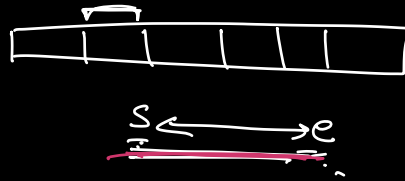


Prefix Sum

Carry forward



Subarray

- Contiguous part of an array
- ⇒ ◦ Complete array is a subarray of itself.
- Single element can also be a subarray.
- Empty array is a subarray.

But we will only be talking about
Non-empty sub-arrays.

A : 3, 4, 5, 6, -2, 8, 10

5, 6, -2 ✓

3, 4, 6, -2 ✗

10 ✓

3, 10 ✗

Ques

[⁰4, ¹2, ²10, ³3, ⁴12, ⁵-2, ⁶15]

s

e

1

1

1

2

1

3

1

6

⇒ 6

No. of Non-empty sub-arrays in an array of size N .

$$\begin{aligned} \# \text{ of sub-arrays} &= \# \text{ subarrays starting from index } 0 = N \\ &+ \\ &\# \text{ subarrays starting from index } 1 = N-1 \\ &+ \\ &\# \text{ subarrays starting from index } 2 = N-2 \\ &+ \\ &\# \text{ subarrays starting from index } 3 = N-3 \\ &\vdots \\ &+ \\ &\# \text{ subarrays starting from index } N-1 = 1 \end{aligned}$$

$$\begin{aligned} \# \text{ No of sub arrays} &= \frac{N \times (N+1)}{2} \\ &\approx O(N^2) \end{aligned}$$

Q Print all values of a subarray.

```
void printSubArray ( A[], s , e ) {  
    for ( i = s; i <= e; i++ ) {  
        Print ( A[i] );  
    }  
}
```

$s \rightarrow 0$
 $e \rightarrow N-1$
TC: $O(N)$

Q Find the sum of a given subarray.

```
int addSubArray ( A[], s, e ) {  
    sum = 0;  
    for (i = s; i <= e; i++) {  
        sum = sum + A[i];  
    }  
    return sum;  
}
```

$s \rightarrow 0$
 $e \rightarrow N-1$
TC: $O(N)$

Ques $A : [4, 2, 10, 3, 12, -2, 15]$

$$N = 7$$

$$\frac{N \times (N+1)}{2} = 7 \times 4 = \underline{28}$$

Q Print all subarrays of a given array (size N)

A : 2, 8, 9

<u>s</u>	<u>e</u>	
0	0	<u>[2]</u>
0	1	[2, 8]
0	2	[2, 8, 9]
1	1	[8]
1	2	[8, 9]
2	2	[9]

[illegible]
$$\text{for } (i=0; i < N; i++) \{$$
$$f_0(j=i; j < N; j++) \{$$

// Subarray from i to j

⇒ Print SubArray (A, $\underset{\substack{\downarrow \\ \text{start}}}{i}, \underset{\substack{\downarrow \\ \text{end}}}{j}$); ⇒ $O(N)$

$$O(N^2)$$

↑ Iterate over
all sub-arrays

TC: $O(N^3)$

 $O(N^2)$

↑
of subarray

 $\times O(N)$

↑
Print

$$\Rightarrow O(N^3)$$

Q Print the sum of every single sub-array.

A : $3, 2, -1, 4$

<u>s</u>	<u>e</u>		
0	0	[3]	→ 3
0	1	[3, 2]	→ 5
0	2	[3, 2, -1]	→ 4
0	3	[3, 2, -1, 4]	→ 8
⋮			⋮

```
for (i=0; i<N; i++) {
    for (j=i; j<N; j++) {
```

$O(N^2)$
↑ Iterate over all sub-array

```
        // subarray from i to j
        [ addSubArray(A,  $\underbrace{i}_{\text{start}}, \underbrace{j}_{\text{end}}$ );  $\Rightarrow O(N)$  ]
    }
```

