WAVEGUIDES

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- Wave guide
- Basic features
- Rectangular Wave guide
- Circular Wave guide
- Applications

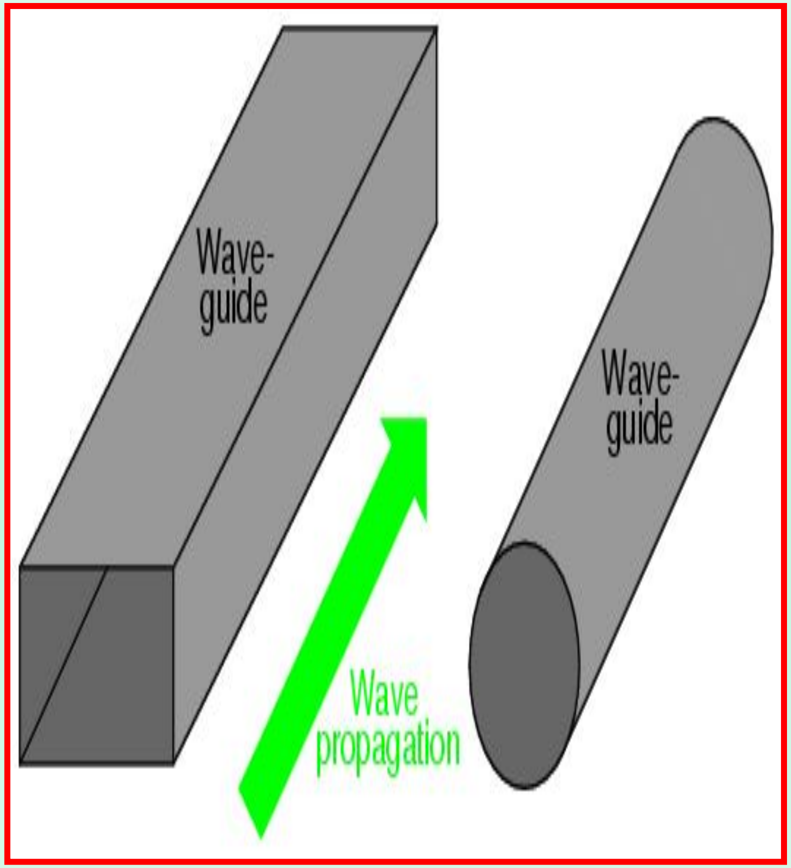
Waveguides

Introduction

- At frequencies higher than 3 GHz, transmission of electromagnetic energy along the transmission lines and cables becomes difficult.
- This is due to the losses that occur both in the solid dielectric needed to support the conductor and in the conductors themselves.
- A metallic tube can be used to transmit electromagnetic wave at the above frequencies

Definition

A Hollow metallic tube of uniform cross section for transmitting electromagnetic waves by successive reflections from the inner walls of the tube is called ***waveguide***.



Basic features

- Waveguides may be used to carry energy between pieces of equipment or over longer distances to carry transmitter power to an antenna or microwave signals from an antenna to a receiver

- Waveguides are made from copper, aluminum or brass. These metals are extruded into long rectangular or circular pipes.

- An electromagnetic energy to be carried by a waveguide is injected into one end of the waveguide.

- The electric and magnetic fields associated with the signal bounce off the inside walls back and forth as it progresses down the waveguide.

ADVANTAGES

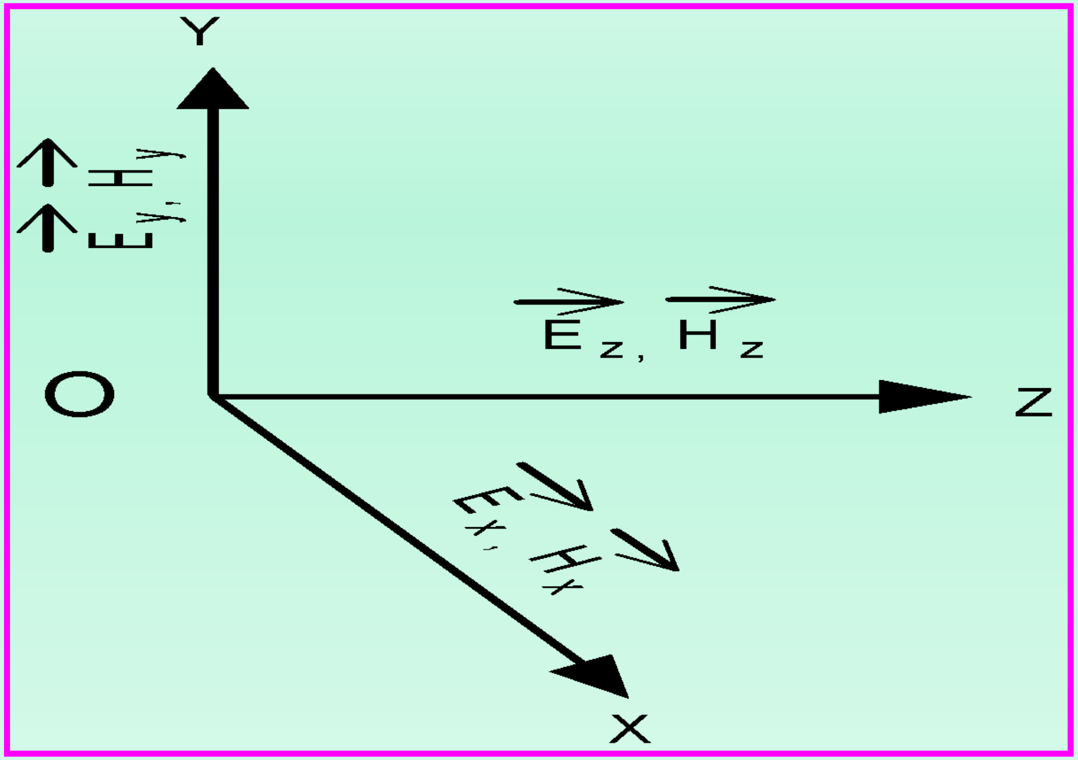
1. Simpler to manufacture than coaxial line
2. Flash over is very less
3. Power handling ability is more than 10 times
4. Power losses lower
5. Mechanical simplicity
6. Much higher operating freq. @325Ghz as compared to 18 GHz

