Homework 4 - Atharva Pandhare

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1 Problem 1

1.1 A. Optimization Comparisions

1.2 Comparing SGD, SGD+Momentum, AdaGrad, RMSprop:

- Conventional Standard Gradient Descent (SGD)
  - Simplest method
  - Updates are solely based on learning rate and the current batch's gradient
  - ***Not Perfect:*** Gradient can get stuck in a local minima or a saddle point

$$x += -lr * dx$$

- Standard Gradient Descent with **Momentum** (SGD+Momentum)
  - Adds a momentum component which can be used to carry some of the velocity of the weights
  - Velocity a decaying accumulation of the gradients
  - Aims to solve issue from SGD of getting stuck in minima or saddle
  - Momentum aspect accelerate convergence, and dampens oscillations.

$$v = \mu * v - lr * dxx += v$$

  - adds v which is the velocity
  - adds μ which is the momentum normally set to ≈ 0.9
- Adaptive Gradient (AdaGrad)
  - Element-wise adaptively adjust effective learning rate
    - for weights with high gradients: reduce
    - for weights with small/infrequent updates: increase

$$cache += dx^2 x += \frac{-lr * dx}{\sqrt{cache} + \epsilon}$$

  - $1e{-8} < \epsilon < 1e{-4}$
  - *cache*: sum of squared gradients

  - ***Not Perfect***: Cache increases causeing the effective learning rate to decreases, potentially causing learningto stop early
- Root Mean Squared Propagation (RMSprop)
  - Builds on AdaGrad
  - Aims to solve gripes of AdaGrad by using an exponetially decaying moving average *cache*

$$cache = decay * cache + (1 - decay) * dx^2 x += \frac{-lr * dx}{\sqrt{cache} + \epsilon}$$

- *decay* is typically $0.9, 0.99, 0.999...$
- Prevents the learning rate from vanishing too quickly, allowing for continued learning

1.3 Popularity of Adam

- Adam's popularity comes because it does the adaptive learning rate from RMSprop and also brings in the momentum aspect from SGD+Momentum.
- These features allow adam to be able to navigate complex loss landscapes
- It converges faster, being more efficient and compute friendly
- These factors and the effectiveness across various tasks such as NLP, and computer vision has led to it wide spread adoption in the field.

1.4 B. Implementations

1.4.1 Imports and datasets

[1]:
```
# Load in relevant libraries, and alias where appropriate
import torch
import torch.nn as nn
import torchvision.datasets as datasets
import torchvision.transforms as transforms
import matplotlib.pyplot as plt
import time

# Define relevant variables for the ML task
batch_size = 256
learning_rate = 0.01
num_epochs = 10 # 20 just gets almost 100 everytime

# Device will determine whether to run the training on GPU or CPU.
device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
# For training data
transform = transforms.Compose([
    transforms.ToTensor(),
    transforms.Normalize((0.1307,), (0.3081,))  # MNIST standard mean and␣
 ↪standard deviation
])

cifar_trainset = datasets.MNIST(root='./data', train=True, download=True,␣
 ↪transform=transform)

# For test data
cifar_testset = datasets.MNIST(root='./data', train=False, download=True,␣
 ↪transform=transform)



# DataLoader for training and test datasets
```

```
trainloader = torch.utils.data.DataLoader(cifar_trainset,␣
 ↪batch_size=batch_size, shuffle=True)
testloader = torch.utils.data.DataLoader(cifar_testset, batch_size=batch_size,␣
 ↪shuffle=False)
```

1.4.2 Model

[2]:
```
class MLP:
    def __init__(self):
        super(MLP, self).__init__()
        self.net = nn.Sequential(
            nn.Flatten(),
            nn.Linear(784, 200),
            nn.ReLU(),
            nn.Linear(200, 50),
            nn.ReLU(),
            nn.Linear(50, 10)
            # nn.Softmax(dim=1) # Not needed for CrossEntropyLoss
        )

    def forward(self, x):
        return self.net(x)

model = MLP().net.to(device)
criterion = nn.CrossEntropyLoss() # Loss function
```

1.4.3 Train and Test Funcitons

[3]:
```
def train(optimizer, name = 'err', scheduler=None, model=model,␣
 ↪trainloader=trainloader, num_epochs=num_epochs):
    start_time = time.time()
    criterion = nn.CrossEntropyLoss()
    # Lists to store losses for plotting
    epoch_losses = []

    # Training loop
    for epoch in range(num_epochs):
        # Set model to training mode
        model.train()

        running_loss = 0.0
        batches_in_epoch = 0

        for i, (inputs, labels) in enumerate(trainloader):
            # Move data to device (CPU/GPU)
            inputs = inputs.to(device)
            labels = labels.to(device)
```

```
            # Zero the parameter gradients
            optimizer.zero_grad()

            # Forward pass
            outputs = model(inputs)
            loss = criterion(outputs, labels)

            # Backward pass and optimize
            loss.backward()
            optimizer.step()

            # Accumulate statistics
            running_loss += loss.item()
            batches_in_epoch += 1

        # Step the scheduler if it exists
        if scheduler is not None:
            scheduler.step()

        # Store average loss for this epoch
        avg_epoch_loss = running_loss / batches_in_epoch
        epoch_losses.append(avg_epoch_loss)
        print(f'Epoch {epoch + 1}, Loss: {avg_epoch_loss:.3f}')

    # Plot the training loss
    plt.figure(figsize=(10, 6))
    plt.plot(range(1, num_epochs + 1), epoch_losses, marker='o')
    plt.title('Training Loss Over Epochs for MLP' + name)
    plt.xlabel('Epochs')
    plt.ylabel('Loss')
    plt.grid(True)
    plt.show()

    end_time = time.time()
    elapsed_time = end_time - start_time
    print(f"Training completed in {elapsed_time:.2f} seconds")


def test(model = model, testloader=testloader):
    start_time = time.time()
    # Testing the best model on test data
    model.eval()
    correct = 0
    total = 0
    with torch.no_grad():
        for data in testloader:
```

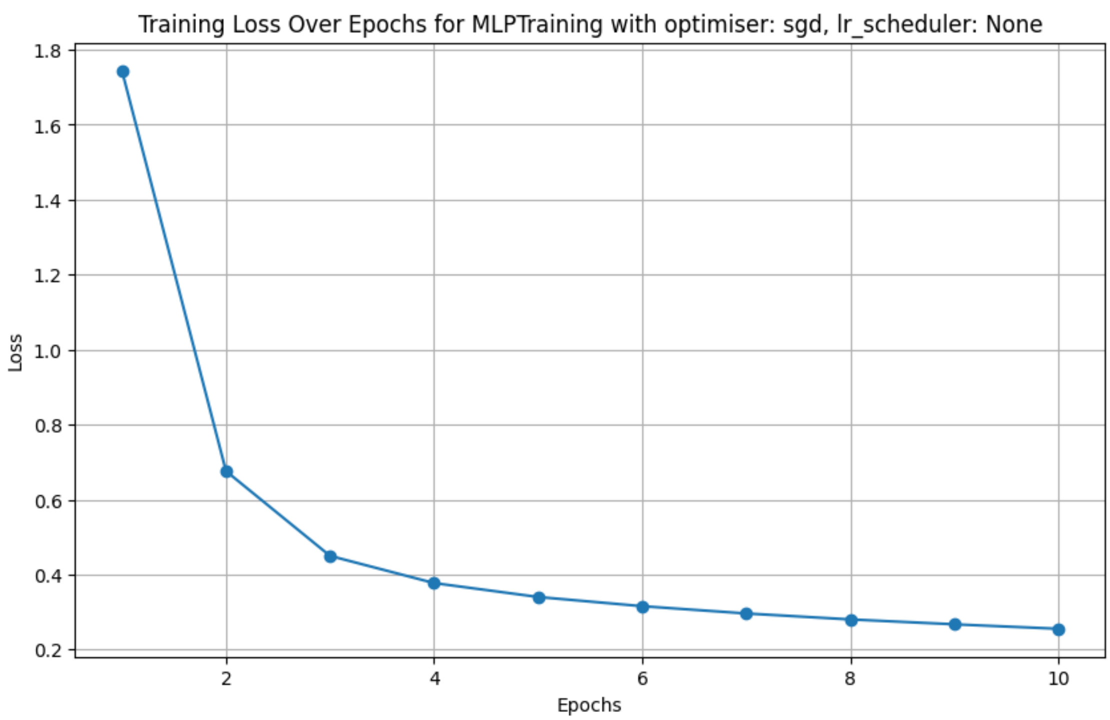
```
            images, labels = data
            images, labels = images.to(device), labels.to(device)
            outputs = model(images)
            _, predicted = torch.max(outputs.data, 1)
            total += labels.size(0)
            correct += (predicted == labels).sum().item()
    accuracy = 100 * correct / total
    end_time = time.time()
    elapsed_time = end_time - start_time
    print(f'testing finished in {elapsed_time:.2f} seconds, Accuracy: {accuracy:
 ↪.2f}%')
    return accuracy

def train_test(optimizer:torch.optim, name = 'err', scheduler=None):
    train(optimizer=optimizer, name=name, scheduler=scheduler)
    return test()
```

