Note Title 12/23/2021

5. Bode plots:

→ Graph:

↓

scale: log. scale:

purpose:

1. To find stability of C.L.T.F.
2. To find Gain Margine (G.M.) & phase margine (P.M)
3. To find Gain cross over freq (ω_{gc}) & phase cross over freq (ω_{pc})

Bode plot :-

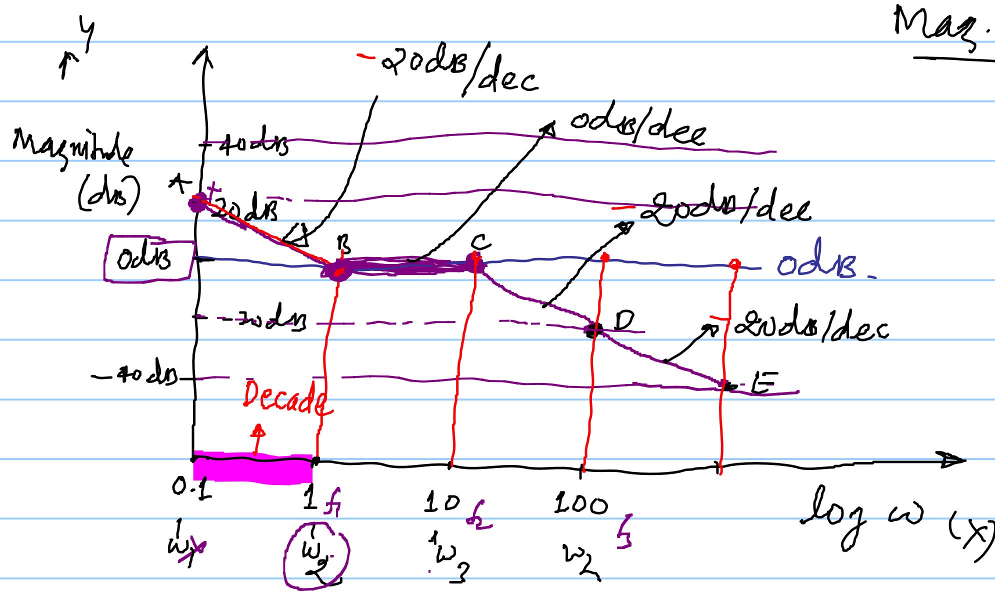
1) Magnitude plot.

2) phase plot.

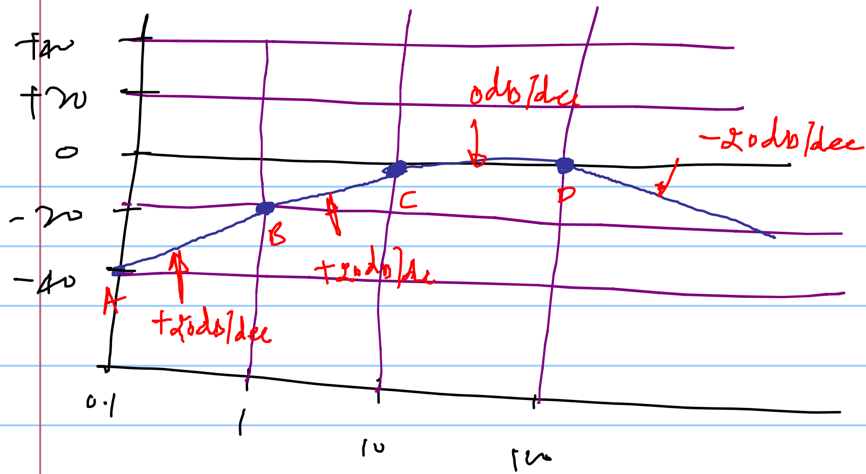
1) Magnitude plot:- It is a Graph betn Magnitude (in dB) and log. freq. i.e. $\log \omega$.

