m1-peer-reviewed

June 27, 2023

1 Module 1 - Peer reviewed

1.0.1 Outline:

In this homework assignment, there are four objectives.

1. To assess your knowledge of ANOVA/ANCOVA models
2. To apply your understanding of these models to a real-world datasets

General tips:

1. Read the questions carefully to understand what is being asked.
2. This work will be reviewed by another human, so make sure that you are clear and concise in what you are attempting to explain or answer.

[1]:
```r
# Load Required Packages
library(tidyverse)
library(ggplot2)
library(dplyr)
```

```
  Attaching packages                                tidyverse
1.3.0

  ggplot2 3.3.0       purrr   0.3.4
  tibble  3.0.1       dplyr   0.8.5
  tidyr   1.0.2       stringr 1.4.0
  readr   1.3.1       forcats 0.5.0

  Conflicts
tidyverse_conflicts()
  dplyr::filter() masks stats::filter()
  dplyr::lag()    masks stats::lag()
```

1.0.2 Problem #1: Simulate ANCOVA Interactions

In this problem, we will work up to analyzing the following model to show how interaction terms work in an ANCOVA model.

$$Y_i = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon_i$$

This question is designed to enrich understanding of interactions in ANCOVA models. There is no additional coding required for this question, however we recommend messing around with the coefficents and plot as you see fit. Ultimately, this problem is graded based on written responses to questions asked in part **(a)** and **(b)**.

To demonstrate how interaction terms work in an ANCOVA model, let's generate some data. First, we consider the model

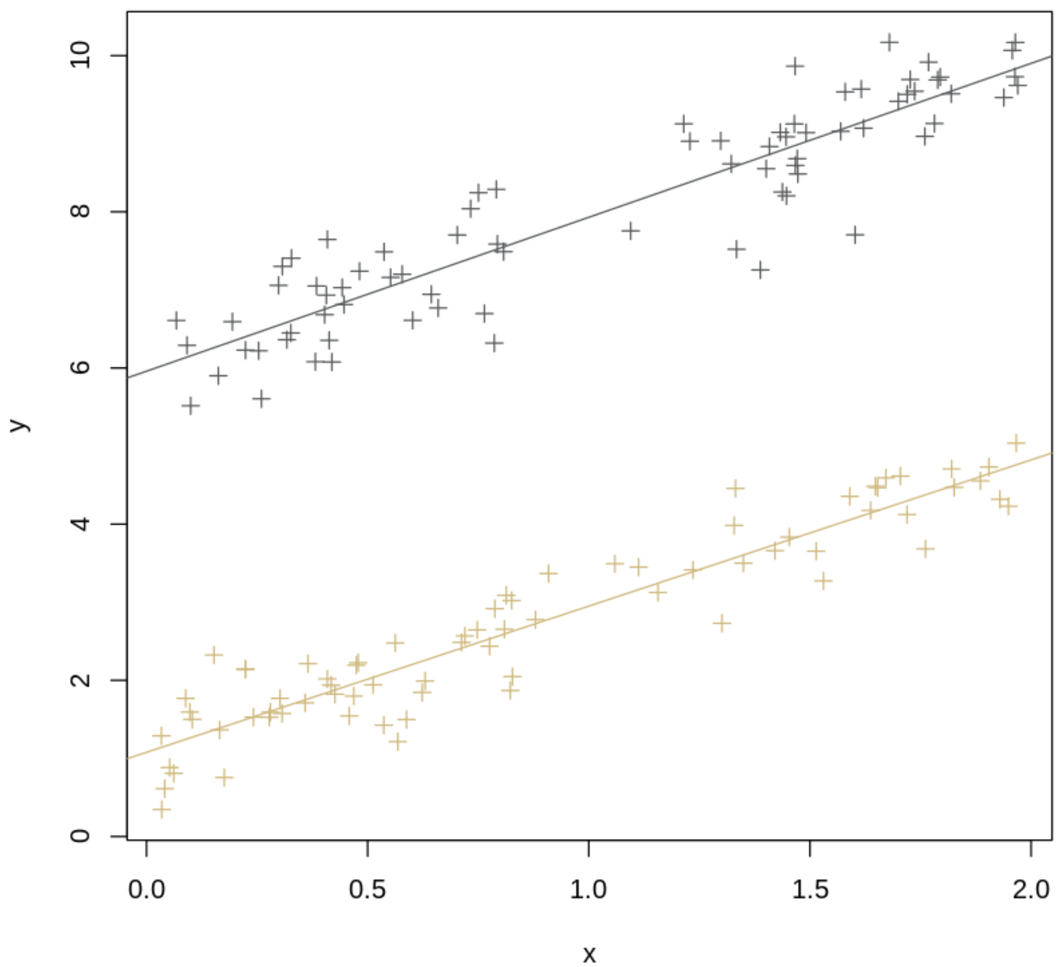
$$Y_i = \beta_0 + \beta_1 X + \beta_2 Z + \varepsilon_i$$

