ROOT LOCUS DIAGRAM

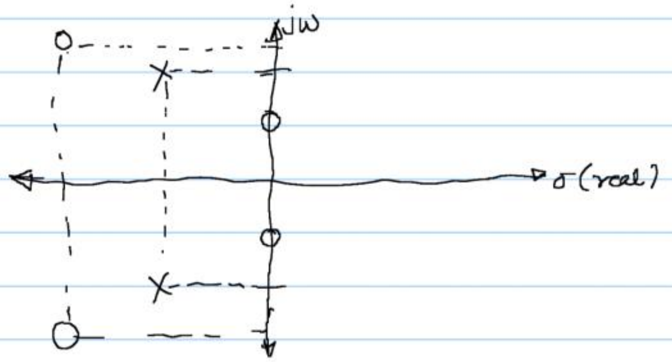
Thursday, November 26, 2020 9:04 AM

Construction Rules for R.L.D.

Note Title 11/10/2020

1. Symmetrical:

↳ The R.L.D is symmetrical about the real axis (σ-axis)



2. Number of Locci or Root locus Branches :-

→ No. of R.L Branches is always depends on No. of poles & zeros.

Case 1: No. of poles > No. of zeros ($P > Z$)

↳ No. of Locci or R.L.Branches = No. of poles.

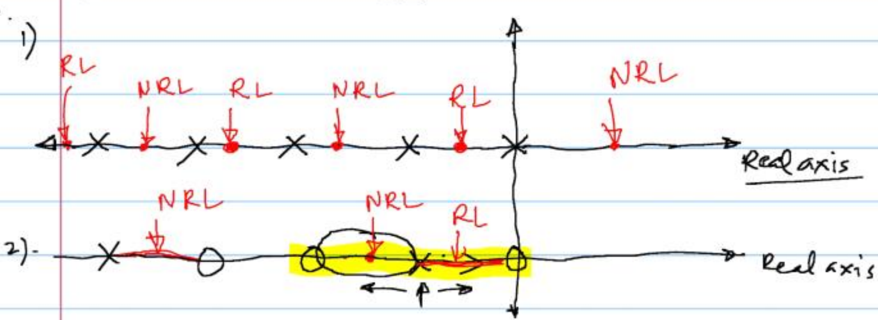
Case 2:- No. of poles < No. of zeros ($P < Z$)

↳ No. of Locci or R.L. Branches = No. of Zeros

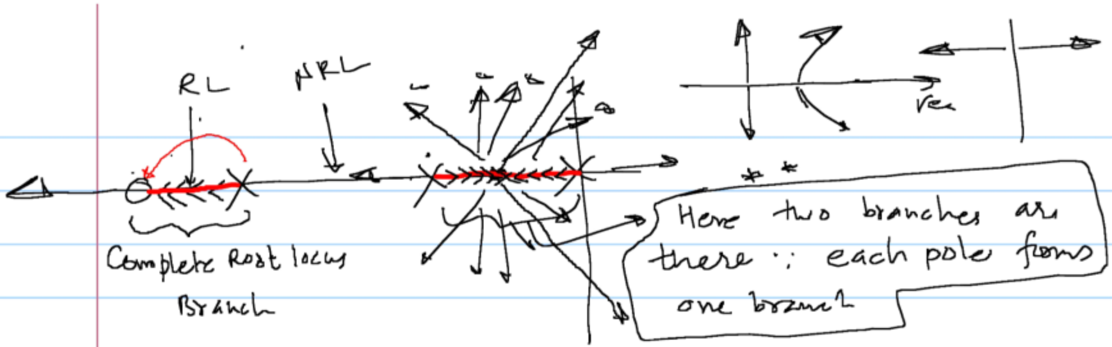
** 3. Real axis Locci or R.L. Branches:

↳ A point exists on Real axis root Locus branches if the sum of the poles & zeros to the right hand side of that point should be "ODD" (NOTE:- DO NOT consider that point)

Ex.