Mag. plot :

slope



here $\boxed{\omega_2 = 10\omega_1}$



2) phase plot: Graph betⁿ phase angles & log ω.

* Procedure to draw Bode plots:

1. 's' is replaced by 'jω'

* 2. Write the magnitude and convert it into dB.

i.e. Magnitude in dB = $20 \log |G(j\omega)|$

3. Write the phase angle by using $\tan^{-1}\left(\frac{\text{Img Part}}{\text{Real part}}\right)$

1st

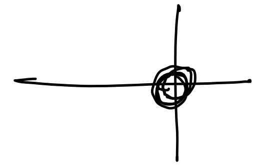
$\boxed{s = \sigma + j\omega}$

$$\boxed{z = x + iy \Rightarrow \theta = \tan^{-1}(y/x)}$$

z = Complex No, x = Real part, y = img part

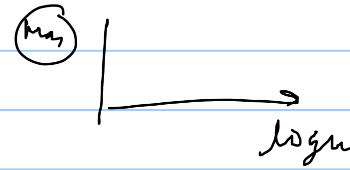
4. Vary 'ω' from minimum to maximum value & draw the [redacted] magnitude & phase plot

(pbm) 1. Draw Bode plot of O.L.T.F. given as follows,



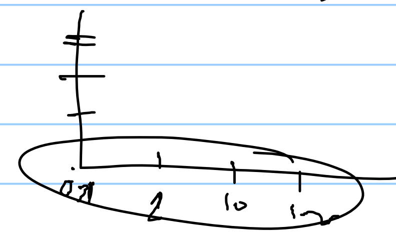
$\boxed{0 + j\omega}$ $\qquad G(s) = s \rightarrow$ ZERO at the origin $\Rightarrow$ $\boxed{\text{differentiator}}$

soln: 1. $G(j\omega) = j\omega \Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{\omega}{0}\right)}$ IP / RP



2. Magnitude in dB $\rightarrow 20 \log |G(j\omega)|$

$= 20 \log \omega$ dB



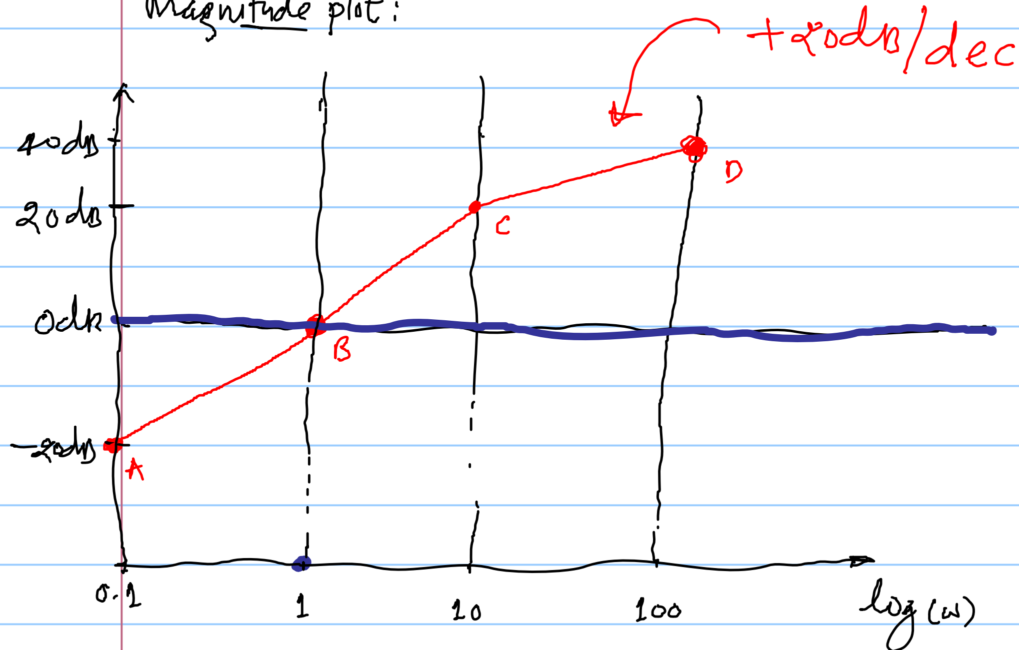
for $\omega = 0.1$ Mag $= 20\log(0.1) = -20$ dB