EM field configuration within the waveguide

- In order to determine the EM field configuration within the waveguide, Maxwell’s equations should be solved subject to appropriate boundary conditions at the walls of the guide.

- Such solutions give rise to a number of field configurations. Each configuration is known as a *mode*. The following are the different modes possible in a waveguide system

Components of Electric and Magnetic Field Intensities in an EM wave



Possible Types of modes

1. Transverse Electro Magnetic (TEM) wave:

Here both electric and magnetic fields are directed components. (i.e.) $E_z = 0$ and $H_z = 0$

Electric field and magnetic field and Direction of Propagation are mutually perpendicular

2. Transverse Electric (TE) wave: Modes in which no component of electric field in Direction of Propagation.

Here only the electric field is purely transverse to the direction of propagation and the magnetic field is not purely transverse. (i.e.) $E_z = 0, H_z \neq 0$

Possible Types of modes

3. Transverse Magnetic (TM) wave:

Modes in which no component of Magnetic field in Direction of Propagation.

Here only magnetic field is transverse to the direction of propagation and the electric field is not purely transverse. (i.e.) $\mathbf{E_z \neq 0, H_z = 0.}$

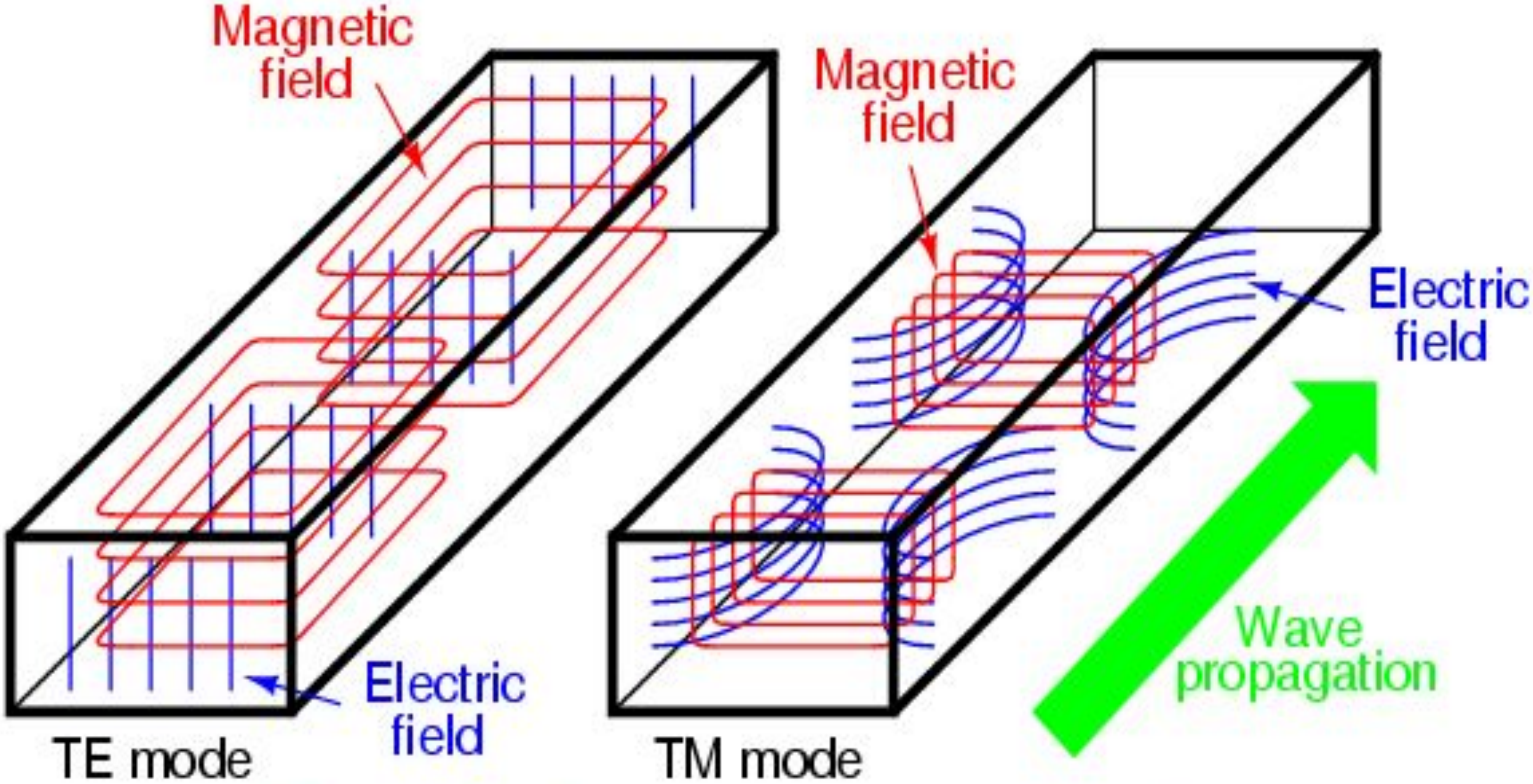
Dominant mode of operation: Natural mode of Operation.
This mode is lowest possible freq. that can be propagated in WG

