PERT and CPM

Introduction

- One of the most challenging jobs that any manager can take on is the management of a large-scale project that requires coordinating numerous activities throughout the organization.

- A myriad of details must be considered in planning how to coordinate all these activities, in developing a realistic schedule, and then in monitoring the progress of the project.

Introduction

- Fortunately, two closely related operations research techniques, PERT (program evaluation and review technique) and CPM (critical path method), are available to assist the project manager in carrying out these responsibilities.

- These techniques make heavy use of networks to help plan and display the coordination of all the activities. They also normally use a software package to deal with all the data needed to develop schedule information and then to monitor the progress of the project. Project management software, such as MS Project is widely available for these purposes.

Introduction

- PERT and CPM are basically time-oriented methods in the sense that they both lead to determination of a time schedule for the project.

- The significant difference between two approaches is that the time estimates for the different activities in **CPM were assumed to be deterministic** while in PERT these are described **probabilistically**. These techniques are referred as **project scheduling techniques**

Applications of CPM / PERT

- Construction of a dam or a canal system in a region
- Construction of a building or highway
- Maintenance or overhaul of airplanes or oil refinery
- Space flight
- Cost control of a project using PERT / COST
- Designing a prototype of a machine
- Development of supersonic planes

Network Diagram Representation

Activity

- Any individual operation which utilizes resources and has an end and a beginning is called activity. An arrow is commonly used to represent an activity with its head indicating the direction of progress in the project. These are classified into four categories

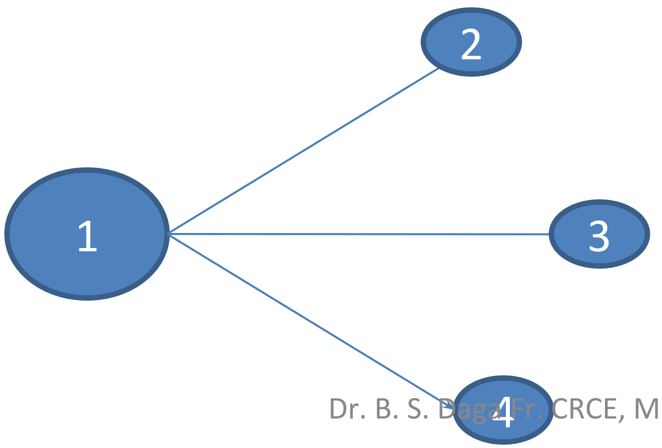
Types of Activity

- **Predecessor activity** – Activities that must be completed immediately prior to the start of another activity are called predecessor activities.

- **Successor activity** – Activities that cannot be started until one or more of other activities are completed but immediately succeed them are called successor activities.

…Types of Activity

- **Concurrent activity** – Activities which can be accomplished concurrently are known as concurrent activities. It may be noted that an activity can be a predecessor or a successor to an event or it may be concurrent with one or more of other activities.



…Types of Activity

- **Dummy activity – An activity which does not consume any kind of resource but merely depicts the technological dependence is called a dummy activity.**

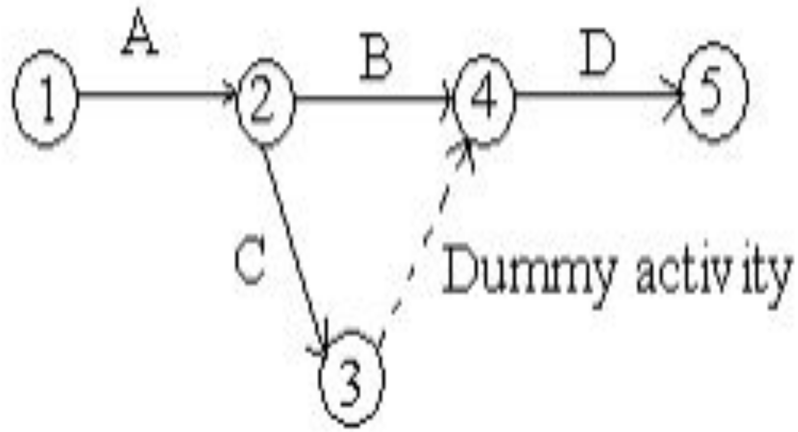
- It is represented by **dotted line arrow**. It is used only when it is necessary , there is no restriction of no. of dummy activity used. There should be no looping and dangling on network diagram.

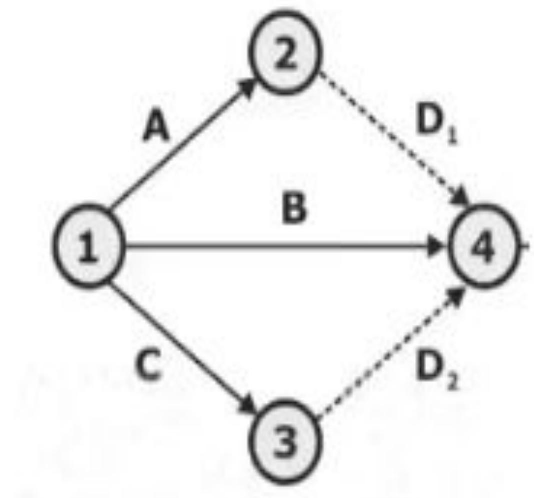
…Types of Activity

- The dummy activity is inserted in the network to ESTABLISH THE given precendence relationship among the activities of the project. It is needed when
- (a) two or more parallel activities in a project have same head and tail events
- (b) two or more activities have some (but not all) of their immediate predecessor activities in common.

…Types of Activity

- For example, consider a situation where A and B are concurrent activities. C is dependent on A and D is dependent on A and B both. Such a situation can be handled by using a dummy activity as shown in the figure.


A
B
D
C
Dummy activity
1
2
4
5
3


A
B
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D_2
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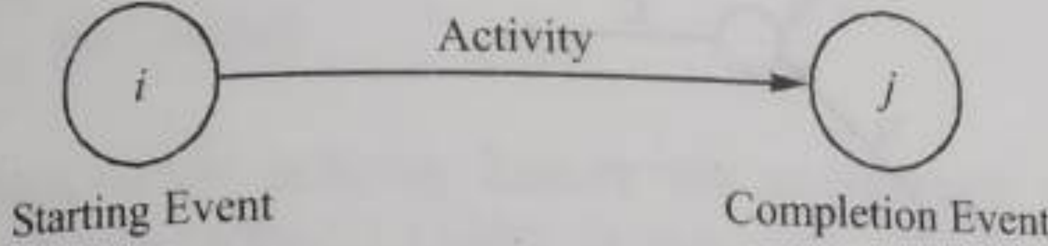
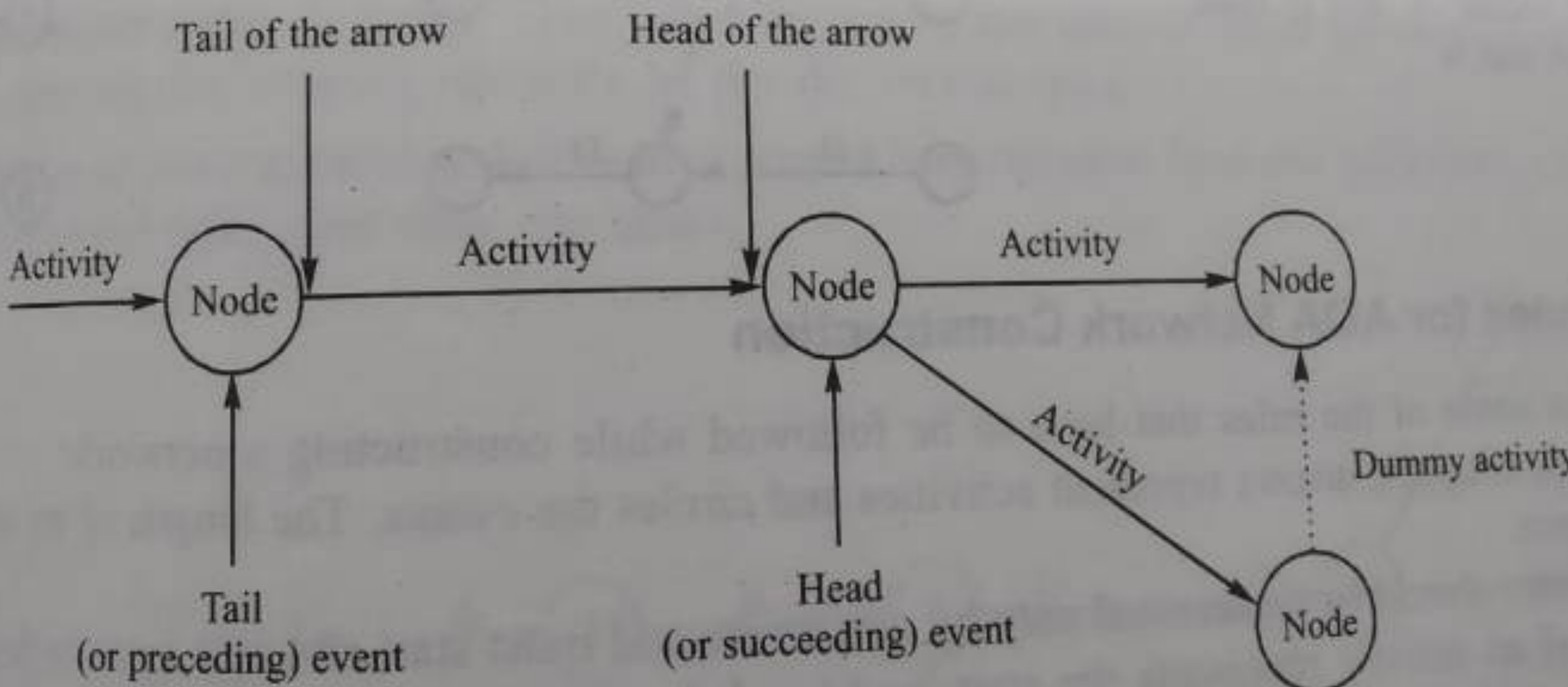
on. A dumm... ...in the network diagram as shown in Fig. 1



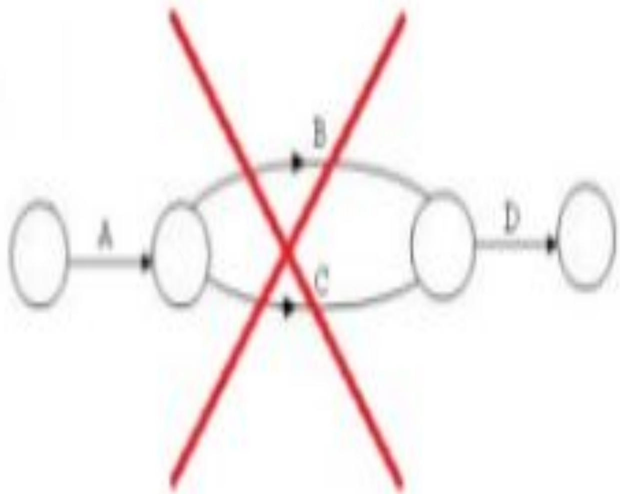
(a) Activity-Node Relationship

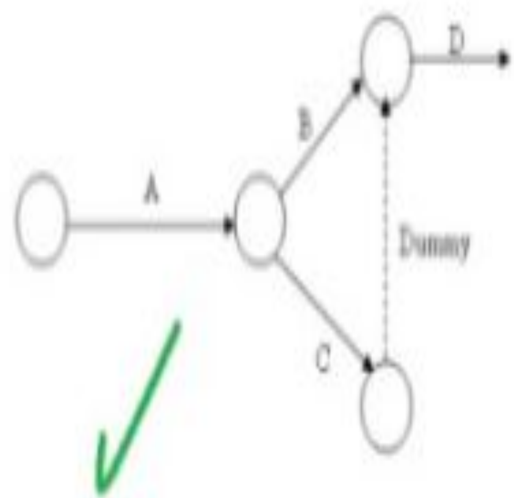


(b) Activity-Node Relationship

...k to show precedence

- no two activities can be identified by the same beginning and end event in such cases a dummy activity is introduced to resolve the problem


A
B
C
D


A
B
C
D
Dummy

Network

- A network is a graphic representation of a project's operations and a composed of activities and events that must be completed to reach the end objective of a project, showing the planning sequence of time accomplishment, their dependence and inter-relationship. The basic components of a network are

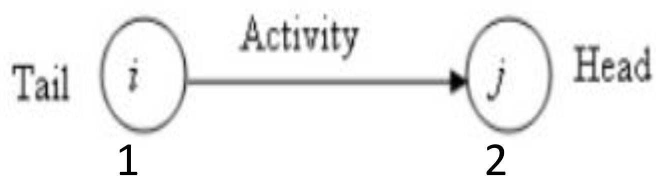
Activity-

- An activity is a task, or item of work to be done, that consume time, effort, money or other resources. An activity is represented by an arrow with its head indicating the sequence in which the events are to occur.

