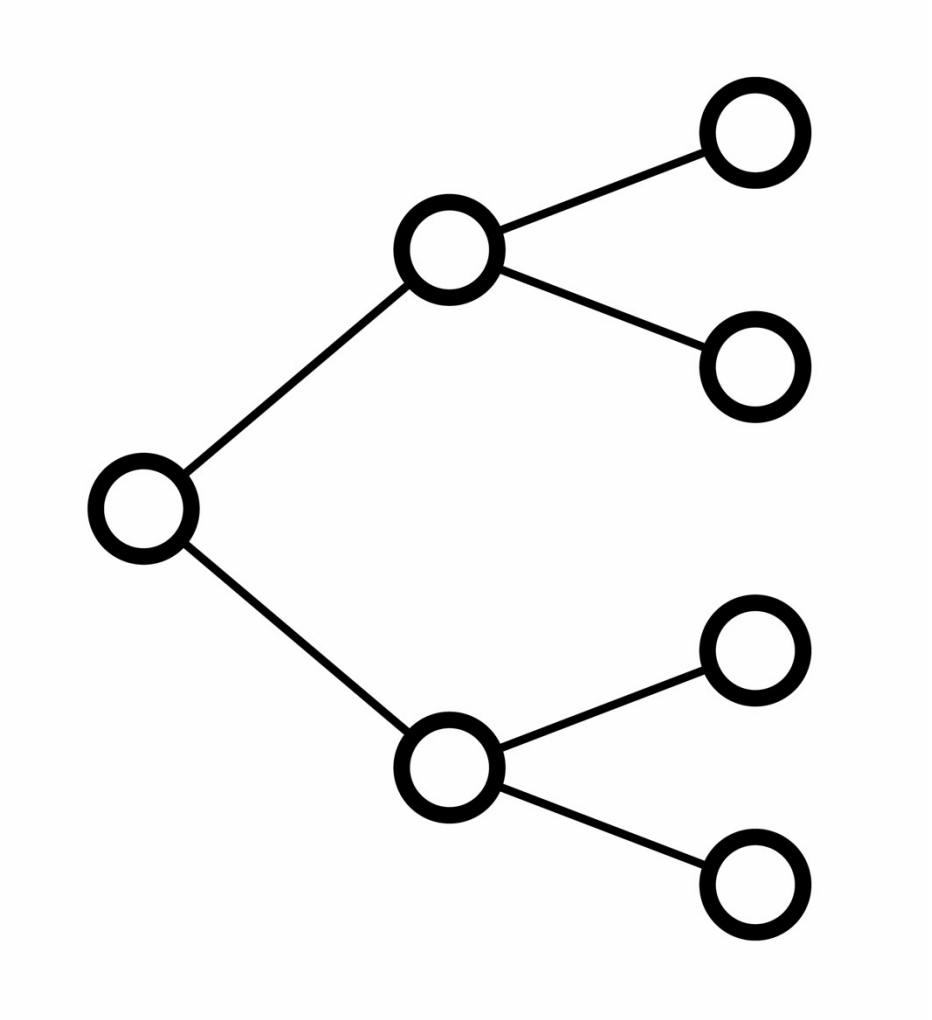
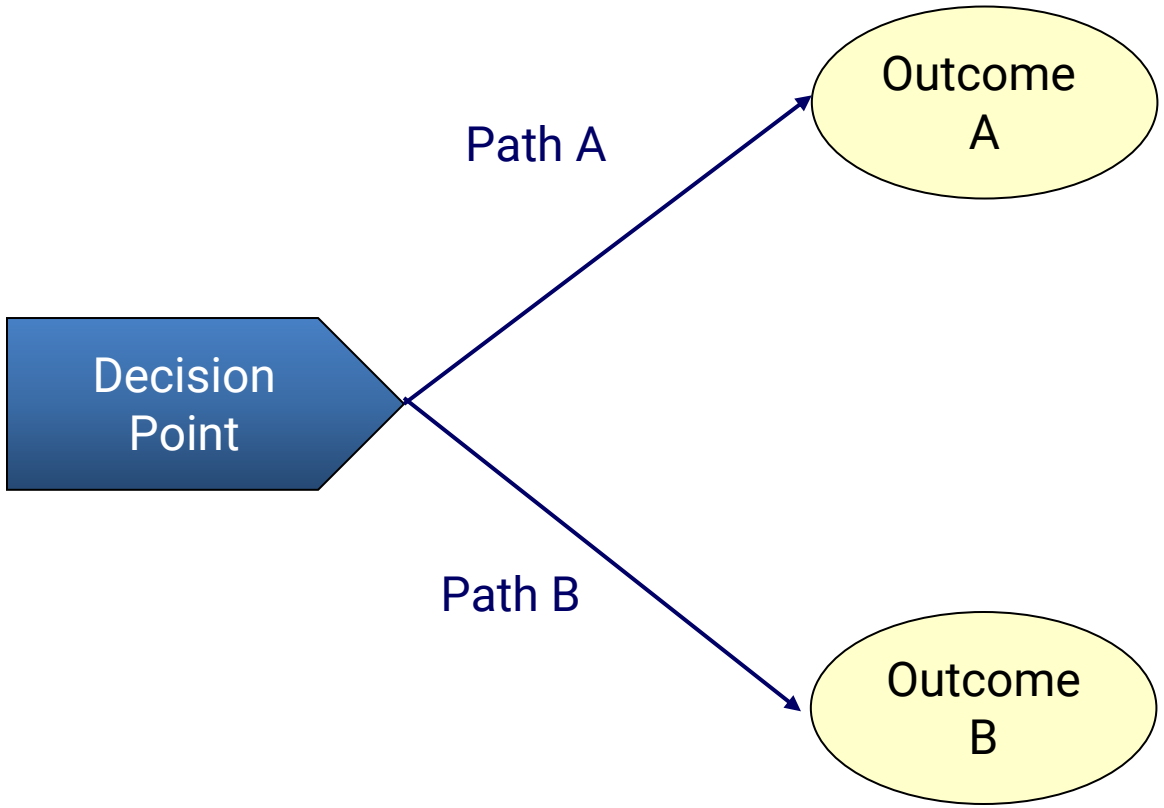
Decision Tree Analysis