1.4.4 Implementations

[4]:
```
optimizers = {
    'sgd': torch.optim.SGD(model.parameters(), lr=learning_rate),
    'sgd_momentum': torch.optim.SGD(model.parameters(), lr=learning_rate,␣
 ↪momentum=0.9),
    'adagrad': torch.optim.Adagrad(model.parameters(), lr=learning_rate,␣
 ↪eps=1e-6),
    'rmsprop': torch.optim.RMSprop(model.parameters(), lr=learning_rate,␣
 ↪alpha=0.9, eps=1e-6),
    'adam': torch.optim.Adam(model.parameters(), lr=learning_rate)
}
results = {}
def try_all(optimizers = optimizers):
    for optimizer in optimizers:
        optim = optimizers[optimizer]
        print_out = f"Training with optimiser: {optimizer}, lr_scheduler: None"
        print(print_out)
        results[print_out] = train_test(optim, name=print_out)
        print_out = f"Training with optimiser: {optimizer}, lr_scheduler:␣
 ↪StepLR"
        print(print_out)
        results[print_out] = train_test(optim, name=print_out, scheduler=torch.
 ↪optim.lr_scheduler.StepLR(optimizers[optimizer], step_size=2, gamma=0.1))
        print_out = f"Training with optimiser: {optimizer}, lr_scheduler:␣
 ↪CosineAnnealingLR"
        print(print_out)
        results[print_out] = train_test(optim, name=print_out, scheduler=torch.
 ↪optim.lr_scheduler.CosineAnnealingLR(optimizers[optimizer], T_max=10))
```

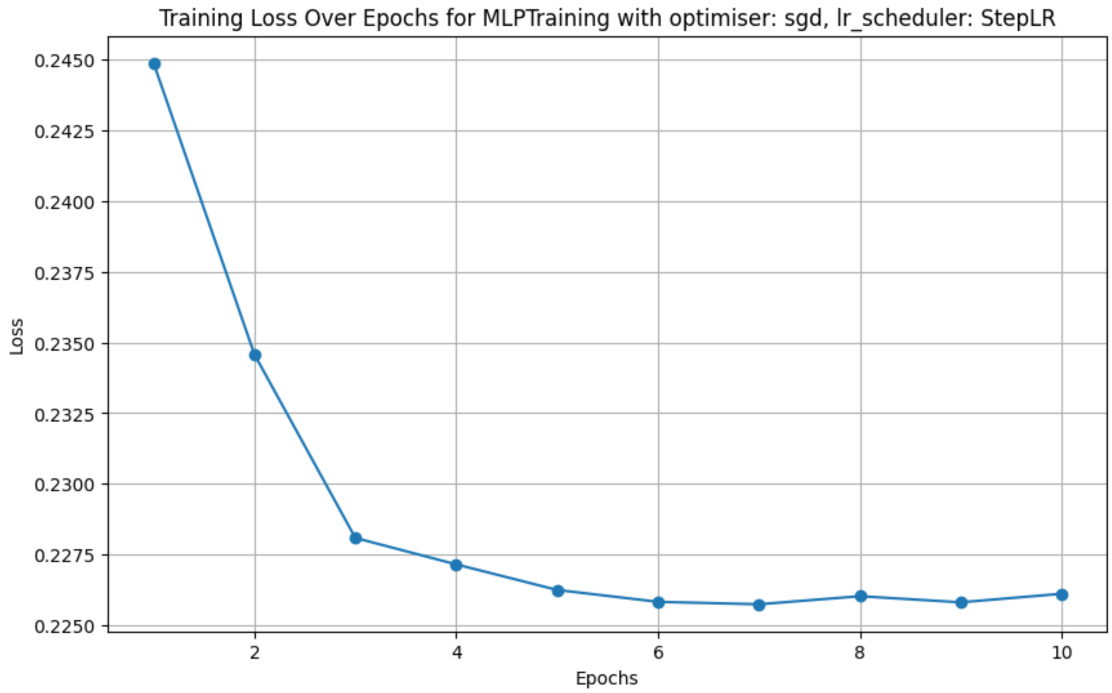
```python
try_all()

for result in results:
    print(f"After {result}, Accuracy: {results[result]}")
```

```
Training with optimiser: sgd, lr_scheduler: None
Epoch 1, Loss: 1.745
Epoch 2, Loss: 0.676
Epoch 3, Loss: 0.450
Epoch 4, Loss: 0.377
Epoch 5, Loss: 0.340
Epoch 6, Loss: 0.316
Epoch 7, Loss: 0.296
Epoch 8, Loss: 0.280
Epoch 9, Loss: 0.267
Epoch 10, Loss: 0.255
```

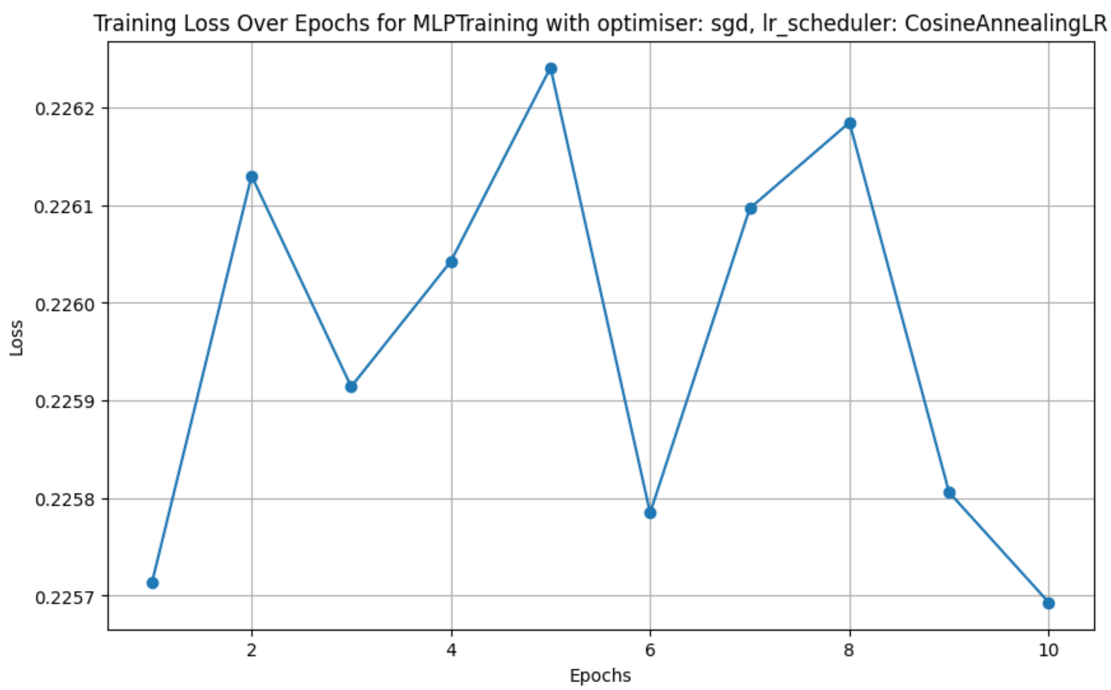



```
Training completed in 40.43 seconds
testing finished in 0.66 seconds, Accuracy: 93.16%
Training with optimiser: sgd, lr_scheduler: StepLR
Epoch 1, Loss: 0.245
Epoch 2, Loss: 0.235
Epoch 3, Loss: 0.228
```