- Event- An event represents the start (beginning) or completion (end) of some activity and as such it consume no time. It has no time duration and does not consume any resources. It is also known as a node. An event is generally represented on the network by a circle.

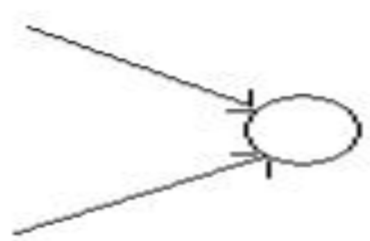
Event (Milestone)

- The beginning and end points of an activity are called as event or nodes. event is a point in time and does not consume any resources. It is represented by a number circle. the head even called as jth event always a number higher than the tale event called the ith eventevent



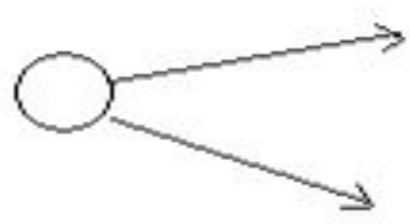
- The events are classified in to three categories

- Merge event: When more than one activity comes and joins an event such an event is known as merge event.



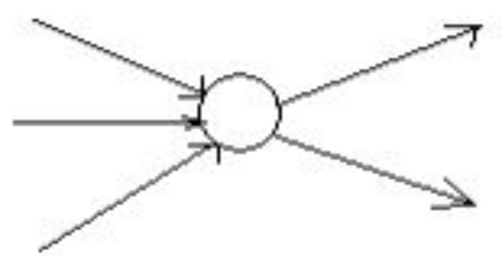
Merge event

- Burst event – When more than one activity leaves an event such an event is known as burst event



Burst event

- Merge and Burst event – An activity may be merge and burst event at the same time as with respect to some activities it can be a merge event and with respect to some other activities it may be a burst event.

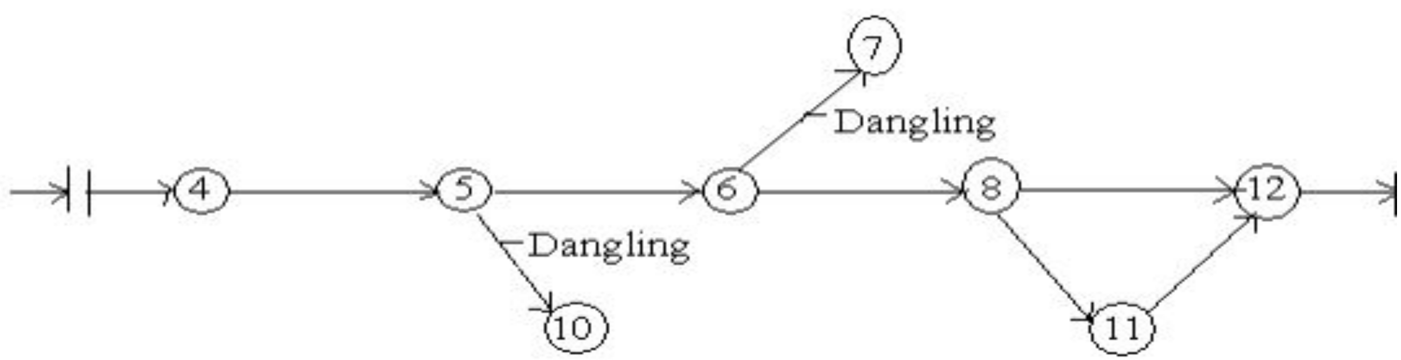


Merge and Burst event

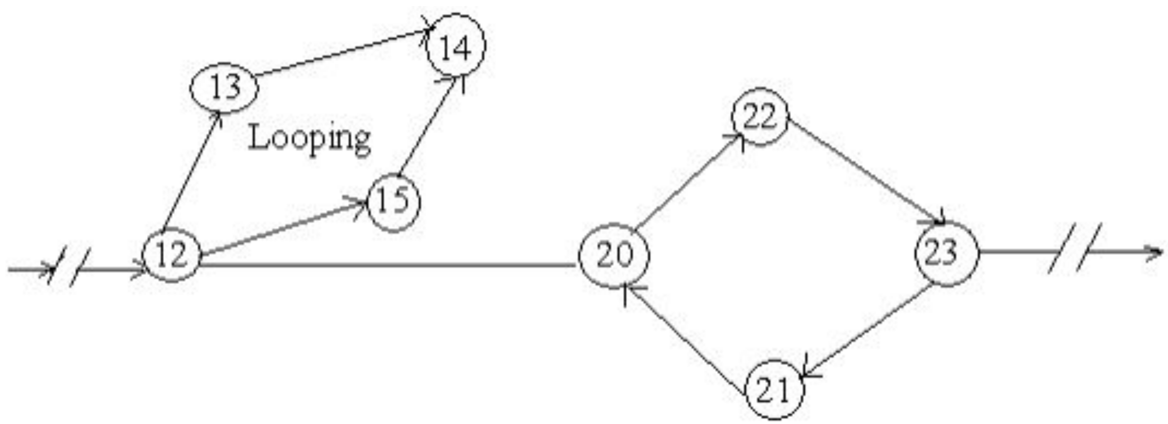
- The activity can be further classified into the following three categories

Common Errors in Drawing Networks

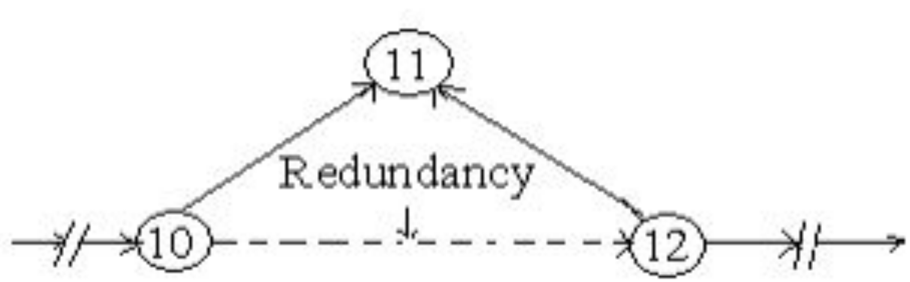
- 1. Dangling
  - To disconnect an activity before the completion of all activities in a network diagram is known as dangling. As shown in the figure activities (5 – 10) and (6 – 7) are not the last activities in the network. So the diagram is wrong and indicates the error of dangling



- Looping or Cycling:
  - Looping error is also known as cycling error in a network diagram. Drawing an endless loop in a network is known as error of looping as shown in the following figure



- Redundancy: Unnecessarily inserting the dummy activity in network logic is known as the error of redundancy as shown in the following diagram



Rules for Network Representation

- Three rules are available for constructing the network

  1. Each activity is represented by one, and only one arrow (arc)

  2. Each activity must be identified by two distinct end nodes & no two or more activities can have the same tail.

  3. To maintain the correct precedence relationships, the following questions must be answered as each is added to network:

  (a) What activities must immediately precede the current activity?

  (b) What activities must follow the current activity?

  (c) What activities must occur concurrently with the current activity? The answer of these questions may require the use of dummy activities to ensure correct precedences among the activities.

Numbering the events- Fulkerson Rule

- After the network is drawn in a logical sequence, every event is assigned a number. The number sequence must be such as to reflect the flow of the network. In event numbering, the following rules should be observed, which is also known as Fulkerson's rule.

  (a) Event numbers should be unique

  (b) Event numbering should be carried out on a sequential basis from left to right

  (c) The initial event which has all outgoing arrows with no incoming arrow is numbered 0 or 1

  (d) The head of an arrow should always bear a number higher than the one assigned at the tail of the arrow

  (e) Gaps should be left in the sequences of event numbering to accommodate subsequent inclusion of activities, if necessary.

CPM/PERT

- CPM/PERT are network based models designed to assist in the planning, scheduling and control of projects.

- Project- A project is defined as a collection of interrelated activities with each activity consuming time and resources

CPM/PERT

- The objective of CPM/PERT is to provide analytic means for scheduling the activities. Followings are the steps of the techniques

CPM/PERT

1. We define the activities of the project, their precedence relationship and their time requirements.
2. The precedence relationship among the activities are represented by a network
3. Specific computations to develop the time schedule for the project. During the actual execution of the project things may not proceed as planned, as some of the activities may be expedited or delayed. When this happens, the schedule must be revised to reflect the realities on the ground. This is the reason for including a feedback loop between the time schedule phase and the network phase, as shown in following diagram.

CPM/PERT

- The two techniques, CPM and PERT, which were developed independently, differ in that CPM assumes deterministic activity duration and PERT assumes probabilistic durations.

CPM

- It is commonly used for those projects which are repetitive in nature & where one has prior experience of handling similar projects. It is a deterministic model and places emphasis on time & cost for activities of a project.

PERT

- PERT (Program evaluation & review Technique)- it is generally used for those projects where time required to complete various activities are not known as a prior. It is probabilistic model & is primarily concerned for evaluation of time. It is event oriented.

PERT/CPM

Advantages

- A PERT/CPM chart explicitly defines and makes visible dependencies (precedence relationships) between the elements,
- PERT/CPM facilitates identification of the critical path and makes this visible,
- PERT/CPM facilitates identification of early start, late start, and slack for each activity,
- PERT/CPM provides for potentially reduced project Duration due to better understanding of dependencies leading to improved overlapping of activities and tasks where feasible.

PERT/CPM disadvantages

- There can be potentially hundreds or thousands of activities and individual dependency relationships,
- The network charts tend to be large and unwieldy requiring several pages to print and requiring special size paper,
- The lack of a timeframe on most PERT/CPM charts makes it harder to show status although colours can help (e.g., specific colour for completed nodes),
- When the PERT/CPM charts become unwieldy, they are no longer used to manage the project.

Rules for AOA network construction

- Following are some of the rules that have to be followed while constructing a network:

1. In network diagram, arrows represent activities and circles the events. The length of an arrow is of no significance.

Rules for AOA network construction

- Each activity should be represented only by one Arrow and must start and end in a circle called event. The tail of an activity represent the start, and head the completion of work

Rules for AOA network construction

- The event numbered 1 denote the start of the project and is called initial event. All activities emerging from event 1 should not be preceded by any other activity or activities. an event carrying the highest number denote the completion event. A network should have only one initial event and only one terminal event

Rules for AOA network construction

- The general rule for numbering the event is that the head even should always be number larger than the number at its tail that is event should be number such that for each activity (I,j), i<j.

