

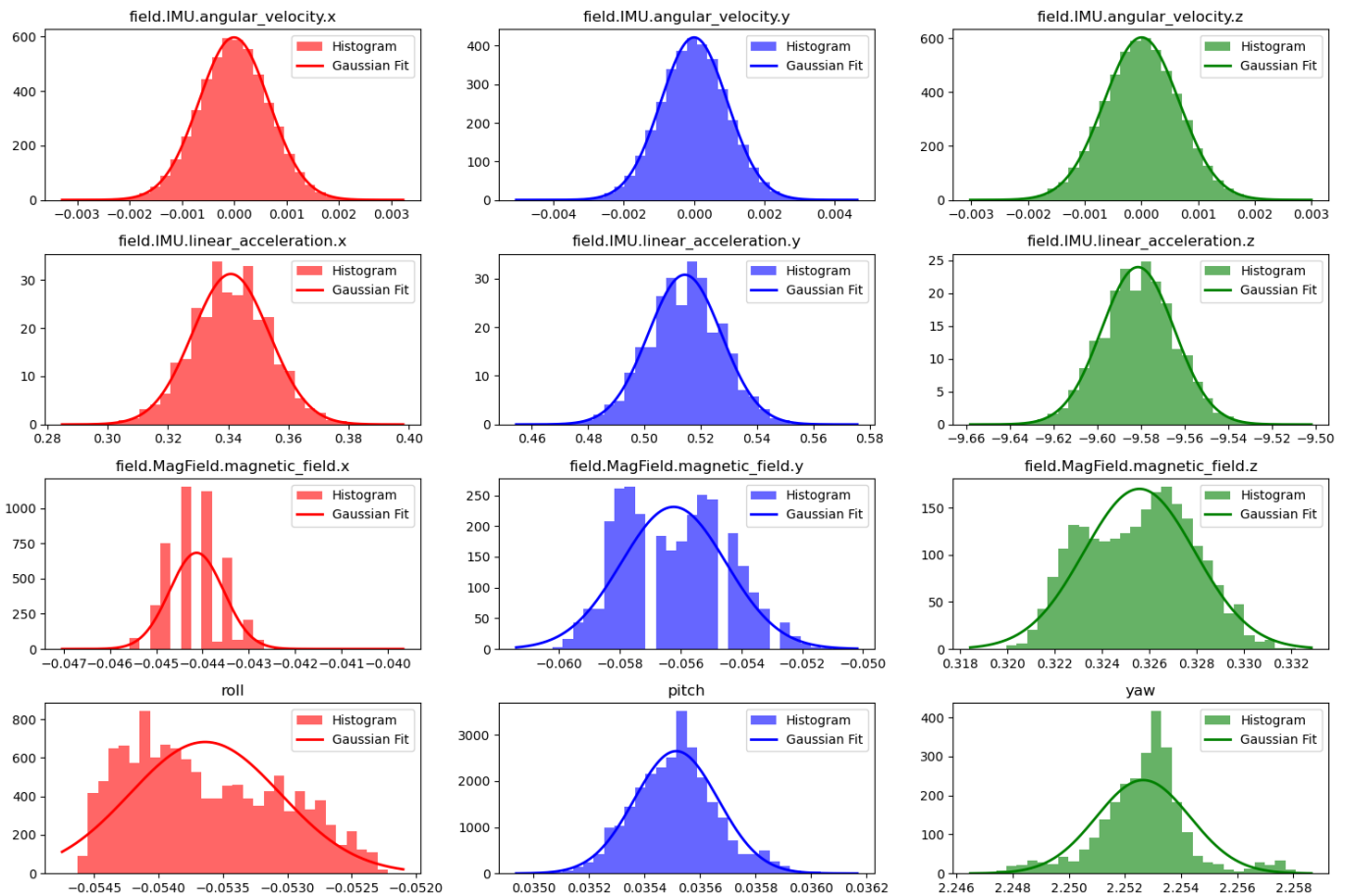
EECE 5554: Lab 4 Report

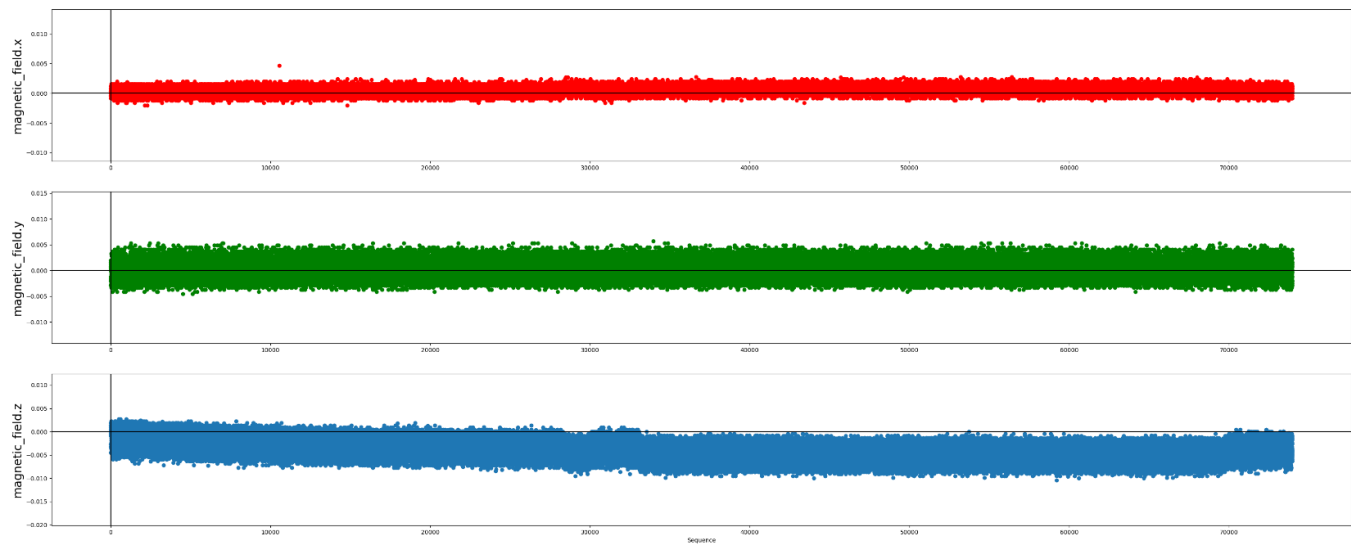