Sub Modes:

1. TEm,n For TE : E field is perpendicular to direction of wave propagation
2. TMm,n For TM : M field is perpendicular to direction of wave propagation

m: no of half wavelengths across WG width

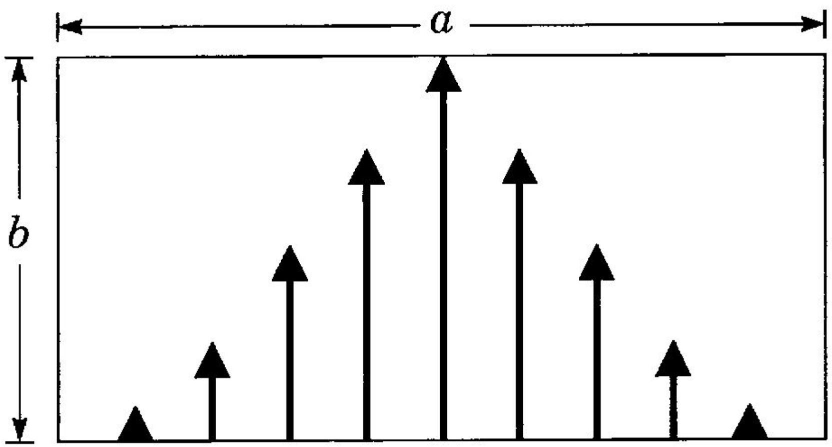
n : no of half wavelengths across WG height



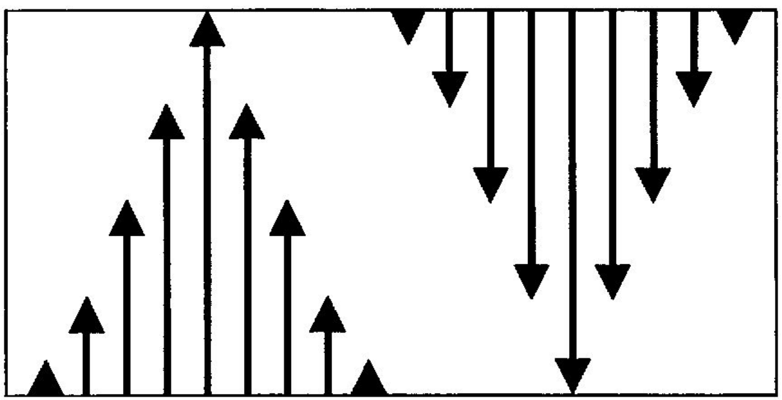
Magnetic flux lines appear as continuous loops

Electric flux lines appear with beginning and end points

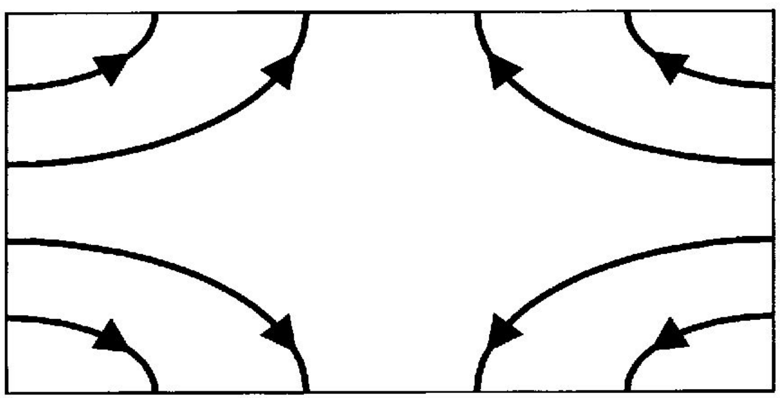
TE Modes in Rectangular Waveguide



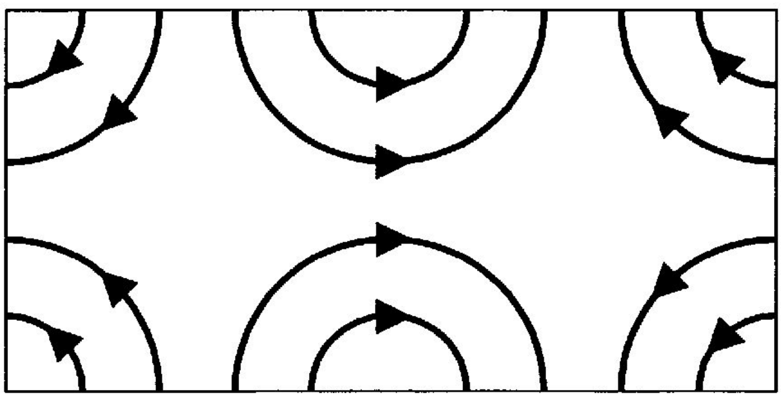
(a) TE_{10}



(b) TE_{20}



(c) TE_{11}



(d) TE_{21}

Transparency 252 (Figure 19.3)

GROUP VELOCITY & PHASE VELOCITY

Phase Velocity:

Any EM wave has two velocities

a) One with which it propagate.

b) Second with which it changes phase.

