

*A Real Engineering Problem**



**Adapted from Newnan et al.*

A Real Engineering Problem

You're an architectural engineer designing a low energy use home for a client. The home will require significant amounts of wall and attic insulation.

The specifications require a maximum heat transfer through the walls of 30,000 Joules/hr/m² when there is a $\Delta T = 30^\circ\text{C}$ between the inside and outside of the wall.

You're exploring 2 insulation materials, each having the following properties and costs:

Insulation Material	Cost, \$/m ³	Thermal Conductivity, J/m/°C/hr
Rock Wool Batting	\$12.50	140
Foamed Insulation	\$14.00	110

Which insulation material do you select?

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How would we solve this to ensure the lowest cost option?

Step 1: Define the equation that governs heat transfer across a thickness of insulated wall.

Step 2: Given the parameters we have, solve for the insulation thickness required for each material.

Step 3: With the material thickness known, determine the cost per unit area of wall.



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Step 1: Define the heat transfer equation:

$$Q = \frac{K \Delta T}{L}$$

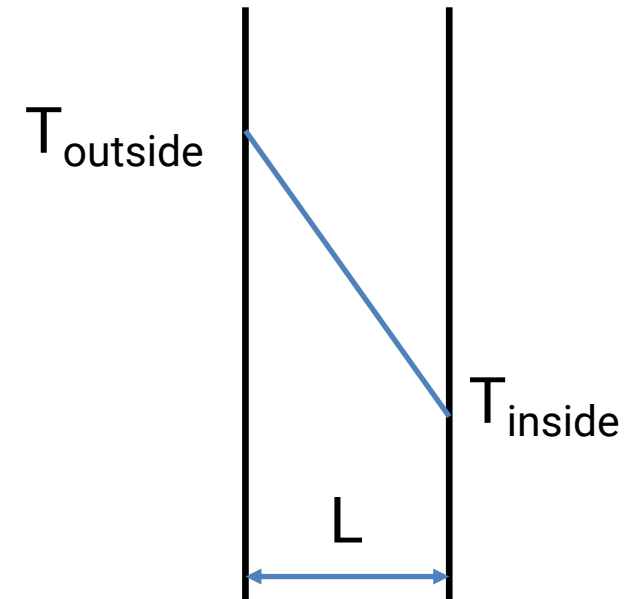
Where:

Q = the amount of heat transfer (J/hr/m²)

K = material thermal conductivity (J/m/°C/hr)

ΔT = the difference in temperature between the inner and outer wall surfaces (°C)

L = thickness between the inner and outer wall surfaces (m)



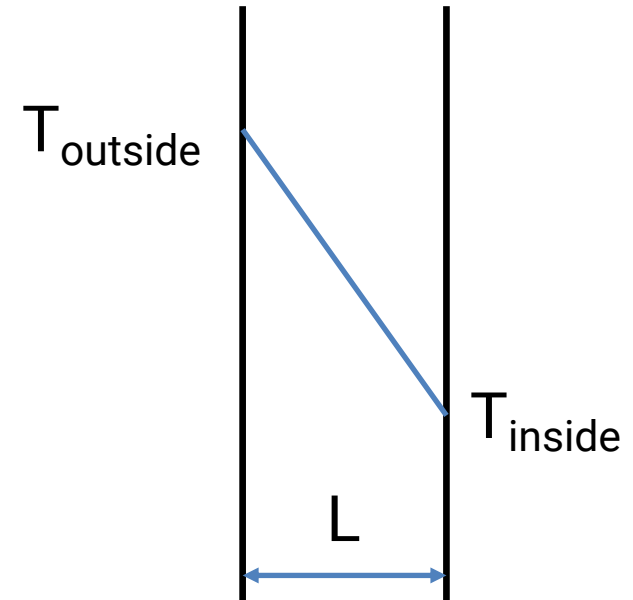
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Step 2: Rearrange our heat transfer equation to identify the thickness of insulation that meets the design criteria:

$$Q = \frac{K \Delta T}{L} \quad \Rightarrow \quad L = \frac{K \Delta T}{Q}$$

$$L_{\text{Rock Wool}} = \frac{\left(140 \frac{\text{J}}{\text{m}^\circ\text{C hr}}\right) (30^\circ\text{C})}{30,000 \frac{\text{J}}{\text{hr m}^2}} = 0.14\text{m}$$

$$L_{\text{Foam}} = \frac{\left(110 \frac{\text{J}}{\text{m}^\circ\text{C hr}}\right) (30^\circ\text{C})}{30,000 \frac{\text{J}}{\text{hr m}^2}} = 0.11\text{m}$$



