

Q3 Solve the following 0/1 knapsack problem
 $n = 4$, $m = 18$, $w = (3, 8, 6, 4)$, $p = (9, 10, 12, 9)$

→

formula

$$\textcircled{i} f_i(y) = \max \begin{cases} f_{i-1}(y) \\ f_{i-1}(y - w_i) + p_i \end{cases}$$

$$\textcircled{ii} f_1(m') = p_1 \quad \text{if } (w_1 \leq m') \\ = 0, \quad \text{otherwise}$$

$$f_1(-m') = -\infty$$

$$p_1 = 9, \quad p_2 = 10, \quad p_3 = 12, \quad p_4 = 9 \\ w_1 = 3, \quad w_2 = 8, \quad w_3 = 6, \quad w_4 = 4$$

$$\begin{aligned} \text{f.f.f.f. } f_4(18) &= \max \begin{cases} f_3(18) \\ f_3(18-4) + 9 \end{cases} \\ &= \max(31, 31) \\ &= 31 \end{aligned}$$

$$\begin{aligned} \text{f.f.f. } f_3(18) &= \max \begin{cases} f_2(18) \\ f_2(18-6) + 12 \end{cases} \\ &= \max(19, 31) \\ &= 31 \end{aligned}$$

$$\begin{aligned} f_3(14) &= \max \begin{cases} f_2(14) \\ f_2(14-6) + 12 \end{cases} \\ &= \max(19, 22) \\ &= 22 \end{aligned}$$

$$\begin{aligned}
 f_1(18) &= \max \begin{cases} f_3(18) \\ f_1(18-8)+10 \end{cases} \\
 &= \max(9, 19) \\
 &= 19
 \end{aligned}$$

$$\begin{aligned}
 f_2(12) &= \max \begin{cases} f_1(12) \\ f_1(12-8)+10 \end{cases} \\
 &= \max(9, 19) \\
 &= 19
 \end{aligned}$$

$$\begin{aligned}
 f_2(14) &= \max \begin{cases} f_1(14) \\ f_2(14-8)+10 \end{cases} \\
 &= \max(9, 19) \\
 &= 19
 \end{aligned}$$

$$\begin{aligned}
 f_2(8) &= \max \begin{cases} f_1(8) \\ f_1(8-0)+10 \end{cases} \\
 &= \max(0, 10) \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \text{Solution vector} &= \{p_4, p_3, p_2, p_1\} \\
 &= \{1, 1, 1, 0\}
 \end{aligned}$$

$$\begin{aligned}
 \text{Total profit} &= p_4 + p_3 + p_2 \\
 &= 10 + 12 + 9 \\
 &= \underline{\underline{31}}
 \end{aligned}$$

$$\begin{aligned}
 \text{weight} &= w_4 + w_3 + w_2 \\
 &= 4 + 6 + 8 \\
 &= \underline{\underline{18}}
 \end{aligned}$$

maximum profit for given knapsack problem is 31.