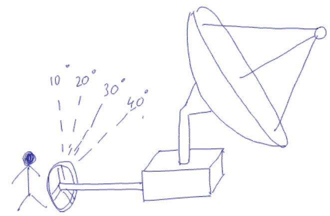
CONTROL SYSTEM

18m ISRO antenna used to track Chandrayaan (part of Indian Deep Space Network)

This antenna needed to track Chandrayaan.

Q. How to change the azimuth angle precisely according to the motion of Chandrayaan ?

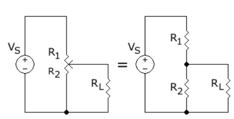
1) Impossible to do manually



Impossible: Too slow and imprecise

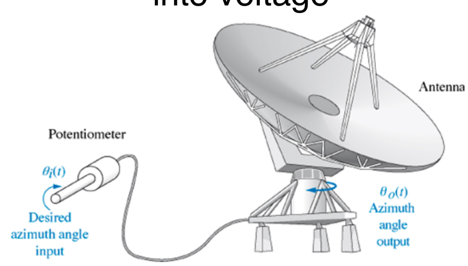
Human inputs can be converted to electrical signals

Potentiometer —— can help.



Converts angle command into voltage

Use POT to convert angle command into voltage

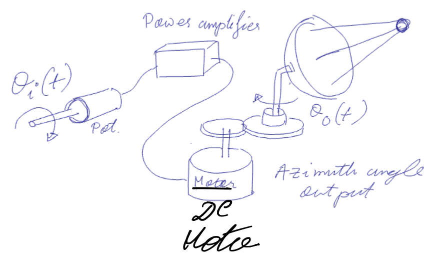


(a)

Power Amplification Required

Unrealistic arrangement! WHY?
Answer: Large motor requires a lot of power.

Use Power amplifier to magnify POT output voltage



STILL SOMETHING MISSING! WHAT?
Answer: Error Correction

→ A simplified model of this setup is given to you.

Problem : 1) Design $\theta_i(t)$ to move the Antenna from any initial angle to 5 rad and hold it there automatically.

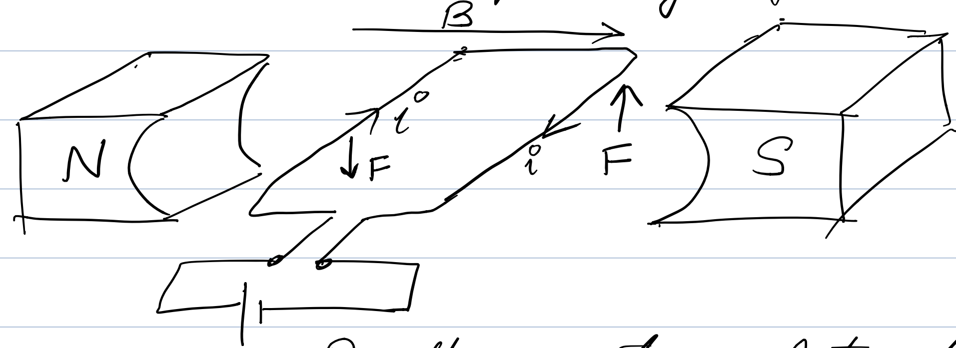
2) how quickly can your control system reach target?

3) how accurately can your control system hold target after sufficient time has passed?

4) What is the maximum deviation from the target that your system is experiencing?

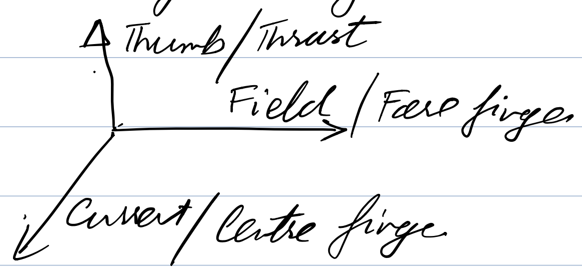
5) The answers to 2, 3, 4 depend on what parameters? How?