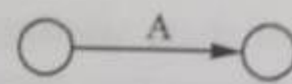
Rules for AOA network construction

- An activity must be uniquely identified by its starting and completion event which implies that
- An event number should not get repeated or duplicated
- two activity should not be identified by the same completion event
- Activities must be represented either by their symbols or by the corresponding ordered pair of starting completion event

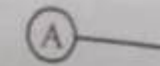
AOA network

AON network

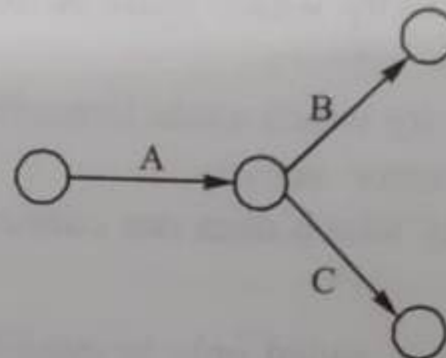
1. Activity A



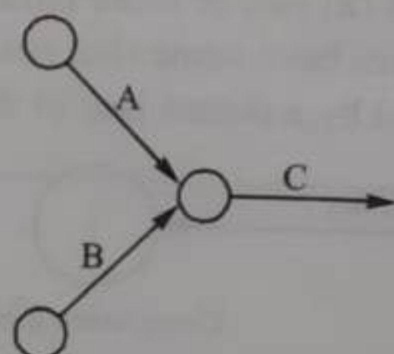
2. B must follow A



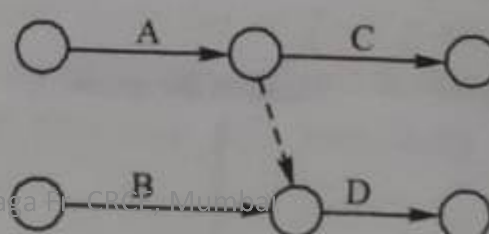
3. B and C must follow A

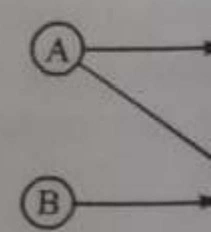


4. C must follow A and B



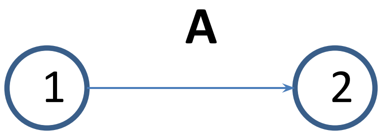
5. C must follow A, and D must follow A and B

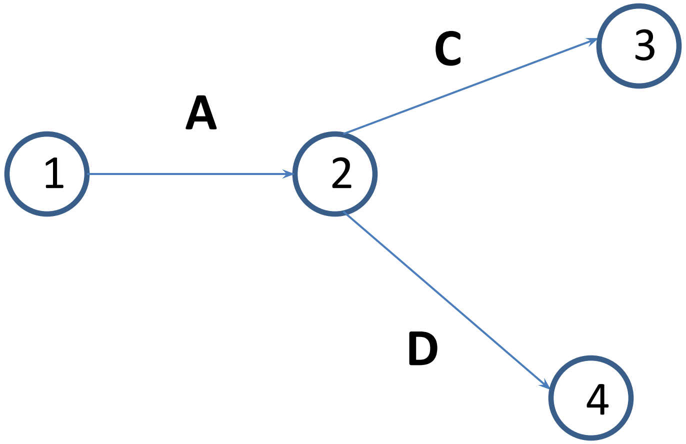


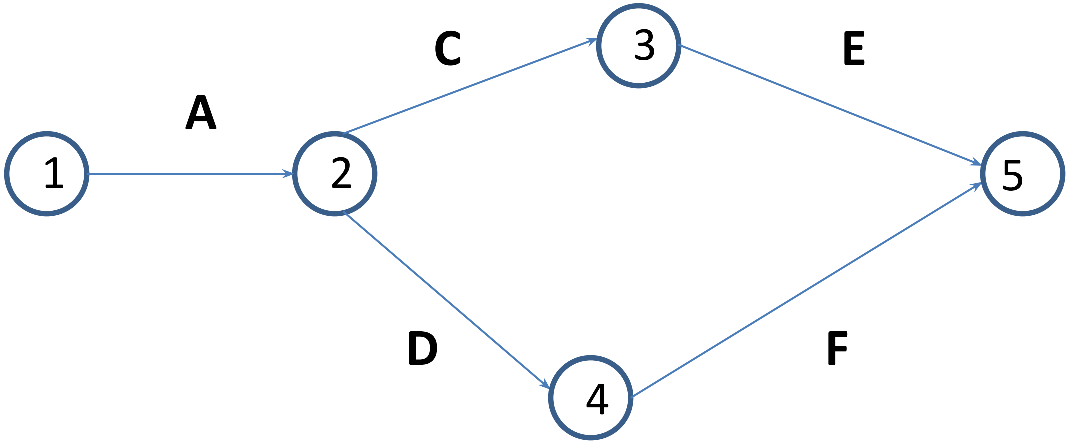


Example

- Draw the logic network for the following:
- Activities C and D both follow A , activity E follows C , activity F follows D , activity E and F precedes B.


A
1
2


A
C
D
1
2
3
4


A
C
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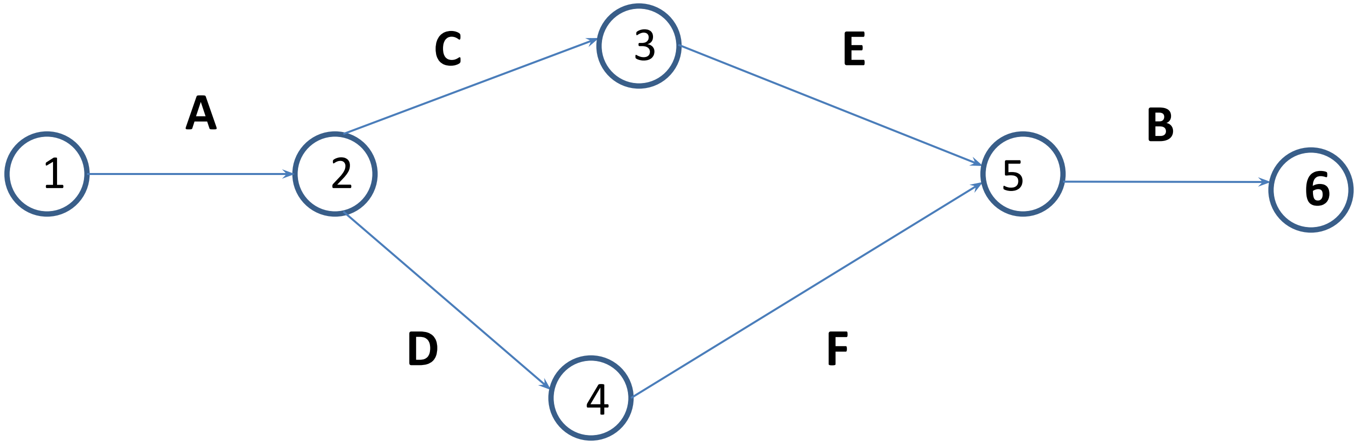
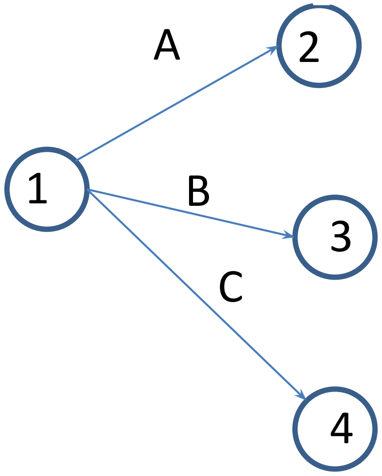

A
C
3
E
B
1
2
5
6
D
4
F

Construct a network for a project whose activities and their predecessor relationship are given in table

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor	-	-	-	A	B	B	C	D	E	H,I	F,G


A
2
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B
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C
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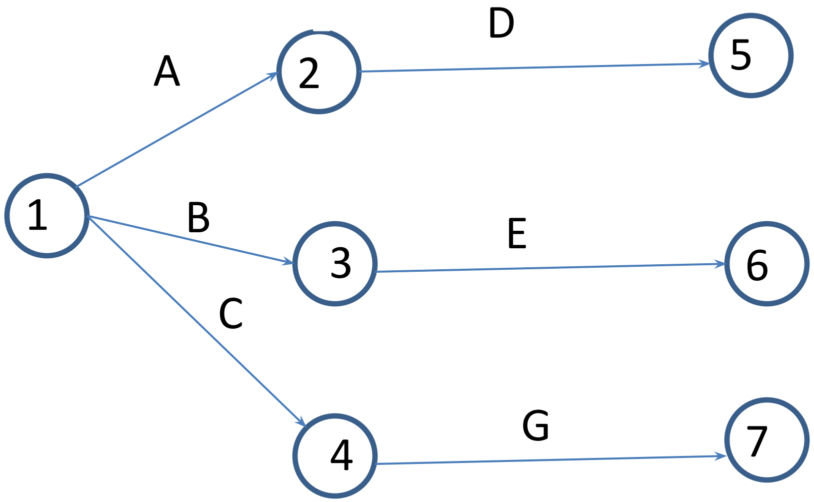
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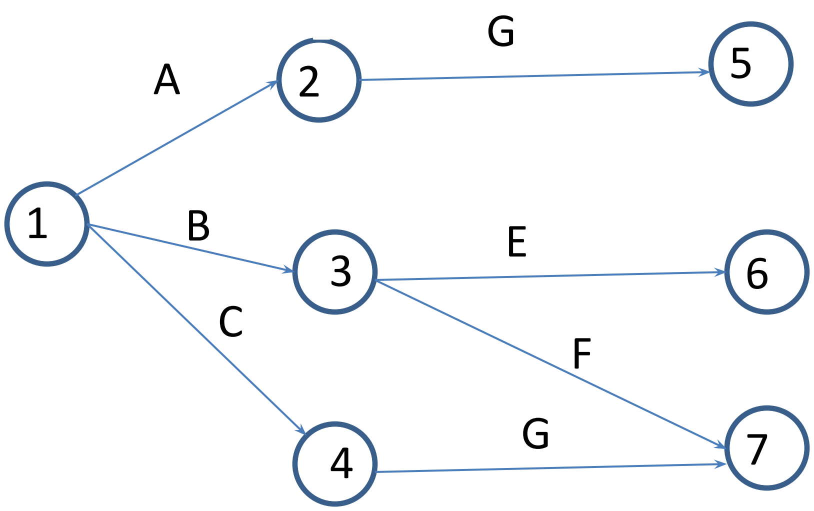
G
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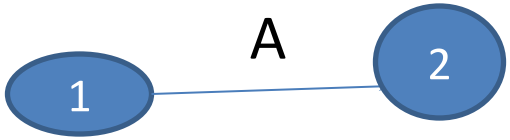
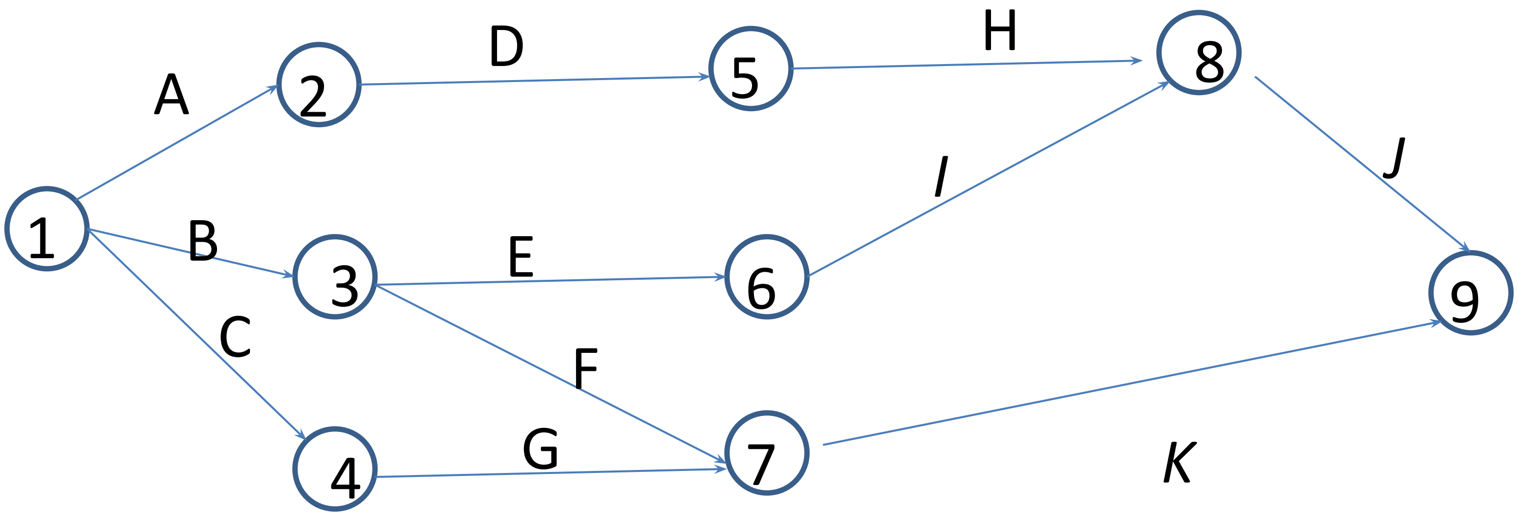
7


