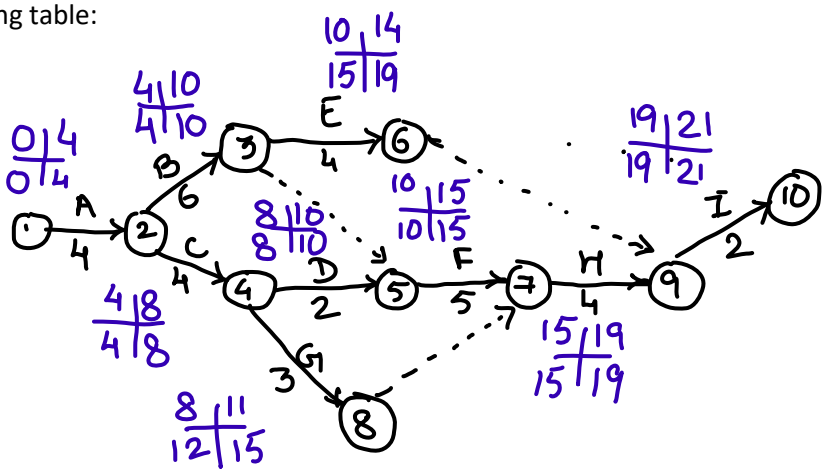
Forward Pass →

ES | EF
LS | LF

← Backward Pass

1. Given the information in the following table:

Activity	Duration	Predecessor
A	4	None
B	6	A
C	4	A
D	2	C
E	4	B
F	5	B,D
G	3	C
H	4	F,G
I	2	E,H



(a) Construct the network diagram.

(b) Find the activity's ES, EF, LS, LF.

(c) Identify all paths. Which path is the critical path?

(d) Calculate the slack for each activity.

(e) How long will it take to complete the project?

21 units

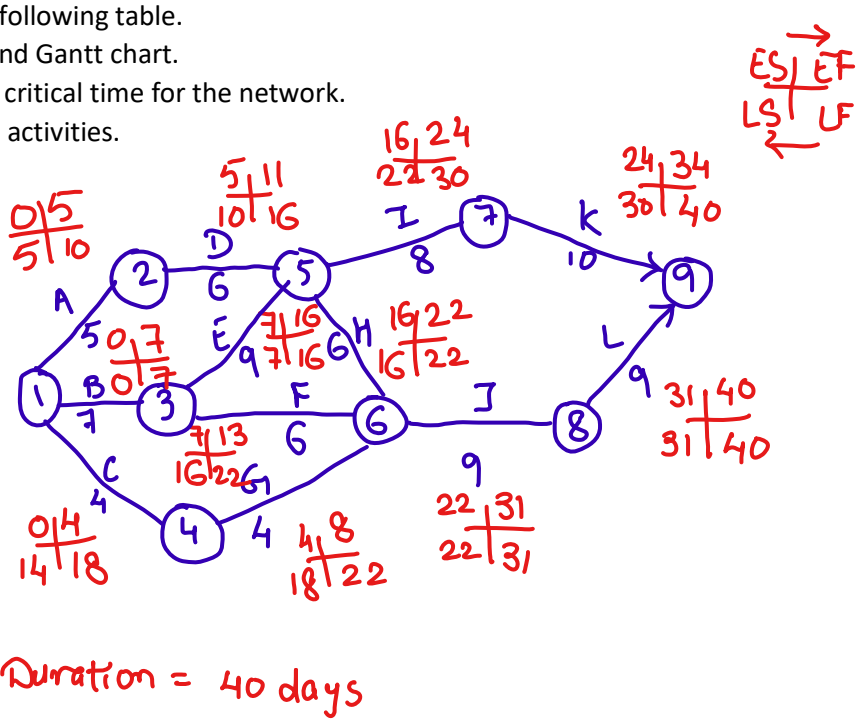
paths: A-B-E-I 16 units
A-B-F-H-I 21 units
A-C-D-F-H-I 21 units
A-C-G-H-I 16 units

Solution:

Activity	Duration	Predecessor	ES	EF	LS	LF	Slack LS-ES
A	4	None	0	4	0	4	0
B	6	A	4	10	4	10	0
C	4	A	4	8	4	8	0
D	2	C	8	10	8	10	0
E	4	B	10	14	15	19	5
F	5	B,D	10	15	10	15	0
G	3	C	8	11	12	15	4
H	4	F,G	15	19	15	19	0
I	2	E,H	19	21	19	21	0

2. Given the information in the following table.
   (a) Draw the AOA network and Gantt chart.
   (b) Find the critical path and critical time for the network.
   (c) Then find the slack for all activities.