$\omega = 1$, mag $= 20\log(1) = 0$ dB

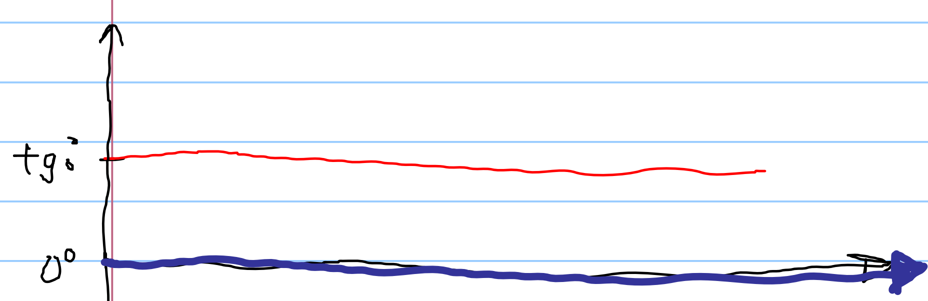
$\omega = 10$, mag $= 20\log(10) = +20$ dB

$\omega = 100$, mag $= 20\log(100) = +40$ dB

Magnitude plot:



3 phase Angle $(\phi) = \tan^{-1}\left(\frac{\omega}{0}\right) = \tan^{-1}(\infty) = +90^\circ$



-90°

Integrator

(2) $G(s) = \frac{1}{s}$

Soln: 1. $G(j\omega) = \frac{1}{j\omega}$

$$\boxed{\log\left(\frac{a}{b}\right) = \log a - \log b}$$

2. Magnitude $= 20\log\left|\frac{1}{j\omega}\right|$

$= \underbrace{\cancel{20\log 1}}_{0} - 20\log\omega$

Magnitude $= \underline{-20\log\omega \text{ dB}}$

$$\boxed{\begin{array}{l} \text{ZERO} = +20\,\text{dB} \\ \text{pole} = -20\,\text{dB} \end{array}}$$