A
1
2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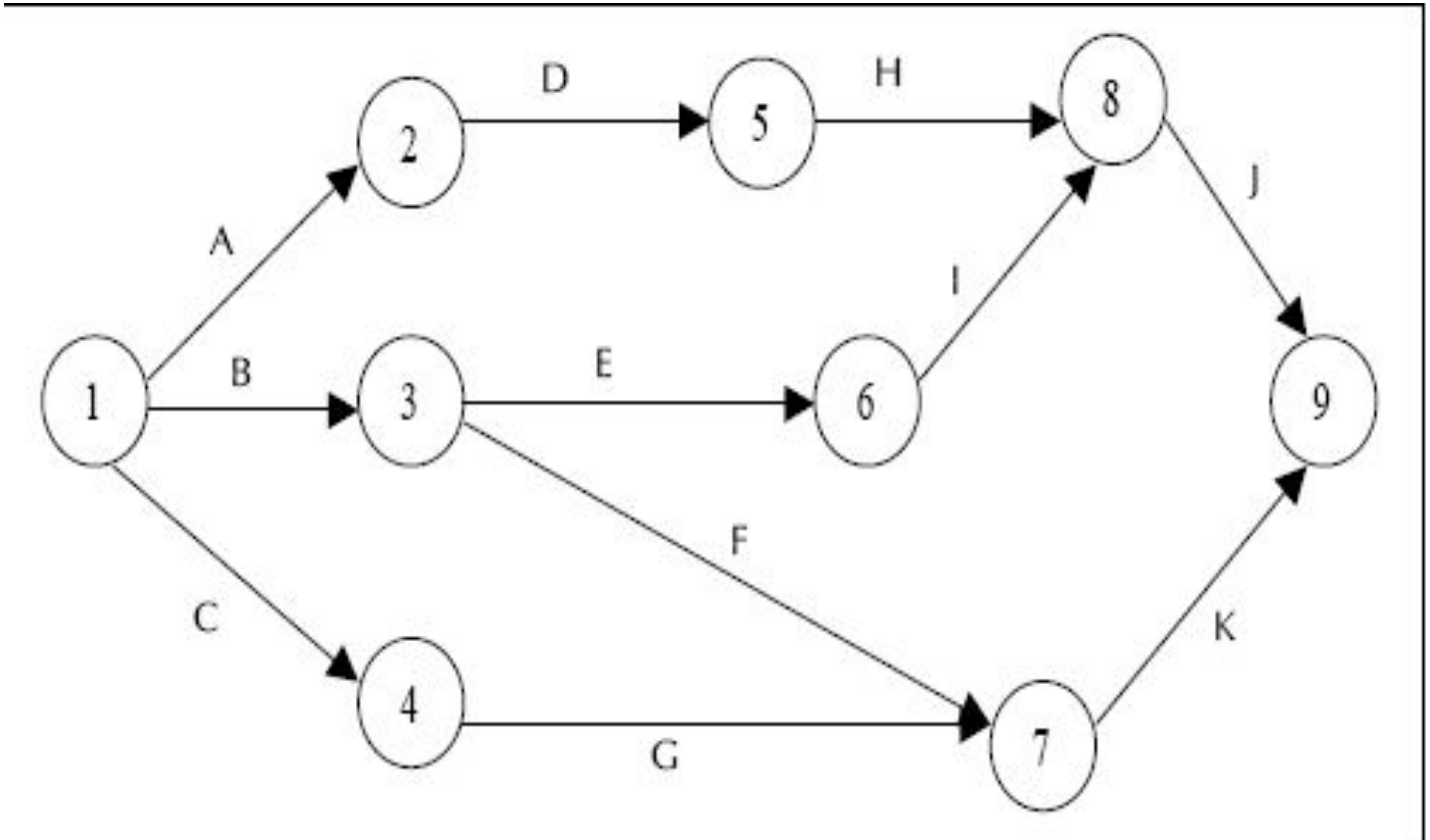
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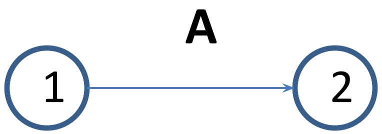
H
I
J
K

Numerical 2

- The sequence of activities together with their predecessor is given below . Draw a network diagram of activities for the project

Activity	Description	Predecessor Activity
A	Open work order	–
B	Get material for *X*	*A*
C	Get material for *Y*	*A*
D	Turn *X* on lathe	*B*
E	Turn *Y* on lathe	*B*, *C*
F	Polish *Y*	*E*
G	Assemble *X* and *Y*	*D*, *F*
H	Pack	*G*


A
1
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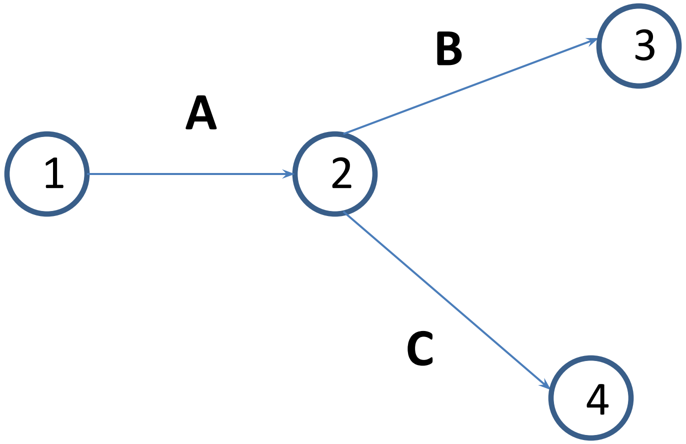
A
B
C
1
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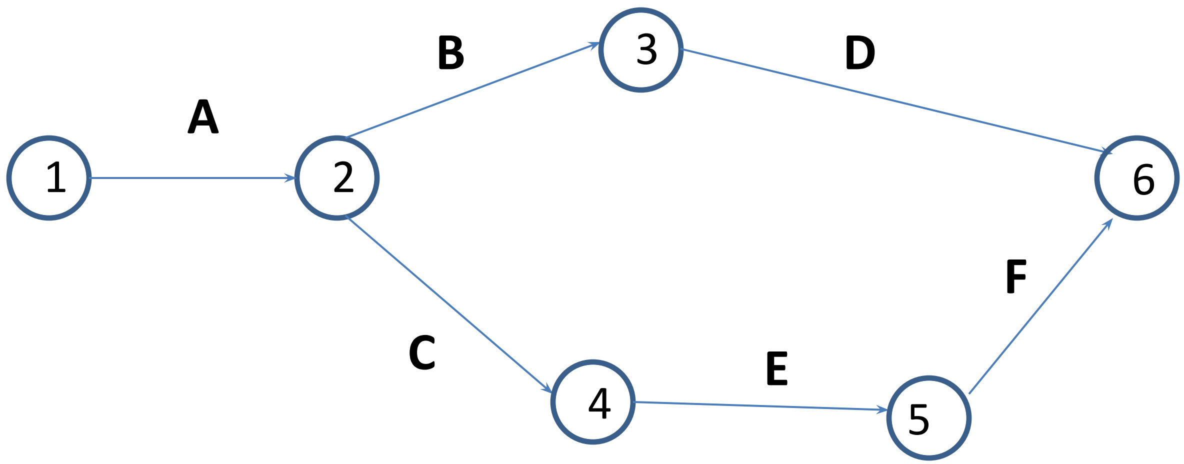
4


A
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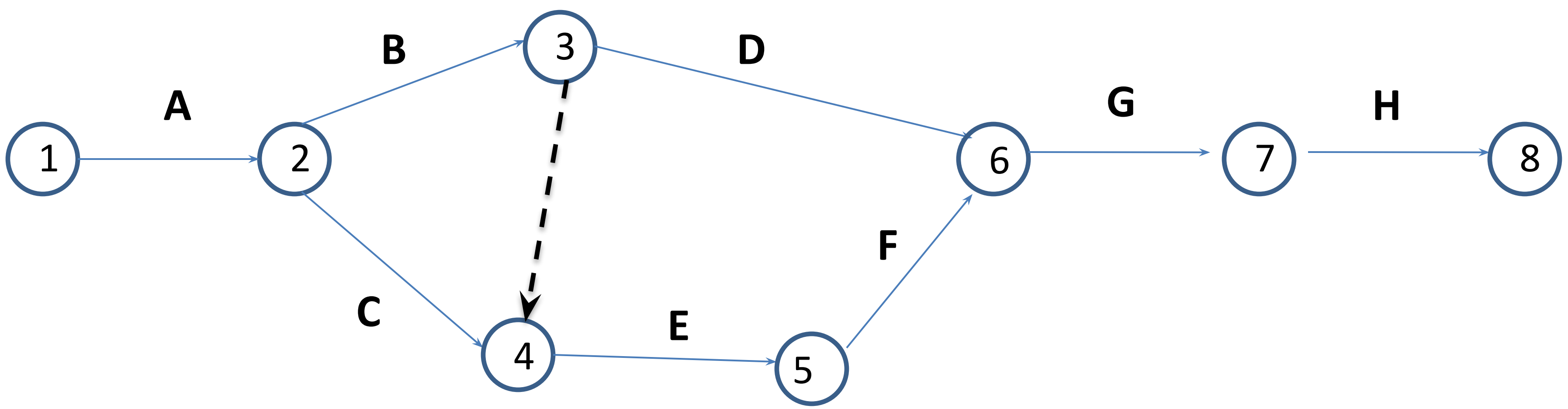
1
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G
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H
8

Critical Path Analysis

- The objective of critical path analysis is to estimate the total project duration and to assign starting and finishing time to all activities involved in the project. This helps to check the actual progress against the scheduled duration of the project.

- Having done this the following factor should be known in order to prepare the project scheduling.

1. Total completion time of the project
2. Earlier and latest start time of each activity
3. Critical activities and critical path

- Float for each activity that is the amount of time by which the completion of non critical activity can be delayed without deleting the total project completion time

Critical Path in Network Analysis

- The notations used are
- (i, j) = Activity with tail event i and head event j
- Ei = Earliest occurrence time of event, i. This is the earliest time for an event to occur immediately after all the preceding activities have been completed without delaying the entire project
- Li = Latest allowable time of event i. This is the latest time at which an event can occur without causing a delay in already determined project completion time

notations

- tij = duration of an activity (i, j)
- ESij = Earliest starting time of activity (i, j). this is the earliest time an activity can possibly start without affecting the project completion.
- (Ef)ij = Earliest finishing time of activity (i, j). this is the earliest time an activity can possibly finish without affecting the project completion
- LSij = Latest starting time of activity (i, j). this is the latest time an activity can possibly start without affecting the project completion.
- (Lf)ij = Latest finishing time of activity (i, j). this is the latest time an activity must finish without delaying the project completion

Forward Pass method (For earliest event time)

- Set the earliest occurrence time of initial event 1 to zero. That is E1 = 0, for i=1

- Calculate the earliest start time for each activity that begins at the even i(=1). This is equal to the earliest ocurrence time of event, i. That is

$$ESij = Ei \text{ for all activities } (i,j) \text{ starting at event } i.$$

- Calculate the earliest finish time for each activity that begins at the even i. This is equal to the earliest start time of the activity + the duration of the activity That is

$$Efij = Esij + tij = Ei + tij, \text{ for all activities } (i,j) \text{ beginning at event } i.$$

Forward Pass method (For earliest event time)

- Calculate the earliest occurrence time for event j. This is the maximum of the earliest finish time of all activities ending into the event that is,

- $E_j = \text{Max}\ (Ef_{ij}) = \text{Max}\ (E_i + t_{ij})$ for all immediate predecessor activities

Backward Pass Method (For latest Allowable Event time)

- Set the latest occurrence time of last event, N equal to its earliest occurrence time (known from forward pass method)

  That is $L_N = E_N$, $j = N$.

- Calculate the latest finish time for each activity that ends at the event j. This is equal to the latest ocurrence time of final event That is

$$Lf_{ij} = L_i, \text{ for all activities (i,j) ending at event j.}$$

- Calculate the latest start time for each activity ending at the even j. This is obtained by subtracting the duration of the activity from the latest time of the activity That is

$$Lf_{ij} = L_j$$

$$LSij = LFij - tij = Lj - tij, \text{ for all activities (i,j) ending at event i.}$$

Backward Pass Method (For latest Allowable Event time)

- Calculate the latest occurrence time of event I ($i<j$). This is the minimum of the latest start time of all activities from the event. That is

- $L_i = \text{Min}\ (LS_{ij}) = \text{Min}\ (L_j - t_{ij})$ for all immediate predecessor activities

Backward Pass Method (For latest Allowable Event time)

- If j =1 (initial event) then the latest finish time for project, i.e. latest occurrence time L1 for the initial event is given as

$$L_1 = Min\ (LS_{ij})$$

$$= Min\ (L_j - t_{ij})\ \text{for all immediate successor activities}$$

Float

- The term “Float” implies “Fluid”, which in turn implies “**Flexibility**“. In Project Scheduling, Float refers to the amount of **scheduling flexibility.** Float is also popularly called “**Slack**“.

Float (Slack) of an Activity

- The float or free time is the length of time in which in non-critical activity and/or of an event can be delayed or extended without delaying the total project completion time.

Slack of an Event

- The slack(s) also called float of an event is the difference between its latest occurrence time and its earliest occurrence time. That is

$$\text{Event float} = L_i - E_i$$

If L=E, for certain events, then such events are called critical events.

