

# Lesson Summary - GCD Algorithms

## 1. Introduction to GCD (HCF)

- Problem: Find the greatest common divisor of two numbers.

## 2. Method 1: List All Factors Method

- List factors of both numbers.
- Find the largest common factor.
- Time complexity:  $O(\sqrt{n} + \sqrt{m})$

## 3. Method 2: Brute Force Linear Check

- Iterate from 1 to  $\min(x, y)$ .
- Time complexity:  $O(\min(x, y))$

## 4. Method 3: Euclidean Algorithm (Subtraction Version)

- Use property:  $\text{GCD}(a, b) = \text{GCD}(a-b, b)$  if  $a > b$ .
- Time complexity:  $O(\max(a, b))$

## 5. Method 4: Euclidean Algorithm (Modulo Version)

- Property:  $\text{GCD}(a, b) = \text{GCD}(b, a \% b)$
- Most efficient for software implementations.
- Time complexity:  $O(\log(\min(a, b)))$

## 6. Method 5: Stein's Algorithm (Binary GCD Algorithm)

- Highly hardware-friendly (bitwise operations only).
- Steps:
  - a. If both  $a$  and  $b$  are even, factor out 2.
  - b. Remove all factors of 2 from both numbers.

c. Use subtraction and bit shifts to reduce.

d. Multiply back common powers of 2.

- Time complexity:  $O(\log(\max(a, b)))$

- Uses only subtraction, bit shifts, no division or modulus.

- Used in embedded systems and cryptography.