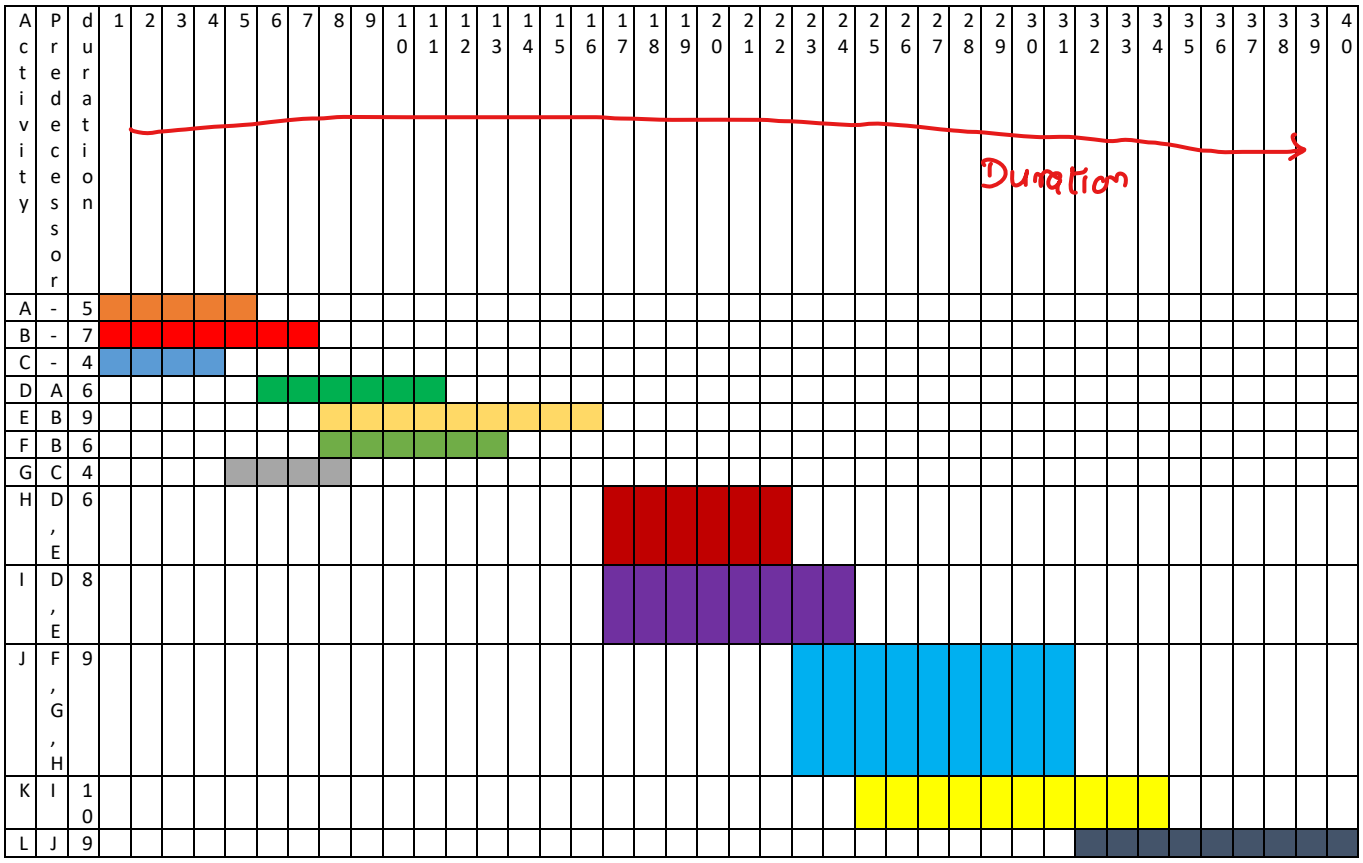
Activity	Predecessor	Duration
A	--	5 days
B	--	7
C	--	4
D	A	6
E	B	9
F	B	6
G	C	4
H	D,E	6
I	D,E	8
J	F,G,H	9
K	I	10
L	J	9



Solution:

Activity	Predecessor	Duration	ES	EF	LS	LF	Slack LS - ES
A	--	5 days	0	5	5	10	5
B	--	7	0	7	0	7	0
C	--	4	0	4	14	18	14
D	A	6	5	11	10	16	5
E	B	9	7	16	7	16	0
F	B	6	7	13	16	22	9
G	C	4	4	8	18	22	14
H	D,E	6	16	22	16	22	0
I	D,E	8	16	24	22	30	6
J	F,G,H	9	22	31	22	31	0
K	I	10	24	34	30	40	6
L	J	9	31	40	31	40	0

Gantt Chart



a = opt b = pess m: most likely

3. In the following table, task durations are given in days.

Activity	Predecessor	a	m	b
A	-	8	10	16
B	A	11	12	14
C	B	7	12	19
D	B	6	6	6
E	B	10	14	20
F	C,D	6	10	10
G	D	5	10	17
H	E,G	4	8	11