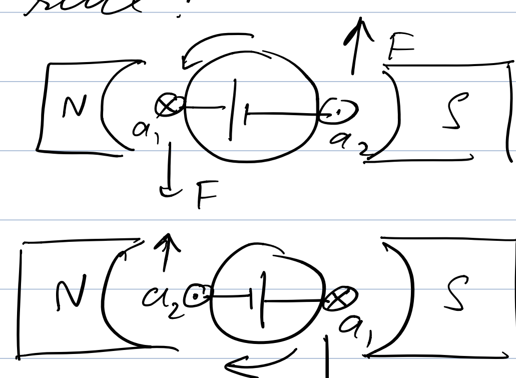
DC - Machine (Principle of Operation)



Resulting motion - Anti-clockwise Rotation.

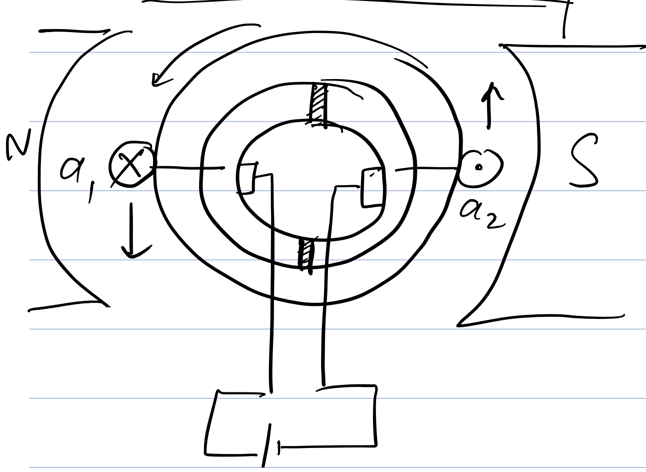
Flemings Left Hand rule:

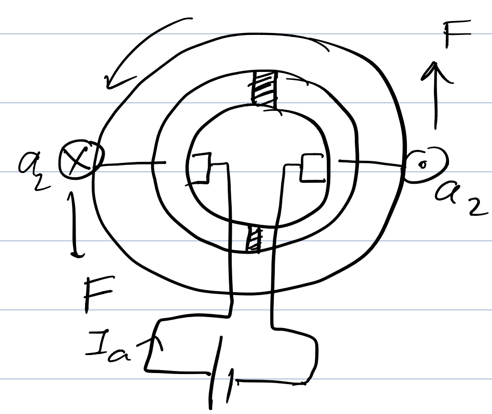




See-saw motion - no continuous rotation!

Commutator trick





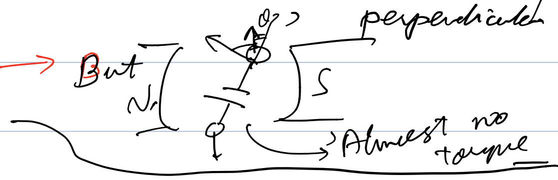
Force Formula : $F = BIL \sin \phi$

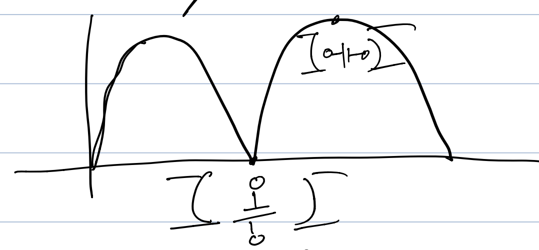
$\phi \rightarrow$ angle between B & $I \cong 90^\circ$

So $F = BIL$ / Torque $\propto F \sin\theta$

Torque $\propto I_a$

→ But



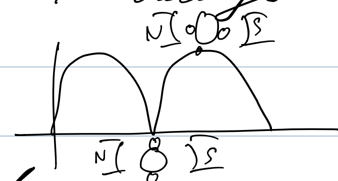


Q. where did the $\sin\theta$ then go?

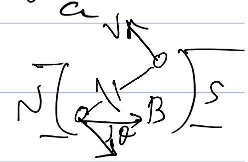
Q. So decrease armature resistance $\Rightarrow I_a \uparrow$

$\Rightarrow$ Torque goes up ??