At origine

4. For $\omega = 0.1$, Mag $= -20\log(0.1) = +20$ dB ✓

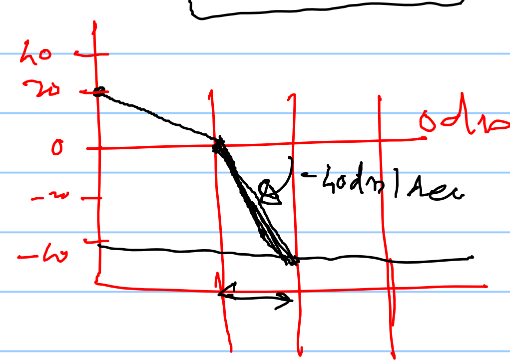
$\omega = 1$, Mag $= -20\log(1) = 0$ dB

$\omega = 10$, Mag $= -20\log(10) = -20$ dB

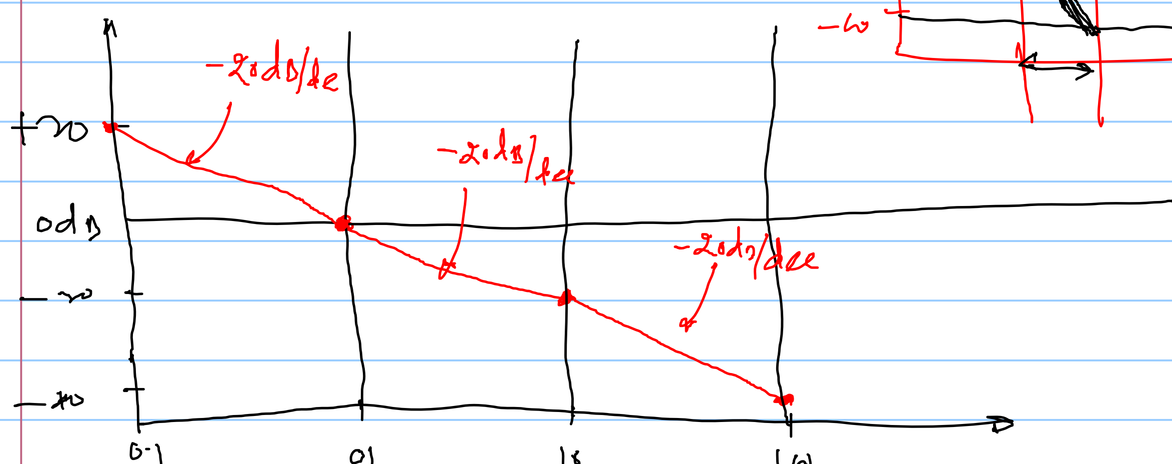
$\omega = 100$, Mag $= -20\log(100) = -40$ dB

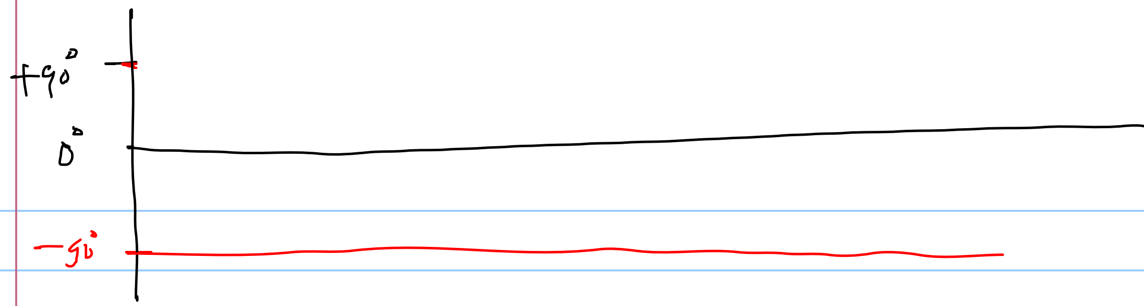
$$\boxed{\begin{array}{l} \frac{1}{j} = -j = -90^\circ \\ j = +90^\circ \end{array}}$$

3. $\boxed{\phi = \angle\left(\frac{1}{j}\right) = -90^\circ}$ ✓



Magnitude plot:





HW

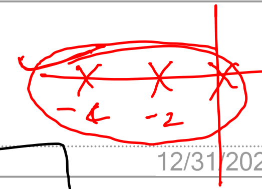
① $\frac{1}{s^2}$ $\downarrow$ 2 poles at origin

② s^2 $\downarrow$ 2 zeros at origin

Note Title 12/31/2021

Bode plot = Magnitude plot + Phase plot

$\boxed{M\,dB \text{ Vs } \log\omega}$ + $\boxed{\text{phase in deg Vs } \log\omega}$



1. Draw Bode plot of O.L.T.F Given and find Gain Margin (GM) & phase margin (PM).

$$G(s) = \frac{1000}{s(s+2)(s+4)}$$

$\boxed{(1+sT)}$

$$T.C = \frac{K(1+sT_1)(1+sT_2)}{s^n(1+sT_3)(1+sT_4)}$$

1. first, Convert the given T.F in time-const form

$$\text{i.e. } G(s) = \frac{1000}{2\times s\left(1+\frac{s}{2}\right)\times 4\left(1+s/4\right)} = \frac{\cancel{1000}\ 125}{\cancel{8}s(1+s/2)(1+s/4)}$$