where X is a continuous covariate, Z is a dummy variable coding the levels of a two level factor, and $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$. We choose values for the parameters below (b0,...,b2).

[2]:
```r
rm(list = ls())
set.seed(99)

#simulate data
n = 150
# choose these betas
b0 = 1; b1 = 2; b2 = 5; eps = rnorm(n, 0, 0.5);
x = runif(n,0,2); z = runif(n,-2,2);
z = ifelse(z > 0,1,0);
# create the model:
y = b0 + b1*x + b2*z + eps
df = data.frame(x = x,z = as.factor(z),y = y)
head(df)

#plot separate regression lines
with(df, plot(x,y, pch = 3, col = c("#CFB87C","#565A5C")[z]))
abline(coef(lm(y[z == 0] ~ x[z == 0], data = df)), col = "#CFB87C")
abline(coef(lm(y[z == 1] ~ x[z == 1], data = df)), col = "#565A5C")
```

A data.frame: 6 × 3

	x	z	y
	<dbl>	<fct>	<dbl>
1	0.09159879	1	6.290179
2	1.96439135	1	10.168612
3	0.57805656	1	7.200027
4	0.03370108	0	1.289331
5	1.82614045	0	4.470862
6	0.71220319	0	2.485743



1. (a) What happens with the slope and intercept of each of these lines? In this case, we can think about having two separate regression lines–one for Y against X when the unit is in group $Z = 0$ and another for Y against X when the unit is in group $Z = 1$. What do we notice about the slope of each of these lines?

When - $Z = 0$ the regression looks like $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ - $Z = 1$ the regression looks like $Y_i = (\beta_0 + \beta_2) + \beta_1 X_i + \varepsilon_i$

so we end up having two regressions with the ***same slope*** and ***different intercepts***.

1. (b) Now, let's add the interaction term (let $\beta_3 = 3$). What happens to the slopes of each line now? The model now is of the form:

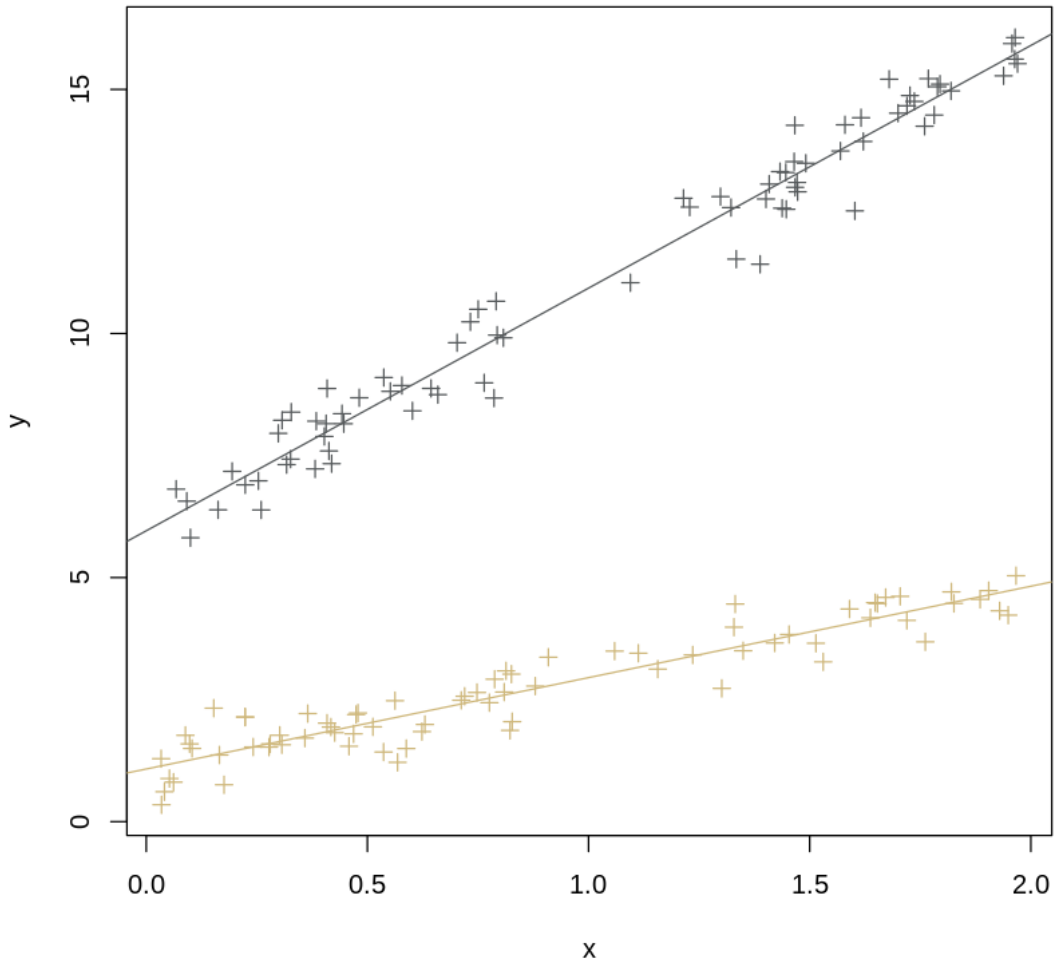
$$Y_i = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ + \varepsilon_i$$

where X is a continuous covariate, Z is a dummy variable coding the levels of a two level factor, and $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$. We choose values for the parameters below (b0,...,b3).

[3]:
```r
#simulate data
set.seed(99)
n = 150
# pick the betas
b0 = 1; b1 = 2; b2 = 5; b3 = 3; eps = rnorm(n, 0, 0.5);

#create the model
y = b0 + b1*x + b2*z + b3*(x*z) + eps
df = data.frame(x = x,z = as.factor(z),y = y)
head(df)

lmod = lm(y ~ x + z, data = df)
lmodz0 = lm(y[z == 0] ~ x[z == 0], data = df)
lmodz1 = lm(y[z == 1] ~ x[z == 1], data = df)
# summary(lmod)
# summary(lmodz0)
# summary(lmodz1)

# lmodInt = lm(y ~ x + z + x*z, data = df)
# summary(lmodInt)

#plot separate regression lines
with(df, plot(x,y, pch = 3, col = c("#CFB87C","#565A5C")[z]))
abline(coef(lm(y[z == 0] ~ x[z == 0], data = df)), col = "#CFB87C")
abline(coef(lm(y[z == 1] ~ x[z == 1], data = df)), col = "#565A5C")
```

A data.frame: 6 × 3

	x <dbl>	z <fct>	y <dbl>
1	0.09159879	1	6.564975
2	1.96439135	1	16.061786
3	0.57805656	1	8.934197
4	0.03370108	0	1.289331
5	1.82614045	0	4.470862
6	0.71220319	0	2.485743



In this case, we can think about having two separate regression lines–one for Y against X when the unit is in group $Z = 0$ and another for Y against X when the unit is in group $Z = 1$. **What do you notice about the slope of each of these lines?**

This time when - $Z = 0$ the regression looks like $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ - $Z = 1$ the regression looks like $Y_i = (\beta_0 + \beta_2) + (\beta_1 + \beta_3)X_i + \varepsilon_i$

and we end up having two regressions with ***different slopes*** and ***different intercepts***.