Decision Tree Analysis

What are the financial ramifications of possible paths to take?



Can we include some probability that Outcome A or B will in fact occur?

The Average Value…

The statistical Average (or Mean) for a sample of size *N*, is:

$$\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$$

Ex: what is the average of the following values: 10, 20, 30, 40, 50?

$$\bar{X} = \frac{10 + 20 + 30 + 40 + 50}{5}$$

$$\bar{X} = \frac{150}{5} = 30$$

The Expected Value…

Expected value E(X) is a *weighted* average based on a value's likelihood of occurring.

The E(X) is just the sum of each possible outcome multiplied by its probability:

$$E(X) = \sum_{i=1}^{N} P(X_{i)}\, X_i$$

Ex: What is the Expected Value based on the following:

Value	Probability of Occurring
10	5%
20	20%
30	30%
40	40%
50	5%

E(X) = 0.05 (10) + 0.20 (20) + 0.30 (30) + 0.40 (40) + 0.05 (50)

E(X) = 0.5 + 4.0 + 9.0 + 16.0 + 2.5

E(X) = 32.0

The Expected Value…

	Optimistic	Most Likely	Pessimistic
Annual benefit	$10,000	$8,000	$5,000
Probability	10%	60%	30%

$$\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$$

Avg Benefit = ($10,000 + $8,000 + $5,000) / 3

Avg. Benefit = $7,667

$$E(X) = \sum_{i=1}^{N} P(X_i) X_i$$

E(Benefit) = (0.10)($10,000) + (0.60)($8,000) + (0.30)($5,000)

E(Benefit) = $7,300

A more statistically relevant estimate; it takes into account probabilities of all possible outcomes

Decision Tree Analysis…

Example: Your company is deciding whether to buy a new thin film coating system or continue to use the older one. A new system is anticipated to save money due to its greater efficiency. Estimates have been made for the annual savings, along with the probabilities that they will occur.

Your company has a discount rate of 18%. The economic analysis period is 6 years.