→ Is that possible. → Faraday's law does not agree!

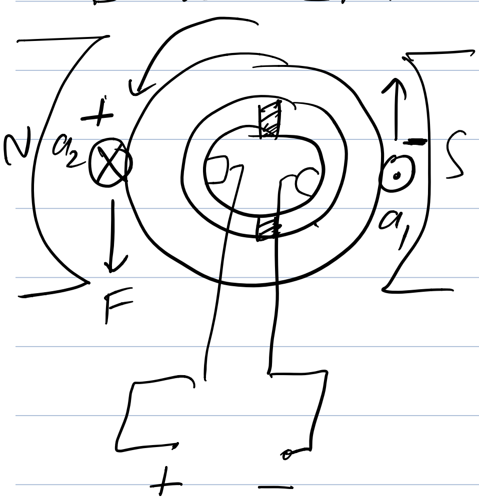


The motion of the single coil in B induces "Back EMF" $E_b = 2BLv\sin\theta$

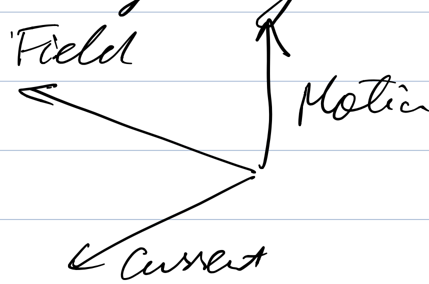


or $E_b = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\theta)$

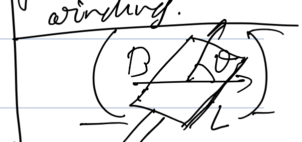
$= -BA\sin\theta \frac{d\theta}{dt} = -BA\omega\sin\theta$



Faradays right hand rule.



Real motor get rid of the $\sin\theta$ term by using multiple poles & clever winding.



$E_b \propto \omega$

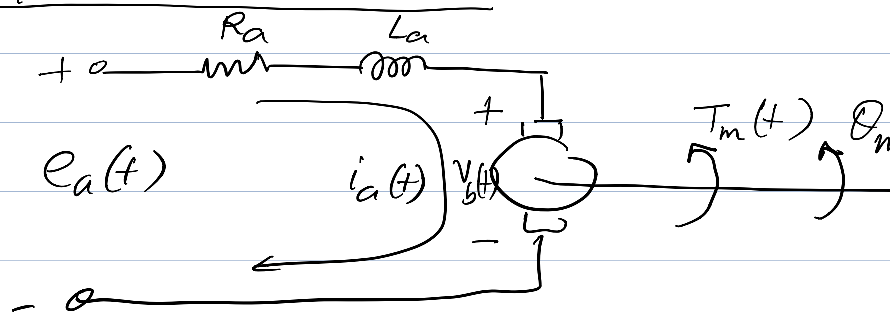
Show Motor Animation

For a real DC machine, poles & conductors are arranged carefully to hold the torque nearly constant as the armature rotates.

$$T_m = K_t I_a \quad \text{——— (1)}$$

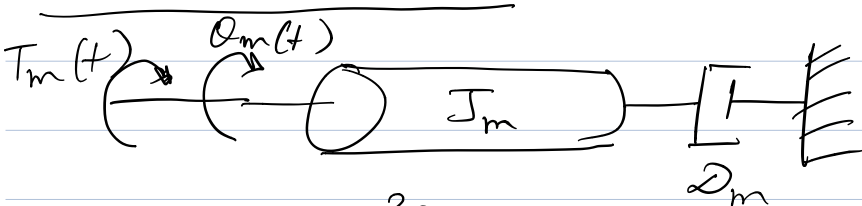
$$V_b = K_b \frac{d\theta_m}{dt} = K_b \omega_m \quad \text{——— (2)}$$