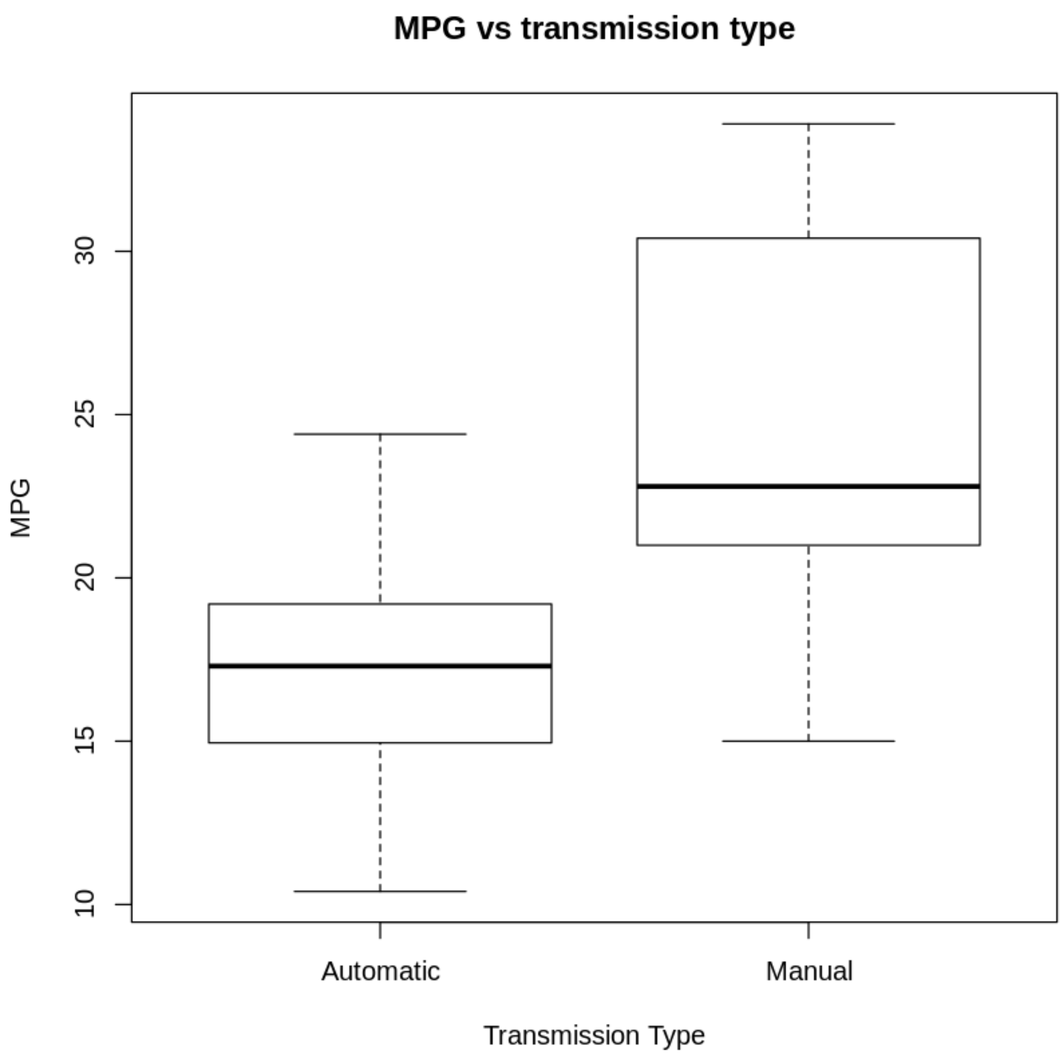
1.1 Problem #2

In this question, we ask you to analyze the `mtcars` dataset. The goal if this question will be to try to explain the variability in miles per gallon (mpg) using transmission type (am), while adjusting for horsepower (hp).

To load the data, use `data(mtcars)`

2. (a) Rename the levels of am from 0 and 1 to "Automatic" and "Manual" (one option for this is to use the revalue() function in the plyr package). Then, create a boxplot (or violin plot) of mpg against am. What do you notice? Comment on the plot

[4]:
```
data(mtcars)

# your code here

df = mtcars[c('mpg', 'hp', 'am')]
df$am = ifelse(df$am == 0, 'Automatic', 'Manual')

boxplot(mpg ~ am, data=df, main="MPG vs transmission type", xlab =␣
 ↪"Transmission Type", ylab = "MPG")
```



We can clearly see that `Manual` transmission yields higher `mpg` when compared to `Automatic`. Medians have sizable separation, Q3 of `Automatic` is lower than Q1 of `Manual`.