$Sum = Ps[j] - Ps[i-1]$

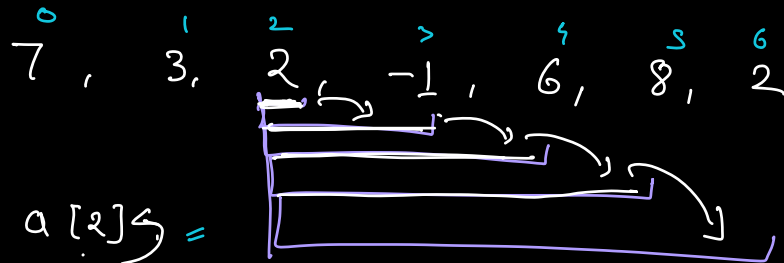
↑ $O(1)$

TC : $O(N^3)$

| use Ps → $O(N^2)$

SC : $O(N) \Leftarrow$ (PS array)
(Extra)

Q Print sum of all the subarrays starting from index $\rightarrow 3$



$$2, 2 \Rightarrow a[2] =$$

$$2, 3 \Rightarrow a[2] + a[3]$$

$$2, 4 \Rightarrow a[2] + a[3] + a[4]$$

$$2, 5 \Rightarrow a[2] + a[3] + a[4] + a[5]$$

$$2, 6 \Rightarrow a[2] + a[3] + a[4] + a[5] + a[6]$$

```
for (i=0; i<N; i++){
```

```
    sum = 0; //
```

```
        for (j=i; j<N; j++){
            sum = sum + a[j];
            Print (sum);
        }
    }
```

Prints the sum of all the subarrays starting from index i

A : 1, 2, 3

i	j	Sum
0	0	1
0	1	3
0	2	6
1	1	2
1	2	5
2	2	3

TC: $O(N^2)$

SC: $O(1)$

(Extra)

Q Given an array. Find the sum of all subarray sums.

Google

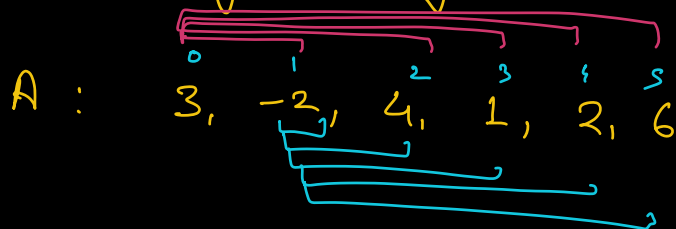
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A : ⁰1, ¹2, ²3

<u>S</u>	<u>e</u>	Sum
0	0	1 → a[0] ←
0	1	3 → a[0] + a[1] ←
0	2	6 → a[0] + a[1] + a[2] ←
1	1	2 → a[1] —
1	2	5 → a[1] + a[2] =
2	2	3 → a[2] —

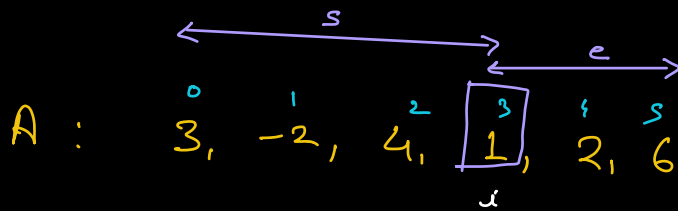
$$\Sigma = 20 \rightarrow 3 \times a[0] + 4 \times a[1] + 3 \times a[2]$$

// In how many sub-array element a[i] will be present.

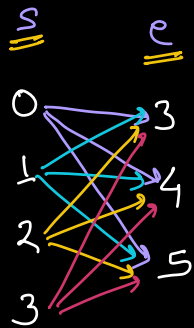


$$i=0 \Rightarrow 6$$

$$i=1 \Rightarrow$$



index i will be present in only those subarrays for which



$$[a, b] = b - a + 1$$

$$s \leq i \rightarrow [0, i] \Rightarrow |s| = i + 1$$

$$e \geq i \rightarrow [i, N-1] \Rightarrow |e| = N - i$$

$$\text{No of subarrays} = |s| \times |e|$$

$$\text{\# of subarrays} = (i+1) \times (N-i)$$

$A :$	⁰ 3	¹ -2	² 4	³ -1	⁴ 2	⁵ 6
$ s :$	1	2	3	4	5	6
$ e :$	6	5	4	3	2	1
$\text{\# of subarray containing ends } i$	6	10	12	12	10	6

$$\sum \text{subarrays} = (6 \times 3) + (-2 \times 10) + (4 \times 12) + (-1 \times 12) + (2 \times 10) + (6 \times 6) = 90$$