Slack of an Activity

- It is the amount of time that an activity can be delayed without delaying project completion, it is calculated as the difference between the latest finish time and the earliest finish time for the activity. in other words,

- the computation of activity float tell us how long an activity time may be increased without increasing the project completion time. mainly 3 types of floats are defined for each non-critical activity of the project.

Total Float

- *That a schedule activity can be delayed or extended from its early start date without delaying the project finish date or violating a schedule constraint.*

- Total float is the amount of time an activity can be delayed **without delaying the project completion date.** This is the type of Float that is commonly referred to as "Float".

Total Float

- Total Float is about **flexibility at the project level**. It is about the flexibility that an activity has in its execution **without delaying the Project finish date.**

Example

- If activity 1 has a duration of **6 days** and is occurring concurrently with activity 2 which has a duration of **9 days,** activity 1 has 3 days of total float. Meaning, it can be delayed up to three days without any effect on the project.
  - However, if activity 1 is delayed by 5 days, there is now a **negative float situation: -2 days.** This reflects the fact that the project will now take two days longer than anticipated.

Total Float

- Total float is calculated by subtracting the Early Start date of an activity from its Late Start date (Late Start date (LS) – Early Start date (ES)), or Early Finish date (EF) from its Late Finish date (LF) (Late Finish date – Early Finish date).

Total Float or Float = LS – ES or **LF – EF**

Total Float

- The time within which an activity must be scheduled computed from LS and ES values for each activities start Event and end event. That is, for each activity (i,j) the total float is equal to the latest allowable time for the event at the end of the activity minus the earliest time for an event at the beginning of the activity minus the activity duration that is

Total Float (TF_{ij}) = (L_j – E_i) – t_{ij} (late start – early start)

- = Ls_{ij} – Es_{ij}
- = Lf_{ij}- Ef_{ij} (late finish – early finish)

Total Float

- The total float is the difference between project completion date and the total duration of critical path activities.

- In other words, you have a project to finish in 25 days. Your calculated critical path activities on the schedule network diagram will take 22 days. So you have a project float of +3 days. Here you can see, afloat can be a positive or negative number.

Free Float

- how much and activities completion time may be delayed without causing any delay in its immediate successor activities

- ***The amount of time – that a schedule activity can be delayed without delaying the early start date of any successor or violating a schedule constraint***

Free Float

- Free Float is about **flexibility at the activity level**. It is about the flexibility that an activity has in its execution **without delaying its successor activity**

Free Float

- consider one activity A, have total duration of 6 days, and its successor activity B is starting 3 days after completing of activity A than the free float between the activities is 3 .means there will not be any impact on activity B even activity A gets delayed by 3 Days.

Free Float

- Free float is calculated by subtracting the Early Finish date of current activity from the Early Start date of its successor activity (ES of successor Activity – EF of current Activity).

- **Free Float = ES (of successor) – EF (of current)**

Free float

- Free float of a non critical activity is defined as the time by which the completion of an activity can be delayed without causing any delay in its immediate succeeding activities. Free float values for each activity (i,j) are computed as

  – Free Float (FF_{ij}) = (E_j-E_i)-t_{ij}

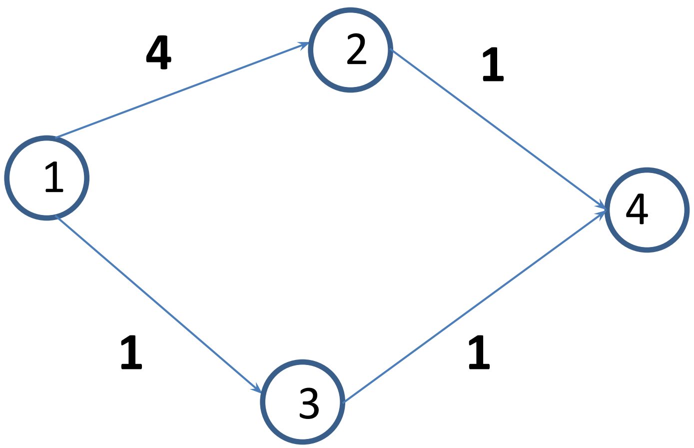
Total Float vs Free float

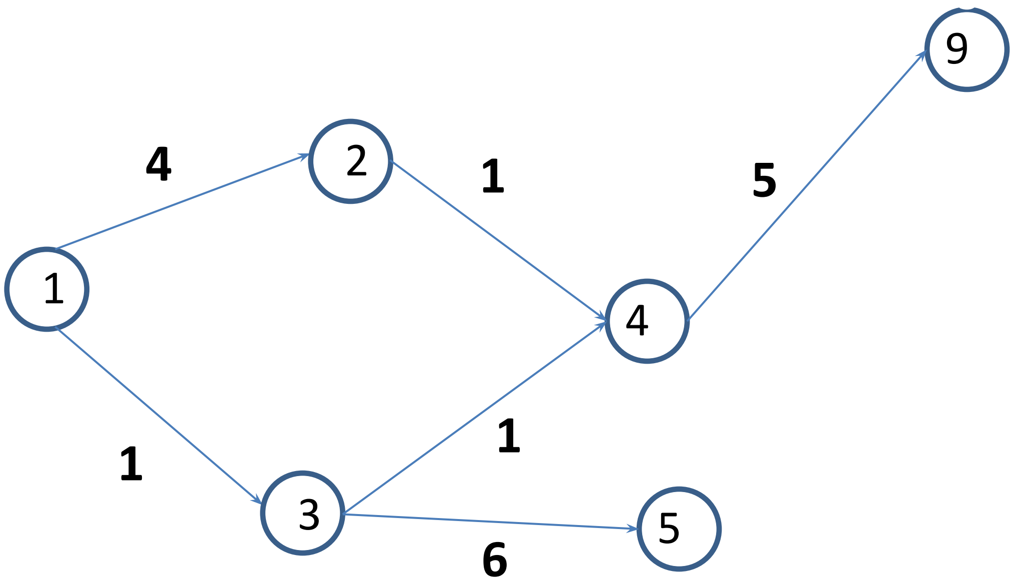
- While **Total Float** is how much an activity can be delayed without affecting the **project Finish date**, **Free Float** is about how much an activity can be delayed without affecting its **successor activity.**

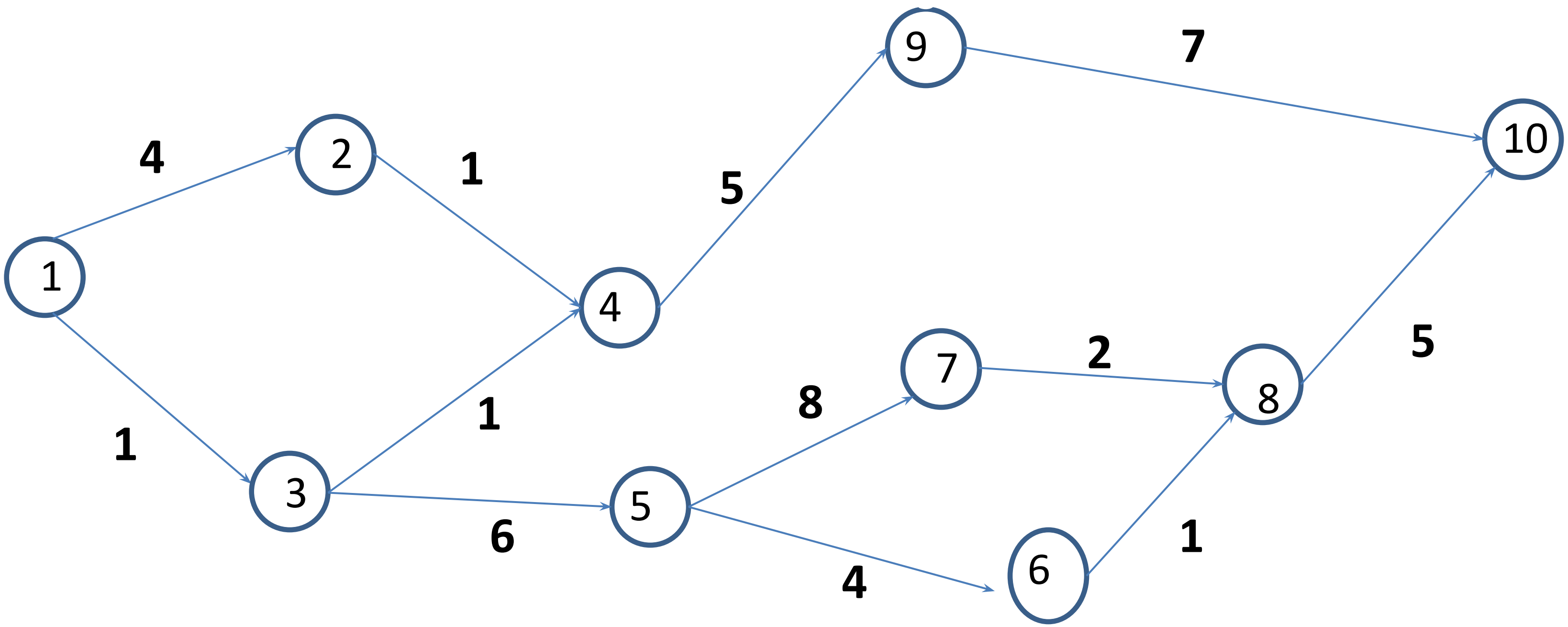
Numerical

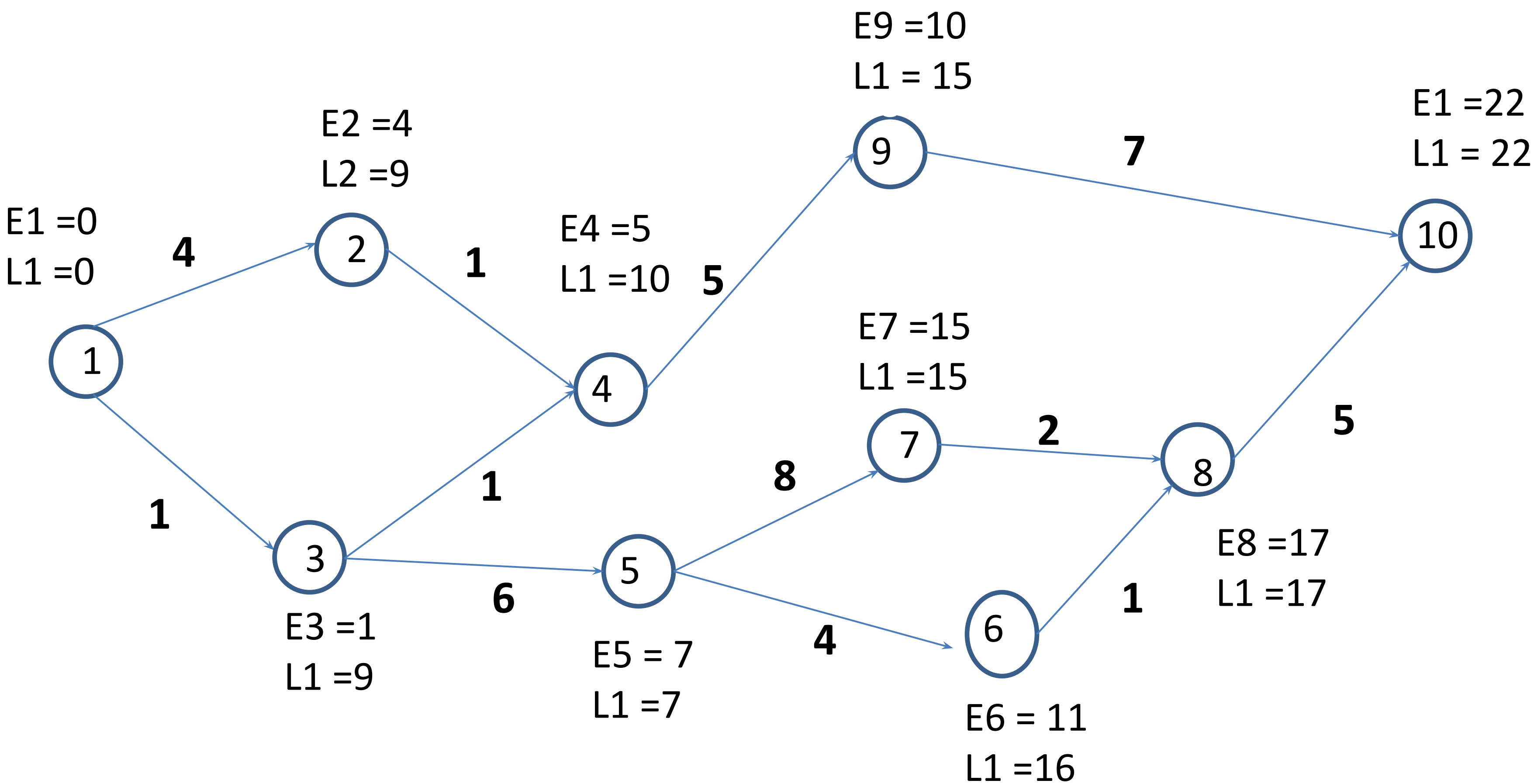
- A project has the following characteristics:

Activity	Days
1-2	4
1-3	1
2-4	1
3-4	1
3-5	6
4-9	5
5-6	4
5-7	8
6-8	1
7-8	2
8-10	5
9-10	7


4
1
1
1
1
2
3
4


4
1
5
1
1
6
1
2
3
4
5
9


1
2
3
4
5
6
7
8
9
10
4
1
1
1
5
6
8
4
7
2
5
1


E9 =10
L1 = 15
E2 =4
L2 =9
E1 =22
L1 = 22
E1 =0
L1 =0
4
2
1
E4 =5
L1 =10
5
9
7
10
1
4
E7 =15
L1 =15
7
2
8
5
1
8
1
3
6
5
4
6
1
E3 =1
L1 =9
E5 = 7
L1 =7
E6 = 11
L1 =16
E8 =17
L1 =17

Numerical 2

Activity	Description	Predecessors	Duration (days)
A	Organize sales office	–	6
B	Hire salesmen	A	4
C	Train salesmen	B	7
D	Select advertising agency	A	2
E	Plan advertising campaign	D	4
F	Conduct advertising campaign	E	10
G	Design package	–	2
H	Setup packaging facilities	G	10
I	Package initial stocks	J, H	6
J	Order stock from manufacturer	–	13
K	Select distributors	A	9
L	Sell to distributors	C, K	3
M	Ship stocks to distributors	I, L	5

- Construct a Network Diagram
- Compute the total float, free float and Independent Float for each activity.
- Find the Critical path and total project duration.

Numerical 2

Activity	Predecessors	Duration (days)
A	-	6
B	A	4
C	B	7
D	A	2
E	D	4
F	E	10
G	-	2
H	G	10
I	J,H	6
J	-	13
K	A	9
L	C,K	3
M	I,L	5

Solution

Solution (a) The arrow diagram for the given project, along with E-values and L-values, is shown in Fig. 13.11. Determine the earliest start time – E_i and the latest finish time – L_j for each event by proceeding as follows:

Forward Pass Method

$E_1 = 0$

$E_3 = E_1 + t_{1,3} = 0 + 2 = 2$

$E_5 = E_2 + t_{2,5} = 6 + 4 = 10$

$E_2 = E_1 + t_{1,2} = 0 + 6 = 6$

$$E_4 = \underset{i=1,3}{\text{Max}} \{E_i + t_{i,4}\} = \text{Max} \{E_1 + t_{14}; E_3 + t_{34}\}$$

$$= \text{Max} \{0 + 13, 2 + 10\} = 13$$

$$E_6 = \underset{i=2,5}{\text{Max}} \{E_i + t_{i,6}\} = \text{Max} \{E_2 + t_{2,6}; E_5 + t_{5,6}\}$$

$$= \text{Max} \{6 + 9; 10 + 7\} = 17$$

$E_7 = E_2 + t_{2,7} = 6 + 2 = 8$

$$E_9 = \underset{i=4,6}{\text{Max}} \{E_i + t_{i,9}\} = \text{Max} \{E_4 + t_{4,9}; E_6 + t_{6,9}\}$$

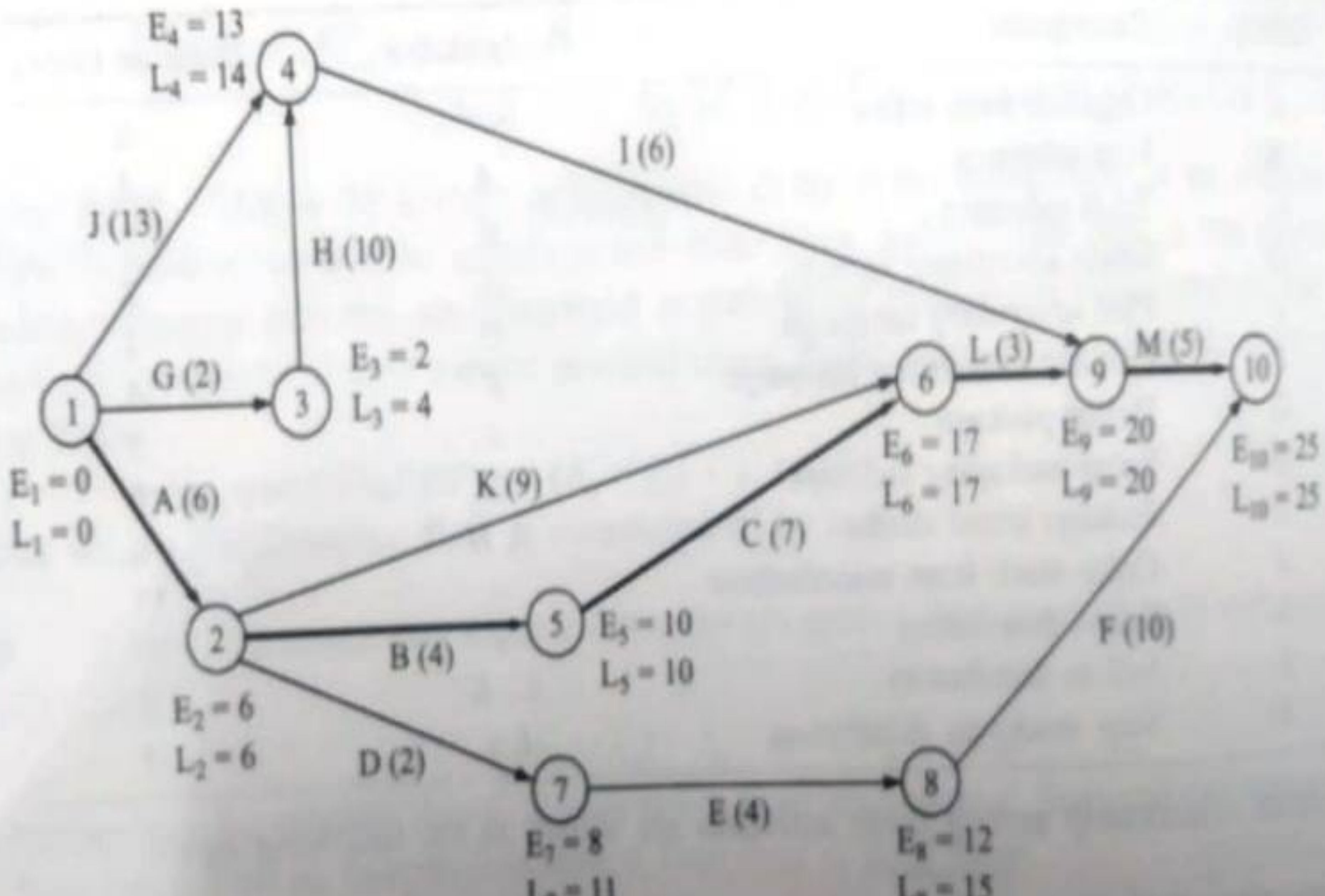
$$= \text{Max} \{13 + 6; 17 + 3\} = 20$$

$E_8 = E_7 + t_{7,8} = 8 + 4 = 12$

$$E_{10} = \underset{i=8,9}{\text{Max}} \{E_i + t_{i,10}\}$$

$$= \text{Max} \{E_8 + t_{8,10}; E_9 + t_{9,10}\}$$

$$= \text{Max} \{12 + 10; 20 + 5\} = 25.$$



E4 = 13
L4 = 14
4
I (6)
J (13)
H (10)
G (2)
3
E3 = 2
L3 = 4
1
E1 = 0
L1 = 0
A (6)
K (9)
6
L (3)
9
M (5)
10
E6 = 17
L6 = 17
E9 = 20
L9 = 20
E10 = 25
L10 = 25
C (7)
2
5
E5 = 10
L5 = 10
B (4)
F (10)
E2 = 6
L2 = 6
D (2)
7
E (4)
8
E7 = 8
L7 = 11
E8 = 12
L8 = 15

Backward Pass Method

$$L_{10} = E_{10} = 25$$

$$L_8 = L_{10} - t_{8,10} = 25 - 10 = 15$$

$$L_6 = L_9 - t_{6,9} = 20 - 3 = 17$$

$$L_4 = L_9 - t_{4,9} = 20 - 6 = 14$$

$$L_2 = \underset{j=5,6,7}{\text{Min}} \{L_j - t_{2,j}\}$$

$$= \text{Min}\{L_5 - t_{2,5}; L_6 - t_{2,6}; L_7 - t_{2,7}\}$$

$$= \text{Min}\{10 - 4; 17 - 9; 11 - 2\} = 6$$

$$L_9 = L_{10} - t_{9,10} = 25 - 5 = 20$$

$$L_7 = L_8 - t_{7,8} = 15 - 4 = 11$$

$$L_5 = L_6 - t_{5,6} = 17 - 7 = 10$$

$$L_3 = L_4 - t_{3,4} = 14 - 10 = 4$$

$$L_1 = \underset{j=2,3,4}{\text{Min}} \{L_j - t_{1,j}\}$$

$$= \text{Min}\{L_2 - t_{1,2}; L_3 - t_{1,3}; L_4 - t_{1,4}\}$$

$$= \text{Min}\{6 - 6; 4 - 2; 14 - 13\} = 0$$

(b) The critical path in the network diagram (Fig. 13.12) has been shown. This has been done by double lines by joining all those events where *E*-values and *L*-values are equal.

The critical path of the project is: 1 – 2 – 5 – 6 – 9 – 10 and critical activities are *A*, *B*, *C*, *L* and *M*. The total project time is 25 weeks.

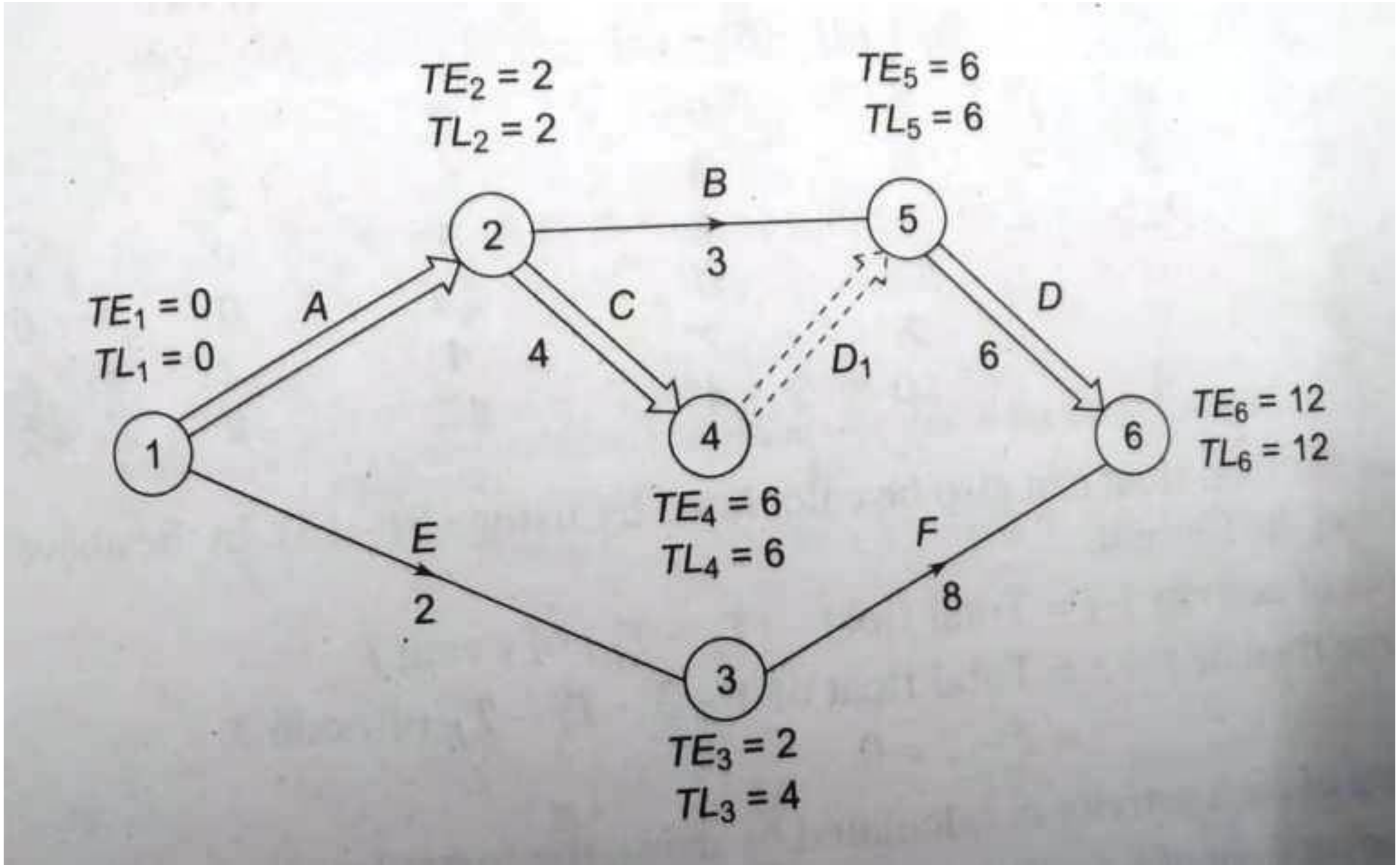
(c) For each non-critical activity, the total float and free float calculations are shown in Table 13.1.