2. (b) Calculate the mean difference in mpg for the Automatic group compared to the Manual group.

[5]:
```
# your code here

mean(df[df$am == 'Manual', ][['mpg']]) - mean(df[df$am == 'Automatic',␣
↪][['mpg']])
```

7.24493927125506

On average cars with manual transmission have 7.24 higher MPG when compared to automatic.

2. (c) Construct three models:

1. An ANOVA model that checks for differences in mean mpg across different transmission types.
2. An ANCOVA model that checks for differences in mean mpg across different transmission types, adjusting for horsepower.
3. An ANCOVA model that checks for differences in mean mpg across different transmission types, adjusting for horsepower and for interaction effects between horsepower and transmission type.

Using these three models, determine whether or not the interaction term between transmission type and horsepower is significant.

[6]:
```
# your code here
model_1 = lm(mpg ~ am, data=df)
summary(model_1)

model_2 = lm(mpg ~ am + hp, data=df)
summary(model_2)

model_3 = lm(mpg ~ am + hp + am:hp, data=df)
summary(model_3)
```

```
Call:
lm(formula = mpg ~ am, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-9.3923 -3.0923 -0.2974  3.2439  9.5077

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   17.147      1.125  15.247 1.13e-15 ***
amManual       7.245      1.764   4.106 0.000285 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.902 on 30 degrees of freedom
Multiple R-squared:  0.3598,Adjusted R-squared:  0.3385
F-statistic: 16.86 on 1 and 30 DF,  p-value: 0.000285



Call:
lm(formula = mpg ~ am + hp, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-4.3843 -2.2642  0.1366  1.6968  5.8657
```

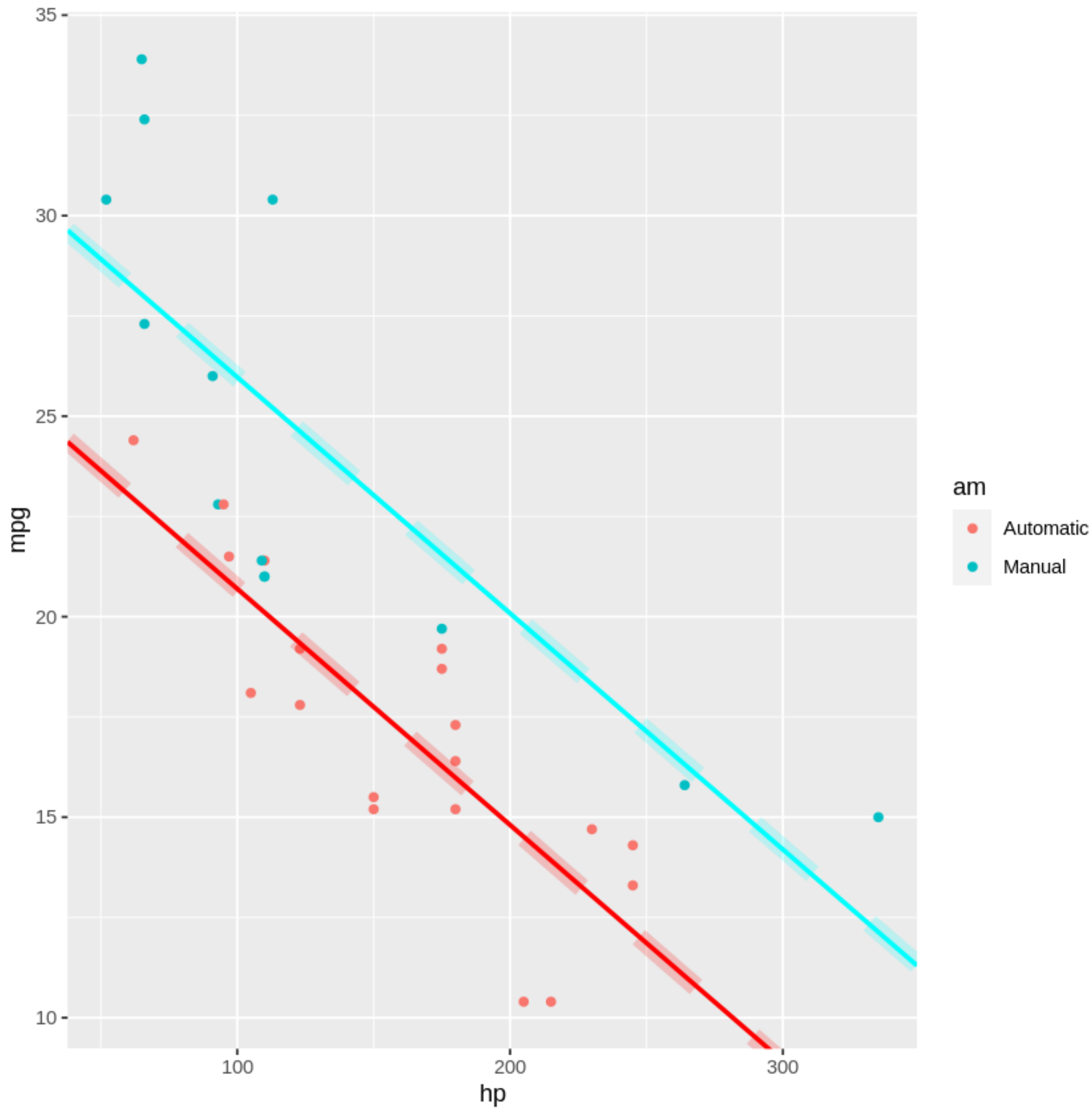
```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 26.584914   1.425094  18.655  < 2e-16 ***
amManual     5.277085   1.079541   4.888 3.46e-05 ***
hp          -0.058888   0.007857  -7.495 2.92e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.909 on 29 degrees of freedom
Multiple R-squared:  0.782,Adjusted R-squared:  0.767
F-statistic: 52.02 on 2 and 29 DF,  p-value: 2.55e-10



Call:
lm(formula = mpg ~ am + hp + am:hp, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-4.3818 -2.2696  0.1344  1.7058  5.8752

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 26.6248479  2.1829432  12.197 1.01e-12 ***
amManual     5.2176534  2.6650931   1.958   0.0603 .
hp          -0.0591370  0.0129449  -4.568 9.02e-05 ***
amManual:hp  0.0004029  0.0164602   0.024   0.9806
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.961 on 28 degrees of freedom
Multiple R-squared:  0.782,Adjusted R-squared:  0.7587
F-statistic: 33.49 on 3 and 28 DF,  p-value: 2.112e-09
```