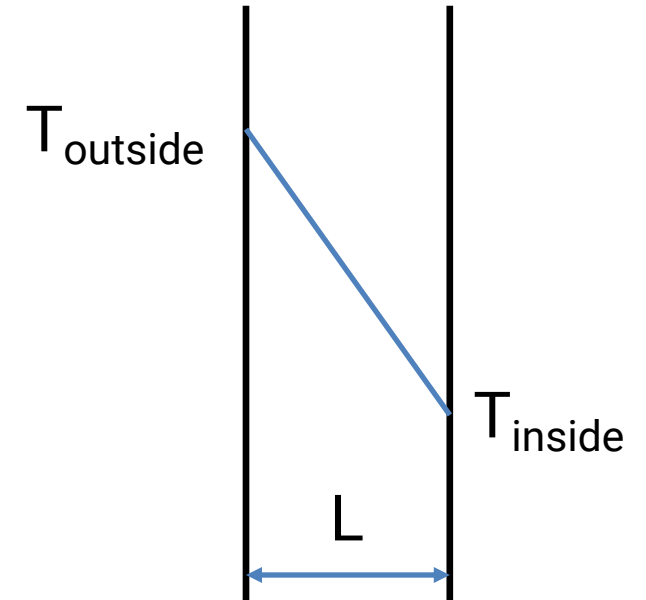
Design Parameters:
when $\Delta T = 30^\circ\text{C}$
 $Q_{\text{max}} = 30,000 \text{ J/hr/m}^2$

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Step 3: Calculate the cost per unit area for each insulation material:

$$\text{Cost}_{\text{Rock Wool}} = 0.14\text{m} \times \$12.50/\text{m}^3 = \$1.75/\text{m}^2$$

$$\text{Cost}_{\text{Foamed}} = 0.11\text{m} \times \$14.00/\text{m}^3 = \$1.54/\text{m}^2$$



Step 4: Evaluate the Results and Make Your Decision

The higher insulation capability of the foamed insulation offsets its higher price - making it the most economical option.

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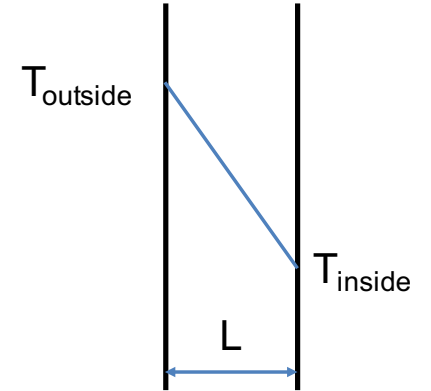
Your decision should include what is likely to happen once in the field!

Have we considered all issues we might encounter?

The thickness that meets the heat transfer-based design criteria:

$$L_{\text{Rock Wool}} = 0.14\text{m} (\sim 5.5'')$$

$$L_{\text{Foamed}} = 0.11\text{m} (\sim 4.3'')$$



What if the walls are framed with 2x6's, which are actually 5.5" wide?
And your foamed insulation contractor simply fills up the space?