Activity I,j	Duration	Earliest Time		Latest Time		Float	
	tij (1)	Start Ei (2)	Finish Ei +tij (3)	Start Lj – tij (4) (5-1)	Finish Lj (5)	Total Float (Lj – tij) - Ei (4-2)	Free Float (Ej -Ei) - tij
1 (i)-3 (j)	2	0	2	2	4	2	0
1-4	13	0	13	1	14	1	0
2-6	9	6	15	8	17	2	2
2-7	2	6	8	9	11	3	0
3-4	10	2	12	4	14	2	1
4-9	6	13	19	14	20	1	1
7-8	4	8	12	11	15	3	0
8-10	10	12	22	15	25	3	3

Numerical 2

Activity	Predecessors	Duration (days)
A	-	2
B	A	3
C	A	4
D	B,C	6
E	-	2
F	E	8

- Construct a Network Diagram
- Compute the total float, free float for each activity.
- Find the Critical path and total project duration.



The critical path is represented by double lines in the network. The project duration is 12 days The various float for each activity are calculated and represented in the following table

- Using forward pass computations , the earliest time Ei is calculated for each node as follows:

- Set E1 = 0
- E2 = E1+2 = 0+2 =2
- E3 = E1+2 = 0+2 =2
- E4 = E2+4 = 2+4 =6
- E5 = Max(E2+3, E4+0) = maX (2+3, 6+0) =6
- E6 = Max(E5+6, E3+8) = maX (6+6, 2+8) =12

- Using BACKWARD pass computations , the LATEST occurrence time Ei is calculated for each node as follows:
- Set L6 = E6 =12
- L5 = L6 – 6 = 12-6= 6
- L3 = L6 – 8 = 12 -8 = 4
- L4 = L5 – 0 = 6
- L2 = Min (L 5 – 3, L4 -4) = (6-3, 6-4) = 2
- L1 = Min (L2 – 2, L3 -2) = (2-2, 4-2) = 0

Activity I,j	Duration	Earliest Time		Latest Time		Float	
	tij	Start Ei	Finish Ei +tij	Start Lj – tij	Finish Lj	Total (Lj – tij) - Ei	Fre (Ej -Ei) - tij
A(1-2)	2	0	2	0	**2**	0	0
B(2-5)	3	2	5	3	**6**	1	1
C(2-4)	4	2	6	2	**6**	0	0
D(5-6)	6	6	12	6	**12**	0	0
E(1-3)	2	0	2	2	**4**	0	0
F(3-6)	8	2	10	4	**12**	2	0

PERT (Program Evaluation and Review Technique)

- PERT was developed to handle project where the time duration for each activity is no longer just a single time estimate that is decision makers best guess but is a random variable that is characterized by some probability distribution usually a **beta distribution.**

- To estimate the parameters of the beta distribution that is **mean and variance** the path model requires three time estimates for each activity. From these time estimates a single value is estimated for future consideration. The three time estimates that are required are as under:

PERT

- **Optimistic time (t_0 or a):** the shortest possible time in which an activity can be performed assuming that everything goes well.
- **Pessimistic time (tp)**: The longest possible time required to perform an activity under extremely bad conditions However such conditions do not include natural calamities like earthquake, flood etc.
- **Most likely time (tm)**: the time that would occur most often to complete an activity if the activity was repeated under exactly the same conditions many time obviously it is the completion time that would occur most frequently

Expected time of an activity $te = \dfrac{to + 4tm + tp}{6}$

t_0==optimistic time, tm: Most likely time, tp = pessimistic time

Variance of an activity $\sigma^2 = \left[\dfrac{t_p - t_o}{6}\right]^2$

- The probability distribution of times for completing an event can be approximated by the normal distribution due to the central limit theorem. Thus the **Probability of completing the project in the scheduled time, Ts is given as**

$$Z = \frac{Ts - Te}{\sigma_i}$$

- Te = Expected Completion time of the project
- $\sigma_i^2 = \sigma_1^2 + \sigma_2^2 + \ldots\ldots\ldots\ldots \sigma_n^2$

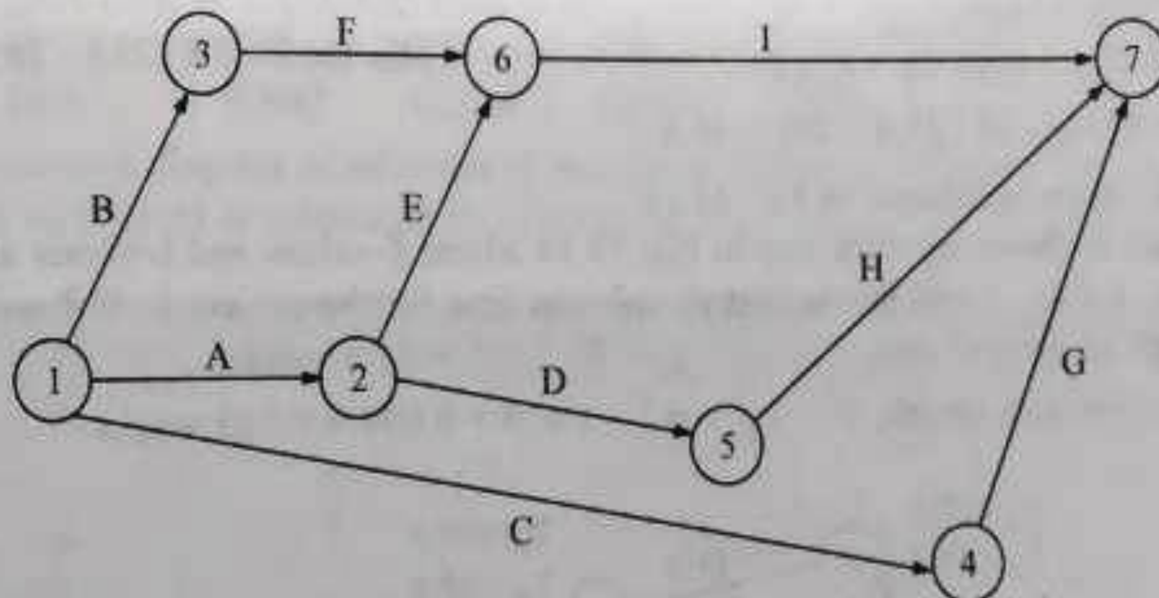
- The desired completion time of the project can be calculated as : Ts = Zσ + Te, where value of Z corresponds to the probability of project completion time.

- The expected completion time of the project is obtained by adding the expected time of each activity lying on the critical path.

- Since it is assumed that the two activities are independent, therefore the variance of the critical path can be known by adding the variance of critical activities.

Example 13.6 The following network diagram represents activities associated with a project:

Activities	A	B	C	D	E	F	G	H	I
Optimistic time	5	18	26	16	15	6	7	7	3
Pessimistic time	10	22	40	20	25	12	12	9	5
Most likely time	8	20	33	18	20	9	10	8	4



Determine the following:

(a) Expected activity time and variance.

(b) The earliest and latest expected completion times of each event.

(c) The critical path.

(d) The probability of expected completion time of the project if the original scheduled time of completing the project is 41.5 weeks.

(e) The duration of the project that will have 95 per cent chance of being completed.

Solution (a) Using the following formula, the expected activity time (t_e or μ) and variance (σ^2) calculations are shown in Table 13.4.

$$t_e = \frac{1}{6}(t_o + 4t_m + t_p) \quad \text{and} \quad \sigma_i^2 = \left\{\frac{1}{6}(t_p - t_o)\right\}^2$$

(b) The earliest and latest expected time for each event will be calculated by considering the expected time of each activity, as shown in Table 13.4.

Activity	t_o	t_p	t_m	$t_e = (t_p + 4t_m + t_p)/6$	$\sigma^2 = [(t_p - t_o)/6]^2$
1 - 2	5	10	8	7.8	0.696
1 - 3	18	22	20	20.0	0.444
1 - 4	26	40	33	33.0	5.429
2 - 5	16	20	18	18.0	0.443
2 - 6	15	25	20	20.0	2.780
3 - 6	6	12	9	9.0	1.000
4 - 7	7	12	10	9.8	0.694
5 - 7	7	9	8	8.0	0.111
6 - 7	3	5	4	4.0	0.111