The interaction term between `am` and `hp` is not significant as if we were to run F-test comparing `model_2` as partial model and `model_3` as full, we wouldn't be able to reject the null that partial model is sufficient since the p-value of that F-test would be exactly the p-value of t-test for `amManual:hp` in `model_3` and this p-value is 0.98.

2. (d) Construct a plot of mpg against horsepower, and color points based in transmission type. Then, overlay the regression lines with the interaction term, and the lines without. How are these lines consistent with your answer in (b) and (c)?

[7]:
```
# your code here


b2 = coef(model_2)
```

```r
b0a2 = b2[1]
b0m2 = b2[1] + b2[2]
b1a2 = b2[3]
b1m2 = b2[3]

b3 = coef(model_3)
b0a3 = b3[1]
b0m3 = b3[1] + b3[2]
b1a3 = b3[3]
b1m3 = b3[3] + b3[4]

p = ggplot(data=df, aes(x=hp, y=mpg, color=am)) +
    geom_point() +
    geom_abline(aes(intercept=b0a2, slope=b1a2), color='red', size=1) +
    geom_abline(aes(intercept=b0m2, slope=b1m2), color='cyan', size=1) +
    geom_abline(aes(intercept=b0a3, slope=b1a3), color='red',␣
 ↪linetype="dashed", size=4, alpha=0.2) +
    geom_abline(aes(intercept=b0m3, slope=b1m3), color='cyan',␣
 ↪linetype="dashed", size=4, alpha=0.2)
print(p)
```



Solid lines represent regression lines from the model without interaction terms (`model_2`) while dashed lines represent regression lines from the model with interaction terms (`model_3`). Colors of the lines are matched to the color of the points.

We can clearly see that the parameter changes due to introduction of interaction terms have pretty much no impact on the resulting fit. The dashed lines align so well with the solid lines so I had to increase the width of the dashed lines to even be able to see them. That supports earlier findings on the lack of statistical significance of the interaction term.