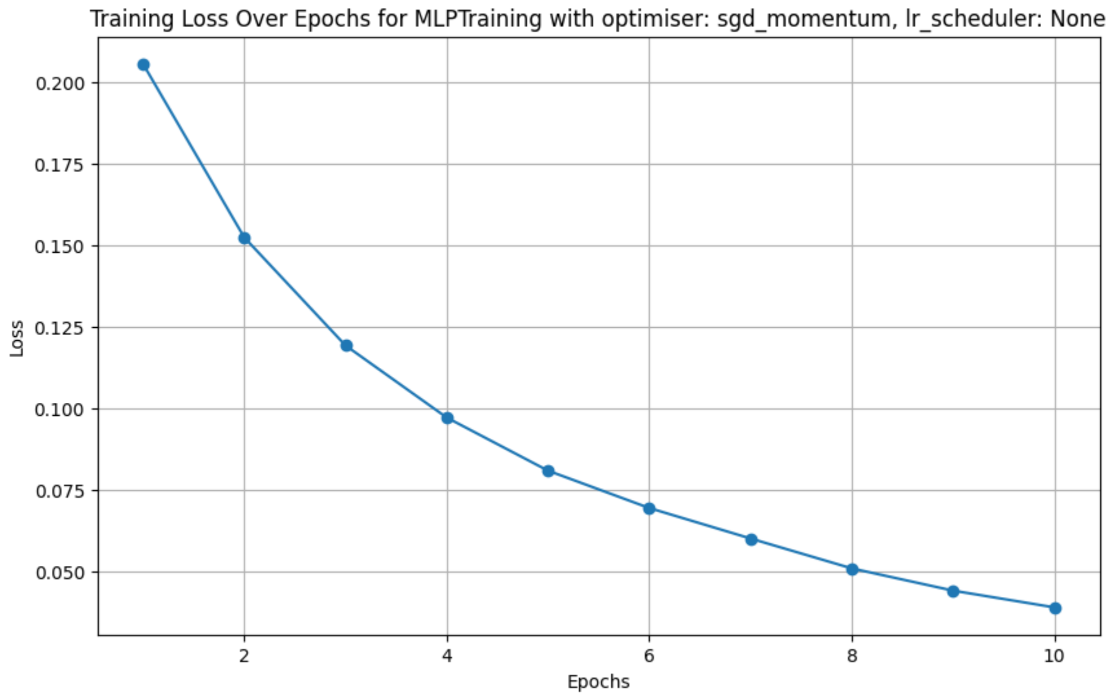
```
Epoch 4, Loss: 0.227
Epoch 5, Loss: 0.226
Epoch 6, Loss: 0.226
Epoch 7, Loss: 0.226
Epoch 8, Loss: 0.226
Epoch 9, Loss: 0.226
Epoch 10, Loss: 0.226
```



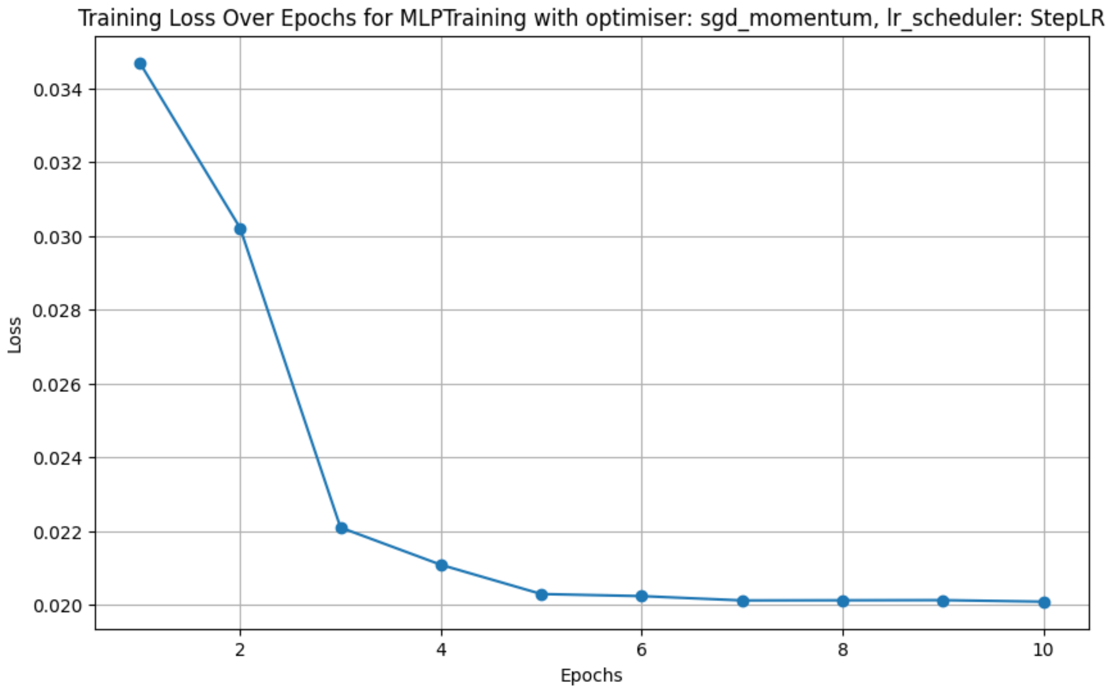

```
Training completed in 37.89 seconds
testing finished in 0.60 seconds, Accuracy: 93.74%
Training with optimiser: sgd, lr_scheduler: CosineAnnealingLR
Epoch 1, Loss: 0.226
Epoch 2, Loss: 0.226
Epoch 3, Loss: 0.226
Epoch 4, Loss: 0.226
Epoch 5, Loss: 0.226
Epoch 6, Loss: 0.226
Epoch 7, Loss: 0.226
Epoch 8, Loss: 0.226
Epoch 9, Loss: 0.226
Epoch 10, Loss: 0.226
```



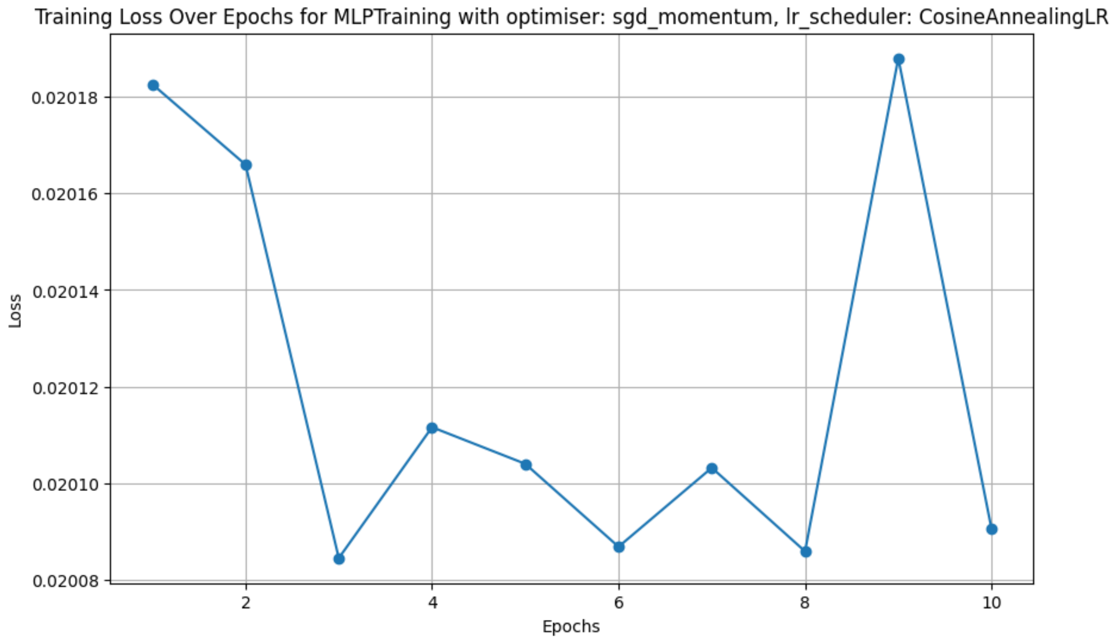

```
Training completed in 36.96 seconds
testing finished in 0.64 seconds, Accuracy: 93.74%
Training with optimiser: sgd_momentum, lr_scheduler: None
Epoch 1, Loss: 0.206
Epoch 2, Loss: 0.152
Epoch 3, Loss: 0.119
Epoch 4, Loss: 0.097
Epoch 5, Loss: 0.081
Epoch 6, Loss: 0.069
Epoch 7, Loss: 0.060
Epoch 8, Loss: 0.051
Epoch 9, Loss: 0.044
Epoch 10, Loss: 0.039
```



