The Normal Distribution

Data Science for Quality Management: Probability and Probability Distributions

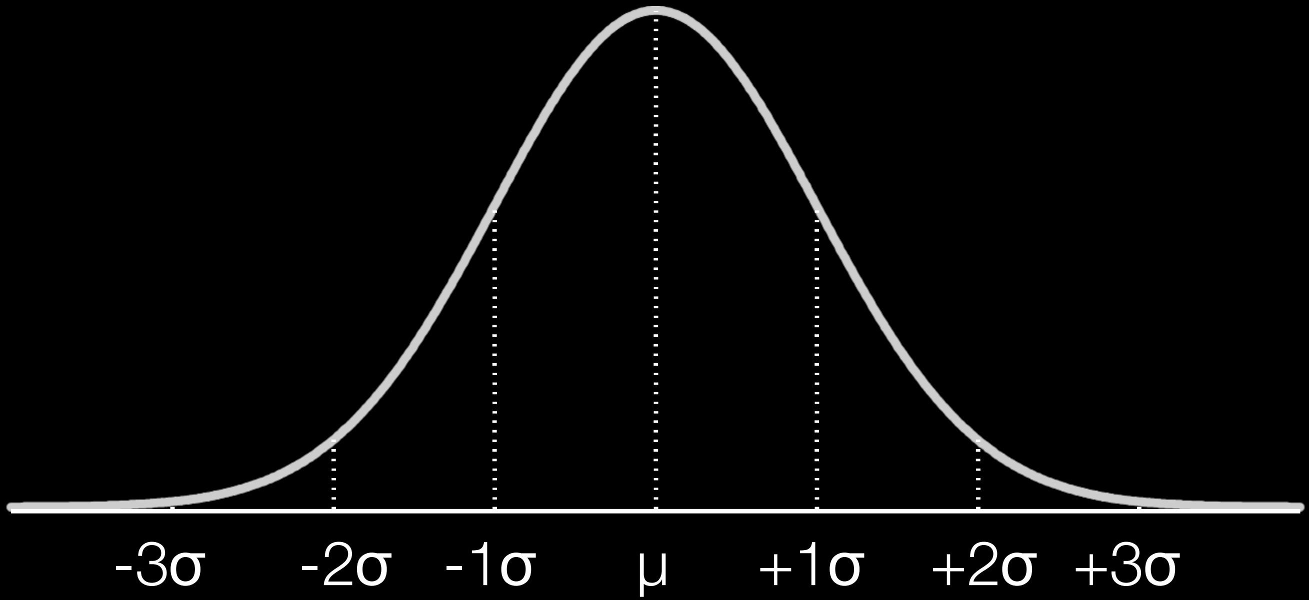
with **Wendy Martin**

Learning objectives:

