The Gaussian (normal) Random Variable Part 2

Probability Theory: Foundation for Data Science

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Random Variables

At the end of this module, students should be able to

- Define a continuous random variable and give examples of a probability density function and a cumulative distribution function.
- Identify and discuss the properties of a uniform, exponential, and **normal random variable**
- Calculate the expectation and variance of a continuous rv.

If $X \sim N(\mu, \sigma^2)$ then

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \text{ for } -\infty < x < \infty$$

If $Z \sim N(0, 1)$ then

$$f_Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \text{ for } -\infty < x < \infty$$

Proposition: If $X \sim N(\mu, \sigma^2)$, then $\dfrac{X - \mu}{\sigma} \sim N(0, 1)$

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Example: If $X \sim N(1, 4)$, (a) find $P(0 \leq X \leq 3.2)$ and find a so that $P(X \leq a) = 0.7$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852

Example: The time that it takes a driver to react to the brake lights on a decelerating vehicle is critical in helping to avoid rear-end collisions. Research suggests that reaction time for an in-traffic response to a brake signal from standard brake lights can be modeled with a normal distribution having mean 1.25 seconds and standard deviation 0.46 seconds. What is the probability that the reaction time is between 1 and 1.75 seconds? What assumptions are you making?

Normal approximation to the binomial distribution

Recall: $X \sim Bin(n, p)$ means that X counts the number of successes in n Bernoulli trials, each with probability of success p. $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k = 0, 1, \ldots, n$, $E(X) = np$ and $V(X) = np(1-p)$.

For large n, X can be approximated by a normal rv with $\mu = np$ and $\sigma^2 = np(1-p)$.

Example: In a given day, there are approximately 1,000 visitors to a website. Of these, 25% register for a service. Estimate the probability that between 200 and 225 people will register for a service tomorrow.