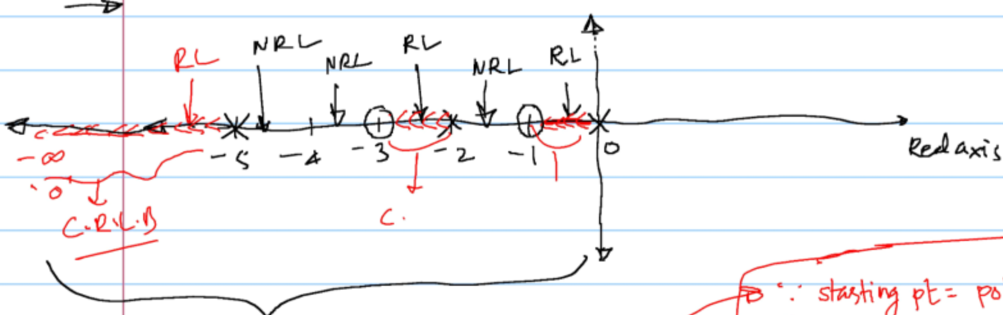


↳ The poles moves only on the RL Branches, & Once pole occupies zero then it is called "Complete Root Locus



Ex. Identify the sections of real axis which belongs to root locus

For $G(s)H(s) = \dfrac{K(s+1)(s+3)}{s(s+2)(s+5)}$



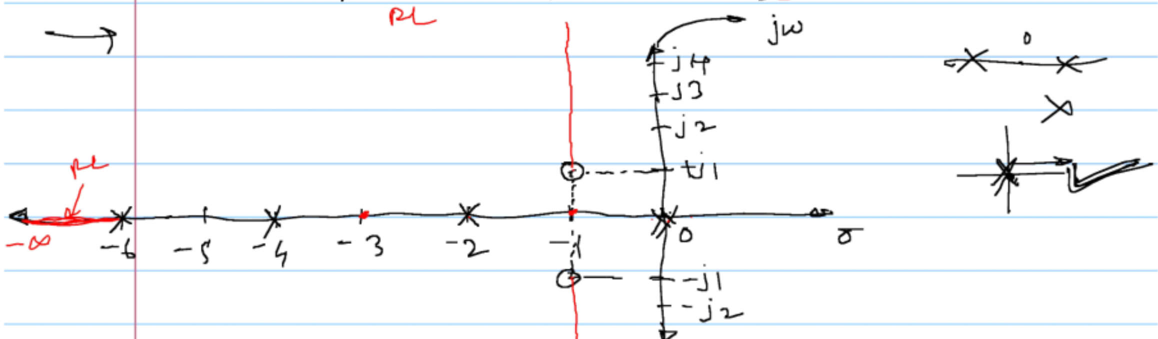
Complete R.L. Diagram as K ↑ 0 to ∞

∵ starting pt = pole & ending pt = zero

2) $G(s)H(s) = \dfrac{K(s^2+2s+2)}{s^2(s+2)(s+4)(s+6)}$ → $-1 \pm j1$ → zeros

i) $s = 0$ NRL ii) $s = -1$ NRL iii) $s = -3$ RL

iv) $s = -\infty$ RL v) $s = -1 + j1$ ← NRL



4) Assymptotes:

↳ Assymptotes are the root locus branches which approches to the '∞'

↳ The no. of assymptotes $= N = P - Z$

P = No. of poles

Z = No. of zeros.

↳ Angle of assymptotes $= \theta = \dfrac{(2q+1)\times 180^\circ}{P-Z}$

where $q = 0, 1, 2, 3 \ldots$

↳ Assymptotes are symmetrical about the real axis.

NOTE: Assymptotes gives the direction zeros when $P > Z$,

5) Centroid: (σ) :

↳ Centroid is nothing but the intersection point of assymptotes on the real axis.

$$\sigma = \frac{\text{Summation of Real part of poles} - \text{Sum}^n \text{ of R.P. of zeros}}{P-Z}$$

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P-Z}$$

NOTE: ↳ Centroid may be located anywhere on the axis, it may or may not be on the root locus branch..

(1) find the angle of assymptotes & centroids to the given system.