Describe the Normal probability distribution

Calculate probabilities using the standard normal distribution

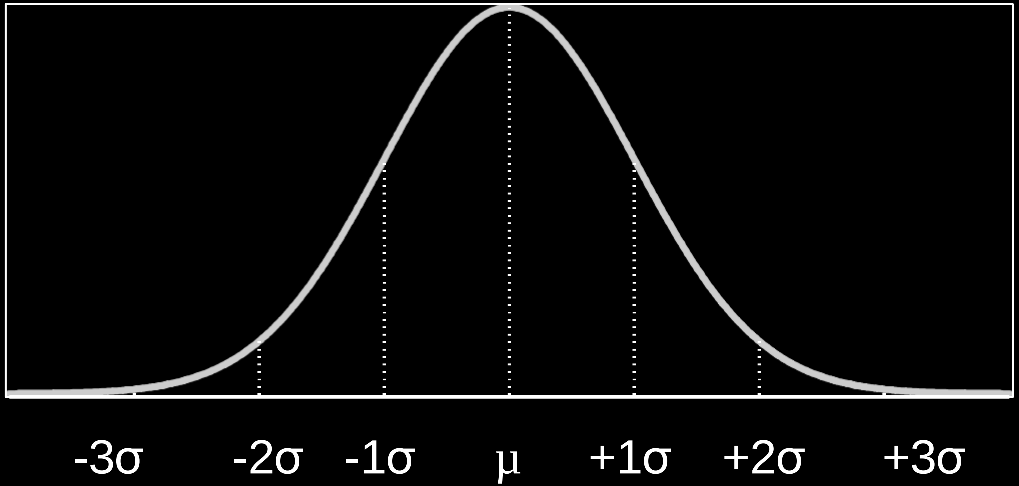
The Normal Distribution



The Normal Distribution

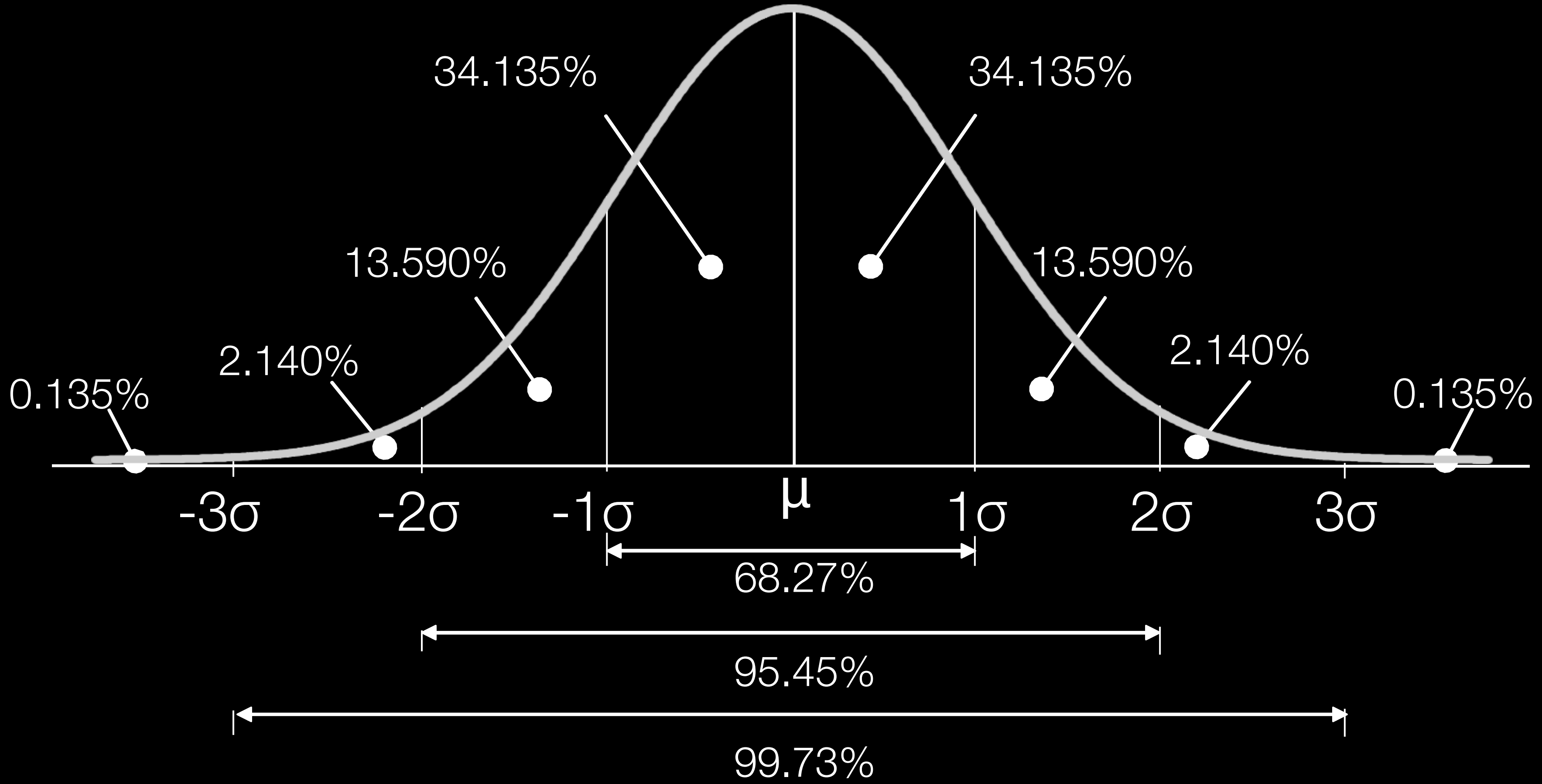
- A theoretical probability distribution for a continuous random variable
- Sometimes (inappropriately) referred to as the bell-shaped curve or distribution
- One of the most important distributions because of its wide range of practical applications