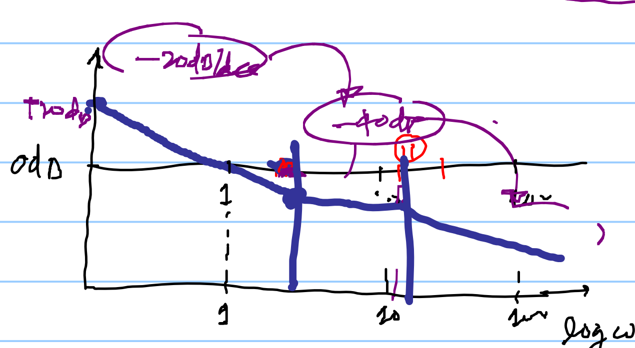
$$\therefore G(s)\Big|_{\text{Time-const form}} = \frac{125}{s\left(1+\frac{s}{2}\right)(1+s/4)} = \frac{125}{s(1+0.5s)(1+0.25s)}$$

$$= \frac{125}{s(1+0.5j\omega)(1+0.25j\omega)}$$

2. find out all the corner frequencies & write them in ascending order.

Cornor freq :- It is the frequency at which slope changes from one level to other level.

- It is nothing but simple pole & zero



Here 5 & 11 are corner frequencies.

$\boxed{\omega c = 1/T}$ ⟹ Now, we have

$$G(s) = \frac{125}{s(1+0.5s)(1+0.25s)}$$

for $(1+0.5s)$, $T = 0.5$ $\therefore \omega_{c_1} = \frac{1}{0.5} = 2$ rad/sec for $(1+0.25s)$, $\tau = 0.25$ $\therefore$ $\omega_{c_2} = \frac{1}{0.25} = 4$ rad/sec

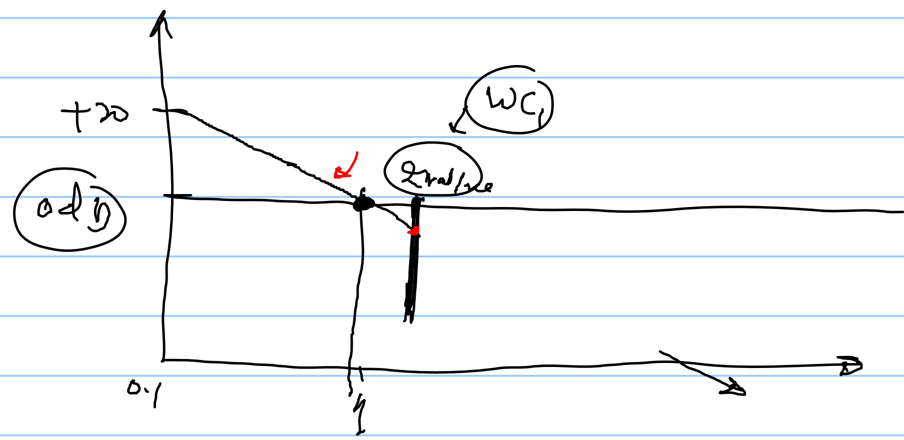
3. Find initial slope of the Magnitude plot:

- Initial slope is given by poles/zeros at the origine.
- i.e if there is a single pole at origine in T.F. then
  initial slope = -20dB/dec
  Similarly for 2 poles at origine = -40dB/dec

1 pole at → ⊖20dB7.

- Whenever the T.F. consists of poles & zeros at the origine, then the magnitude plot starts with a magnitude of opposite sign of slope at a freq of 0.1 rad/sec & it should pass through 0dB line at $\omega = 1$ rad/sec & extended upto 1st corner freq if present otherwise extend it to ∞.