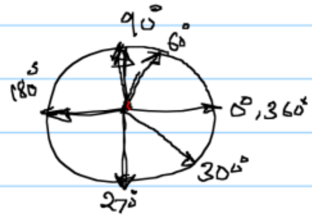
$$G(s)H(s) = \frac{K}{s(s+5)(s+10)}$$

Sol^n: No. of poles i.e. $P = 3$

— " — zeros $Z = 0$

$\therefore N = P - Z = \underline{3}$ ✓

$$\text{Angle of assymptotes} = \theta = \frac{(2q+1)\times 180^\circ}{p-z}$$

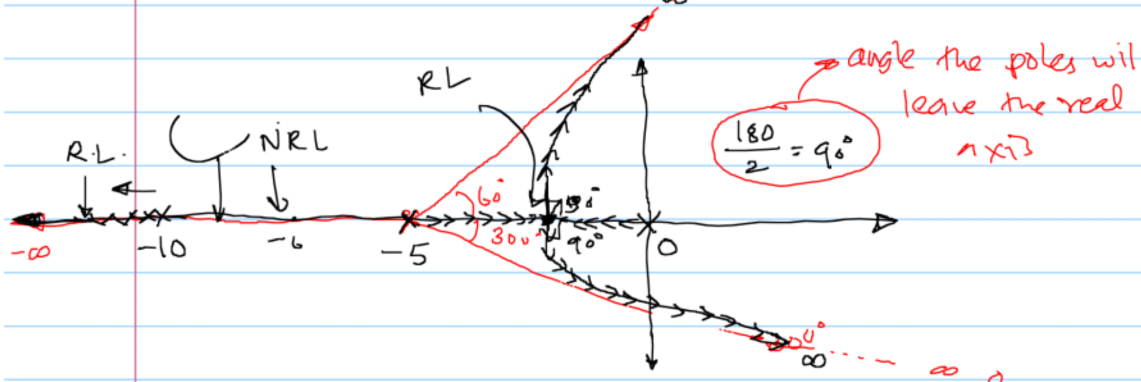


$$= \frac{180}{3} = 60^\circ \quad (\text{For } q=0)$$

$$= \frac{(2+1)\times 180}{3} = 180^\circ \quad (\text{For } q=1)$$

$$= \frac{(2\times 2+1)\times 180^\circ}{3} = 300^\circ \quad (\text{For } q=2)$$

* Centroid $= \sigma = \dfrac{0-5-10}{3} = \dfrac{-15}{3} = -5$



② $G(s)H(s) = \dfrac{K(s+10)}{s^2(s+1)}$

→ $N = p - z =$

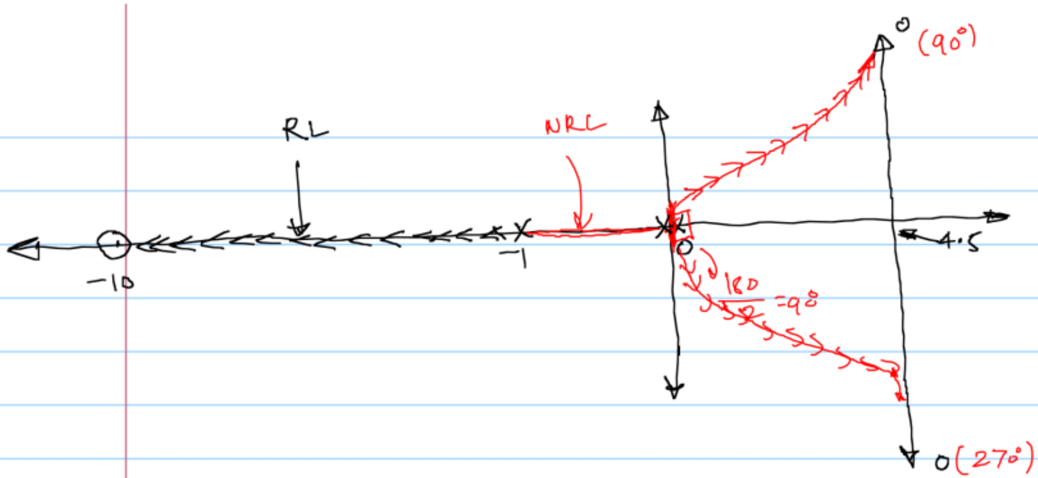
$N = 3 - 1 = 2$ ✓

$$\text{Angle of assymptotes} = \theta = \frac{(2q+1)\times 180^\circ}{p-z}$$

✓ For $q=0 \Rightarrow \theta = \dfrac{180^\circ}{2} = 90^\circ$.