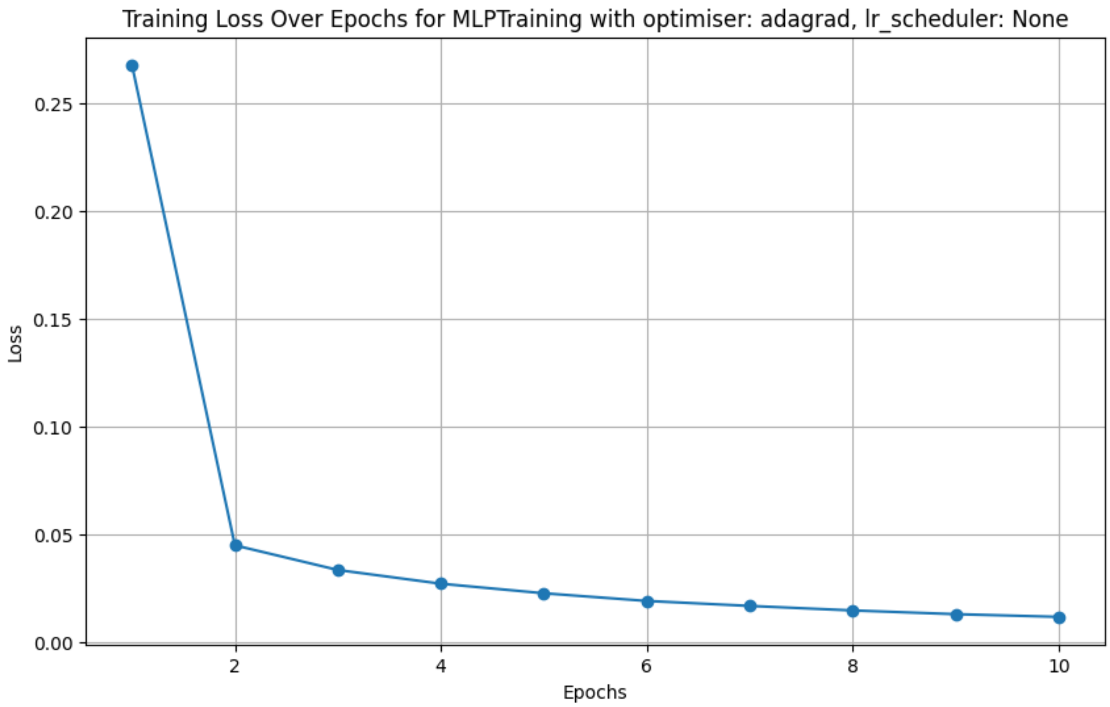

```
Training completed in 40.76 seconds
testing finished in 0.61 seconds, Accuracy: 97.76%
Training with optimiser: sgd_momentum, lr_scheduler: StepLR
Epoch 1, Loss: 0.035
Epoch 2, Loss: 0.030
Epoch 3, Loss: 0.022
Epoch 4, Loss: 0.021
Epoch 5, Loss: 0.020
Epoch 6, Loss: 0.020
Epoch 7, Loss: 0.020
Epoch 8, Loss: 0.020
Epoch 9, Loss: 0.020
Epoch 10, Loss: 0.020
```



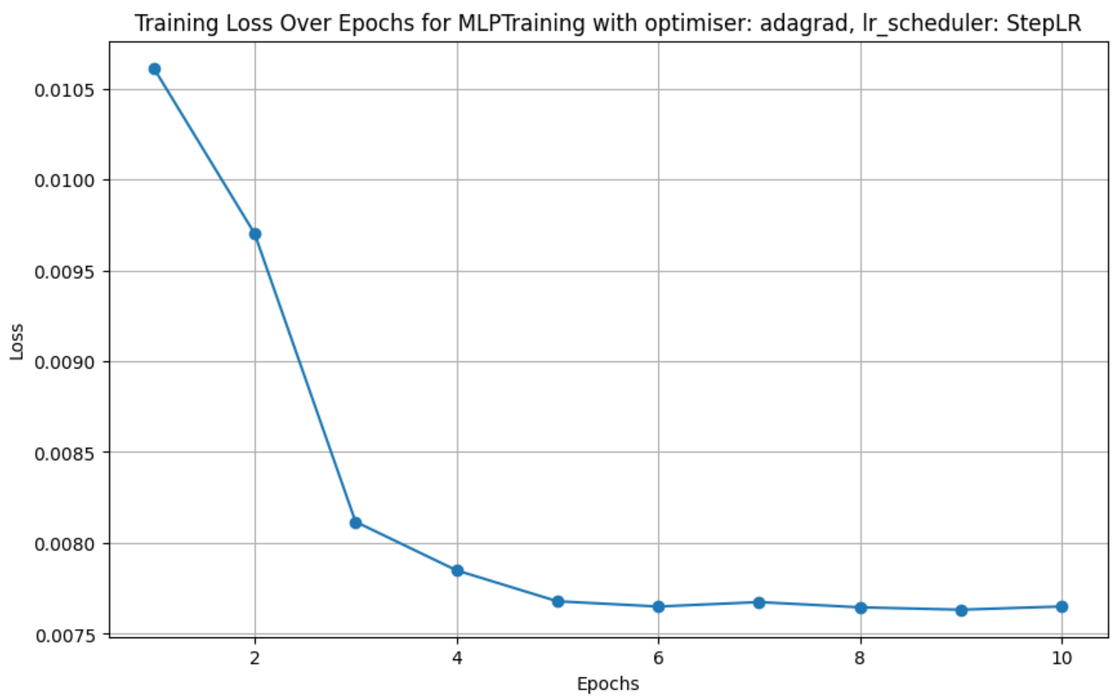

```
Training completed in 38.91 seconds
testing finished in 0.60 seconds, Accuracy: 98.03%
Training with optimiser: sgd_momentum, lr_scheduler: CosineAnnealingLR
Epoch 1, Loss: 0.020
Epoch 2, Loss: 0.020
Epoch 3, Loss: 0.020
Epoch 4, Loss: 0.020
Epoch 5, Loss: 0.020
Epoch 6, Loss: 0.020
Epoch 7, Loss: 0.020
Epoch 8, Loss: 0.020
Epoch 9, Loss: 0.020
Epoch 10, Loss: 0.020
```



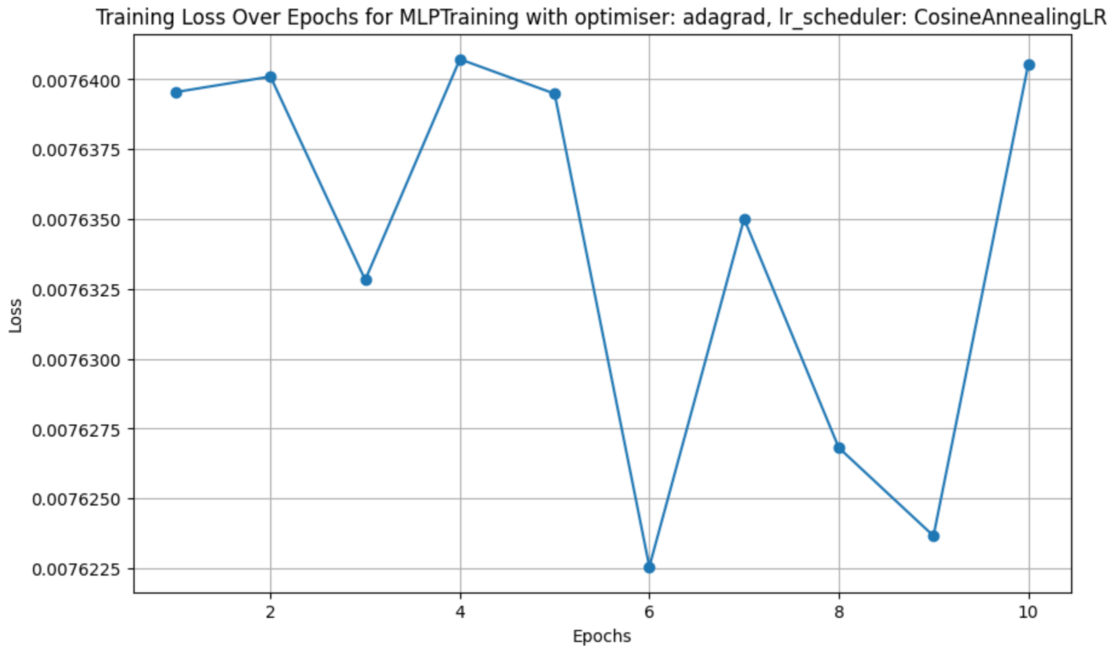

```
Training completed in 38.83 seconds
testing finished in 0.59 seconds, Accuracy: 98.03%
Training with optimiser: adagrad, lr_scheduler: None
Epoch 1, Loss: 0.268
Epoch 2, Loss: 0.045
Epoch 3, Loss: 0.034
Epoch 4, Loss: 0.027
Epoch 5, Loss: 0.023
Epoch 6, Loss: 0.019
Epoch 7, Loss: 0.017
Epoch 8, Loss: 0.015
Epoch 9, Loss: 0.013
Epoch 10, Loss: 0.012
```