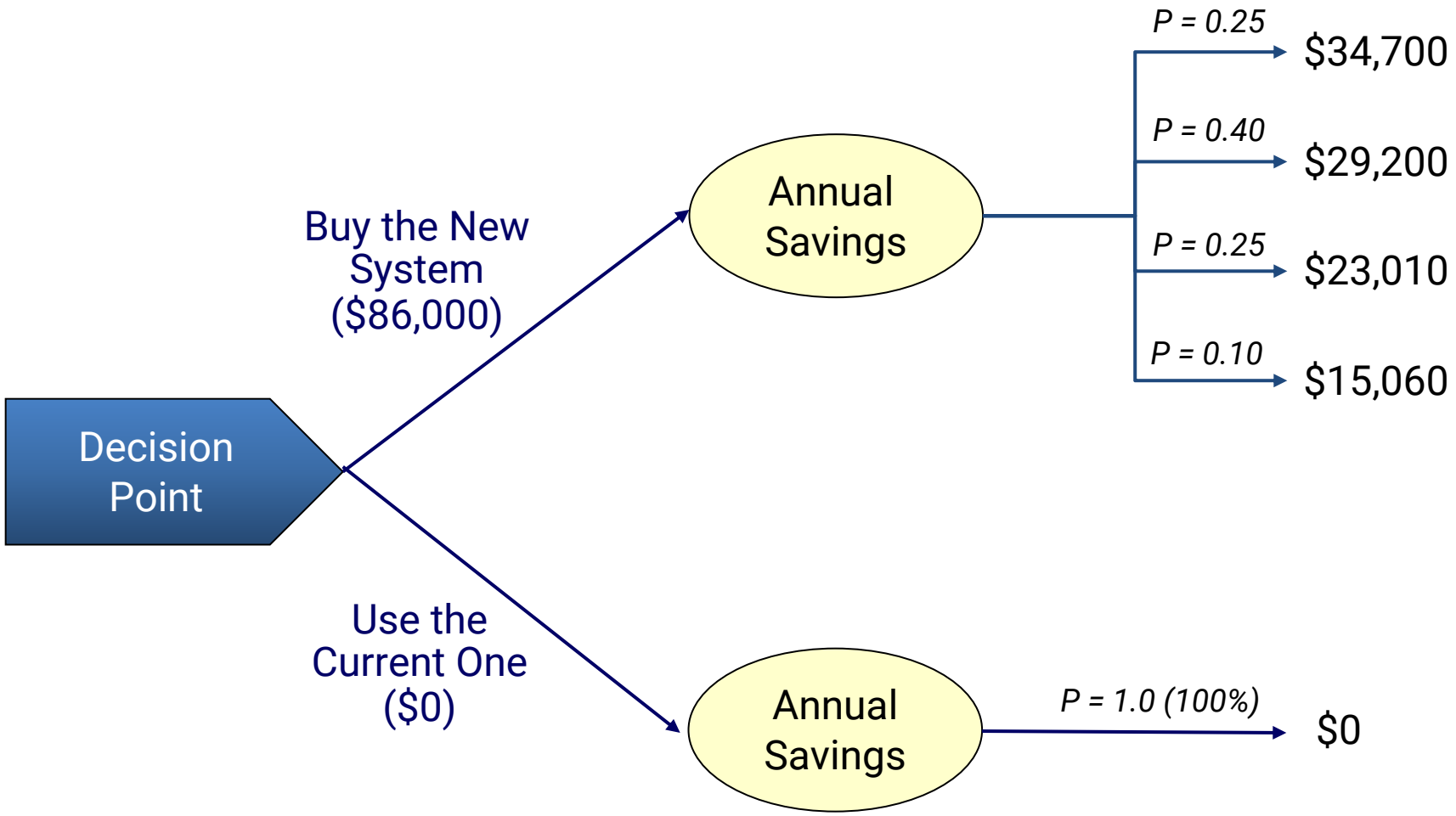
Cost of New System: $86,000

Probability of Annual Savings

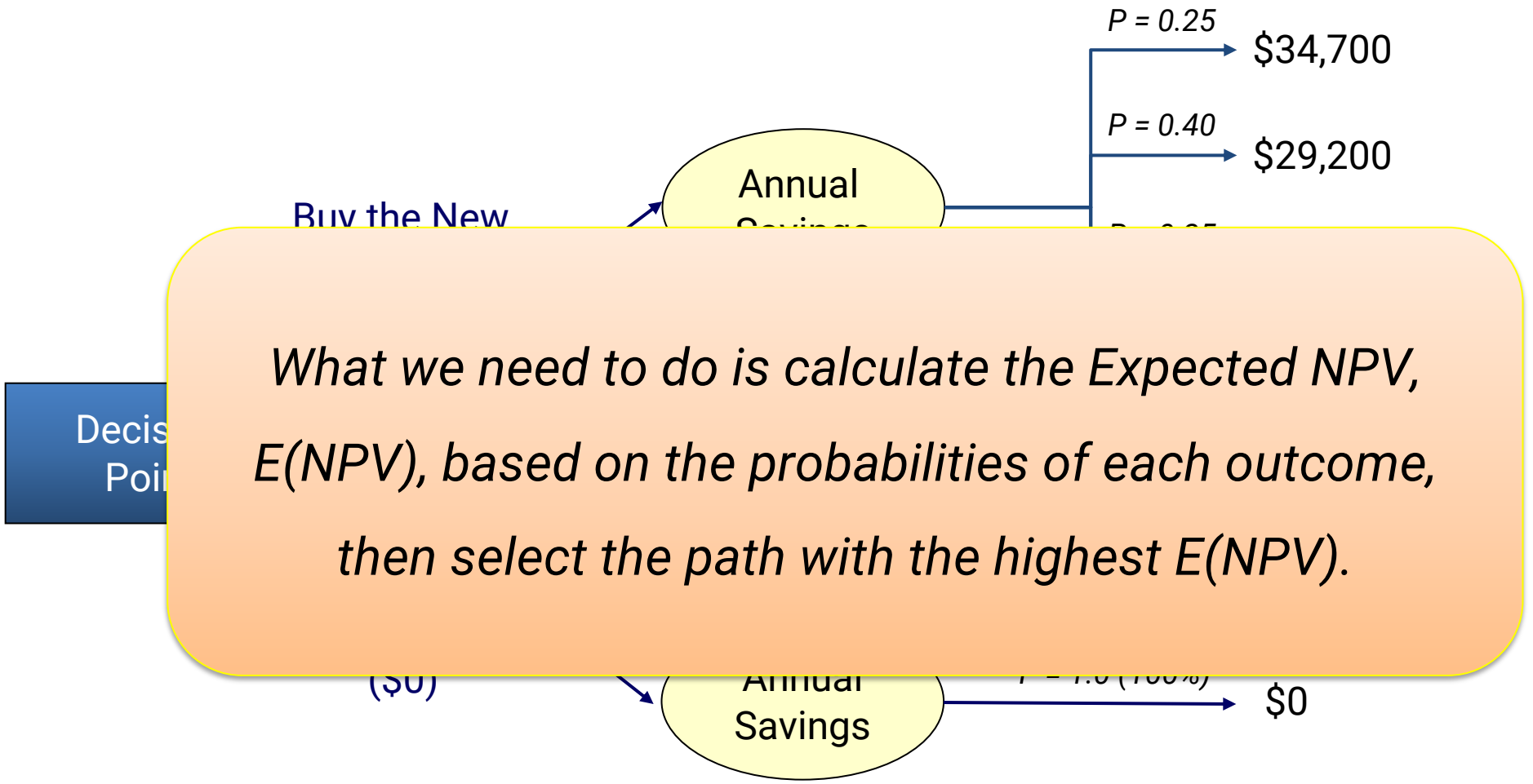
- $34,700 P = 0.25 (25%)
- $29,200 P = 0.40 (40%)
- $23,010 P = 0.25 (25%)
- $15,060 P = 0.10 (10%)

Should your company buy the new system or not?

Decision Tree Analysis…