In free space both are same and it is called as velocity of light v_c.

The product of wavelength and frequency => velocity

Therefore $v_c = f\lambda = 3 * 10$ m/sec. in free space.

-**Phase velocity is velocity with which the wave changes phase in direction parallel to conducting surface.**

Cutoff wavelength λ_0

It is smallest free space wavelength that is just unable to propagate in the waveguide under given conditions:

⇨ Larger free space wavelength certainly can not propagate. But all smaller can be propagated.

Mathematically it is given by: $\lambda 0 = 2a/m$

Where,

λ_0= cut off wavelength,

a= distance btw walls or width of the waveguide,

m= no. of half wavelength btw the walls.

Largest value of cutoff wavelength is 2a when m=1.
It means, longest λ_0 that the signal may have and till be capable of wg is just less than twice the wall separation. When m is unity the signal is said to be propagated in dominant mode which is the method of propagation that yields the longest cutoff wavelength of the guide.

Cont.

Phase wavelength:
Mathematically it can be given as:
$\lambda_p = \lambda/\sqrt{1-(\lambda/\lambda_0)^2}$

Cutoff frequency: frequency below which there wont be any propagation.

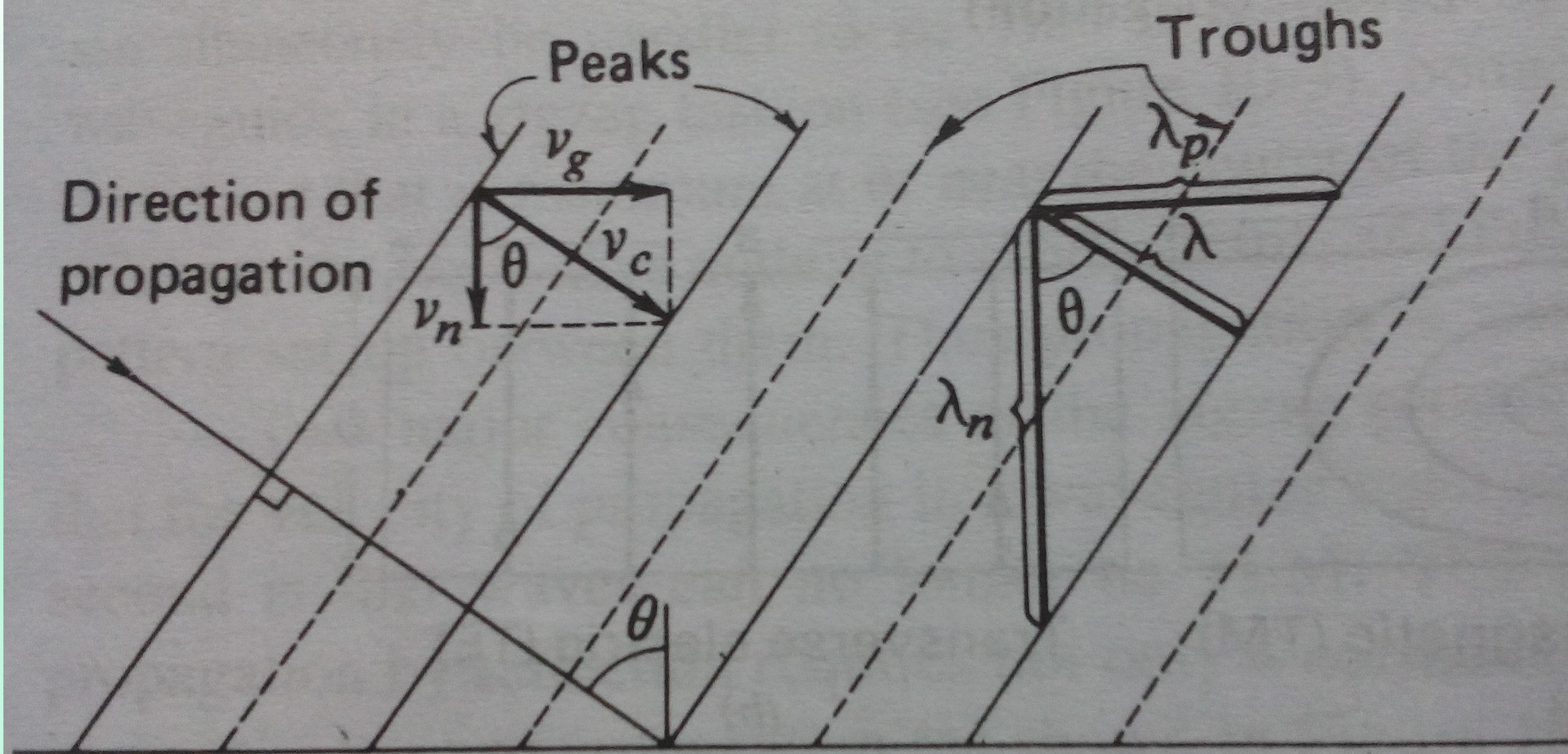


FIGURE 10-7 Plane waves at a conducting surface.