Equivalent Circuit:



$$R_a i_a(t) + L_a \frac{d i_a(t)}{dt} + V_b(t) = e_a(t) \quad \text{——— (3)}$$

Mechanical Load



$$T_m(t) = J_m \frac{d^2\theta_m}{dt^2} + D_m \frac{d\theta_m}{dt} \quad \text{——— (4)}$$

Using (1) and (2) in (3),

$$\frac{R_a}{K_t} T_m(t) + \frac{L_a}{K_t} \frac{d}{dt}\left[T_m(t)\right] + K_b \frac{d\theta_m(t)}{dt} = e_a(t)$$

Assume L_a to be small,

$$\frac{R_a}{K_t} T_m(t) + K_b \frac{d\theta_m(t)}{dt} = e_a(t) \quad \text{—— (5)}$$

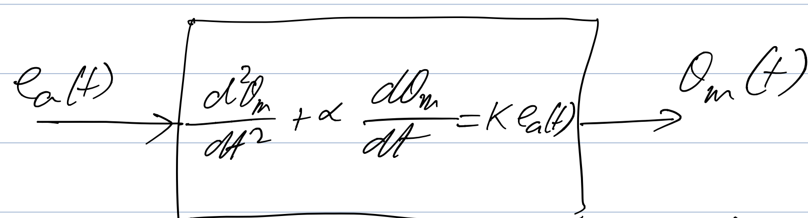
Using (4) in (5),

$$\frac{R_a}{K_t}\left[J_m \frac{d^2\theta_m}{dt^2} + D_m \frac{d\theta_m}{dt}\right] + K_b \frac{d\theta_m}{dt} = e_a(t)$$

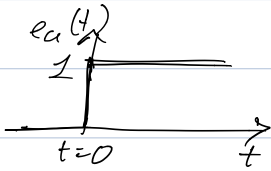
$$\frac{R_a J_m}{K_t} \frac{d^2\theta_m}{dt^2} + \left[\frac{R_a D_m}{K_t} + K_b\right] \frac{d\theta_m}{dt} = e_a(t)$$

Let $K = \frac{K_t}{R_a J_m}$; $\alpha = \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)$

Then $$\frac{d^2\theta_m}{dt^2} + \alpha \frac{d\theta_m}{dt} = K . e_a(t)$$

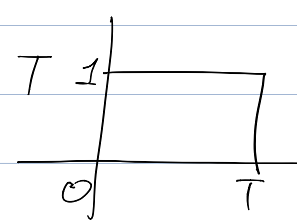


1) For a step voltage, $e_a(t) =$



$$\theta_m(t) = A + Bt + Ce^{-\alpha t}$$

2) For a square pulse of duration T

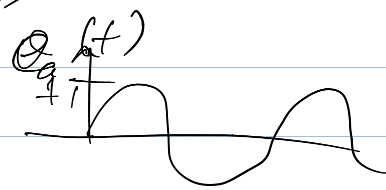


— Check in the simulator

$$\theta_m(t) = At - Bu(t-T)\left[\frac{A}{B}(t-T) + De^{-\alpha(t-T)} + E\right] + BE + Ge^{-\alpha t}$$

so $\theta_m(t) \approx \theta_m(T)$ for $t \geq T$.

3) For sinusoidal input:



$$\theta_m(t) = A + B\sin\omega t + C\cos\omega t - De^{-\alpha t}$$

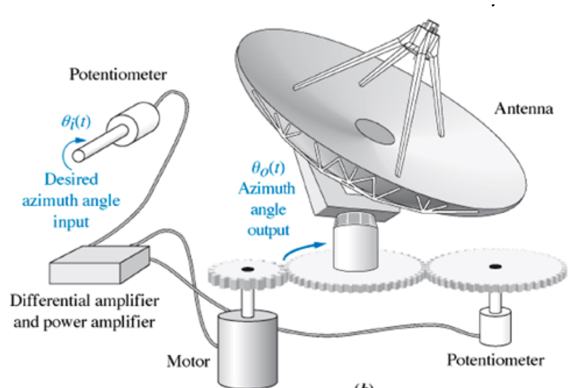
FACT: These calculations are not enough to successfully "point" the antenna at 5 rad.