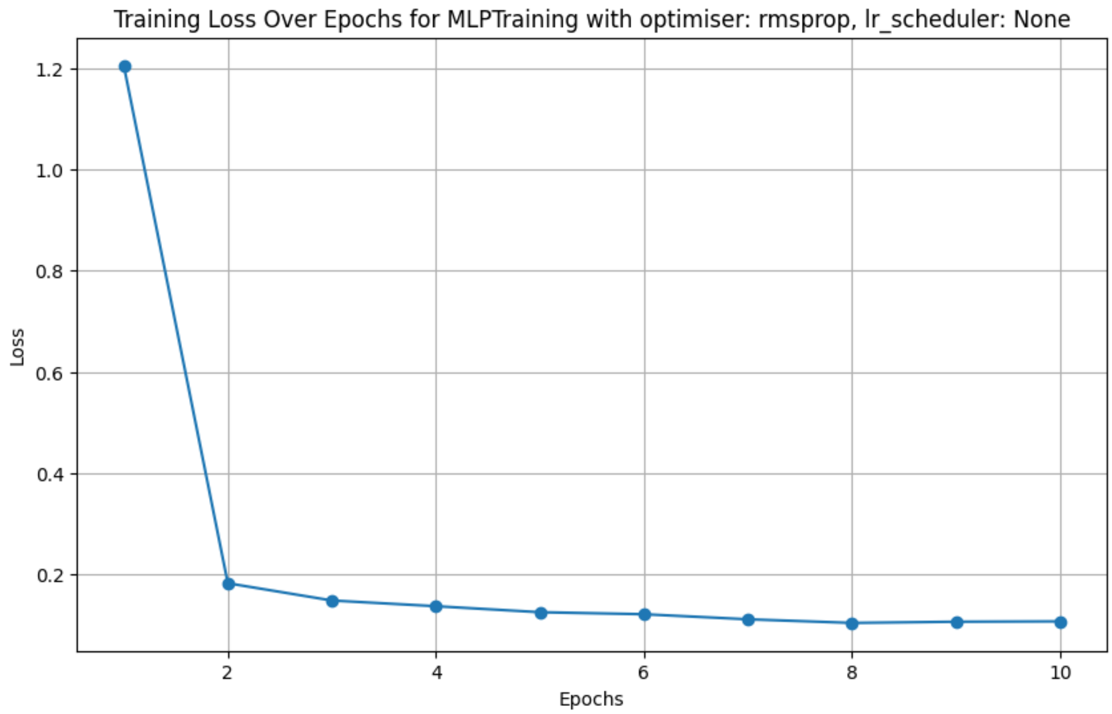

```
Training completed in 40.98 seconds
testing finished in 0.63 seconds, Accuracy: 98.13%
Training with optimiser: adagrad, lr_scheduler: StepLR
Epoch 1, Loss: 0.011
Epoch 2, Loss: 0.010
Epoch 3, Loss: 0.008
Epoch 4, Loss: 0.008
Epoch 5, Loss: 0.008
Epoch 6, Loss: 0.008
Epoch 7, Loss: 0.008
Epoch 8, Loss: 0.008
Epoch 9, Loss: 0.008
Epoch 10, Loss: 0.008
```



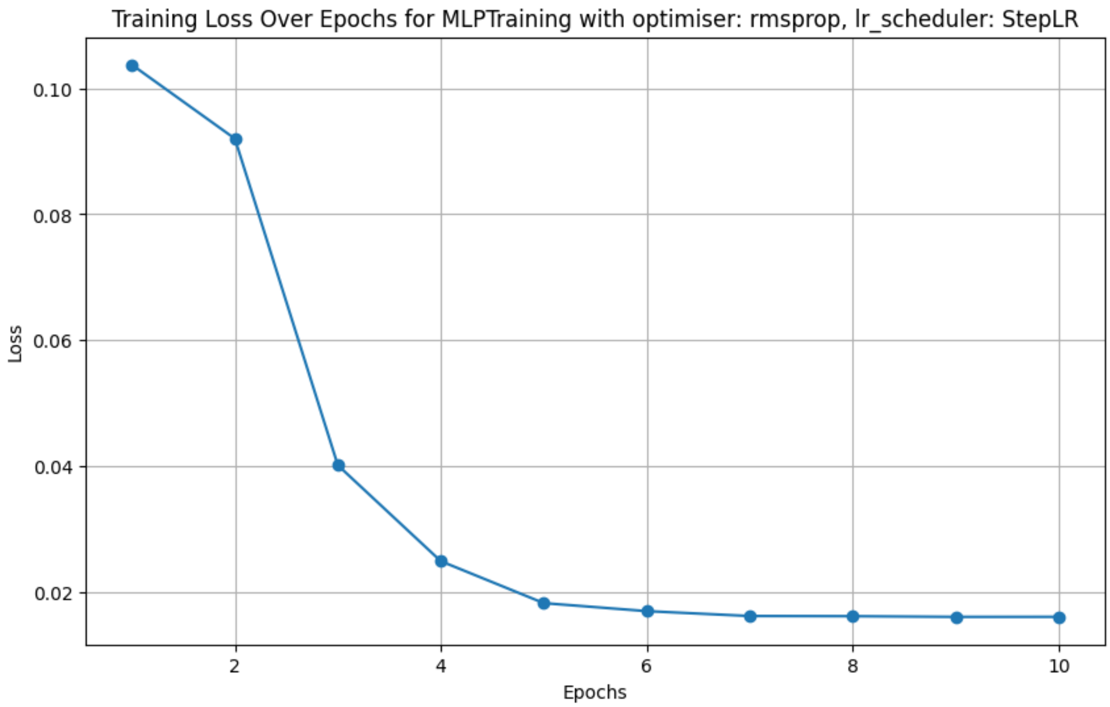

```
Training completed in 40.36 seconds
testing finished in 0.58 seconds, Accuracy: 98.13%
Training with optimiser: adagrad, lr_scheduler: CosineAnnealingLR
Epoch 1, Loss: 0.008
Epoch 2, Loss: 0.008
Epoch 3, Loss: 0.008
Epoch 4, Loss: 0.008
Epoch 5, Loss: 0.008
Epoch 6, Loss: 0.008
Epoch 7, Loss: 0.008
Epoch 8, Loss: 0.008
Epoch 9, Loss: 0.008
Epoch 10, Loss: 0.008
```



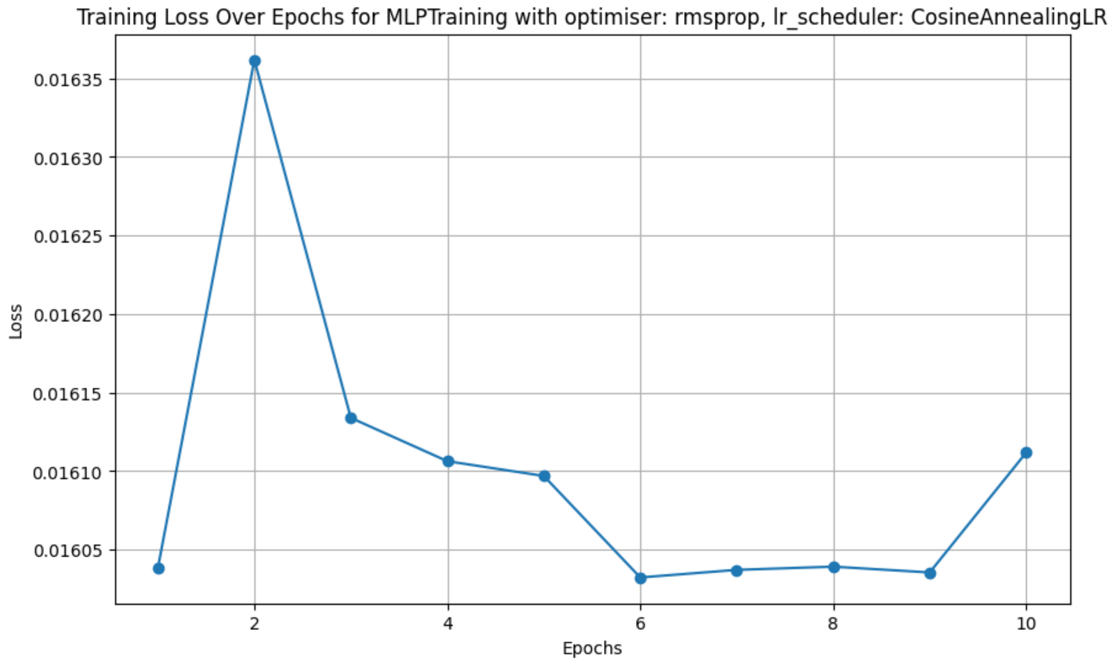

```
Training completed in 38.25 seconds
testing finished in 0.63 seconds, Accuracy: 98.13%
Training with optimiser: rmsprop, lr_scheduler: None
Epoch 1, Loss: 1.205
Epoch 2, Loss: 0.183
Epoch 3, Loss: 0.149
Epoch 4, Loss: 0.138
Epoch 5, Loss: 0.126
Epoch 6, Loss: 0.122
Epoch 7, Loss: 0.112
Epoch 8, Loss: 0.105
Epoch 9, Loss: 0.107
Epoch 10, Loss: 0.108
```