Pole at org.
S = $\boxed{-20\text{dB/dec}}$
T.F



*4. Find the shift to get resultant magnitude plot:-

$$\boxed{\text{shift} = 20\log(k)}$$

Here $k = 125$

$\therefore$ shift $= 20\log(125) = 42$ dB

5. Form the magnitude table:

S.N.	Parameter	corner freq	slope	Resultant slope
1.	125 (42dB)	–	0dB/dec	0dB/dec
2.	$\frac{1}{s}$ (pole at origine)	–	−20dB/dec	−20dB/dec (initial slope) ✓
3.	$\frac{1}{1+0.5s}$ (simple pole)	2 rad/sec	−20dB/dec	−40dB/dec ✓
4.	$\frac{1}{1+0.2s}$ (simple pole)	4 rad/sec	−20dB/dec	−60dB/dec

6. Form a phase table:

We have, $G(s) = \dfrac{125}{s(1+0.5s)(1+0.25s)}$

pole at origine (−90°)

simple pole $\tan^{-1}(0.5\omega)$

$\tan^{-1}(0.25\omega)$

$$\phi = -90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.25\omega)$$

S.N.	ω	ϕ
1.	✓ 0.1	-95.71°
2.	1	-135°
3.	2	-153.43°
4.	4	-165.96°
5.	5 ✓	-168.69°
6.	10 ✓	-174.28°
7.	100	-179.42°

7. find Gain Cross over frequency from magnitude plot & phase cross over frequency from phase plot.

1. Gain Cross over freq: (ω_{gc}) :- It is the intersection pt of Magnitude plot with 0dB line

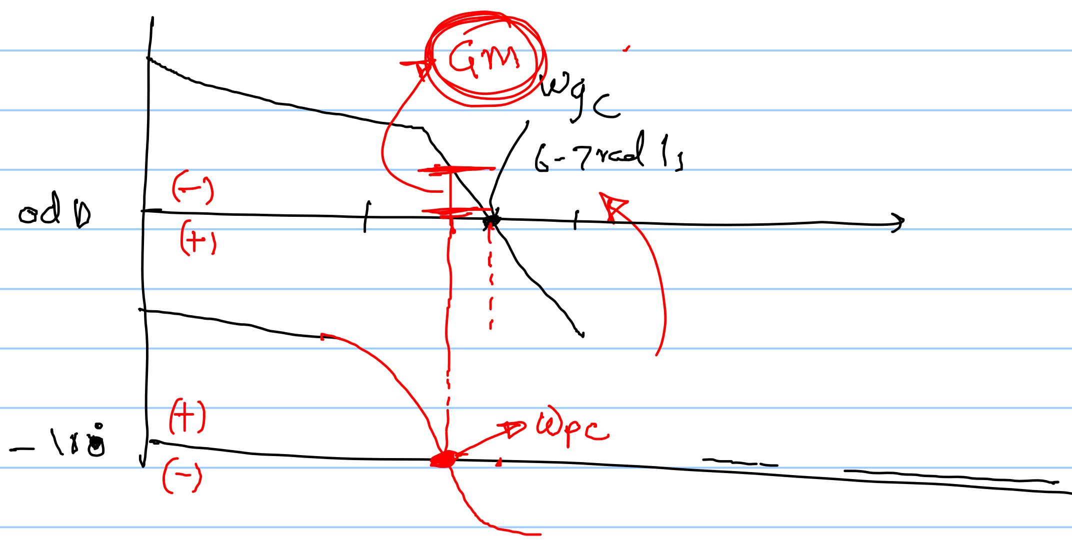
→ Here $\boxed{\omega_{gc} = 6 \text{ rad/sec}}$

Phase Margin (PM):

2. Phase Cross over frequency: (ω_{pc}): it is the intersection pt of phase plot with $-180°$ line

→ Here $\boxed{\omega_{pc} = \infty}$

Gain Margin (GM):



8. Conditions for stability:

1) if $\omega_{pc} > \omega_{gc}$ → stable
2) If $\omega_{pc} = \omega_{gc}$ → Marginally stable
3) if $\omega_{pc} < \omega_{gc}$ → Unstable