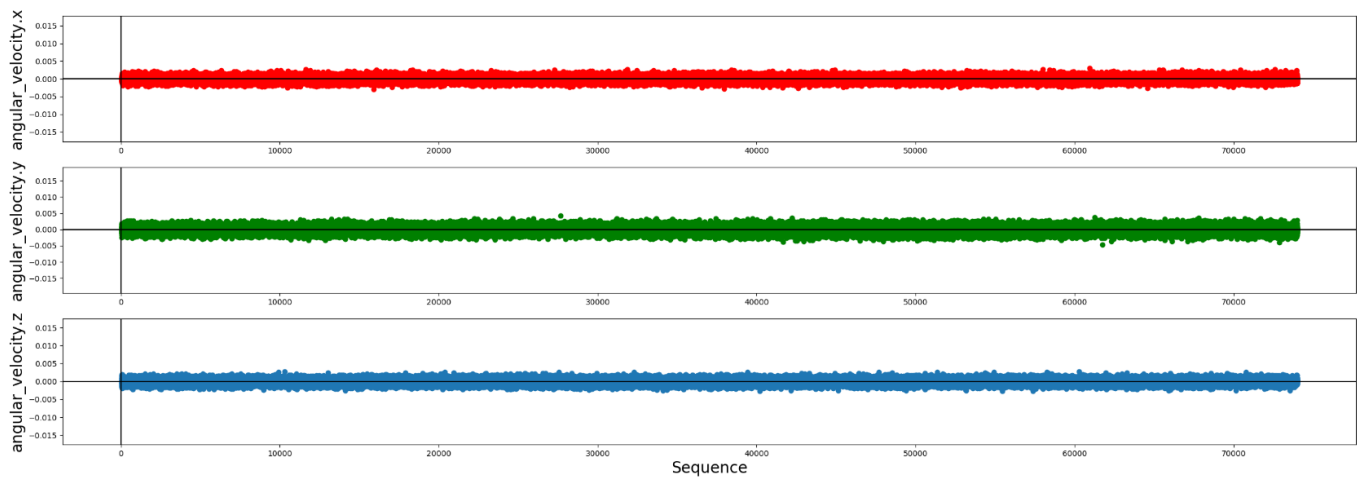
GPS & IMU Pose Estimation for a Vehicle