✓ For $q=1 \Rightarrow \theta = \dfrac{3\times 180^\circ}{2} = 270^\circ$ ✓

Centroid $= \sigma = \dfrac{-1-(-10)}{2} = \dfrac{-1+10}{2} = \dfrac{9}{2} = 4.5$

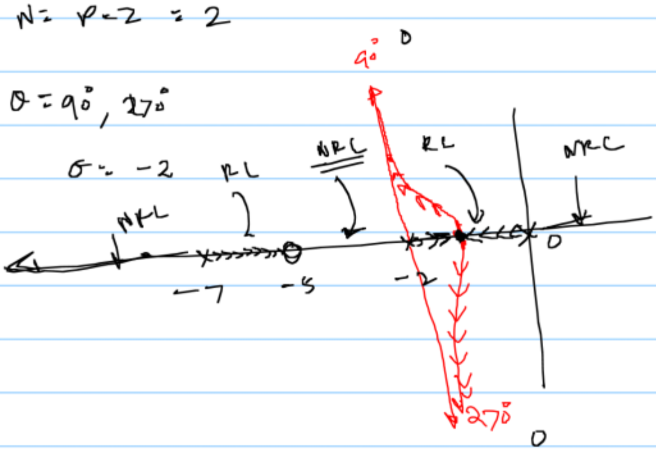


(3) $G(s)H(s) = \dfrac{k(s+5)}{s(s+2)(s+7)}$ (HW)

Sol^n:- $N = P - Z = 2$

$\theta = 90°, 270°$

$\sigma = -2$



(4) $G(s)H(s) = \dfrac{k(s+10)(s+5)}{s^2(s+2)(s+3)(s+4)}$

(5) $G(s)H(s) = \dfrac{k(s+9)}{s(s+2)(s+3)(s+7)}$

6. Breakpoint:

The pt. at which two or more root locus branches meets.

Break away point:- The pt at which the root locus branches leaves the real axis.

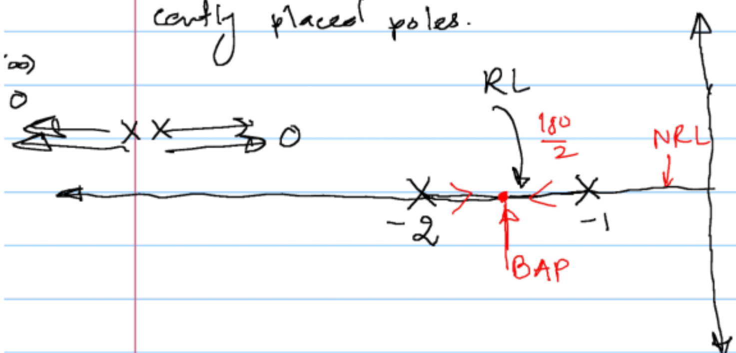
Break in point:- The pt. at which the root locus branches enters into real axis.

Whenever the root locus branches enters or leaves the real axis they always do so at an angle of $\left(\pm\frac{180^\circ}{n}\right)$

where 'n' is the no. of poles or no. of root locus branches at that Break Point.

finding the existance of Break points:-

Case1: Whenever there exist two adjecently placed poles and in betn there exist root locus branch then there should exist at least one Break away point in betn adjecently placed poles.



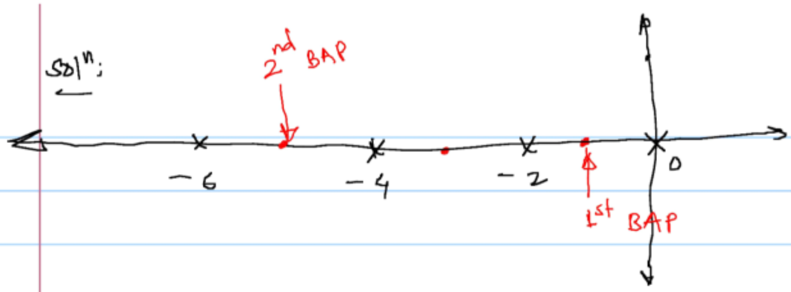
Ex(1) find the no. of Break away points.

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)(s+6)}$$

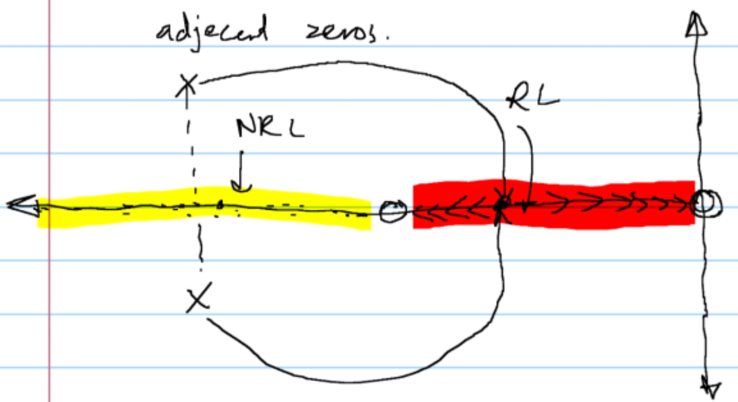
0, -2, -4, -6

2 BAP

Sol^n:



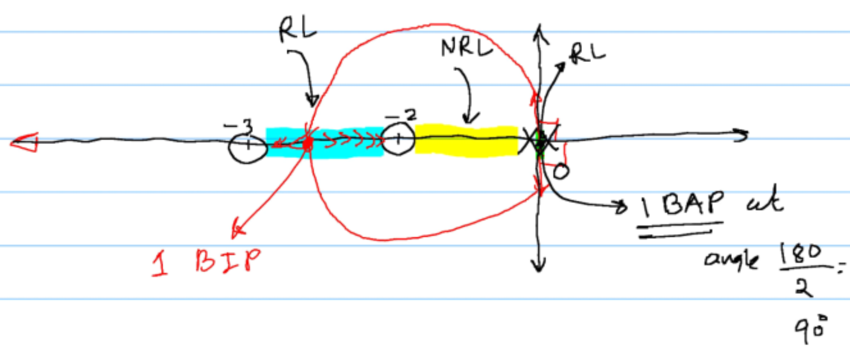
Case 2) Whenever there exist two adjecently placed zeros, and in bet^n them there exist a root locus branch then there should exist min one **Break In point** in bet^n adjecent zeros.



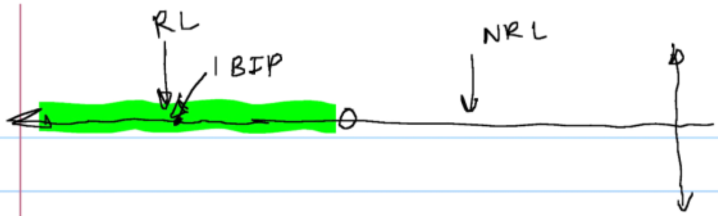
Ex ① find out the Break points: (HW)

$$G(s)H(s) = \frac{K(s+2)(s+3)}{s^2}$$

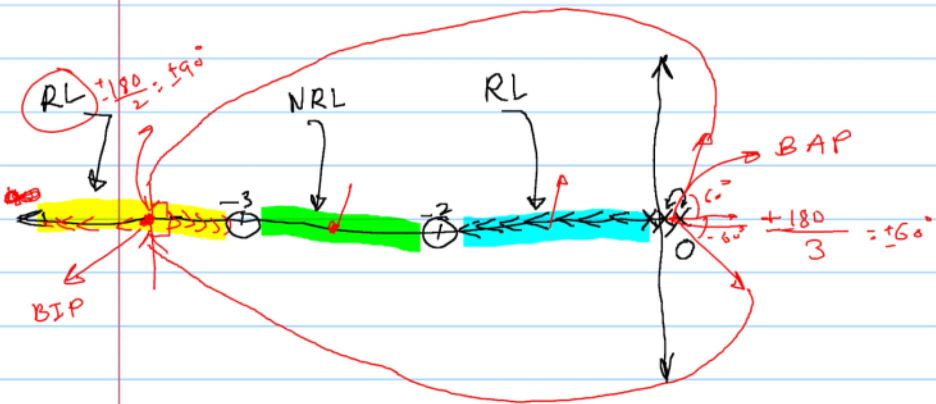
1 → BAP
1 → BIP



Case 3): Whenever there exist the left most side zero, and to the left most side of that zero if there exist a root locus branch then there should be at least one **Break in point** to the left most side of that zero.



Ex ① $G(s)H(s) = \dfrac{K(s+2)(s+3)}{s^3}$



→ finding the co-ordinates of Break points :-

Procedure :

1. Form the characteristic eq^n. ✓
2. Rearrange the above C.E. in the form of $K = f(s)$
3. Differentiate K. w.r.t. s & make is equal to 0

   i.e. $\dfrac{dk}{ds} = 0$

4. The roots of $\dfrac{dk}{ds} = 0$ will give valid or invalid break points.

NOTE: The valid break point must lie on the Root locus branch & for valid break point K value is always +ve.

NOTE: Above procedure is to be followed when there are poles as well as zeros in transfer function. If there are only poles then just expand the denominator & differentiate it & the roots will give valid or invalid Break points.