From fig→ The velocity of propagation in direction to parallel of conducting surface is

$\sin\Theta = v_g/v_c$

$v_g = v_c \sin\Theta$

Wavelength in this direction

$\lambda_p = \lambda/\sin\Theta$.............................1

The phase velocity is given by product of two i.e. f and λ_p

Therefore

$v_p = f\lambda_p$..............................2

Put eqn 2 in 1

$v_p = f\lambda/\sin\Theta = v_c/\sin\Theta$...............3

v_p is phase velocity

Group Velocity: It is velocity of wave in direction parallel to conducting surface and is given by

$v_g = v_c \sin\Theta$……………………..4

Multiply eqn 3 and 4

$v_g v_p = v_c / \sin\Theta \ * \ v_c \sin\Theta$

Hence $v_g v_p = v_c^2$

Therefore the product of group velocity and phase velocity of a signal propagating in WG is square of velocity of light in free space.

Cutoff Wavelength: It is smallest free space wavelength that is just unable to propagate in WG under given condition.

V_g and V_p

in Terms of cutoff wavelength

We know, $v_g v_p = v_c^2$ ....(1)

$V_p = f\,(\lambda/\sqrt{1-(\lambda/\lambda_0)^2})$

$V_p = V_c/\sqrt{1-(\lambda/\lambda_0)^2}$ ....(2)

Substituting (2) in (1)

$V_g = V_c^2/V_p$

$V_g = V_c^2((\sqrt{1-(\lambda/\lambda_0)^2})/V_c)$

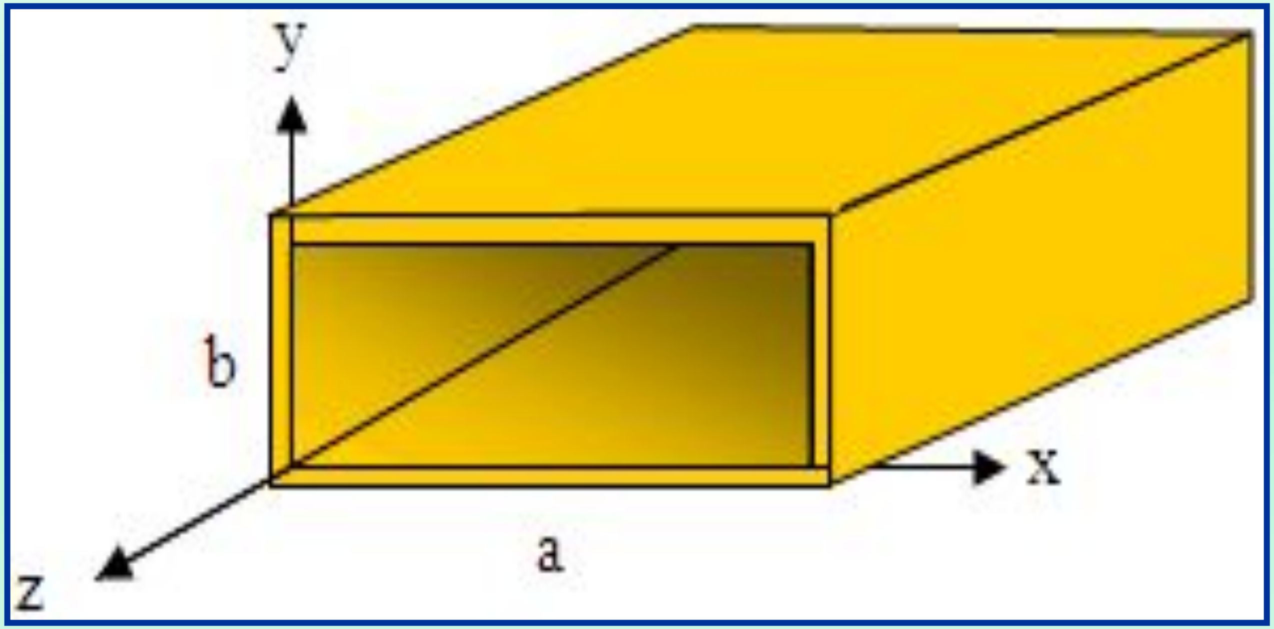
$V_g = V_c(\sqrt{1-(\lambda/\lambda_0)^2})$ ....(3)

Rectangular Waveguides

- Any shape of cross section of a waveguide can support electromagnetic waves of which rectangular and circular waveguides have become more common.

- A waveguide having rectangular cross section is known as *Rectangular waveguide*

Rectangular waveguide



Dimensions of the waveguide which determines the operating frequency range

Dimensions of the waveguide which determines the operating frequency range:

1. The size of the waveguide determines its operating frequency range.
2. The frequency of operation is determined by the dimension ‘a’.
3. This dimension is usually made equal to one – half the wavelength at the lowest frequency of operation, this frequency is known as the waveguide *cutoff frequency.*
4. At the cutoff frequency and below, the waveguide will not transmit energy. At frequencies above the cutoff frequency, the waveguide will propagate energy.