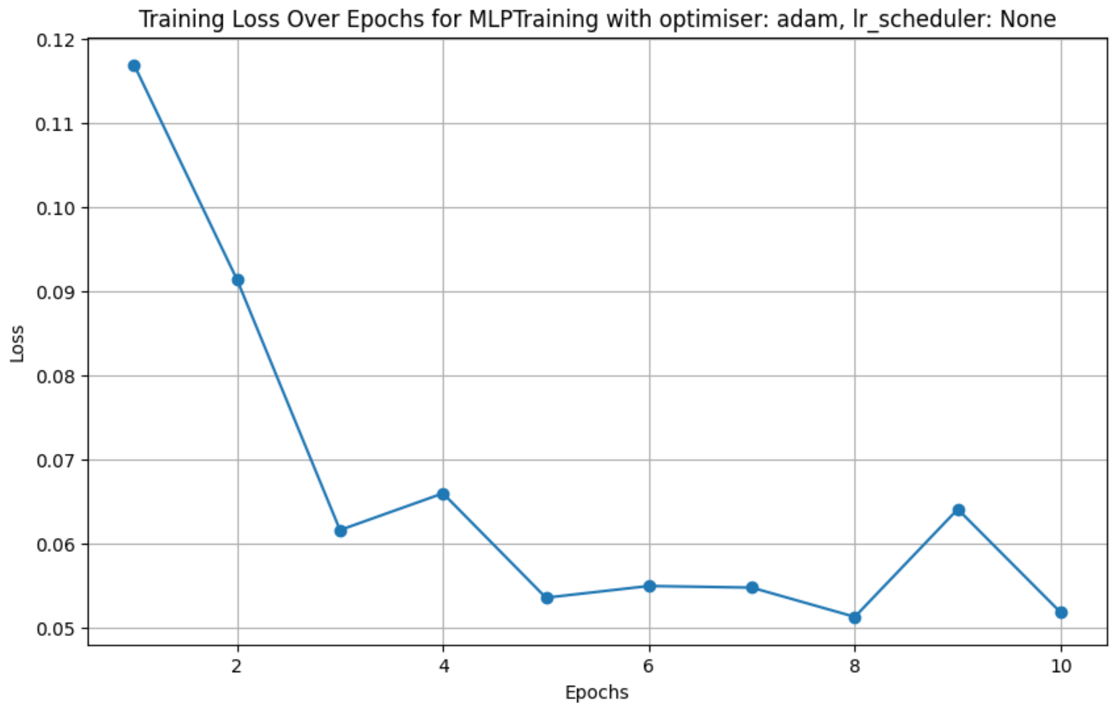

```
Training completed in 38.78 seconds
testing finished in 0.60 seconds, Accuracy: 95.33%
Training with optimiser: rmsprop, lr_scheduler: StepLR
Epoch 1, Loss: 0.104
Epoch 2, Loss: 0.092
Epoch 3, Loss: 0.040
Epoch 4, Loss: 0.025
Epoch 5, Loss: 0.018
Epoch 6, Loss: 0.017
Epoch 7, Loss: 0.016
Epoch 8, Loss: 0.016
Epoch 9, Loss: 0.016
Epoch 10, Loss: 0.016
```



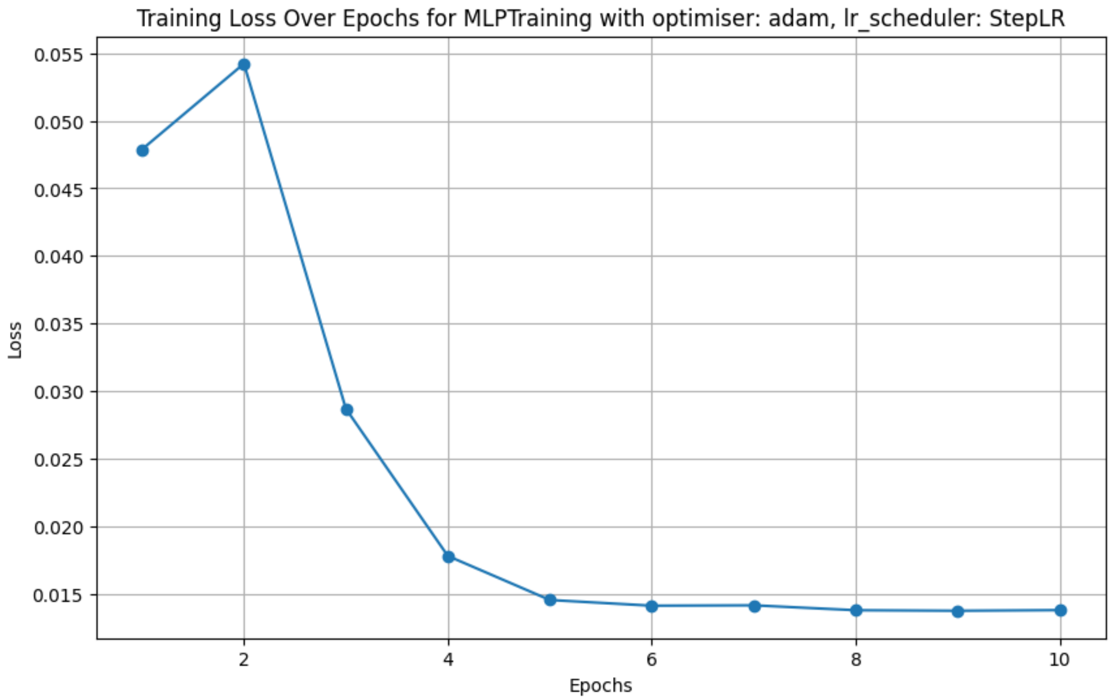

```
Training completed in 37.89 seconds
testing finished in 0.61 seconds, Accuracy: 97.87%
Training with optimiser: rmsprop, lr_scheduler: CosineAnnealingLR
Epoch 1, Loss: 0.016
Epoch 2, Loss: 0.016
Epoch 3, Loss: 0.016
Epoch 4, Loss: 0.016
Epoch 5, Loss: 0.016
Epoch 6, Loss: 0.016
Epoch 7, Loss: 0.016
Epoch 8, Loss: 0.016
Epoch 9, Loss: 0.016
Epoch 10, Loss: 0.016
```



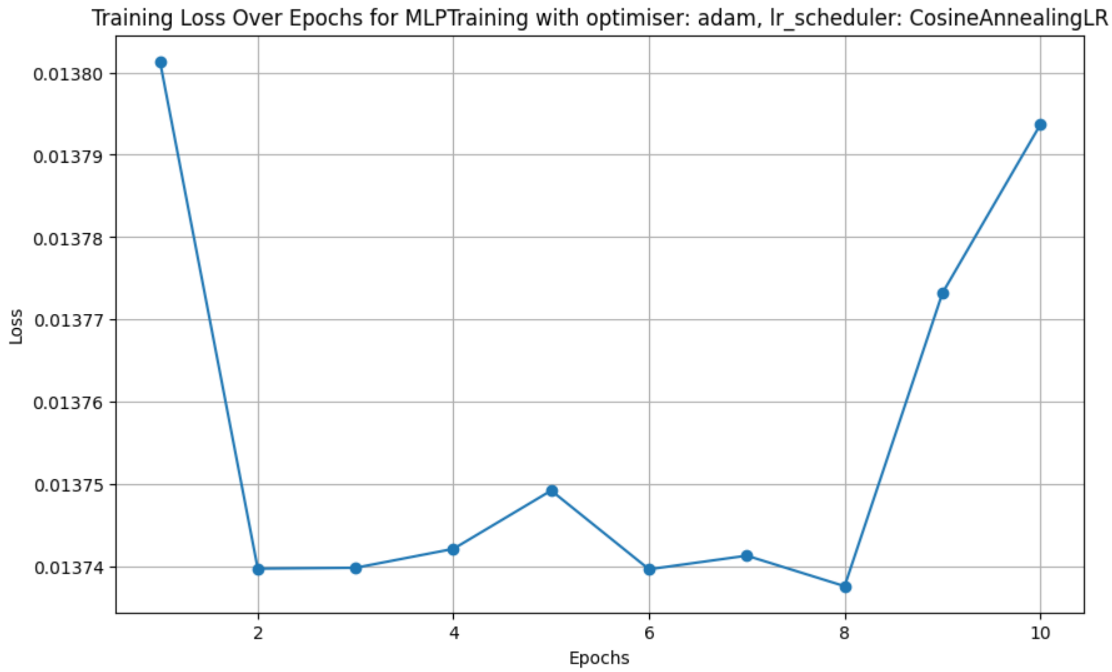

```
Training completed in 36.50 seconds
testing finished in 0.58 seconds, Accuracy: 97.87%
Training with optimiser: adam, lr_scheduler: None
Epoch 1, Loss: 0.117
Epoch 2, Loss: 0.091
Epoch 3, Loss: 0.062
Epoch 4, Loss: 0.066
Epoch 5, Loss: 0.054
Epoch 6, Loss: 0.055
Epoch 7, Loss: 0.055
Epoch 8, Loss: 0.051
Epoch 9, Loss: 0.064
Epoch 10, Loss: 0.052
```




```
Training completed in 39.33 seconds
testing finished in 0.60 seconds, Accuracy: 97.30%
Training with optimiser: adam, lr_scheduler: StepLR
Epoch 1, Loss: 0.048
Epoch 2, Loss: 0.054
Epoch 3, Loss: 0.029
Epoch 4, Loss: 0.018
Epoch 5, Loss: 0.015
Epoch 6, Loss: 0.014
Epoch 7, Loss: 0.014
Epoch 8, Loss: 0.014
Epoch 9, Loss: 0.014
Epoch 10, Loss: 0.014
```




```
Training completed in 40.14 seconds
testing finished in 0.68 seconds, Accuracy: 97.59%
Training with optimiser: adam, lr_scheduler: CosineAnnealingLR
Epoch 1, Loss: 0.014
Epoch 2, Loss: 0.014
Epoch 3, Loss: 0.014
Epoch 4, Loss: 0.014
Epoch 5, Loss: 0.014
Epoch 6, Loss: 0.014
Epoch 7, Loss: 0.014
Epoch 8, Loss: 0.014
Epoch 9, Loss: 0.014
Epoch 10, Loss: 0.014
```




```
Training completed in 40.64 seconds
testing finished in 0.66 seconds, Accuracy: 97.59%
After Training with optimiser: sgd, lr_scheduler: None, Accuracy: 93.16
After Training with optimiser: sgd, lr_scheduler: StepLR, Accuracy: 93.74
After Training with optimiser: sgd, lr_scheduler: CosineAnnealingLR, Accuracy:
93.74
After Training with optimiser: sgd_momentum, lr_scheduler: None, Accuracy: 97.76
After Training with optimiser: sgd_momentum, lr_scheduler: StepLR, Accuracy:
98.03
After Training with optimiser: sgd_momentum, lr_scheduler: CosineAnnealingLR,
Accuracy: 98.03
After Training with optimiser: adagrad, lr_scheduler: None, Accuracy: 98.13
After Training with optimiser: adagrad, lr_scheduler: StepLR, Accuracy: 98.13
After Training with optimiser: adagrad, lr_scheduler: CosineAnnealingLR,
Accuracy: 98.13
After Training with optimiser: rmsprop, lr_scheduler: None, Accuracy: 95.33
After Training with optimiser: rmsprop, lr_scheduler: StepLR, Accuracy: 97.87
After Training with optimiser: rmsprop, lr_scheduler: CosineAnnealingLR,
Accuracy: 97.87
After Training with optimiser: adam, lr_scheduler: None, Accuracy: 97.3
After Training with optimiser: adam, lr_scheduler: StepLR, Accuracy: 97.59
After Training with optimiser: adam, lr_scheduler: CosineAnnealingLR, Accuracy:
97.59
```