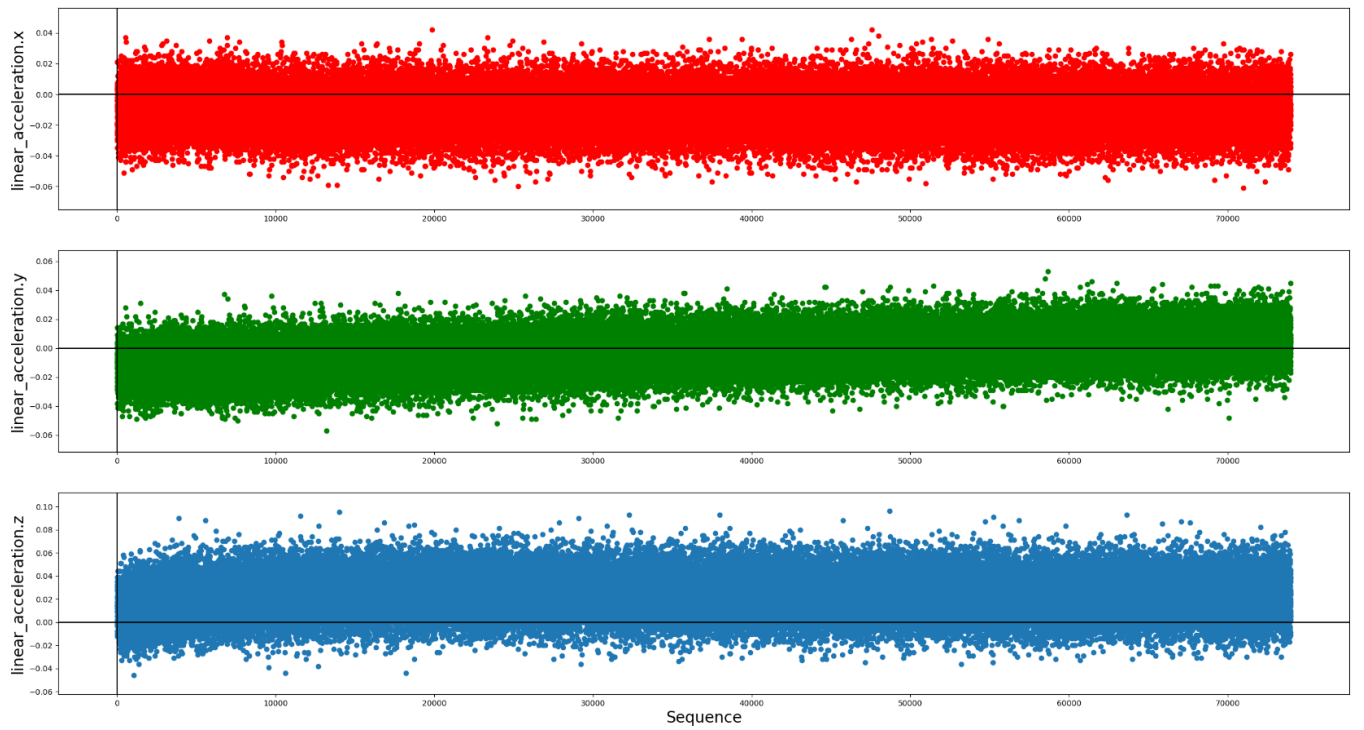
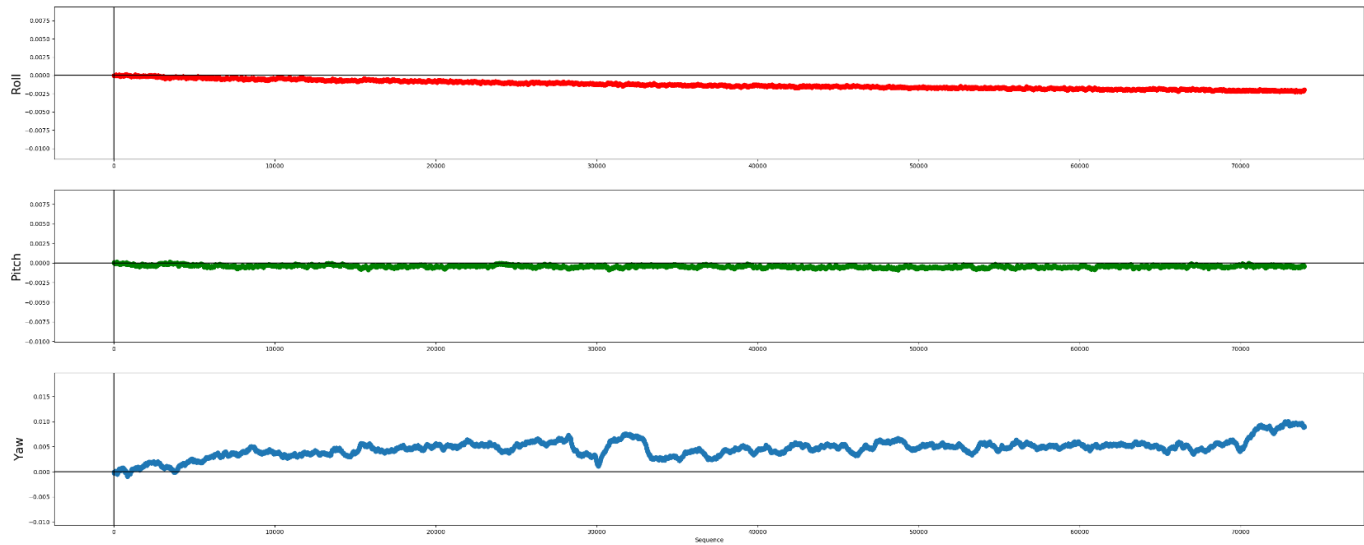
Aim:

1. To collect short-term IMU data at a stationary pose and analyze the error.
2. To collect moving data in a vehicle and perform the following tasks:
 - a. Perform Magnetometer Data Calibration and calculate Yaw estimate.
 - b. Calculate Forward Velocity from Linear Acceleration
 - c. Perform Dead Reckoning and plot the trajectory of the vehicle.

Part 1: Stationary Data Analysis







Column	Range	Mean	Standard Deviation	Drift (per hour)
angular_velocity.x	0.005924	-0.000008	0.000668	0.000120
angular_velocity.y	0.008824	0.000001	0.000946	-0.000044
angular_velocity.z	0.005469	0.000003	0.000661	0.000012
linear_acceleration.x	0.103000	0.340901	0.012759	-0.002971
linear_acceleration.y	0.11	0.514315	0.012946	0.038681
linear_acceleration.z	00.142000	-9.581304	0.016636	0.003671
magnetic_field.x	0.0067	-0.044128	0.000583	0.000871
magnetic_field.y	0.1021	-0.056231	0.001725	0.000240
magnetic_field.z	0.0131	0.325593	0.002345	-0.006147
roll	0.002409	-0.053638	0.000585	-0.003869
pitch	0.001117	0.035515	0.000151	-0.000346
yaw	0.011048	2.252649	0.001668	0.006994

- The drift is calculated by calculating the slope of the data column by performing linear regression.
- The mean can be seen as the offset for the angular velocity fields as they have the true value 0.
- The angular velocity fields have Gaussian Distribution.
- The linear acceleration fields have a close resemblance to Gaussian Distribution.
- Roll has a higher drift than Yaw and Pitch, as seen in the graph and the Drift value in the table.

Part 2: Moving Data Analysis

I] Magnetometer Calibration for Yaw Estimate:

- The vehicle had initially moved in circles to perform calibration for the magnetometer.
- The two sources of distortion present in the data were due to Soft Iron sources and Hard Iron sources present in the environment (inside the vehicle).
- Hard Iron errors are offsets that are caused by the presence of external magnetic fields that are not part of the Earth's magnetic field.
- The presence of ferrous materials causes soft iron errors in magnetometer readings.
- Therefore, the presence of other electronic systems and ferrous materials in the vehicle may have caused these distortions.
- After plotting the circular data, the offset from the origin (Hard Iron distortion) and distorted ellipse shape (Soft Iron Distortion) can be observed.

