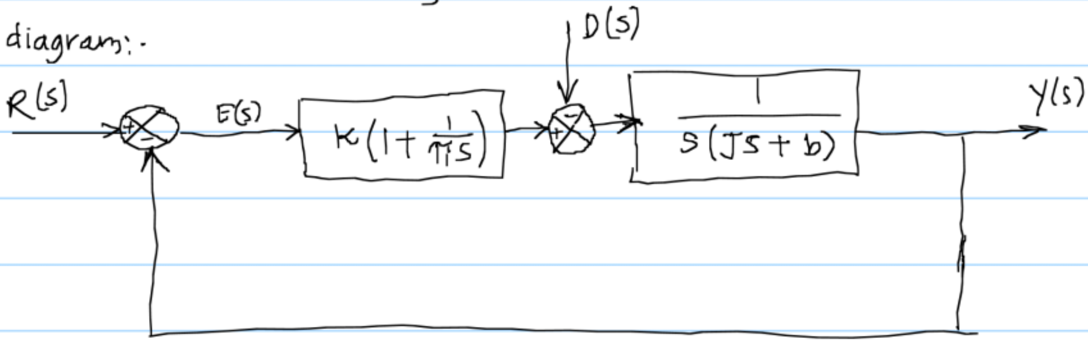
PI AND PD Controller

Friday, January 8, 2021 1:26 PM

3. Proportional + Integral controller / PI controller

$$K + \frac{K}{s}$$

Block diagram:-



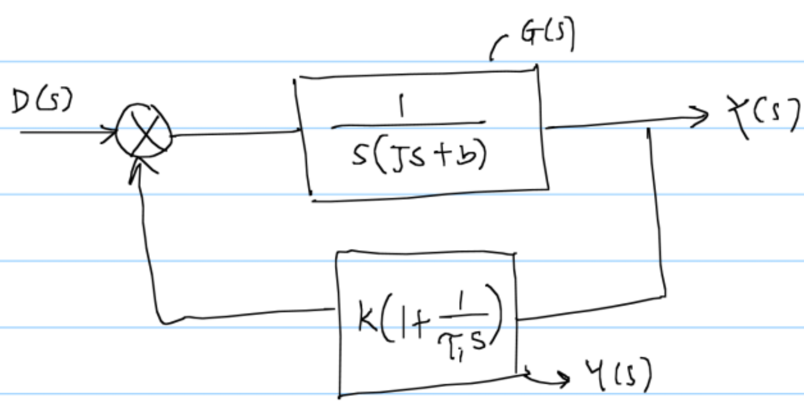
where

E(s): Error signal, D(s): Disturbance

R(s): Input, Y(s): o/p

let $D(s) = T \cdot \frac{1}{s} = \frac{T}{s}$

let R(s) = 0 then Block diagram would look like



$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{1}{s(Js+b)}}{1+\frac{1}{s(Js+b)}\cdot K\left(1+\frac{1}{T_i s}\right)}$$

$$= \frac{\frac{1}{\cancel{s(Js+b)}}}{\frac{s(Js+b)+1\left(K+\frac{K}{T_i s}\right)}{\cancel{s(Js+b)}}}$$

$$= \frac{1}{s(Js+b)+1\left(K+\frac{K}{T_i s}\right)}$$

$$= \frac{1}{s(Js+b)+\frac{KT_i s+K}{T_i s}}$$

$$= \frac{1}{\frac{T_i s\left(s(Js+b)\right)+KT_i s+K}{T_i s}}$$

$$= \frac{T_i s}{T_i J s^3 + b s^2 T_i + T_i s K + K}$$

Taking T_i common from N^r & D^r

$$\frac{Y(s)}{D(s)} = \frac{s}{Js^3+bs^2+Ks+\frac{K}{T_i}} \quad \text{--- (1)}$$

Now, $E(s) = R(s) - Y(s)$

$= 0 - Y(s)$

$$\boxed{E(s) = -Y(s)}$$

put $Y(s) = -E(s)$ in (1)

$$\frac{E(s)}{D(s)} = \frac{-Y(s)}{D(s)} = \frac{-s}{Js^3 + bs^2 + ks + K/T_i}$$

$$\therefore E(s) = \frac{-s}{Js^3 + bs^2 + ks + \frac{K}{T_i}} \times \frac{T}{s}$$

$$\therefore e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot E(s)$$

$$= \lim_{s \to 0} \frac{-s^2}{Js^3 + bs^2 + ks + \frac{K}{T_i}} \times \frac{T}{s}$$

$$\boxed{e_{ss} = 0}$$

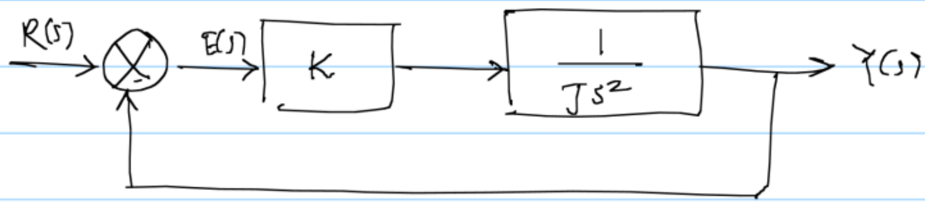
Observation:-

Thus PI controller eliminates the steady state error.

4. Derivative Controller

Consider the plant $G(s) = \dfrac{1}{Js^2}$

let's first study the effect of P controller on above plant



Now, $G(s) = \dfrac{K}{Js^2}$

$$\frac{C(s)}{R(s)} = \text{C.L.T.F} = \frac{K}{Js^2+K} \quad \to \quad \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

} unstable

$$C.E \to Js^2+K=0$$

$$\therefore Js^2 = -K$$

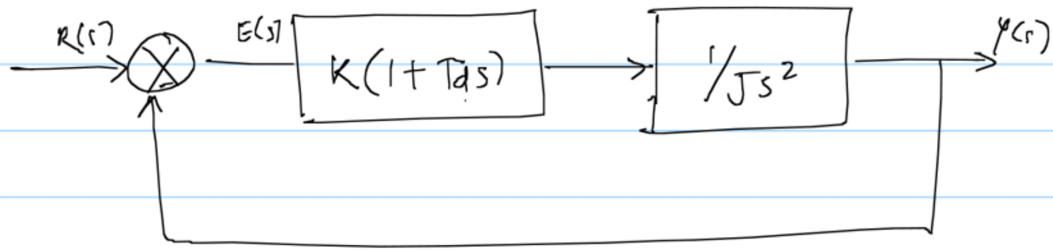
$$s^2 = \frac{-K}{J}$$

$$S = \sqrt{\frac{-K}{J}}$$

$$\boxed{S = \pm i\sqrt{\frac{K}{J}}}$$

- As, the poles are on img axis $\to$ G(s) is unstable
- Thus to make the system relatively stable we need to add 's' coee i.e. damping. This can be achieved by derivative Controller

Block diagram of PD controller



$$\frac{Y(s)}{R(s)} = \frac{K(1+T_d s)}{Js^2 + KT_d s + K} \rightarrow \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Observation:

→ Thus Derivative controller used introduces coeff of 's' which will provide damping & make system more relatively stable

→ Thus we have damped the oscillations of unstable system $\frac{1}{Js^2}$ by using derivative controller