= characteristic wave impedance for $TE_{m,0}$ mode

$$Z_0 = \frac{Z}{\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}$$

Z = characteristic imp of space

$= 377\,\Omega = 120\pi$

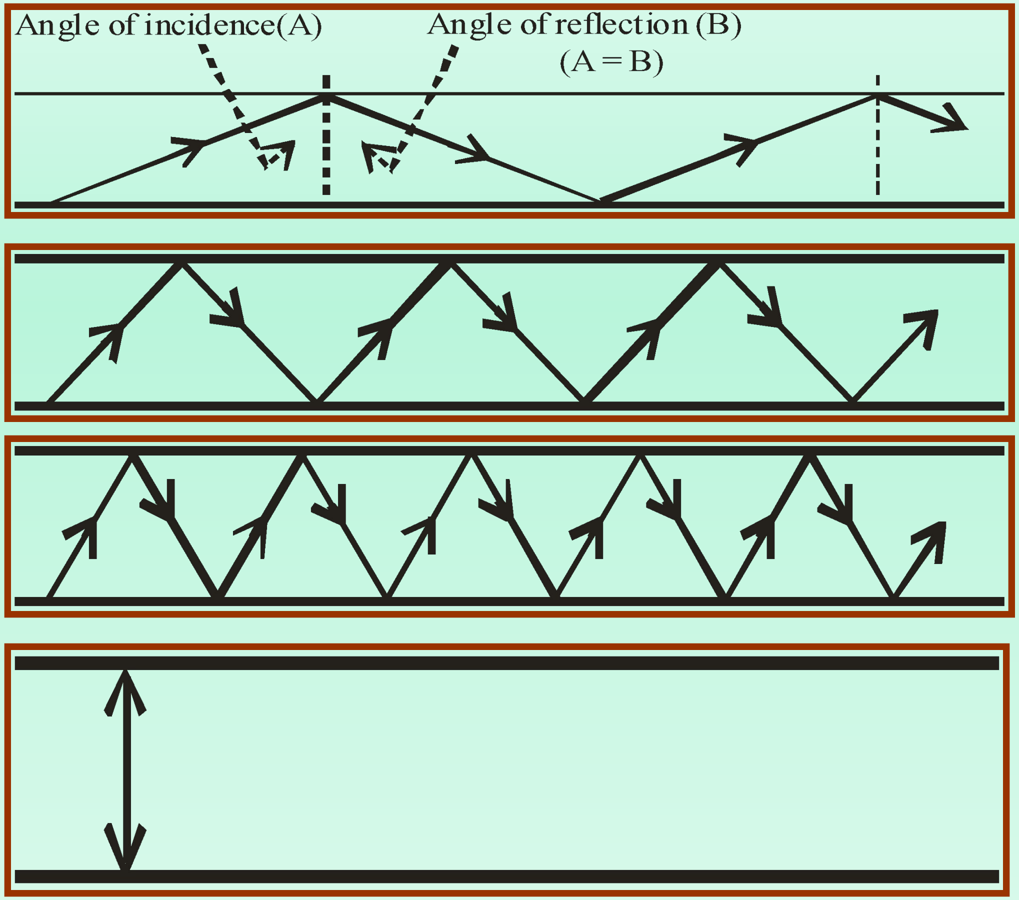
$TM_{m,n} = Z$

$$Z_0 = Z\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}$$

$$\lambda_0 = \frac{2a}{m}$$

Cutoff $\lambda_0 = \dfrac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}$ for TM mode

Wave paths in a waveguide at various frequencies



(a) At low frequency

(b) At medium frequency

(c) At high frequency

(d) At cutoff frequency

Wave propagation

- When a probe launches energy into the waveguide, the electromagnetic fields bounce off the side walls of the waveguide as shown in the above diagram.

- The angles of incidence and reflection depend upon the operating frequency. At high frequencies, the angles are large and therefore, the path between the opposite walls is relatively long as shown in Fig.

- At lower frequency, the angles decrease and the path between the sides shortens.

- When the operating frequency is reaches the cutoff frequency of the waveguide, the signal simply bounces back and forth directly between the side walls of the waveguide and has no forward motion.

- At cut off frequency and below, no energy will propagate.

Cut off frequency

- The exact size of the wave guide is selected based on the desired operating frequency.
- The size of the waveguide is chosen so that its rectangular width is greater than one – half the wavelength but less than the one wavelength at the operating frequency.
- This gives a cutoff frequency that is below the operating frequency, thereby ensuring that the signal will be propagated down the line.

Representation of modes

- The general symbol of representation will be $TE_{m,n}$ or $TM_{m,n}$ where the subscript *m* indicates the number of half wave variations of the electric field intensity along the *b* (wide) dimension of the waveguide.
- The second subscript *n* indicates the number of half wave variations of the electric field in the *a* (narrow) dimension of the guide.
- The $TE_{1,0}$ mode has the longest operating wavelength and is designated as the dominant mode. It is the mode for the lowest frequency that can be propagated in a waveguide.

Expression for cut off wavelength

- For a standard rectangular waveguide, the cutoff wavelength is given by,
- $$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Where a and *b* are measured in centimeters

Q. A wave propagated in parallel plane wg with frequency 6GHz and plane separation is 3cm. Calculate:

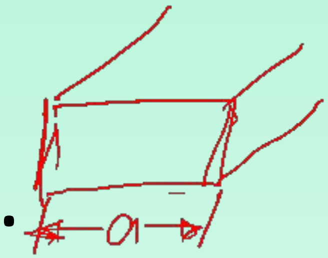
$TM_{m,n}$

$TE_{m,n}$ $TE_{1,0}$ $m=1$

a) cutoff wavelength for dominant mode.

b) wavelength in the wg, also for dominant mode.

c) group velocity and phase velocity.