The Vector Nav Website has provided a calibration for the Magnetometer on their website^[2]. The calibration wasn't applicable as the vehicle provided a new environment with different Soft and Hard Iron sources.

The method used to calibrate Magnetometer data is to fit an ellipse to the data after removing the hard iron offset^[1]. Then, the ellipse was transformed into a circle and given a rotation to match the initial yaw estimate from the IMU.

The Mathematical Representation of the Correction for the Magnetometer is given by:

$$Mag_{cal} = R_{corr}(Mag_{raw} - Mag_{hard})$$

where,

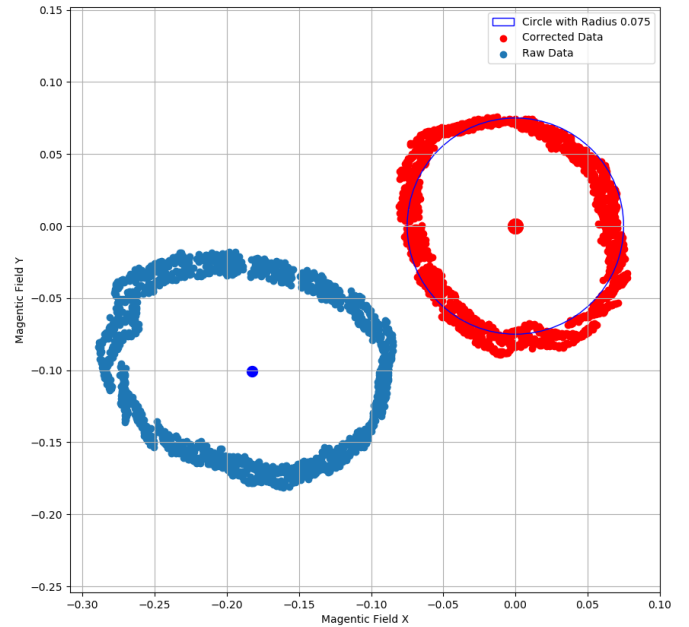
Mag_{cal} – Calibrated Magnetic Field

Mag_{raw} : Calibrated Magnetic Field

Mag_{hard} : Hard Iron Correction

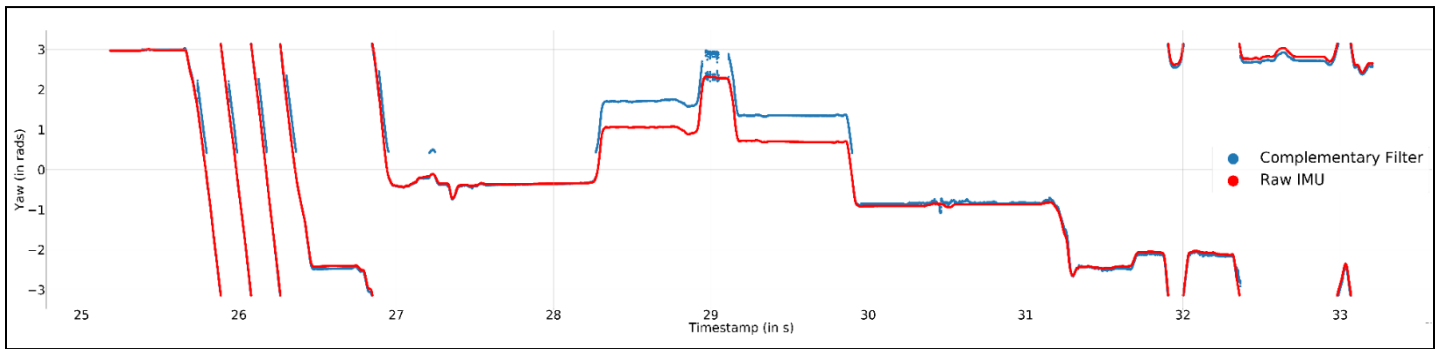
R_{corr} = Soft Iron Correction Matrix

$$\theta_{mag} = \arctan(Mag_x/Mag_y)$$



The complementary filter was implemented on the Magnetometer Yaw and the estimate obtained by integrating the gyroscope Z reading. The filter has a bias of ($\alpha=0.9$) in favor of the Gyroscope reading.

Before passing the Magnetometer data, a low pass filter was applied to smoothen the curve.