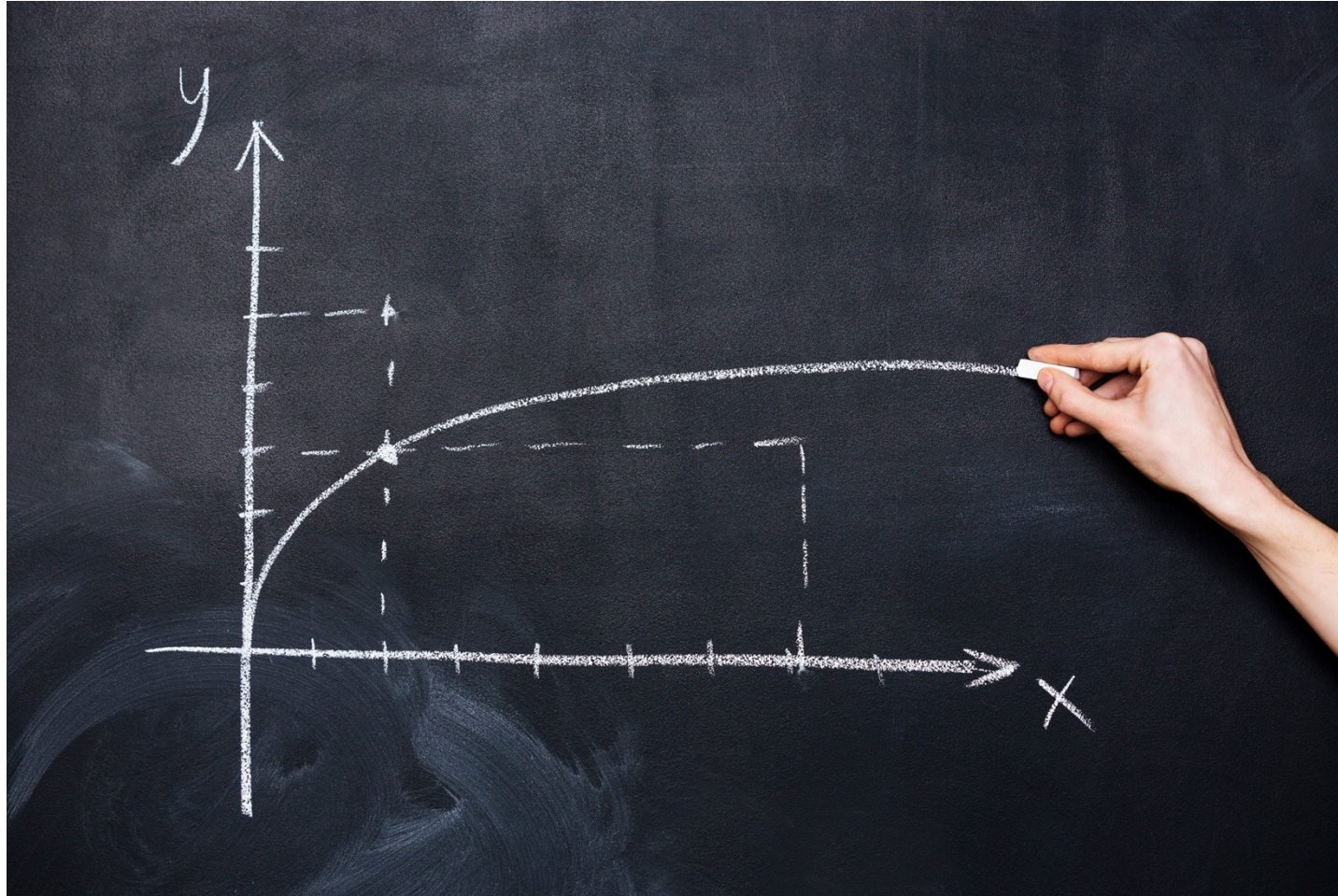
$$\text{Cost}_{\text{Rock Wool}} = 0.14\text{m} \times \$12.50/\text{m}^3 = \$1.75/\text{m}^2$$

$$\text{Cost}_{\text{Foamed}} = 0.14\text{m} \times \$14.00/\text{m}^3 = \$1.96/\text{m}^2$$



Next Time...

Continuous Cost Models



Credits & References

Slide 1: Passive solar house concept by mipan, Adobe Stock (305245605.jpeg). This example has been adapted from "Engineering Economic Analysis, 14th edition", by D.G. Newnam, T.G. Eschenbach, J.P Lavelle and N.A. Lewis. Oxford University Press (2020).

Slide 3: Passive solar house concept by mipan, Adobe Stock (305245605.jpeg).

Slide 7: New construction Rough Framing with bay Window by Michael, Adobe Stock (291705631.jpeg).

Slide 8: Hand drawing graph of mathematical function parabola on blackboard by Drobot Dean, Adobe Stock (106199736.jpeg).