Given ⇒ $f = 6\,GHz$, $a = 3\,cm$, $m = 1$

$$\lambda_c = \frac{2a}{m} = \frac{2\times 3}{1} = 6\,cm$$

$$\lambda_0 = \frac{v_c}{f} = \frac{3\times10^8}{6\times10^9} = 0.05\,m = 5\,cm$$

6 cm

$$v_g = v_c \sqrt{1 - \left(\frac{\lambda}{\lambda_0}\right)^2}$$

$$= \sqrt{1 - \left(\frac{5}{6}\right)^2} = 0.55$$

$$v_p = \frac{v_c}{\sqrt{1-\left(\frac{\lambda}{\lambda_0}\right)^2}}$$

metre

$$\lambda_p = \frac{\lambda}{}$$

$$= 3\times10^8\,m/s \times 0.55 = 1.65\times10^8\,m/s$$

$$v_g = 3\times10^8 \sqrt{1-\left(\frac{0.05}{0.06}\right)^2}$$

$$1.65\times10^{10}\,cm/s$$

Q. It is necessary to propagate 12GHz signal in a wg whose wall separation is 6cm. what is the greatest no. of half waves of electric intensity which it will be possible to establish b/w two walls. (largest value of m?) calculate the guide wavelength for this propagation.

10

$f = 10 \times 10^9$, $a = 6cm$

$\lambda = \frac{v_c}{f} = \frac{3 \times 10^{10}}{10 \times 10^9} = 0.025$ m $\sim$ 3cm

2.5cm

→ When $m = 1$ $\lambda_0 =$

$\lambda_0 = \frac{2a}{m} = \frac{2 \times 6}{1} = 12\,cm$ This mode propagate

$m = 2 = 2 = 12/2 = 6\,cm$ —do—

$m = 3 = 4\,cm$ —do—

$m = 4\,n = 3\,cm$ This mode not propagate

$m = 5$

$\lambda_p = \frac{3}{\sqrt{1 - \left(\frac{3}{4}\right)^2}} = 4.43\,cm$

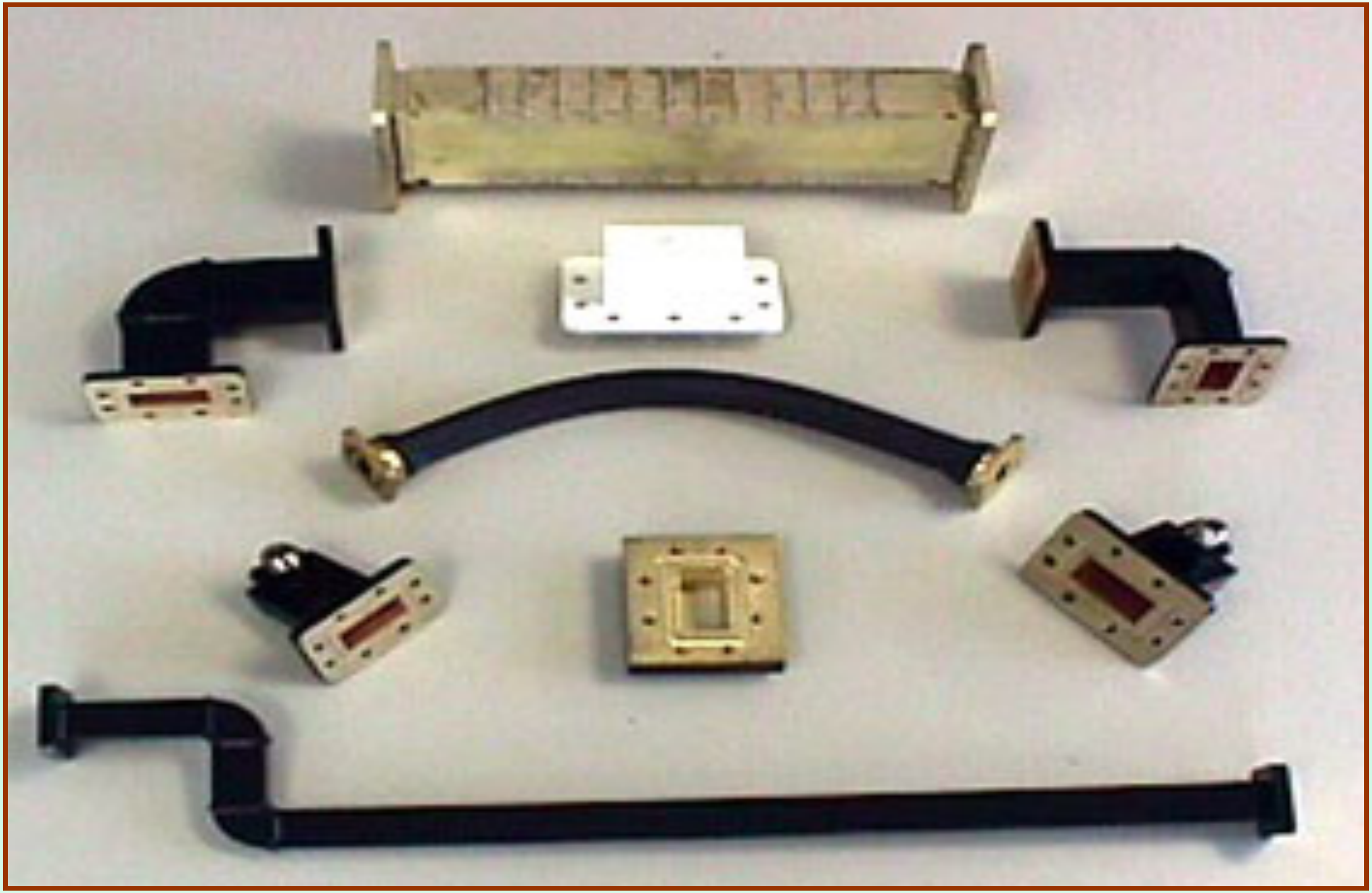
Q. A rectangular wg measures 3 x 4.5 cm internally and has 9GHz signal propagation in it. Calculate cutoff wavelength, guide wavelength, group and phase velocity and characteristic wave impedence for a) TE_{10} b) TM_{11} mode.

