Continuous Random Variables

Probability Theory: Foundation for Data Science

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Random Variables

At the end of this module, students should be able to

- **Define a continuous random variable and give examples of a probability density function and a cumulative distribution function.**
- Identify and discuss the properties of a **uniform**, exponential, and normal random variable.
- Calculate the expectation and variance of a continuous rv.

Continuous Random Variables

Definition: A random variable is **continuous** if possible values comprise either a single interval on the number line or a union of disjoint intervals.

Examples:

- In the study of the ecology of a lake, a rv X could be the depth measurements at randomly chosen locations. $X \in [0, \text{maximum depth of lake}]$.
- In a study of a chemical reaction, Y could be the concentration level of a particular chemical in solution.
- In a study of customer service, W could be the time a customer waits for service.

Note: If X is continuous, $P(X = x) = 0$ for any x! Why?

Motivating example: Suppose a train is scheduled to arrive at 1 pm. Let X be the minutes past the hour that it arrives and $X \in \{0, 1, 2, 3, 4, 5\}$.

x	0	1	2	3	4	5
p(x)	.1	.15	.3	.25	.15	.05

Properties of the probability density function

For any continuous rv X with probability density function (pdf) f we have:

- The probability density function $f : (-\infty, \infty) \to [0, \infty)$, so $f(x) \geq 0$.
- $P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x)\, dx = 1$
- $P(a \leq X \leq b) = \int_a^b f(x)\, dx$

Note: $P(X = a) = \int_a^a f(x)\, dx = 0$ for all real numbers a.

Cumulative Distribution Function

Definition The cumulative distribution function (cdf) for a continuous rv X is given by $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$

Uniform Random Variable

Definition A random variable X has the **uniform distribution** on the interval $[a, b]$ if its density function is given by

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{else} \end{cases}$$

Notation: $X \sim U[a, b]$

Example: Random number generators select numbers uniformly from a specific interval, usually $[0, 1]$.

Example: Suppose the diameter of aerosol particles in a particular application is uniformly distributed between 2 and 6 nanometers. Find the probability that a randomly measured particle has diameter greater than 3 nanometers.

Example: You throw a dart at a dartboard. The radial distance of the dart from the x-axis can be modeled by a uniform random variable.

Expectation and variance for a continuous rv X:

Compute expectation and variance for $X \sim U[a, b]$