The Normal Distribution

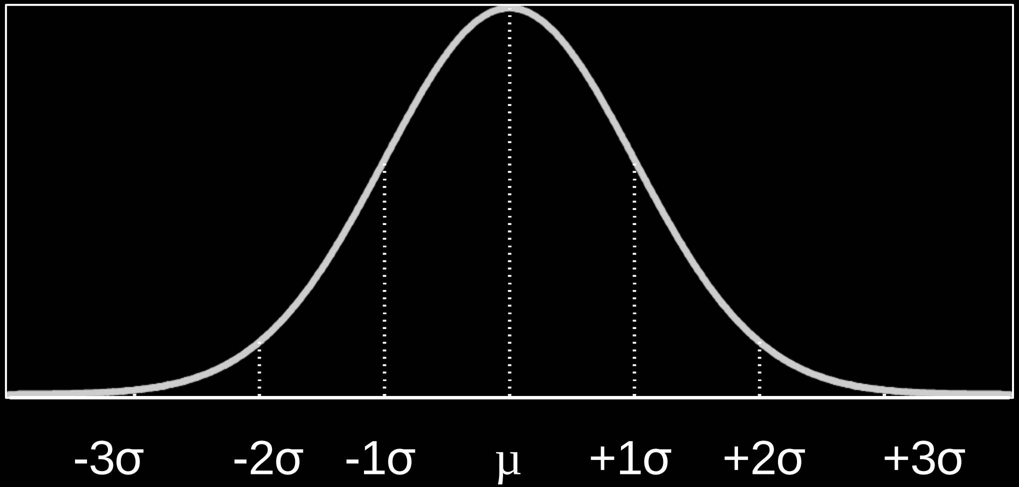


1. Mean = Median = Mode
2. Symmetrical around μ
3. Tails extend to ∞ but never touch the horizontal axis
4. $\gamma_3 = 0.00$
5. $\gamma_4 = 0.00$
6. Areas under the curve are predictable based upon standard deviation values.

Areas Under the Normal Curve



The Normal Distribution



$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{(X-\mu)^2}{2\sigma^2}\right]$$

Area Calculations

- The area corresponding to any score value may be found using a z-score, where

$$z = \frac{X - \mu}{\sigma}$$

- Z is the number of standard deviation units from X to μ

Example

- To date, tooling used on a particular drilling process has lasted an average of 180 hours before requiring replacement, with a standard deviation of 5 hours.