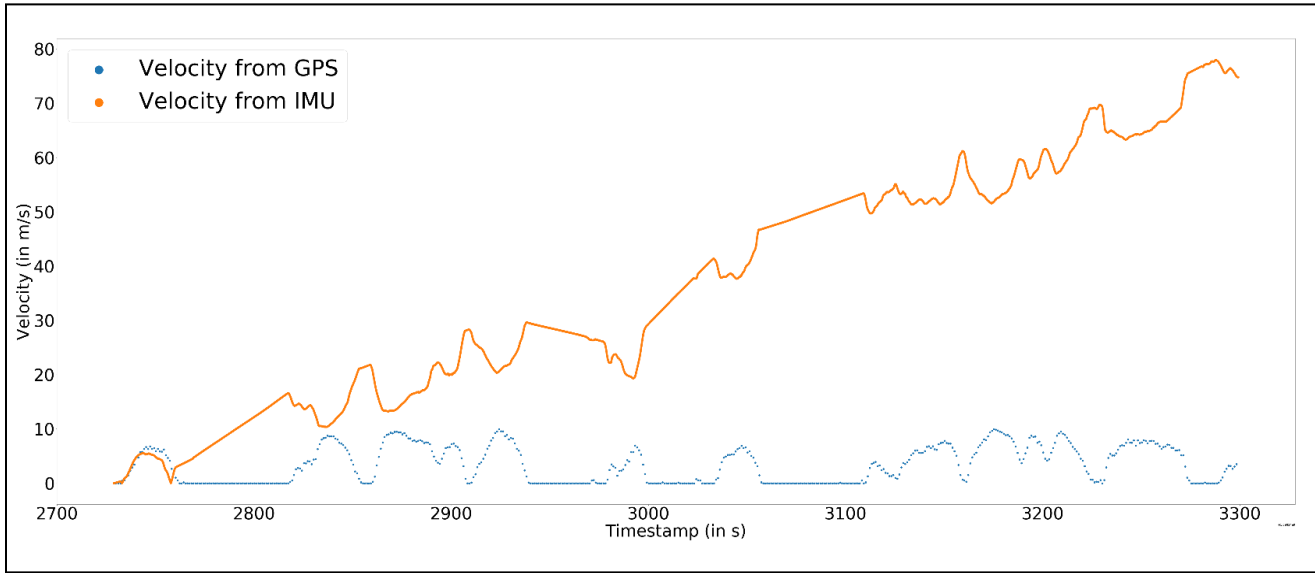
The Gyroscope reading is way more reliable when the vehicle moves (rotating). The Magnetometer is important for initializing the value and correcting it when it stops at junctions. To get a good estimate, both estimates must be combined so that their strong points are given priority. A speed estimate can be added for better results that factor in the Rotation rate.

II] Forward Velocity:

The forward velocity estimate is found from the IMU by Integrating the Linear Acceleration X field, as the IMU was aligned with the vehicle's heading

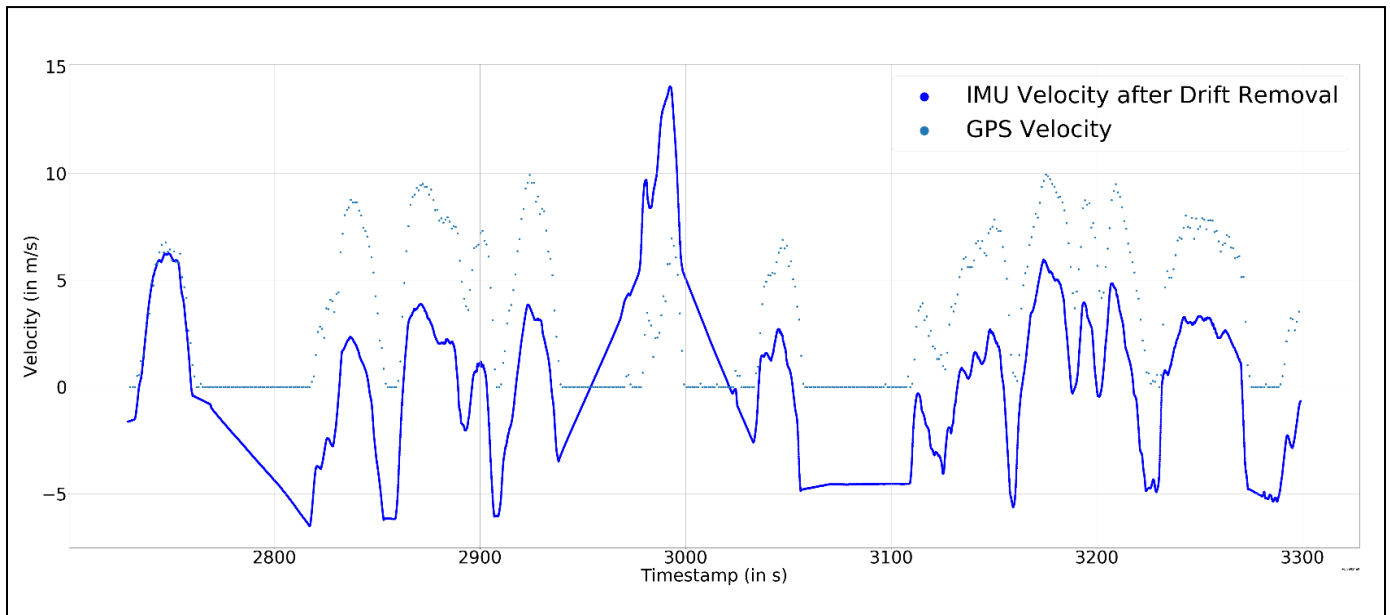
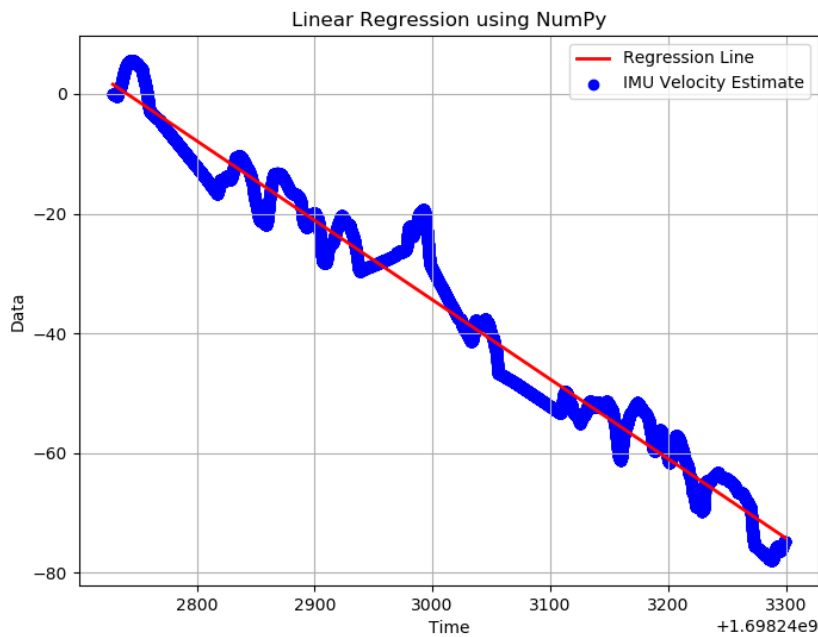
$$V_x = \int_0^t a_x dt$$