Decision Tree Analysis…



What we need to do is calculate the Expected NPV, E(NPV), based on the probabilities of each outcome, then select the path with the highest E(NPV).

Decision Tree Analysis…

This path is easy to figure out…

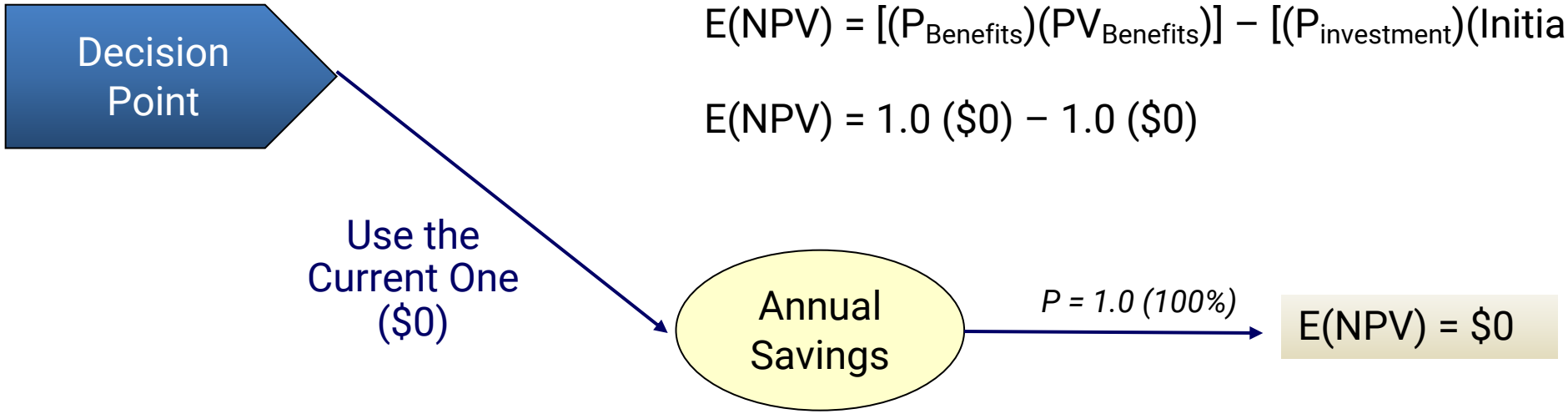
Initial Investment = $0 (with 100% Certainty)