Ex ① find the location of break pts.

$$G(s)H(s) = \frac{K}{s(s+2)}$$

As No zeros, Expanding the denominator,

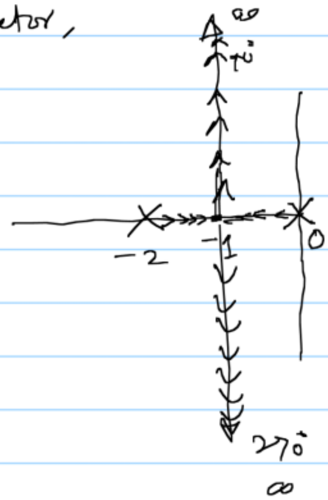
$$s(s+2) = 0$$

$$s^2 + 2s = 0$$

diff. → $2s + 2 = 0$

$$2s = -2$$

$$\boxed{s = -1}$$



② find the co-ordinates of Breakpoints

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$

Solⁿ: No zeros, expand the Dnʳ

$$s(s+2)(s+4) = 0$$

$$(s^2+2s)(s+4) = 0$$

$$s^3 + 4s^2 + 2s^2 + 8s = 0$$

$$s^3 + 6s^2 + 8s = 0$$

diff → $\underset{a}{3}s^2 + \underset{b}{12}s + \underset{c}{8} = 0$ → in calci (fx991)

1) mode set up

2) press Eqⁿ (5)

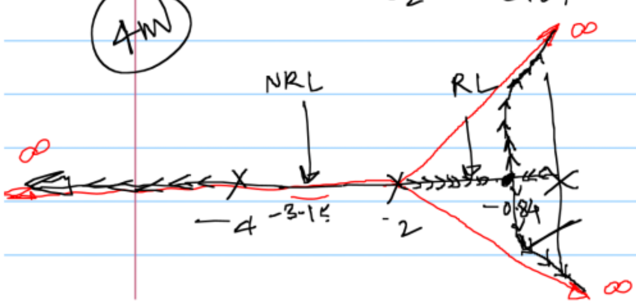
3) Press (3) → $ax^2+bx+c=0$

4) put values of a, b & c & press equal to (=) sign.

$S_1 = -0.84$ (valid)

$S_2 = -3.15$, (Invalid)

4M



$\theta = 60°, 180°, 300°$

$\sigma = -2$

(3) find the co-ordinates of Break point.

$$G(s)H(s) = \frac{K(s+4)}{s(s+2)}$$

Sol^n: As poles & zeros both are present, follow the procedure.

NOTE: For C.E → 1st convert the O.L.T.F. into C.L.T.F.

1st step → $s^2 + 2s + K(s+4) = 0$

2nd step → $K = \dfrac{-s^2 - 2s}{s+4}$

3rd step → $\dfrac{dK}{ds} = 0$; $\dfrac{(s+4)(-2s-2) - (-s^2-2s)1}{(s+4)^2} = 0$

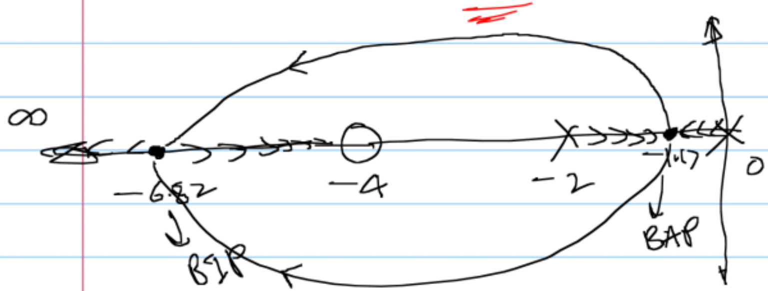
→ $(s+4)(-2s-2) - (-s^2-2s) = 0$

→ $-2s^2 - 8s - \cancel{2s} - 8 + s^2 + \cancel{2s} = 0$

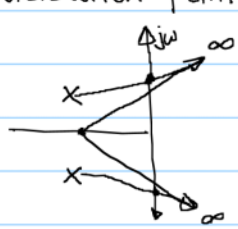
→ $-s^2 - 8s - 8 = 0$

4th step → $s_1 = -1.17$, $s_2 = -6.82$

valid valid



7. Intersection point with imaginary axis:-



→ The intersection pt. with imaginary axis is found out by RH criteria.