Example

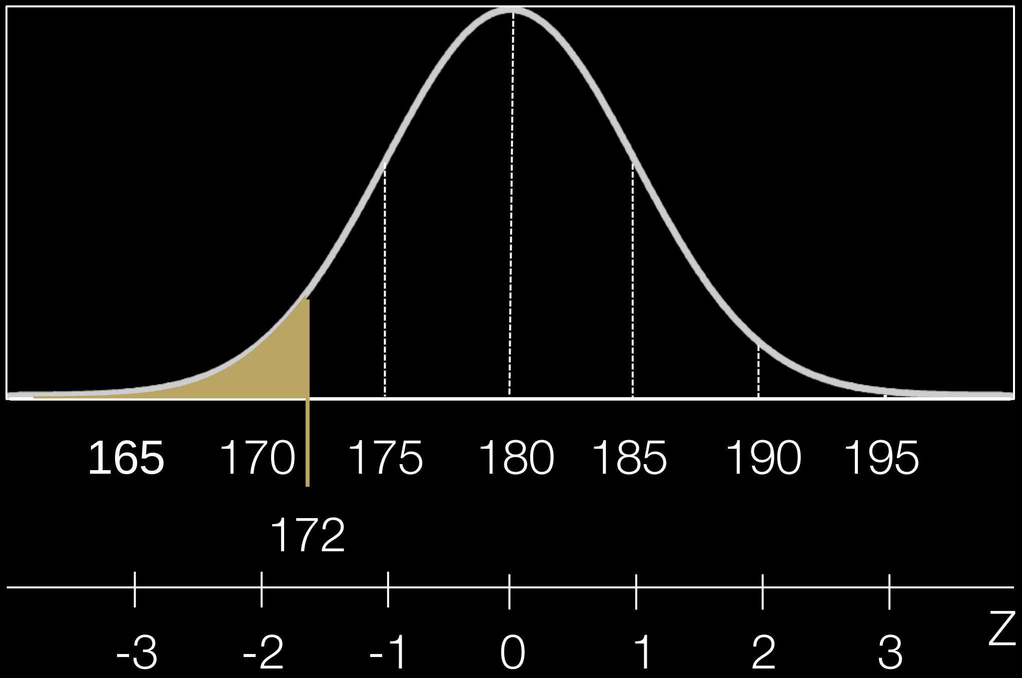
- What is the probability that a tool selected at random from the tool crib will last less than 172 hours before replacement is required?