The initial velocity is zero as the vehicle starts from rest for the considered data section. The acceleration is integrated using the cumtrapz function.



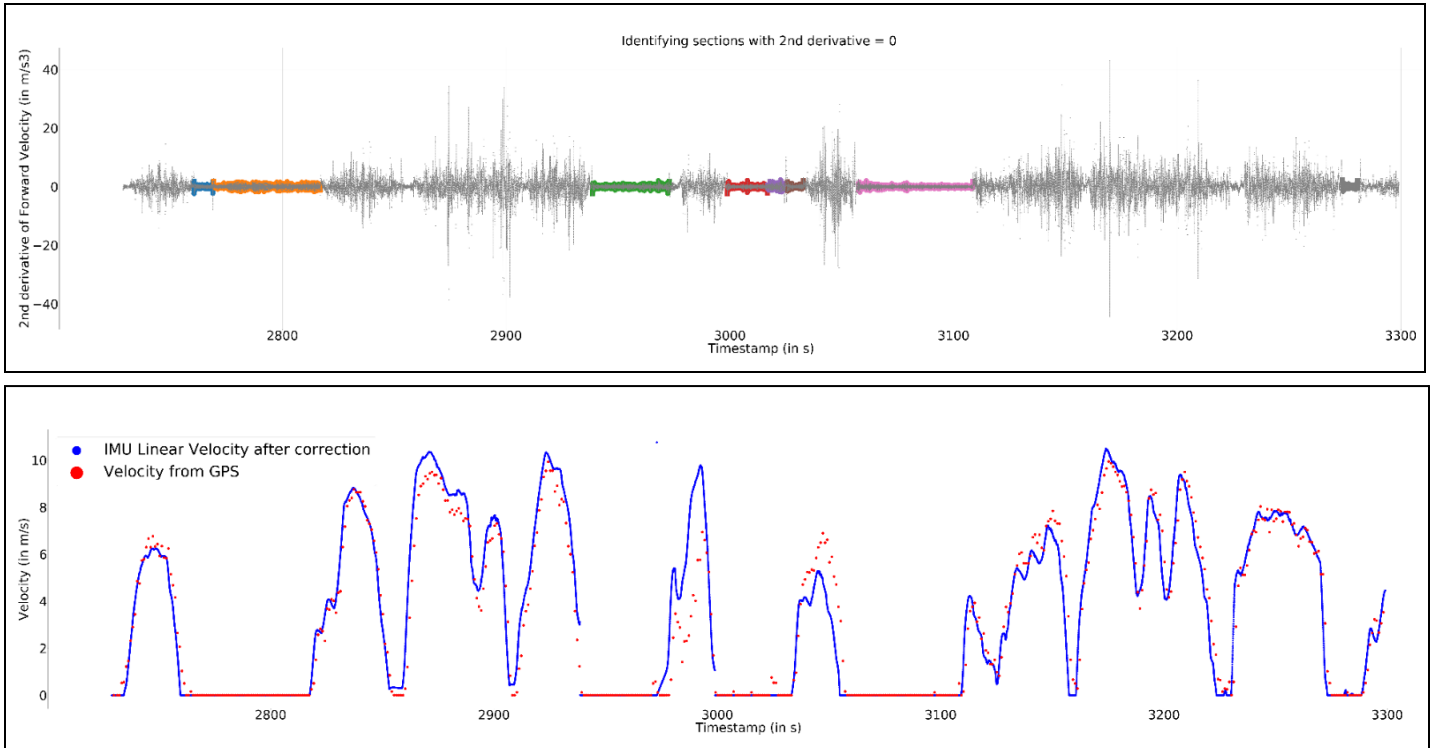
The above figure shows the velocity estimate after integration. Another velocity estimate is derived from GPS data by differentiating the distance traveled per timestamp. The absolute velocities have been shown for easier visualization.

A clear drift in the data is visible. This is due to the presence of additive noise in the Linear Acceleration field in the IMU. To remove this drift, a linear regression best fit was subtracted from the Velocity estimate.



After removing the drift, there were still discrepancies between the velocity estimate of GPS and IMU. There are certain sections in the plot where the vehicle is at rest, but still, the velocity is increasing for the IMU estimate. It can be observed that the drifts in these sections have constant slopes.

To identify these drifts, the 2nd derivate of the velocity estimate was calculated, and these sections' offsets and drifts were eliminated. The below figure shows the sections of interest by highlighting them with different colors.