Thus $G(s) = \dfrac{1080}{s(s+2)(s+4)}$ is stable

NOTE: → To draw Bode plot in Sci-lab use "bode(G)" → Hz

"bode(G,'rad')" → rad/sec

→ For GM & PM :-

syntax: For GM → [GM, PCF] = g_margin(G)

GM =

PCF (ω_{pc}) (Hz) = ≅ 9.2 rad/sec

For PM → [PM, GCF] = p_margin(G)

PM =

GCF (ω_{gc}) (Hz) =

1.5 Hz → rad/sec

HW — $G(s) = \dfrac{400(s+1)}{s(s+2)}$

ZERO unstability

K = 400

ω_{gc} = 10.3 rad/sec

ω_{pc} = 1.94 rad/c

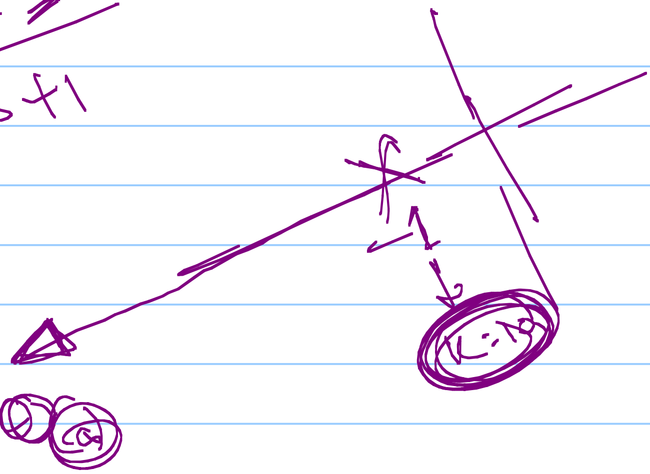
$\omega_{pc} < \omega_{gc}$ US

ZO

(s+1)

② For Root locus use "evans(G)"

$$\frac{10}{s+1}$$



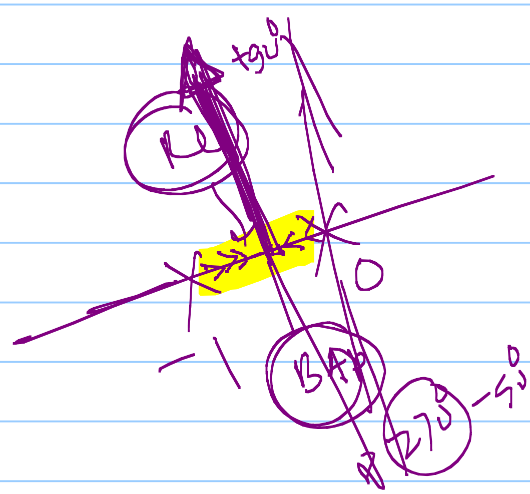
$P = 1$

$Z = 0$

$N = 1$

-180

$$\frac{1000}{s(s+1)}$$



Note Title 1/13/2022

* Gain Cross Over frequency (ω_{gc}) : The frequency at which the magnitude is 0dB, is called gain crossover frequency.

* Phase crossover frequency (ω_{pc}): The frequency at which the phase angle is $-180°$ is called phase cross-over frequency.

* Gain Margin (GM): It is the factor by which the system gain is increased to bring the system to the verge of stability i.e. Marginaly stable.
  - It is the reciprocal of magnitude at ω_{pc}

$$\therefore \boxed{GM = \frac{1}{|GH(j\omega)|}\bigg|_{\omega=\omega_{pc}} \quad \text{in linear}}$$

$$\boxed{GM\big|_{dB} = -20 \log |G(j\omega)|\big|_{\omega=\omega_{pc}}}$$

* Phase Margin (PM): phase margin is the additional phase lag required to add to the system to bring the system to the verge of stability i.e. (MS)

$$\boxed{PM = 180° + \angle GH\big|_{\omega=\omega_{gc}}}$$