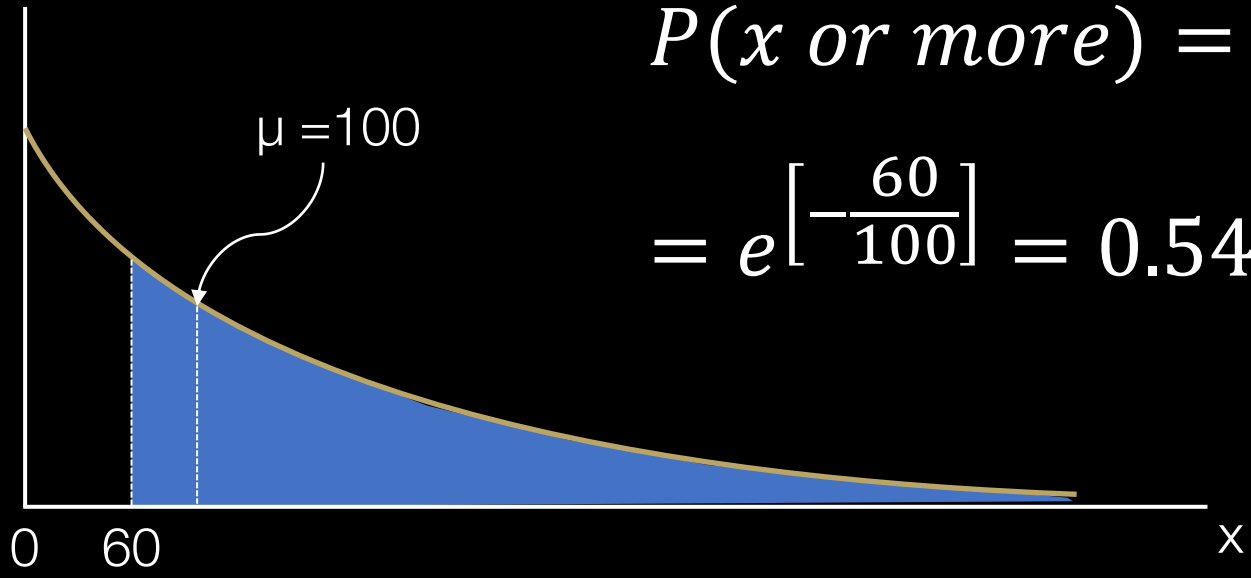
Example 1

- An in-plant study has shown that an engine control module laboratory tester is capable of operating on an average of 100 hours between breakdowns (MTBF).

Example 1

- What is the probability that the tester will run for at least 60 successive hours without a breakdown (assuming that the time to failure pattern is distributed exponentially)?

Example 1



$$P(x \text{ or more}) = e^{\left[-\frac{x}{\mu}\right]}$$

$$= e^{\left[-\frac{60}{100}\right]} = 0.5488$$

Exponential Distribution in RStudio

- pexp(q, rate, lower.tail)

Example 2

- The distribution of time for a particular grinding machine is characterized by the exponential distribution.
- The mean time between breakdowns has been established at 50 minutes.

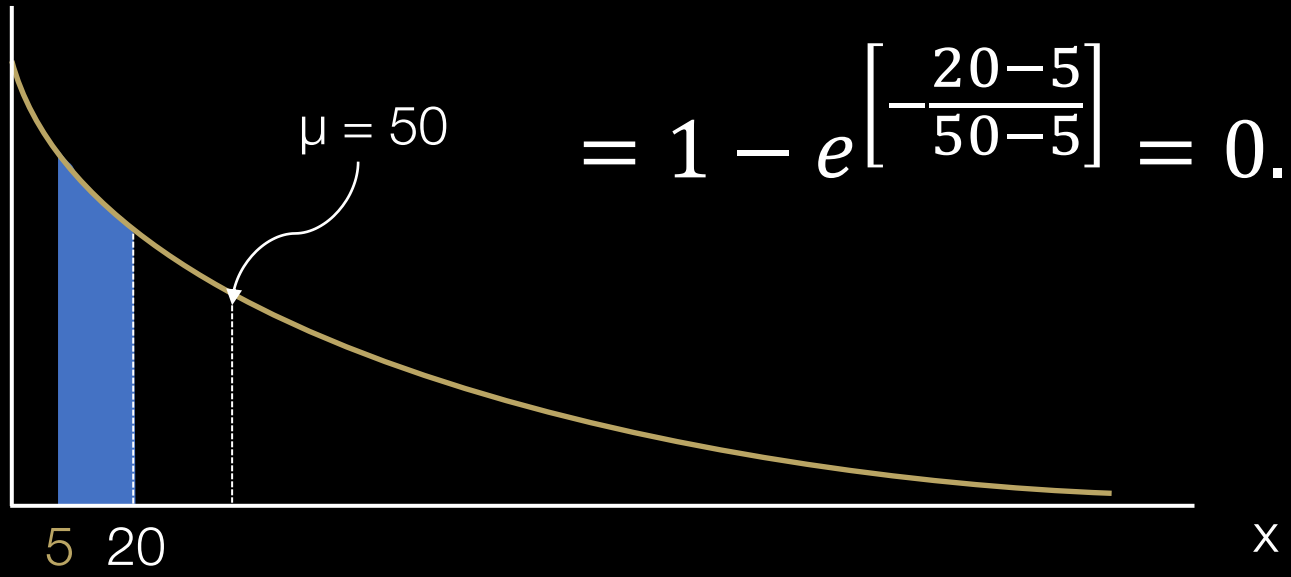
Example 2

- The origin parameter (x_{min}) is estimated to be 5 minutes.
- What is the probability of this machine running 20 minutes or less before a breakdown?

Example 2

$$P(x\ or\ less) = 1 - e^{\left[-\frac{x - x_{min}}{\mu - x_{min}}\right]}$$

$$= 1 - e^{\left[-\frac{20-5}{50-5}\right]} = 0.2835$$



Exponential Distribution in RStudio

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Testing for Exponentiality

- When $n \leq 100$, use the Shapiro-Wilk test
- When $n > 100$, use the Epps and Pulley test

Testing for Exponentiality

In R / Rstudio

> shapiro.wilk.exponentiality.test( )

> shapetest.exp.epps.pulley.1986( )

Sources

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982