$te: \frac{a+4m+b}{6}$

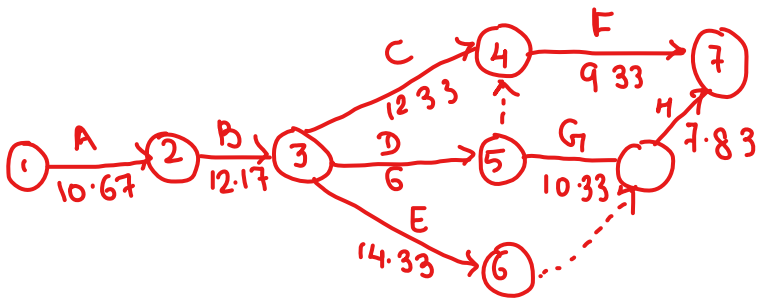
$V: \left(\frac{b-a}{6}\right)^2$

a. Find the expected time and variance for each activity.
b. Draw the network diagram and find the critical path and time.
c. Find the probability that the critical path will be completed in 50 days.
d. Find the probability that the other main path will be completed in 50 days.
e. What is the probability that the entire network will be completed in 50 days?
f. How many days are required for the critical path to have a 0.95 probability of completion?

Solution:

Activity	Predecessor	a	m	b	Duration	Variance
A	-	8	10	16	10.67	1.78
B	A	11	12	14	12.17	0.25
C	B	7	12	19	12.33	4.00
D	B	6	6	6	6	0.00
E	B	10	14	20	14.33	2.78
F	C,D	6	10	10	9.33	0.44
G	D	5	10	17	10.33	4.00
H	E,G	4	8	11	7.83	1.36

Network Diagram:



critical path and time:

Paths:- A-B-C-F 44.5 days
A-B-D-F 38.17 days
A-B-D-G-H 47 days ✓
A-B-E-H 45 days ✓

Critical Path = A-B-D-G-H

Duration = 47 days

T_e = 47 days V_p = 1.39

The probability that the critical path will be completed in 50 days:

T_s = 50 days

$$P(x \leq 50) = F(50) = \phi\left(\frac{T_s - T_e}{\sqrt{V_p}}\right) = \phi\left(\frac{50-47}{\sqrt{7.39}}\right) = \phi(1.10)$$

$$= 0.8643$$

$$= 86.43\%$$

The probability that the other main path will be completed in 50 days

Other main path = A - B - E - H $T_e = 45$ days

$V_p = V_A + V_B + V_E + V_H$

$= 6.17$

$P(X \le 50) = F(50) = \phi\left(\frac{T_s - T_e}{\sqrt{V_p}}\right) = \phi\left(\frac{50-45}{\sqrt{6.17}}\right) = \phi(2.01)$

$= 0.9778$

$= 97.78\%$

The probability that the entire network will be completed in 50 days

A-B-C-F $T_e = 44.5$ $V_p = 6.47$

A-B-D-F $T_e = 38.17$ $V_p = 2.47$

$P(X \le 50) = F(50) = \phi\left(\frac{T_s - T_e}{\sqrt{V_p}}\right) = \phi\left(\frac{50-44.5}{\sqrt{6.47}}\right) = \phi(2.16) = 0.9846$

$= 98.46\%$

$P(X \le 50) = F(50) = \phi\left(\frac{T_s - T_e}{\sqrt{V_p}}\right) = \phi\left(\frac{50-38.17}{\sqrt{2.47}}\right) = \phi(7.52) = 1$

Prob. that entire n/w will be completed in 50 days

$0.8643 \times 0.9778 \times 0.9846 \times 1$

$= 0.832 \quad = 83.2\%$

How many days are required for the critical path to have a 0.95 probability of completion?

$\phi\left(\frac{T_s - T_e}{\sqrt{V_p}}\right) = 0.95 \Rightarrow z = \frac{T_s - T_e}{\sqrt{V_p}}$

$\therefore 1.64 = \frac{T_s - T_e}{\sqrt{V_p}}$

$= \frac{T_s - 47}{\sqrt{7.39}}$

$\therefore T_s = 1.64 \times \sqrt{7.39} + 47$

$= 51.45$ days

4. In the following table, task durations are given in days.

Activity	Predecessor	a	m	b
A	-	2	4	6
B	-	3	5	9
C	A	4	5	7
D	A	4	6	10
E	B,C	4	5	7
F	D	3	4	8
G	E	3	5	8

a. Find the expected time and variance for each activity.
b. Draw the network diagram and find the critical path and time.
c. Find the probability that the critical path will be completed in 23 days.
d. Find the probability that the other main path will be completed in 23 days.
e. What is the probability that the entire network will be completed in 23 days?
f. How many days are required for the critical path to have a 0.9 probability of completion?

Solution:

Activity	Predecessor	a	m	b	Duration	Variance
A	-	2	4	6		
B	-	3	5	9		
C	A	4	5	7		
D	A	4	6	10		
E	B,C	4	5	7		
F	D	3	4	8		
G	E	3	5	8		

Network Diagram:

critical path and time:

The probability that the critical path will be completed in 23 days:

The probability that the other main path will be completed in 23 days

The probability that the entire network will be completed in 23 days

How many days are required for the critical path to have a 0.9 probability of completion?