Mathematics

(Chapter – 8) (Introduction to Trigonometry)
(Class X)

Exercise 8.4

Question 1:

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Answer 1:

We know that, $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1+\cot^2 A}$$

$$\sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\sin A = \pm\frac{1}{\sqrt{1+\cot^2 A}}$$

Therefore, $\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \dfrac{\sin A}{\cos A}$

However, $\cot A = \dfrac{\cos A}{\sin A}$

Therefore, $\tan A = \dfrac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Question 2:

Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer 2:

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Question 4:

Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

(A) 0 (B) 1 (C) 2 (D) −1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) secA (B) sinA (C) cosecA (D) cosA

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$ (B) −1 (C) $\cot^2 A$ (D) $\tan^2 A$

Answer 4:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9(1)$ [As $\sec^2 A - \tan^2 A = 1$]

$= 9$

Hence, alternative (B) is correct.

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$= \frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

Hence, alternative (C) is correct.

(iii) $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$= \cos A$

Hence, alternative (D) is correct.

(iv) $$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}} = \frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer 5:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.} = (\operatorname{cosec}\theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \text{R.H.S.}$$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$

$$= \text{R.H.S.}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{\sin\theta\cos\theta}\right]$$

$$= \frac{(1+\sin\theta\ \cos\theta)}{(\sin\theta\ \cos\theta)}$$

$= \sec\theta\ \text{cosec}\ \theta + 1 = \text{R.H.S.}$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$$\text{L.H.S.} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}} = (\cos A+1)$$

$$= \frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A} \quad = \text{R.H.S}$$

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec} A + \cot A$

Using the identity $\text{cosec}^2 A = 1 + \cot^2 A$

L.H.S = $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \text{cosec} A}{\cot A + 1 - \text{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \text{cosec} A)\}\{(\cot A) - (1 - \text{cosec} A)\}}{\{(\cot A) + (1 - \text{cosec} A)\}\{(\cot A) - (1 - \text{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \text{cosec} A)^2}{(\cot A)^2 - (1 - \text{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \text{cosec}^2 A - 2\cot A - 2\text{cosec} A + 2\cot A \,\text{cosec} A}{\cot^2 A - (1 + \text{cosec}^2 A - 2\text{cosec} A)}$$

$$= \frac{2\text{cosec}^2 A + 2\cot A \,\text{cosec} A - 2\cot A - 2\text{cosec} A}{\cot^2 A - 1 - \text{cosec}^2 A + 2\text{cosec} A}$$

$$= \frac{2\text{cosec} A(\text{cosec} A + \cot A) - 2(\cot A + \text{cosec} A)}{\cot^2 A - \text{cosec}^2 A - 1 + 2\text{cosec} A}$$

$$= \frac{(\text{cosec} A + \cot A)(2\text{cosec} A - 2)}{-1 - 1 + 2\text{cosec} A}$$

$$= \frac{(\text{cosec} A + \cot A)(2\text{cosec} A - 2)}{(2\text{cosec} A - 2)}$$

= cosec A + cot A

= R.H.S

(vi) $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

$$\text{L.H.S.} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} \qquad = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} \qquad = \sec A + \tan A$$

$$= \text{R.H.S.}$$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta \times (1-2\sin^2\theta)}{\cos\theta \times \{2(1-\sin^2\theta)-1\}}$$

$$= \frac{\sin\theta \times (1-2\sin^2\theta)}{\cos\theta \times (1-2\sin^2\theta)}$$

$$= \tan\theta = \text{R.H.S}$$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\text{L.H.S} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

$$= (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A + \sec^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$$

$$= (1) + (1 + \cot^2 A + 1 + \tan^2 A) + (2) + (2)$$

$$= 7 + \tan^2 A + \cot^2 A$$

$$= \text{R.H.S}$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2=\frac{1+\tan^2 A\ -\ 2\tan A}{1+\cot^2 A-2\cot A}$$

$$=\frac{\sec^2 A-\ 2\tan A}{\operatorname{cosec}^2 A-2\cot A}$$

$$=\frac{\dfrac{1}{\cos^2 A}-\dfrac{2\sin A}{\cos A}}{\dfrac{1}{\sin^2 A}-\dfrac{2\cos A}{\sin A}}=\frac{\dfrac{1-2\sin A\cos A}{\cos^2 A}}{\dfrac{1-2\sin A\cos A}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$