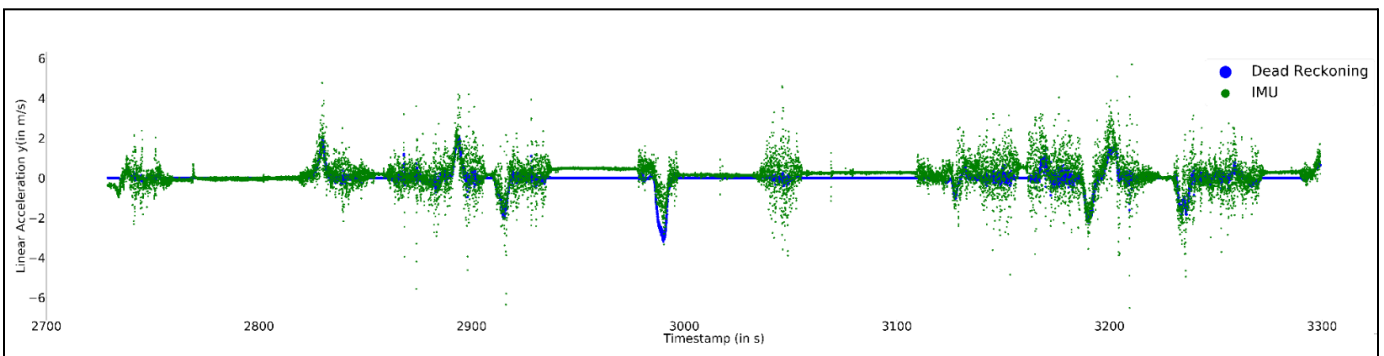


III] Dead Reckoning:

The relation between the Linear acceleration Y observed in IMU and the linear velocity is given by:

$$\dot{Y}^{**} = \omega X^*$$

The linear velocity from the calculations in section II was combined with the Angular velocity Z from the IMU, and then it was compared with IMU Linear Acceleration Z .



Observations:

- The calculated estimate matches the IMU values well.
- The differences can be observed in the smoothness of the two plots. The IMU data has a high frequency, which makes it sensitive to small changes. These changes were smoothed out when we calculated the acceleration by the above method.

- If we pass the IMU data through a low pass filter, the two sets of data will agree with each other.

The vehicle's trajectory is obtained by transforming the forward velocity from the IMU frame to the GNSS frame. The velocity has two components: one in northing direction, the other in easting direction. The integration of these velocities over time gives us the trajectory.

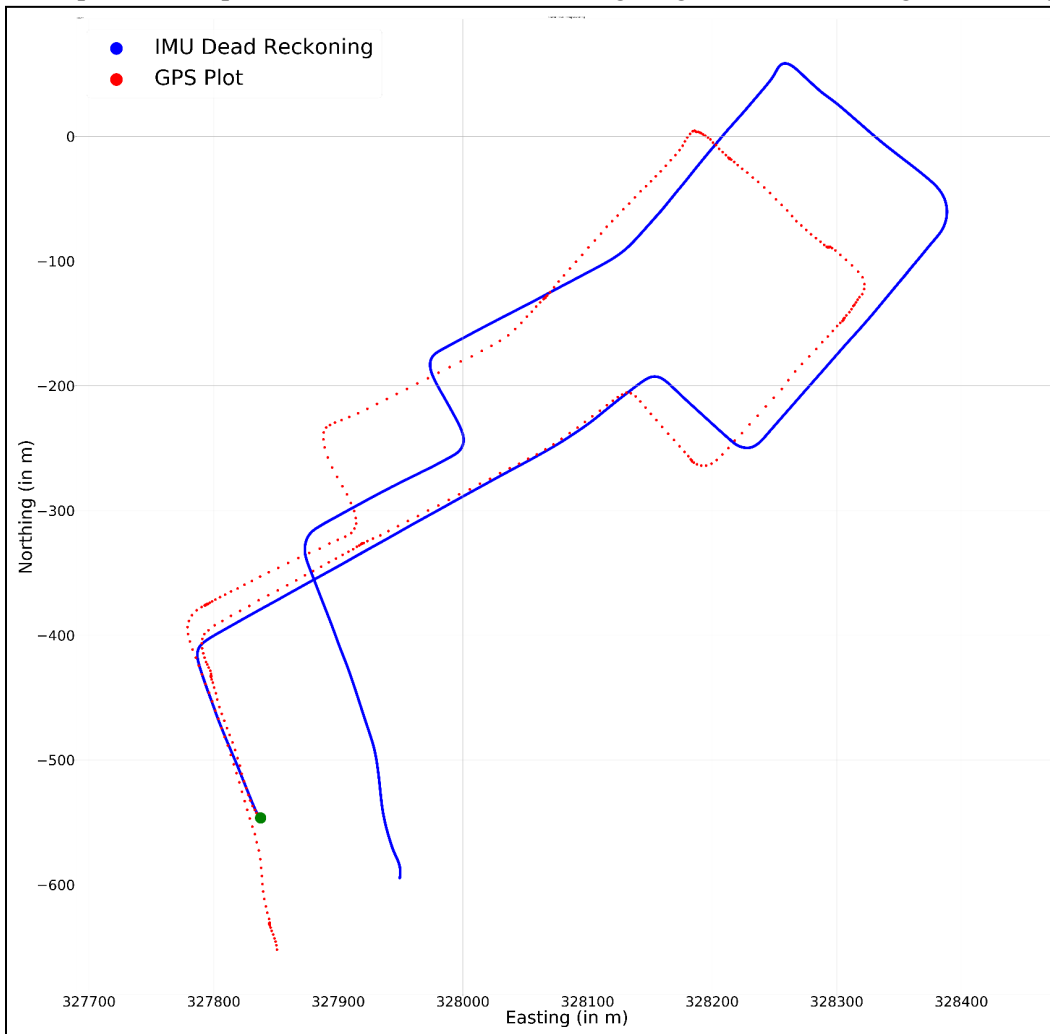
To transform the velocity between the frames:

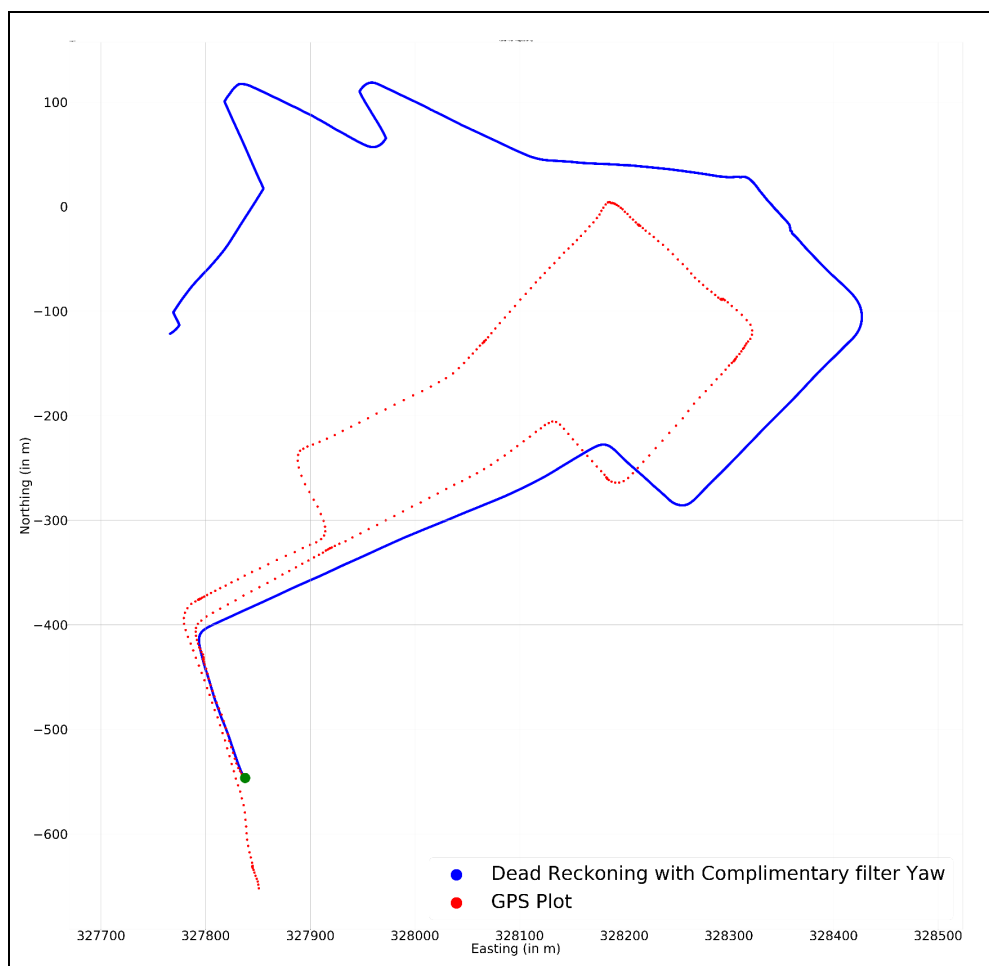
$$V_e = V_x \sin(yaw)$$

$$V_n = V_x \cos(yaw)$$

The trajectories calculated with (i) the IMU Yaw and (ii) the Complementary Filter Yaw have been plotted in the two figures below. The plots have been compared with the coordinates obtained from GPS.

The IMU Yaw calculations give us a better estimate of the trajectory, which further reiterates the stance of the development of a speed-sensitive filter for combining Magnetometer and Angular velocity yaw estimates.





Bonus Part:

$$\begin{aligned}\ddot{x}_{obs} &= \ddot{x} - \omega \dot{y} - \omega^2 x_c \\ \ddot{y}_{obs} &= \ddot{y} + \omega \dot{x} + \dot{\omega} x_c\end{aligned} \quad \left. \vphantom{\begin{aligned}\ddot{x}_{obs} &= \ddot{x} - \omega \dot{y} - \omega^2 x_c \\ \ddot{y}_{obs} &= \ddot{y} + \omega \dot{x} + \dot{\omega} x_c\end{aligned}} \right\} \text{Given equations}$$

Assuming no skidding $\therefore \dot{y} = 0$

$$\therefore \ddot{y} = \frac{d(\dot{y})}{dt} = 0$$

$$\therefore \ddot{y}_{obs} = \dot{\omega} x_c + \omega \dot{x}$$

$$\ddot{x}_{obs} = \ddot{x} - \omega^2 x_c$$

$$\ddot{y}_{obs} = \dot{\omega} x_c + \omega \dot{x}$$

$$\dot{\omega} = \frac{d(\omega)}{dt}$$

(Assuming negligible pitch & roll)

$$x_c = \frac{\ddot{y}_{obs} - \omega \dot{x}}{\dot{\omega}}$$

\ddot{y}_{obs} = Linear accⁿ of y' from IMU.

\dot{x} = Forward velocity (from Part 2.2 of Analysis)

ω = Angular velocity z' from IMU.

The value of x_c can be averaged over the time when the car is changing its heading ~~over~~ at a non-constant rate ($\dot{\omega} \neq 0$).

References:

- [1] [Fierce Electronics: Compensating for Tilt, Hard-Iron, and Soft-Iron Effects by Christopher Konvalin.](#)
- [2] [VectorNav: MAGNETOMETER HARD & SOFT IRON CALIBRATION](#)