$PV_{Benefits}$ = $0 (with 100% Certainty)

$E(NPV) = E(PV_{Benefits}) - E\ (Initial\ Investment)$

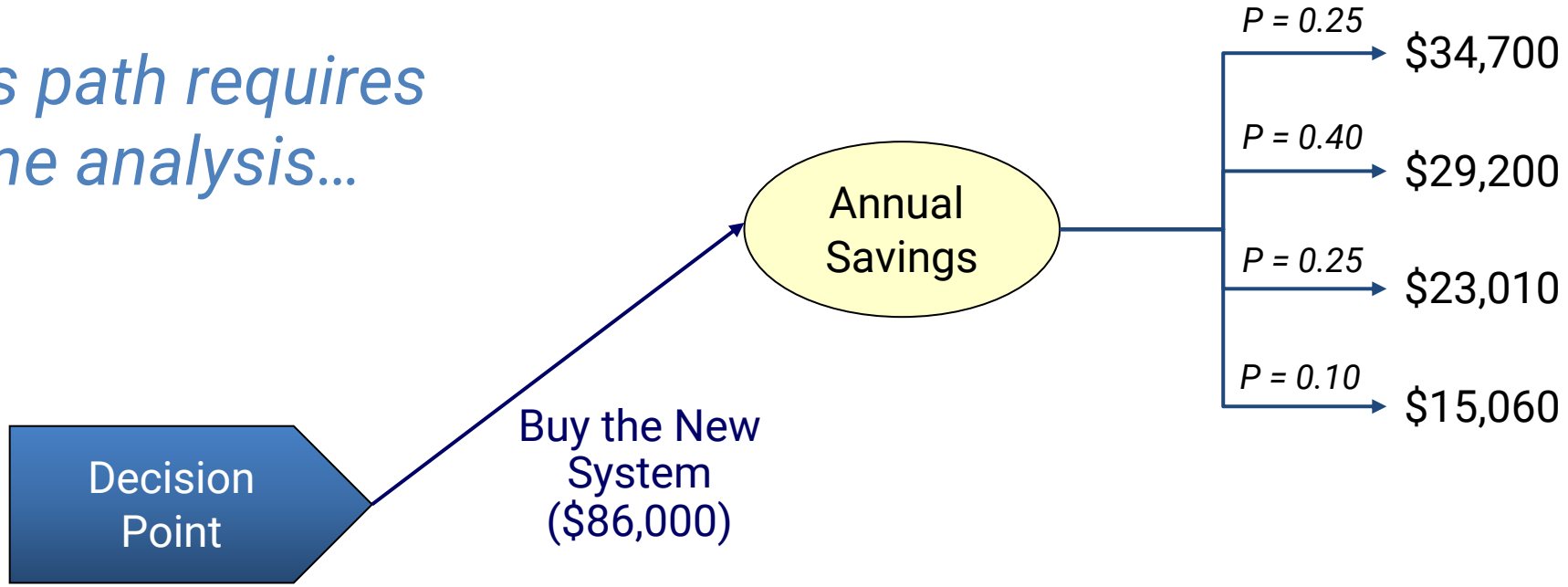
$E(NPV) = [(P_{Benefits})(PV_{Benefits})] - [(P_{investment})(Initial\ Investment)]$

$E(NPV) = 1.0\ (\$0) - 1.0\ (\$0)$



Decision Tree Analysis…

This path requires some analysis…



Initial Investment = $86,000 (with 100% Certainty, meaning E(Investment) = $86,000)

$PV_{Benefits}$ is probability dependent; we calculate the PV of each possible outcome… and then take their probabilities into account…

Decision Tree Analysis…

$$PV_{Benefits} = \text{Annual Savings}\left[\frac{(1+i)^n - 1}{i\,(1+i)^n}\right]$$

$$PV_{P=0.25} = \$34{,}700\left[\frac{(1+0.18)^6 - 1}{0.18\,(1+0.18)^6}\right]$$

$PV_{P=0.25}$ = \$34,700 (3.4976)