Circular wave guide

A Hollow metallic tube of uniform circular cross section for transmitting electromagnetic waves by successive reflections from the inner walls of the tube is called ***Circular waveguide***.

Circular wave guide

- The circular waveguide is used in many special applications in microwave techniques.
- It has the advantage of greater power – handling capacity and lower attenuation for a given cutoff wavelength. However, the disadvantage of somewhat greater size and weight.
- The polarization of the transmitted wave can be altered due to the minor irregularities of the wall surface of the circular guide, whereas the rectangular wave guide the polarization is fixed



Description

- The wave of lowest frequency or the dominant mode in the circular waveguide is the TE_{11} mode.
- The first subscript *m* indicates the number of full – wave variations of the radial component of the electric field around the circumference of the waveguide.
- The second subscript *n* indicates the number of half – wave variations across the diameter.
- The field configurations of TE_{11} mode in the circular waveguide is shown in the diagram below

Cut off wavelength

- The cutoff wavelength for dominant mode of propagation TE_{11} in circular waveguide of radius '*a*' is given by

$$\lambda_c = \frac{2\pi a}{1.814}$$

- The cutoff wavelength for dominant mode of propagation TM_{01} in circular waveguide of radius '*a*' is given by

$$\lambda_c = \frac{2\pi a}{2.405}$$

Applications of circular waveguide

- Rotating joints in radars to connect the horn antenna feeding a parabolic reflector (which must rotate for tracking)
- TE_{01} mode suitable for long distance waveguide transmission above 10 GHz.
- Short and medium distance broad band communication (could replace / share coaxial and microwave links)

Worked Example *2.4*

- ***The dimensions of the waveguide are 2.5 cm × 1 cm. The frequency is 8.6 GHz. Find (i) possible modes and (ii) cut – off frequency for TE waves.***

Solution:

Given a = 2.5 cm , b = 1 cm and f = 8.6 GHz
Free space wavelength

$$\lambda_0 = \frac{C}{f} = \frac{3 \times 10^{10}}{8 \times 10^{9}} = 3.488 \ \text{cm}$$

Solution

The condition for the wave to propagate is that

$\lambda_C > \lambda_0$

For TE_{01} mode

$$\lambda_C = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}} = \frac{2ab}{\sqrt{a^2}} = 2b = 2 \times 1 = 2 \text{ cm}$$

Since $\lambda_C < \lambda_0$, TE_{01} *does not* propagate

- For TE_{10} mode, $\lambda_C = 2a = 2 \times 2.5 = 5$ cm
- Since $\lambda_C > \lambda_0$, TE_{10} mode is a possible mode.

Cut – off frequency = $f_C = \dfrac{C}{\lambda_C} = \dfrac{3 \times 10^{10}}{5} = 6 \text{ GHz}$

Cut-off wavelength for TE_{11} mode $= \dfrac{2ab}{\sqrt{a^2 + b^2}}$

$$= \frac{2 \times 2.5 \times 1}{\sqrt{(2.5)^2 + (1)^2}} = 1.856 \text{ cm}$$

For TE_{11} $\lambda_C < \lambda_0$, TE_{11} is not possible.

The possible mode is TE_{10} mode.

The cut – off frequency = 6 GHz

Worked Example 2.5

- ***For the dominant mode propagated in an air filled circular waveguide, the cut – off wavelength is 10 cm. Find (i) the required size or cross sectional area of the guide and (ii) the frequencies that can be used for this mode of propagation***

The cut – off wavelength = λ_C = 10 cm

The radius of the circular waveguide ,

$$r = \frac{10 \times 1.841}{2\pi} = 2.93\ \mathrm{cm}$$

Solution

- **Area of cross section =** $\pi r^2 = \pi(2.93)^2 = 26.97 \text{ cm}^2$

The cut – off frequency

= $f_c = \frac{C}{\lambda_c} = \frac{3\times 10^{10}}{10}$ = 3 GHz

Therefore the frequency above 3 GHz can be propagated through the waveguide.

Area of cross section = 26.97 cm^2

Cut – off frequency = 3 GHz

Exercise Problem *2.2*

- ***A rectangular waveguide has a = 4 cm and b = 3 cm as its sectional dimensions. Find all the modes which will propagate at 5000 MHz.***

Hint:

The condition for the wave to propagate is that $\lambda_C > \lambda_0$
Here $\lambda_0 = 6$ cm ; λ_C for TE_{01} mode = 6 cm
Hence λ_C is not greater than free space wavelength λ_0 .
TE_{01} mode is not possible.

Exercise problem 2.3

- ***For the dominant mode of operation is an air filled circular waveguide of inner diameter 4 cm. Find (i) cut – off wavelength and (ii) cut – off frequency.***

Hint: λ_C = 6.8148 cm and f_c = 4.395 GHz

Thanks…