Steps →

1. Form the C.E.
2. Write the Routh tabular form.
3. Find the $K_{marginal}$ value.
4. Form Auxillary Eq^n & the roots of Auxillary eq^n will give valid & invalid intersection pts.

NOTE: for valid intersection pt, $K_{marginal}$ should be +ve.

Ex ① :- find the intersection pt. with imaginary axis.

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$

sol^n:-

$$s(s+2)(s+4) + K = 0$$

$$(s^2+2s)(s+4) + K = 0$$

$$s^3 + 2s^2 + 4s^2 + 8s + K = 0$$

$$s^3 + 6s^2 + 8s + K = 0$$

For system to be marginally stable

internal prod = ext. prod

i.e. $\boxed{48 = K_{marginal}}$ ✓

Routh Tabular form :-

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 6 & K \\ s^1 & \frac{48-K}{6} & \\ s^0 & K & \end{array}$$

A. Eq^n = $6s^2 + K = 0$

$6s^2 + 48 = 0$

$s^2 = -\frac{48}{6}$

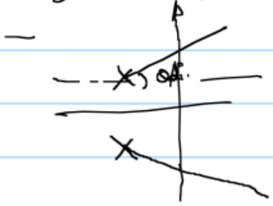
$s^2 = -8$

$\boxed{s = \pm j\sqrt{8}}$

$\boxed{\text{Valid int. pt}}$

$\therefore K = +ve$

8. Angle of Departure and Angle of arrival :-



- Angle of departure (ϕ_d) is calculated at complex conjugate poles and angle of arrival (ϕ_a) is calculated at complex conjugate zero

Angle of Departure (ϕ_d) :-

It gives that with what angle the pole departs or leaves from initial position.

$$\boxed{\phi_d = 180^\circ + \angle G(s)H(s)\Big|_{\text{at the img complex pole.}}}$$

or.

$$\phi_d = 180^\circ - \phi$$

where $\phi = \left(\Sigma \phi_p - \Sigma \phi_z\right)$

Angle of arrival (ϕ_a):

It gives that with what angle the pole arrives or terminates near the complex zero

$$\boxed{\phi_a = 180^\circ - \angle G(s)H(s)\Big|_{\text{at the img complex zero.}}}$$

or

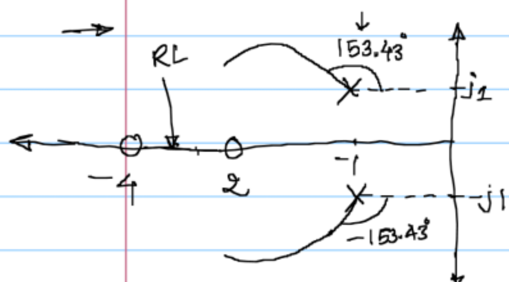
$$\phi_a = 180 + \phi$$

$$\phi = \Sigma\phi_p - \Sigma\phi_z$$

Ex ①: Calculate the angle of departure at a complex pole.

$$G(s)H(s) = \frac{K(s+2)(s+4)}{(s^2+2s+2)}$$

$\longrightarrow (s+1+j)(s+1-j)$



$$\angle G(s)H(s)\Big|_{s=-1+j1} = \frac{\angle K \ \angle(s+2)\ \angle(s+4)}{\angle(s+1+j)\ \angle(s+1-j)}$$

$$\phi_{01}(3,0) = 0^\circ = \quad = \frac{0^\circ \;\angle(-1+j1+2)\;\angle(-1+j1+4)}{\angle(-\cancel{1}+j1+\cancel{1}+j1)\;\angle(-\cancel{1}+\cancel{j1}+\cancel{1}-\cancel{j1})}$$

$$= \frac{0^\circ\;\angle(1+j1)\;\angle(3+j1)}{\angle 2j1 \quad 0^\circ}$$

$$= \frac{\angle 0^\circ + \angle 45^\circ + \angle 18.43^\circ}{\angle 90^\circ + \angle 0^\circ}$$

$$= \frac{\angle 63.43}{\angle 90^\circ}$$

$$= \angle 63.43^\circ - \angle 90^\circ$$

$$\boxed{\angle G(s)H(s)\Big|_{s=-1+j1} = -26.57^\circ}$$

Now,

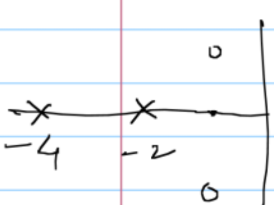
$$\phi_d = 180^\circ + \angle G(s)H(s)$$