Example

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{172 - 180}{5}$$

$z =$ -1.60



Normal Distribution in RStudio

In R / Rstudio

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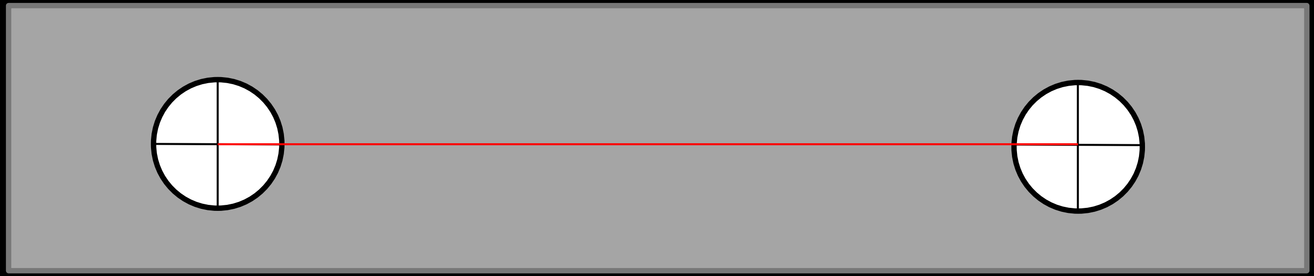
> pnorm(q, mean, sd, lower.tail)
```

Example 2

- A stamping operation has been running consistently, punching two holes in sheet metal.

Example 2

- The center-to-center distance between the two holes has been an average (μ) of 5.20mm, with a standard deviation (σ) of 0.05mm.



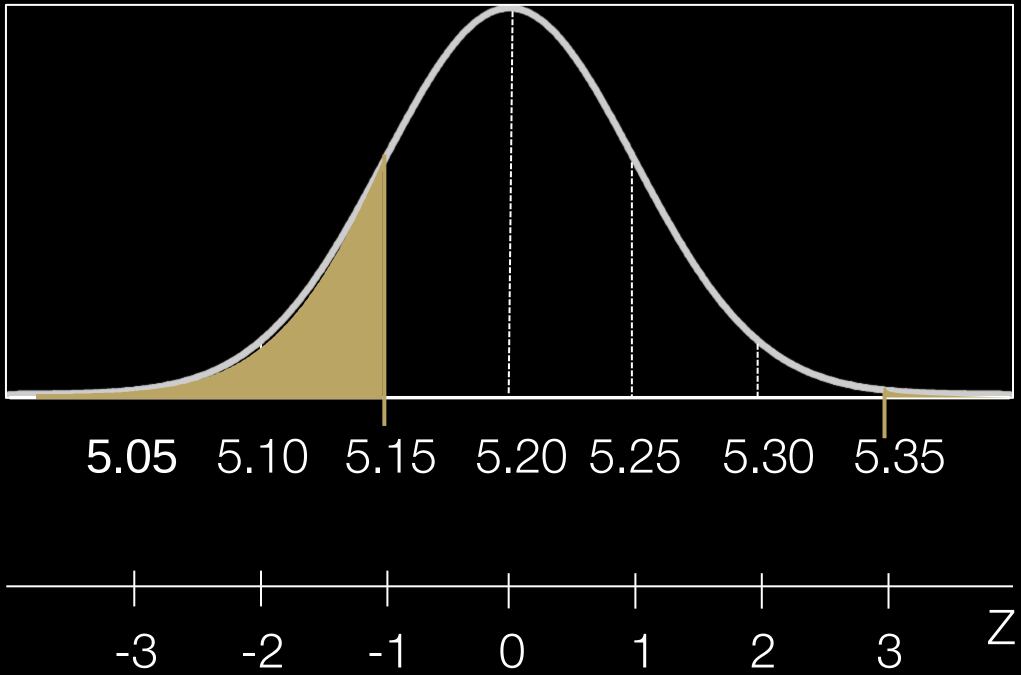
Example 2

- The process produces center-to-center distances that can be modeled with a normal distribution.

Example 2

- The specifications for these parts require a maximum, or upper (USL), limit of 5.35mm and a minimum, or lower (LSL), limit of 5.15mm.
- What percentage of the manufactured parts are likely to fall outside of the specifications?

Example



$$z = \frac{X - \mu}{\sigma}$$

Normal Distribution in RStudio

In R / Rstudio

```
> pnorm(q, mean, sd, lower.tail)
```

Testing for Normality

- When $n < 25$, use the Anderson-Darling test for normality (double check with Shapiro-Wilk test).
- When $n \geq 25$, use the skewness and kurtosis tests (D'Agostino).

Testing for Normality in RStudio

In R / Rstudio

> anderson.darling.normality.test( )
shapiro.wilk.normality.test( ) or
summary.continuous( )

> dagostino.normality.omnibus.test( ) or
summary.continuous( )

Sources

The material used in the PowerPoint presentations associated with this course was drawn from a number of sources. Specifically, much of the content included was adopted or adapted from the following previously-published material:

- Luftig, J. An Introduction to Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1982
- Luftig, J. Advanced Statistical Process Control & Capability. Luftig & Associates, Inc. Farmington Hills, MI, 1984.
- Luftig, J. A Quality Improvement Strategy for Critical Product and Process Characteristics. Luftig & Associates, Inc. Farmington Hills, MI, 1991
- Luftig, J. Guidelines for Reporting the Capability of Critical Product Characteristics. Anheuser-Busch Companies, St. Louis, MO. 1994
- Spooner-Jordan, V. Understanding Variation. Luftig & Warren International, Southfield, MI 1996
- Luftig, J. and Petrovich, M. Quality with Confidence in Manufacturing. SPSS, Inc. Chicago, IL 1997
- Littlejohn, R., Ouellette, S., & Petrovich, M. Black Belt Business Improvement Specialist Training, Luftig & Warren International, 2000
- Ouellette, S. Six Sigma Champion Training, ROI Alliance, LLC & Luftig & Warren, International, Southfield, MI 2005