1.4.5 Write-up

I wrote a loop to run through all the models, and per model it runs every learning rate scheduler. I tried a few different values for num_epochs, I found that using 20 epochs always ended up giving really high accuracy so I was not able to tell a difference. I stuck with 10 to create more of a difference.

Findings Comparing the optimizers I found that SGD (Stochastic Gradient Descent) performed relatively poorly with an accuracy of around 93-94%. Adding momentum to SGD (SGD with momentum) significantly improved the results, reaching around 97-98% accuracy. Adagrad did even better, consistently achieving about 98.13% accuracy. RMSProp and Adam fell somewhere in between, with accuracies around 95-98%.

Comparing the learning rate schedulers, StepLR and CosineAnnealingLR generally helped improve or maintain the accuracy compared to having no scheduler. However, with optimizers like Adagrad, the scheduler didn't seem to make a difference. Overall, using SGD with momentum or Adagrad seemed to yield the best results, especially when combined with a scheduler like StepLR or CosineAnnealingLR.

2 Problem 2

[5]:
```
# Import necessary libraries
import numpy as np
import torch
import torch.nn as nn
import torch.optim as optim
import matplotlib.pyplot as plt

# Set seed for reproducibility
torch.manual_seed(0)
np.random.seed(0)

# Define the target function as the sum of 3 sinusoids
def target_function(x):
    return np.sin(2 * np.pi * 1.0 * x) + 0.5 * np.sin(2 * np.pi * 3.0 * x) + 0.
↪25 * np.sin(2 * np.pi * 5.0 * x)

# Generate dataset with 4000 points
x = np.linspace(0, 1, 4000)  # 4000 points between 0 and 1
y = target_function(x)

# Convert to PyTorch tensors
x_train = torch.tensor(x, dtype=torch.float32).view(-1, 1)
y_train = torch.tensor(y, dtype=torch.float32).view(-1, 1)

# Define the original 3-layer MLP model
class MLP(nn.Module):
    def __init__(self):
```

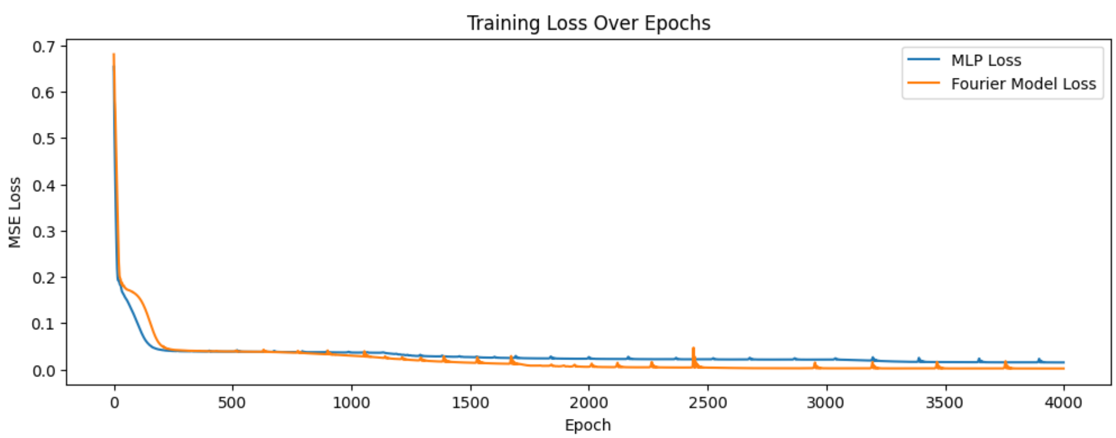
```python
        super(MLP, self).__init__()
        self.fc1 = nn.Linear(1, 100)
        self.fc2 = nn.Linear(100, 100)
        self.fc3 = nn.Linear(100, 1)
        self.activation = nn.ReLU()

    def forward(self, x):
        x = self.activation(self.fc1(x))
        x = self.activation(self.fc2(x))
        return self.fc3(x)

# Define the LFF (Learnable Fourier Feature) layer
class LFF(nn.Module):
    def __init__(self, in_features, out_features, scale=1.0, init="iso", 
 ↪sincos=False):
        super().__init__()
        self.in_features = in_features
        self.sincos = sincos
        self.out_features = out_features
        self.scale = scale
        if self.sincos:
            self.linear = nn.Linear(in_features, out_features // 2)
        else:
            self.linear = nn.Linear(in_features, out_features)
        if init == "iso":
            nn.init.normal_(self.linear.weight, 0, scale / in_features)
            nn.init.normal_(self.linear.bias, 0, 1)
        else:
            nn.init.uniform_(self.linear.weight, -scale / in_features, scale / 
 ↪in_features)
            nn.init.uniform_(self.linear.bias, -1, 1)
        if self.sincos:
            nn.init.zeros_(self.linear.bias)

    def forward(self, x):
        x = np.pi * self.linear(x)
        if self.sincos:
            return torch.cat([torch.sin(x), torch.cos(x)], dim=-1)
        else:
            return torch.sin(x)

# Define the FourierModel with the LFF layer
class FourierModel(nn.Module):
    def __init__(self, state_dim, action_dim, hidden_dim):
        super(FourierModel, self).__init__()
        self.input_layer = LFF(state_dim, hidden_dim, scale=0.1, init="iso", 
 ↪sincos=False)
```