$PV_{P=0.25}$ = \$121,367

$$PV_{P=0.40} = \$29{,}200\left[\frac{(1+0.18)^6 - 1}{0.18\,(1+0.18)^6}\right]$$

$PV_{P=0.40}$ = \$29,200 (3.4976)

$PV_{P=0.40}$ = \$102,130

$$PV_{P=0.25} = \$23{,}010\left[\frac{(1+0.18)^6 - 1}{0.18\,(1+0.18)^6}\right]$$

$PV_{P=0.25}$ = \$23,010(3.4976)

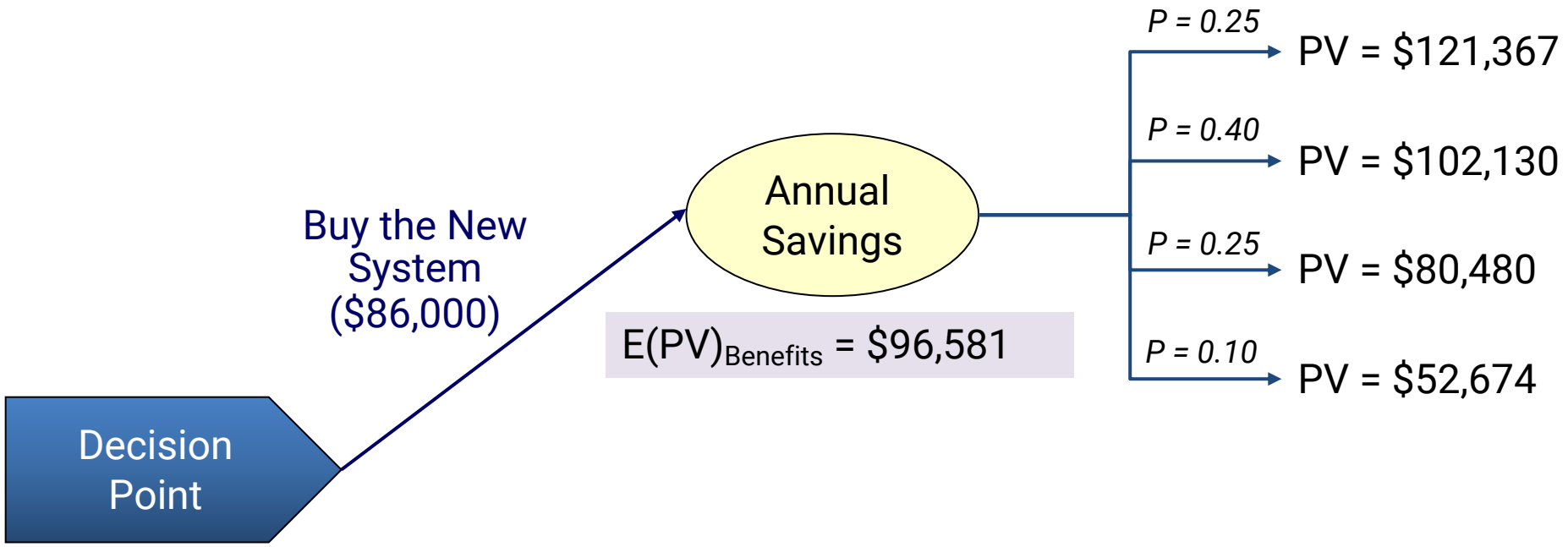
$PV_{P=0.25}$ = \$80,480

$$PV_{P=0.10} = \$15{,}060\left[\frac{(1+0.18)^6 - 1}{0.18\,(1+0.18)^6}\right]$$

$PV_{P=0.10}$ = \$15,060 (3.4976)