Table 13

Forward Pass Method

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 7.8 = 7.8$

$E_3 = E_1 + t_{1,3} = 0 + 20 = 20$

$E_4 = E_1 + t_{1,4} = 0 + 33 = 33$

$E_5 = E_2 + t_{2,5} = 7.8 + 18 = 25.8$

$E_6 = \text{Max}\{E_i + t_{i,6}\} = \text{Max}\{E_2 + t_{2,6}; E_3 + t_{3,6}\}$

$= \text{Max}\{7.8 + 20; 20 + 9\} = 29$

$E_7 = \text{Max}\{E_i + t_{i,7}\}$

$= \text{Max}\{E_4 + t_{4,7}; E_5 + t_{5,7}; E_6 + t_{6,7}\}$

$= \text{Max}\{33 + 9.8; 25.8 + 8; 29 + 4\} = 42.8$

Backward Pass Method

$$L_7 = E_7 = 42.8$$

$$L_5 = L_7 - t_{5,7} = 42.8 - 8 = 34.8$$

$$L_3 = L_6 - t_{3,6} = 38.8 - 9 = 29.8$$

$$L_2 = \text{Min } \{L_j - t_{2,j}\}$$

$$= \text{Min } \{L_5 - t_{2,5} \,;\, L_6 - t_{2,6}\}$$

$$= \text{Min } \{34.8 - 18 \,;\, 38.8 - 20\} = 16.8$$

$$L_6 = L_7 - t_{6,7} = 42.8 - 4 = 38.8$$

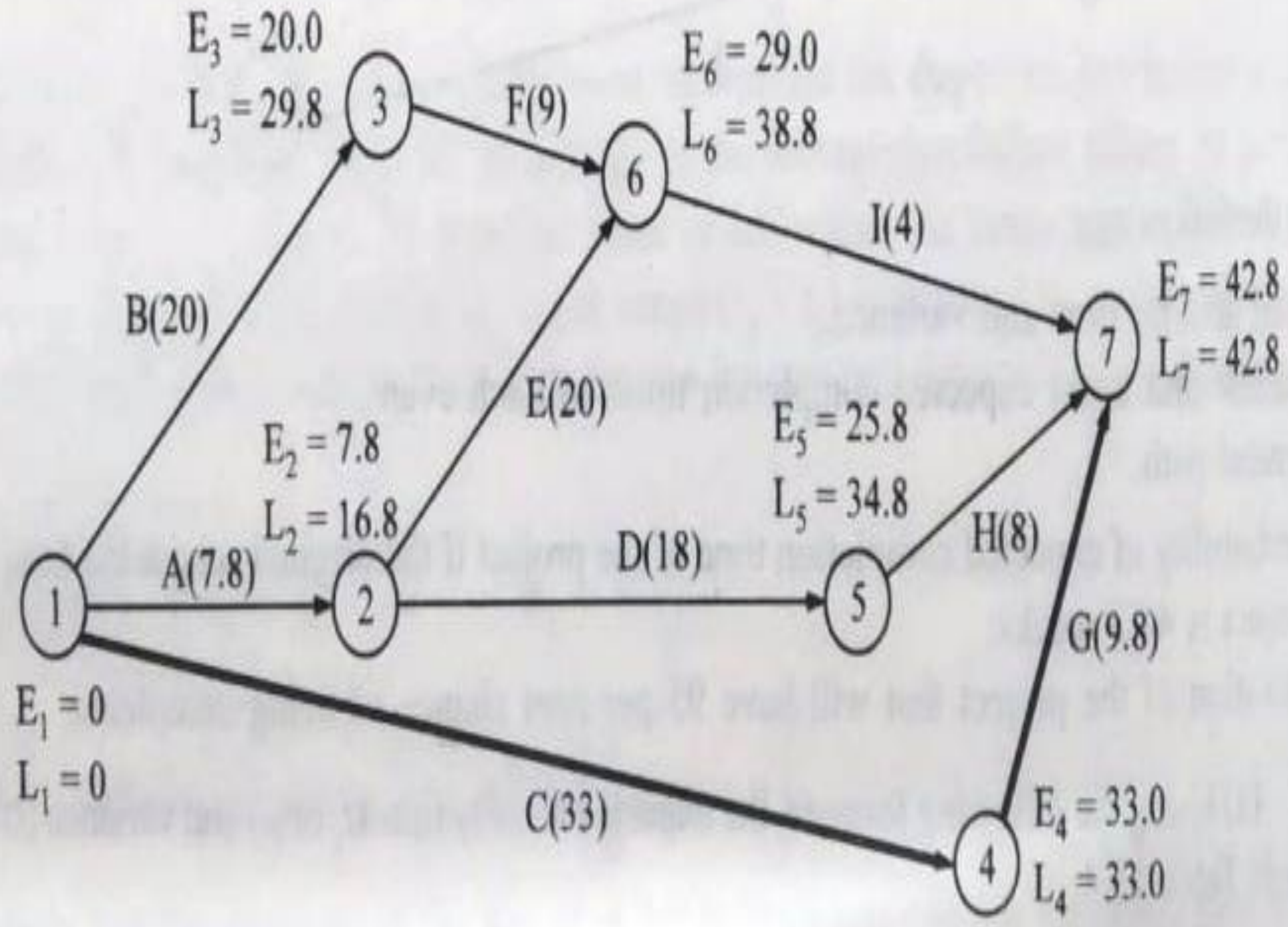
$$L_4 = L_7 - t_{4,7} = 42.8 - 9.8 = 33$$

$$L_1 = \text{Min } \{L_j - t_{1j}\}$$

$$= \text{Min } \{L_2 - t_{1,2}; L_3 - t_{1,3}; L_4 - t_{1,4}\}$$

$$= \text{Min } \{16.8 - 7.8 \,;\, 29.8 - 20 \,;\, 33 - 33\} = 0$$

variance of critical path length,

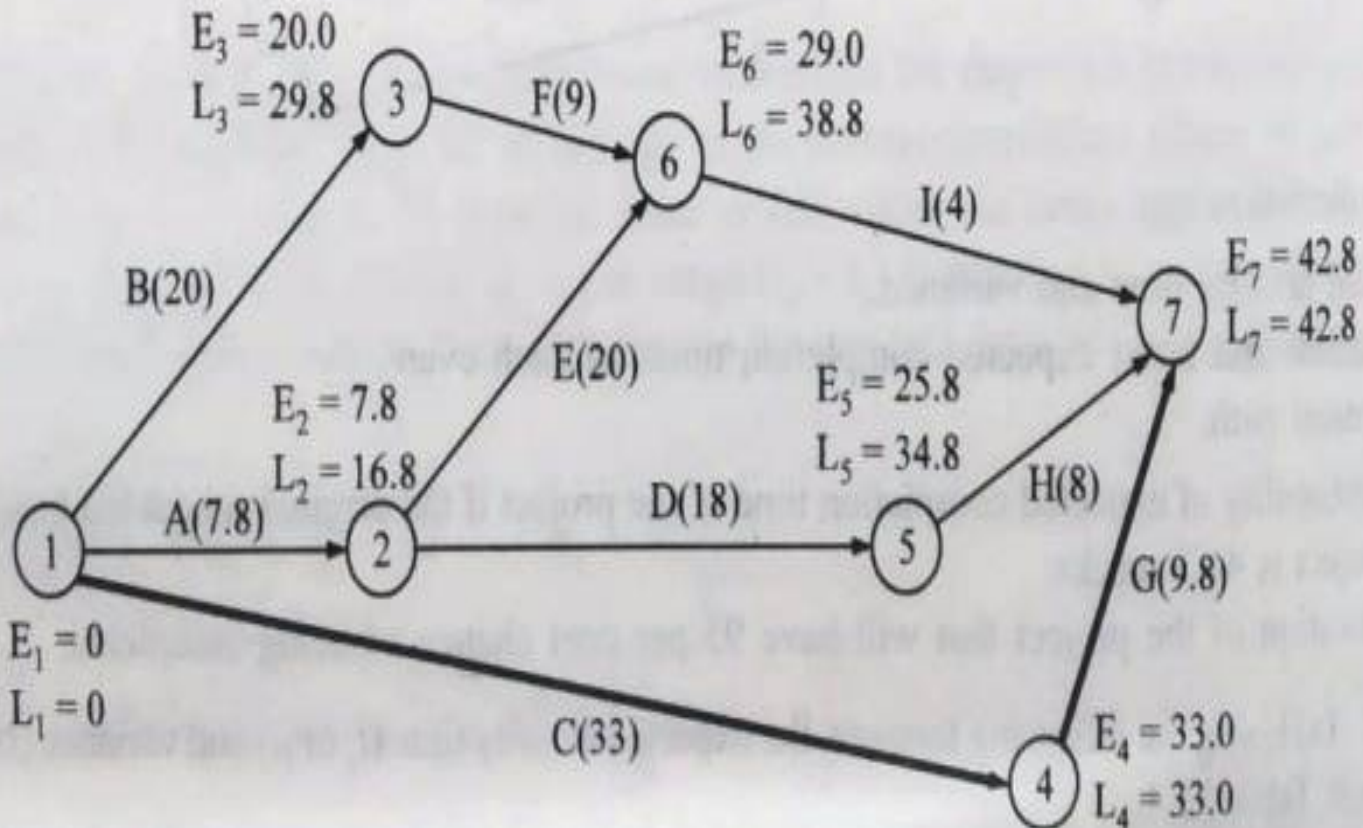


The E-value and L-values are shown in Fig. 13.14.

(c) The critical path is shown by thick line in Fig. 13.14 where E-values and L-values are the same. The critical path is: 1 – 4 – 7 and the earliest completion time for the project is 42.8 weeks.

(b) Expected length of critical path, $T_e = t_C + t_G = 33 + 9.8 = 42.8$ weeks.

Variance of critical path length, $\sigma^2 = \sigma_C^2 + \sigma_G^2 = 5.429 + 0.694 = 6.123$ weeks.

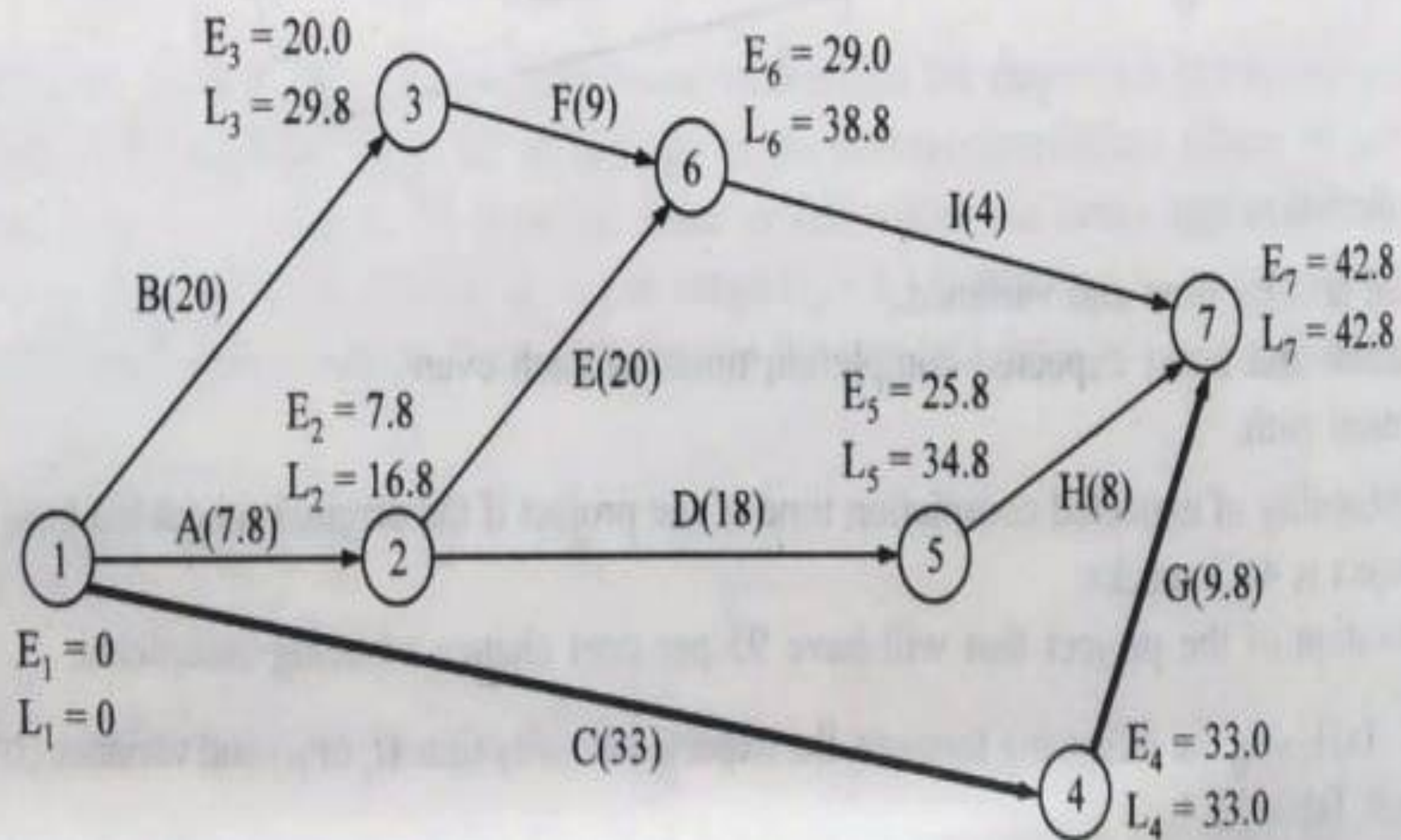


The E-value and L-values are shown in Fig. 13.14.

(c) The critical path is shown by thick line in Fig. 13.14 where E-values and L-values are the same. The critical path is: 1 – 4 – 7 and the earliest completion time for the project is 42.8 weeks.

(b) Expected length of critical path, $T_e = t_C + t_G = 33 + 9.8 = 42.8$ weeks.

Variance of critical path length, $\sigma^2 = \sigma_C^2 + \sigma_G^2 = 5.429 + 0.694 = 6.123$ weeks.



Since $T_s = 41.5$, $T_e = 42.8$ and $\sigma = \sqrt{6.123} = 2.474$, the probability of meeting the schedule time is given by:

$$\text{Prob}\left(Z \le \frac{T_s - T_e}{\sigma}\right) = P\left(Z \le \frac{41.5 - 42.8}{2.474}\right) = \text{Prob}\,(Z \le -0.52)$$

$$= 0.5 - 0.1952 = 0.3048 \text{ (from normal distribution table)}$$

Thus, the probability that the project can be completed in less than or equal to 41.5 weeks is 0.3048. In other words, the probability that the project will get delayed beyond 41.5 weeks is 0.6952.

Given that

$$P\left(Z \le \frac{T_s - T_e}{\sigma}\right) = 0.95$$

But $Z_{0.95} = 1.64$, from normal distribution table. Thus,

$$1.64 = \frac{T_s - 42.8}{2.47} \quad \text{or} \quad T_s = 1.64 \times 2.474 + 42.8 = 46.85 \text{ weeks.}$$

Example 13.7 A small

expected completion time of the project is obtained by adding the expected time of each activity lying on the critical path since it is assumed that the two activities are independent there for the variance of the critical path can be known by adding the variance of selectivity

refernces

- https://www.srividyaengg.ac.in/coursematerial/CSE/104745.pdf