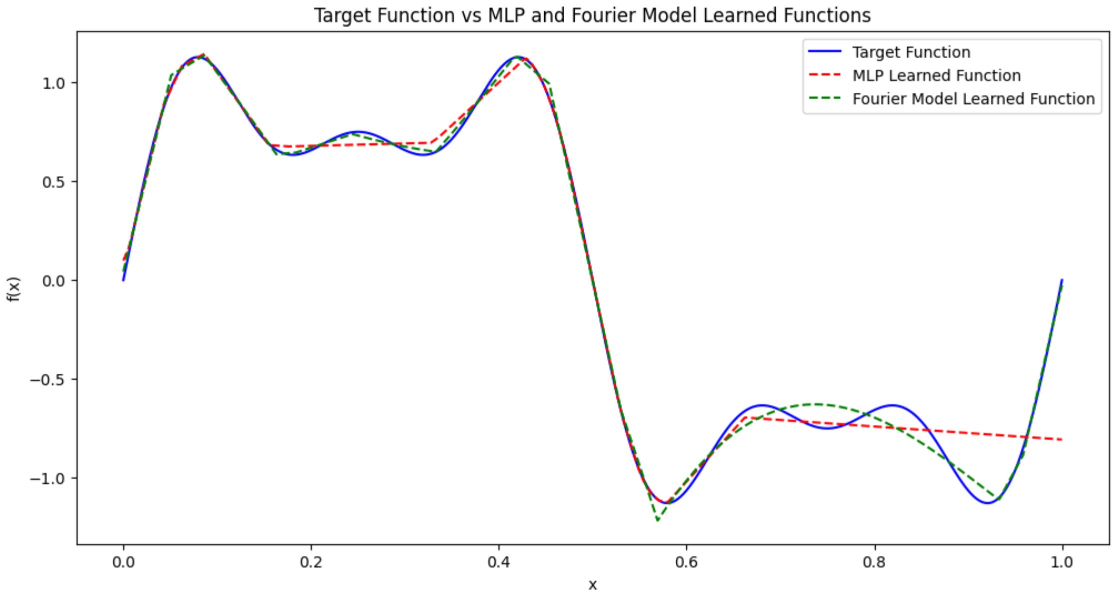
```python
        self.mid_layer = nn.Linear(hidden_dim, hidden_dim)
        self.relu2 = nn.ReLU()
        self.output = nn.Linear(hidden_dim, action_dim)

    def forward(self, x):
        x = self.input_layer(x)
        x = self.mid_layer(x)
        x = self.relu2(x)
        return self.output(x)

# Initialize models, loss function, and optimizers
mlp_model = MLP()
fourier_model = FourierModel(state_dim=1, action_dim=1, hidden_dim=100)

criterion = nn.MSELoss()
mlp_optimizer = optim.Adam(mlp_model.parameters(), lr=0.002)
fourier_optimizer = optim.Adam(fourier_model.parameters(), lr=0.002)

# Training loop for both models
num_epochs = 4000
mlp_losses = []
fourier_losses = []

for epoch in range(num_epochs):
    # Train MLP model
    mlp_model.train()
    mlp_optimizer.zero_grad()
    mlp_y_pred = mlp_model(x_train)
    mlp_loss = criterion(mlp_y_pred, y_train)
    mlp_loss.backward()
    mlp_optimizer.step()
    mlp_losses.append(mlp_loss.item())

    # Train Fourier model
    fourier_model.train()
    fourier_optimizer.zero_grad()
    fourier_y_pred = fourier_model(x_train)
    fourier_loss = criterion(fourier_y_pred, y_train)
    fourier_loss.backward()
    fourier_optimizer.step()
    fourier_losses.append(fourier_loss.item())

    # Print loss every 100 epochs
    if (epoch + 1) % 100 == 0:
        print(f"Epoch [{epoch+1}/{num_epochs}], MLP Loss: {mlp_loss.item():.
↪4f}, Fourier Loss: {fourier_loss.item():.4f}")
```

```python
# Plot the training loss for both models
plt.figure(figsize=(12, 4))
plt.plot(mlp_losses, label="MLP Loss")
plt.plot(fourier_losses, label="Fourier Model Loss")
plt.title("Training Loss Over Epochs")
plt.xlabel("Epoch")
plt.ylabel("MSE Loss")
plt.legend()
plt.show()

# Plot the target function vs. the learned functions
with torch.no_grad():
    mlp_y_learned = mlp_model(x_train).numpy()
    fourier_y_learned = fourier_model(x_train).numpy()

plt.figure(figsize=(12, 6))
plt.plot(x, y, label="Target Function", color='blue')
plt.plot(x, mlp_y_learned, label="MLP Learned Function", color='red',␣
 ↪linestyle='--')
plt.plot(x, fourier_y_learned, label="Fourier Model Learned Function",␣
 ↪color='green', linestyle='--')
plt.title("Target Function vs MLP and Fourier Model Learned Functions")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.legend()
plt.show()

def count_parameters(model):
    return sum(p.numel() for p in model.parameters() if p.requires_grad)
```

```
Epoch [100/4000], MLP Loss: 0.1005, Fourier Loss: 0.1595
Epoch [200/4000], MLP Loss: 0.0425, Fourier Loss: 0.0520
Epoch [300/4000], MLP Loss: 0.0393, Fourier Loss: 0.0407
Epoch [400/4000], MLP Loss: 0.0397, Fourier Loss: 0.0395
Epoch [500/4000], MLP Loss: 0.0385, Fourier Loss: 0.0388
Epoch [600/4000], MLP Loss: 0.0381, Fourier Loss: 0.0382
Epoch [700/4000], MLP Loss: 0.0379, Fourier Loss: 0.0373
Epoch [800/4000], MLP Loss: 0.0375, Fourier Loss: 0.0356
Epoch [900/4000], MLP Loss: 0.0381, Fourier Loss: 0.0382
Epoch [1000/4000], MLP Loss: 0.0369, Fourier Loss: 0.0298
Epoch [1100/4000], MLP Loss: 0.0362, Fourier Loss: 0.0266
Epoch [1200/4000], MLP Loss: 0.0324, Fourier Loss: 0.0223
Epoch [1300/4000], MLP Loss: 0.0299, Fourier Loss: 0.0220
Epoch [1400/4000], MLP Loss: 0.0281, Fourier Loss: 0.0188
Epoch [1500/4000], MLP Loss: 0.0267, Fourier Loss: 0.0146
Epoch [1600/4000], MLP Loss: 0.0262, Fourier Loss: 0.0131
Epoch [1700/4000], MLP Loss: 0.0264, Fourier Loss: 0.0126
```

```
Epoch [1800/4000], MLP Loss: 0.0239, Fourier Loss: 0.0083
Epoch [1900/4000], MLP Loss: 0.0234, Fourier Loss: 0.0073
Epoch [2000/4000], MLP Loss: 0.0265, Fourier Loss: 0.0057
Epoch [2100/4000], MLP Loss: 0.0226, Fourier Loss: 0.0050
Epoch [2200/4000], MLP Loss: 0.0224, Fourier Loss: 0.0047
Epoch [2300/4000], MLP Loss: 0.0221, Fourier Loss: 0.0047
Epoch [2400/4000], MLP Loss: 0.0220, Fourier Loss: 0.0042
Epoch [2500/4000], MLP Loss: 0.0218, Fourier Loss: 0.0040
Epoch [2600/4000], MLP Loss: 0.0216, Fourier Loss: 0.0033
Epoch [2700/4000], MLP Loss: 0.0218, Fourier Loss: 0.0027
Epoch [2800/4000], MLP Loss: 0.0214, Fourier Loss: 0.0026
Epoch [2900/4000], MLP Loss: 0.0213, Fourier Loss: 0.0026
Epoch [3000/4000], MLP Loss: 0.0212, Fourier Loss: 0.0025
Epoch [3100/4000], MLP Loss: 0.0207, Fourier Loss: 0.0024
Epoch [3200/4000], MLP Loss: 0.0190, Fourier Loss: 0.0034
Epoch [3300/4000], MLP Loss: 0.0171, Fourier Loss: 0.0023
Epoch [3400/4000], MLP Loss: 0.0169, Fourier Loss: 0.0022
Epoch [3500/4000], MLP Loss: 0.0159, Fourier Loss: 0.0024
Epoch [3600/4000], MLP Loss: 0.0157, Fourier Loss: 0.0022
Epoch [3700/4000], MLP Loss: 0.0155, Fourier Loss: 0.0022
Epoch [3800/4000], MLP Loss: 0.0154, Fourier Loss: 0.0023
Epoch [3900/4000], MLP Loss: 0.0164, Fourier Loss: 0.0021
Epoch [4000/4000], MLP Loss: 0.0152, Fourier Loss: 0.0021
```





- What is the input and output of the model? What does this code do?
  - **MLP Model**
    - **Input:** Takes in a sample of shape: 1
    - **Output:** ouptuts a sample of shape: 1
  - **Fourier Loss**
    - **Input:** Takes in a sample of shape: 1

    - **Output:** ouptuts a sample of shape: 1

      The code trains the aforementioned neural networks to approximate a target function. it then compares their losses, then compares the learned functions to the target function.

- What a role does the LFF layer shown in the code play? Explain briefly.

  The LFF (Learnable Fourier Feature) layer transforms the input into a higher-dimensional space using sine functions (since sincos=False). This transformation helps the model represnt the sample's in higher dimensionality, this allows the model to train on the frequencies and amplitudes generated. Here `sincos` is set to `False` without this it would generate both $[sin(W_n X_n), cos(W_n x_n)]$, using only $sin(W_n x_n)$ is beneficial for simplicity and can be effective for capturing periodic patterns in the dataset.

- Run the code and describe what you observe. How do the results relate to the Universal Function Approximation Theorem of neural networks?

  Observations

  We can observe that both the learned functions are very similar to the target function. Although the LFF funciton is much closer to the target funciton. This behavior might be because of the fourier features being in a higher dimentionality, allowing for better feature detection for samples with lower complexities.

  universal approximation theorem states

The *universal approximation theorem states* that any continuous function f can be approximated arbitrarily well by a neural network with at least 1 hidden layer with a finite number of weights [source].

How Observations relate to UAT

The similarity between the observations and the UAT demonstrate the theorem in practice. increaseing the number of weights and representing the samples in a higher dimentionality can prove to be effective for funciton approximators.