$PV_{P=0.10}$ = \$52,674

Decision Tree Analysis…

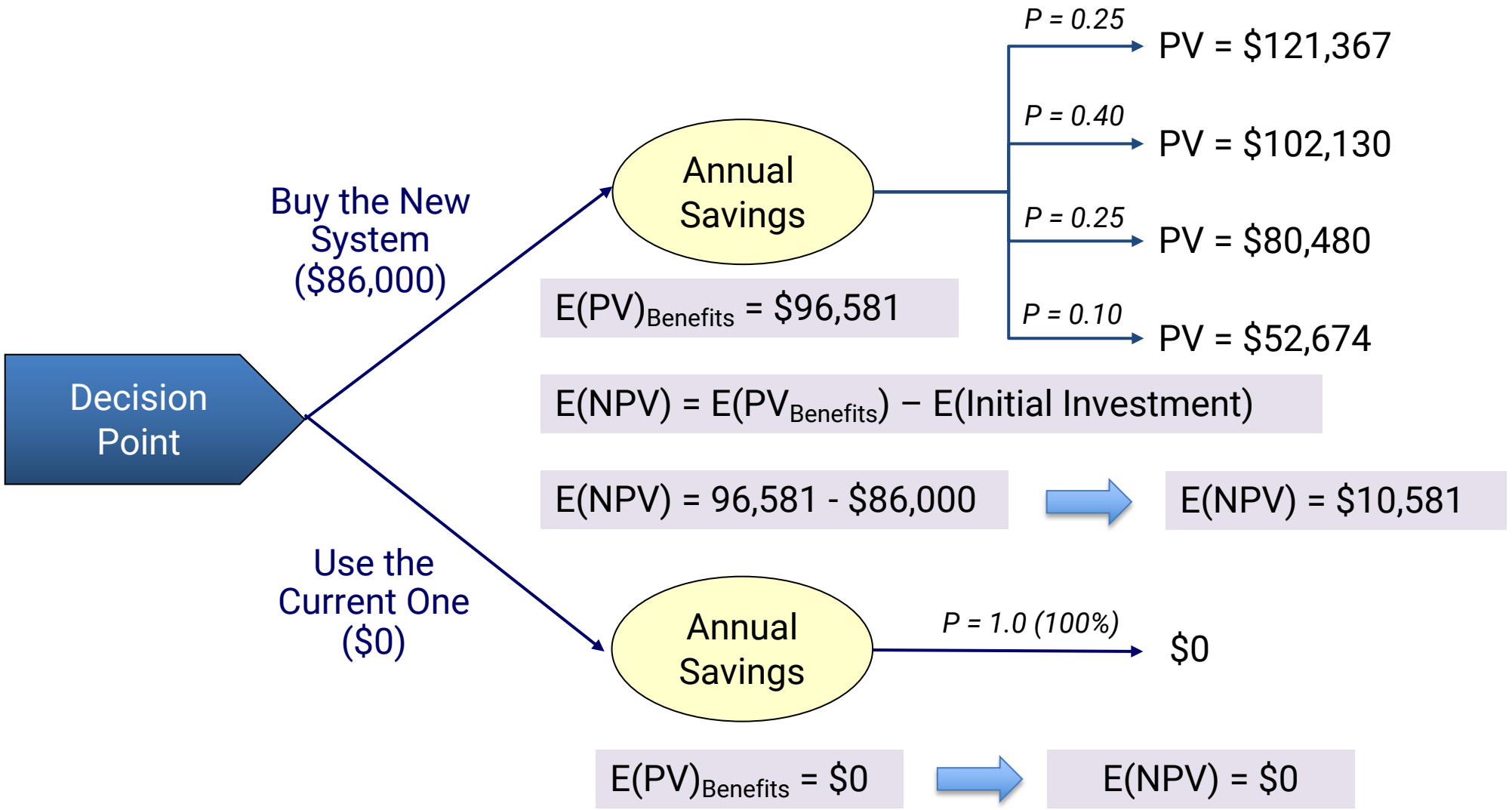


Now we determine the E(PV) of the benefits:

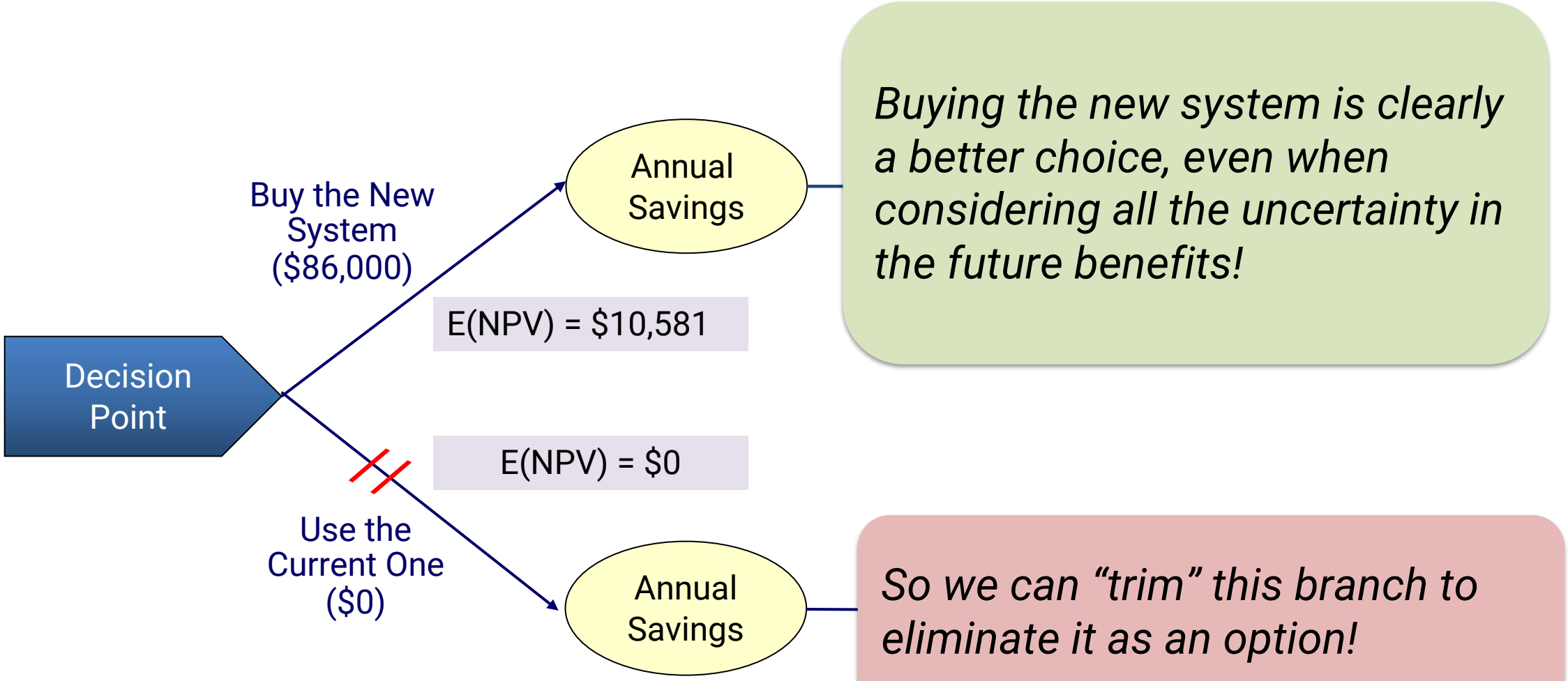
$$E(PV)_{Benefits} = 0.25\ (\$121{,}367) + 0.40\ (\$102{,}130) + 0.25\ (\$80{,}480) + 0.10\ (\$52{,}674)$$

$$E(PV)_{Benefits} = \$96{,}581$$

Decision Tree Analysis…



Decision Tree Analysis…



Buying the new system is clearly a better choice, even when considering all the uncertainty in the future benefits!

So we can "trim" this branch to eliminate it as an option!

Main Takeaways…

Decision Trees define all the various paths you can take, with each path representing a different scenario.

Analyzing some of the branch pathways enables you can ”trim” the branches of the tree, eliminating certain decision paths.

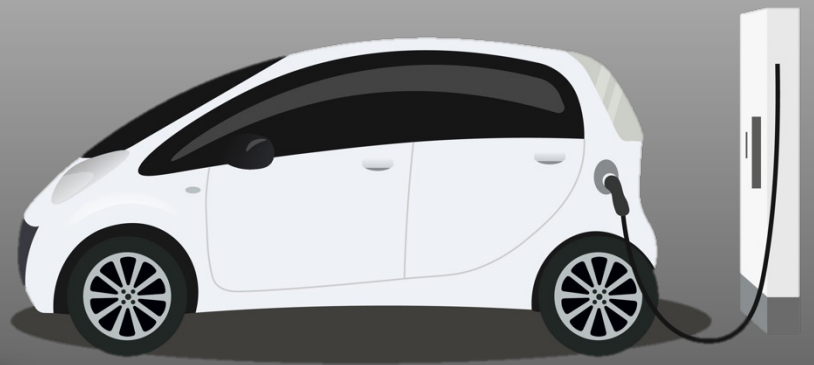
Using a few concepts from probability theory, one can calculate values such as the “Expected NPV” of various outcomes, thereby identifying the best option, even when there is uncertainty in your estimates.

Next Time…

Multi-Attribute Analysis

Practical and Reliable…

Sporty and Eco-Friendly…



Credits & References

Slide 1: Accounting report and financial statement on desk by Leonid, Adobe Stock.

Slide 16: Minivan, isolated by algre, Adobe Stock (48343629.jpeg). Flat vector illustration of a white electric car suv charging at the white charger station by Мария Запеченко, Adobe Stock (419653028.jpeg). Puzzled confused woman has hesitant expression by Wayhome Studio, Adobe Stock (254158203.jpeg).