$$= 180^\circ + (-26.57^\circ)$$

$$\boxed{\phi_d = 153.43^\circ}$$

2 What if $\phi_a\Big|_{s=-1+j1}$ = zero, $\rightarrow G(s)H(s) = \dfrac{k(s^2+2s+2)}{(s+2)(s+4)}$

$$\angle G(s)H(s)\Big|_{s=-1+j1} = \frac{\angle k\;\angle(s+1-j)(s+1+j)}{\angle(-1+j1+2)\;\angle(-1+j1+4)}$$



$$\angle G(s)H(s)\Big|_{s=-1+j1} = 26.57^\circ$$

$$\phi_a = 180 - \angle G(s)H(s)$$

$$= 180 - 26.57$$

$$\boxed{\phi_a = 153.43^\circ}$$

NOTE: Thus whenever, all the poles & zeros are interchanged, we have $\boxed{\text{angle of arrival} = \text{Angle of departure}}$

$$\boxed{\phi_d = \phi_d}$$

1) Draw the root locus diagram for $G(s)H(s) = \frac{k}{s(s+2)}$

Sol^n:- Procedure to draw R.L.D.

1. Identify the real axis root locus branches (poles) and break pts.
2. find the angle of assymptotes & centroid. (If required, find ϕd & ϕa)
3. Vary the k value from 0 to ∞ & identify the path from pole to zero such that root locus diagram is symmetrical @ real axis & all poles must reach to their respective zeros.

1) No. of assymptotes = $N = P - Z = 2$

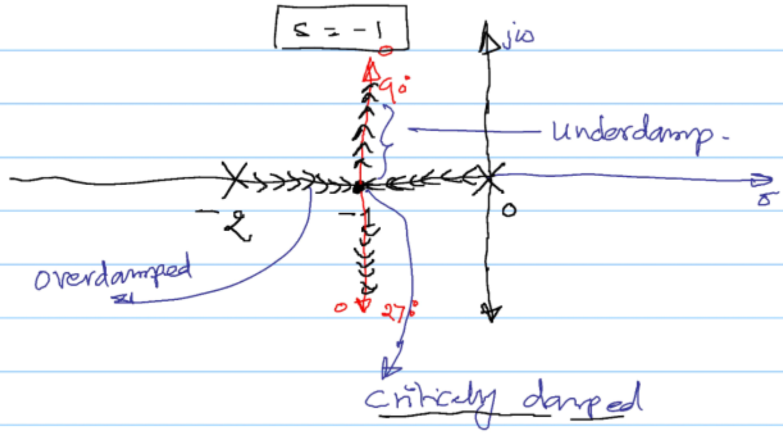
Angle of assymptotes = $\left(\frac{2q+1}{P-Z}\right)180 = 90°, 270°$

2) $\sigma = \frac{-2-0}{2} = -1$ ✓

3) Break pts:- $s^2 + 2s = 0$

diff → $2s + 2 = 0$

$\boxed{s = -1}$



To get the k value for different nature of the system we require to find k value at break point.

$$\left|\frac{k}{s(s+2)}\right|_{s=-1} = 1$$

$$\left|\frac{k}{-1(-1+2)}\right| = 1$$

$$\boxed{K = 1}$$

At break pt, $K = 1$,

put $K = 1$ in O.L.T.F. $= \dfrac{1}{s(s+2)}$

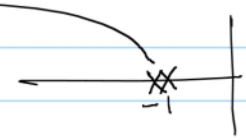
$\downarrow$

C.L.T.F. $= \dfrac{1}{s(s+2)+1}$

$= \dfrac{1}{s^2+2s+1}$

C.L.T.F. $= \dfrac{1}{(s+1)^2} \Rightarrow s = -1, -1$

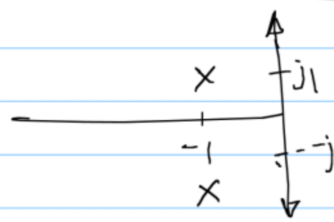
C.D.



put $K > 1$ in O.L.T.F. $= \dfrac{2}{s(s+2)}$

C.L.T.F. $= \dfrac{2}{s^2+2s+2}$

roots



Underdamped

HW (1) $G(s)H(s) = \dfrac{K}{s(s^2+2s+2)}$ ✓

(2) $G(s)H(s) = \dfrac{k}{s(s+4)(s+7)}$ ✓