- especially if there are disturbances.

— Must know where we are at any pt

— Then only we can keep correcting.

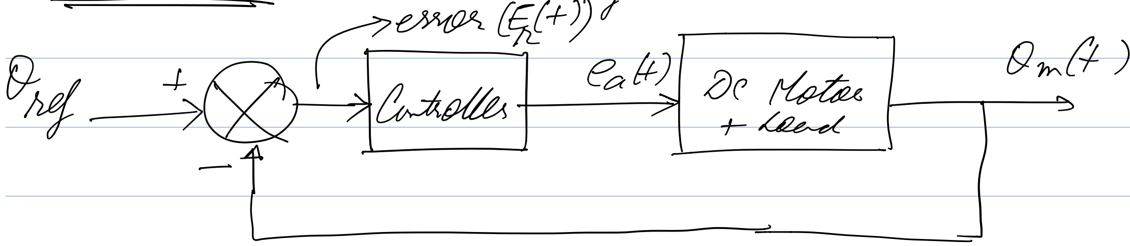
The central theme: feedback



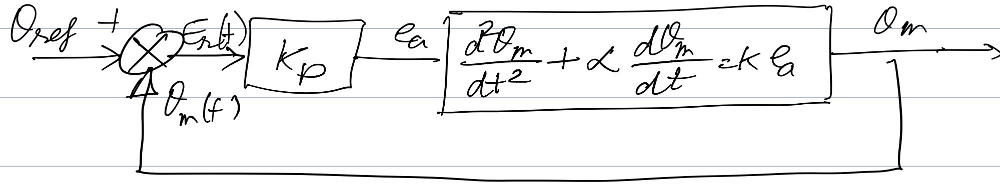
(b)

- Use another gear/pot to check whether actually the output is following the input.
- If not then use the difference/error to drive the syste

Feedback: The basis of automatic control



Lets try with the simplest possible controller : Just a gain → 'K_p'

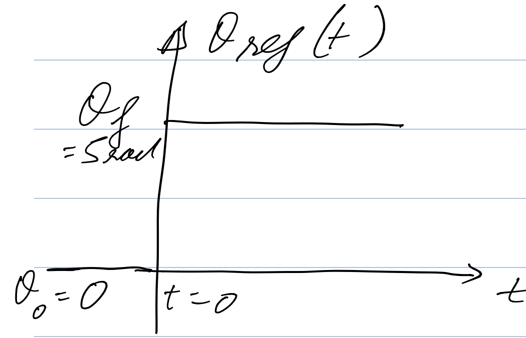


$$\left.\begin{aligned} E_r(t) &= \theta_{ref} - \theta_m \\ e_a(t) &= K_p\, E_r(t) \end{aligned}\right] \longrightarrow \frac{e_a}{K_p} = \theta_{ref} - \theta_m$$

$$\Rightarrow \frac{1}{KK_p}\left[\frac{d^2\theta_m}{dt^2} + \alpha \frac{d\theta_m}{dt}\right] = \theta_{ref} - \theta_m$$

$$\Longleftrightarrow \boxed{\frac{d^2\theta_m}{dt^2} + \alpha \frac{d\theta_m}{dt} + (KK_p)\,\theta_m = KK_p\,\theta_{ref}}$$

Various $\theta_{ref}(t)$ are possible, lets try with a step input (corr. to our problem of starting from θ_0 & bringing θ_m to θ_f as quickly as possible)

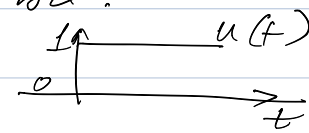


Q. What will the response $\theta_m(t)$ look like?

A: We would like $\theta_m(t)$ to be exactly like $\theta_{ref}(t)$.

— But that is impossible.

unit step



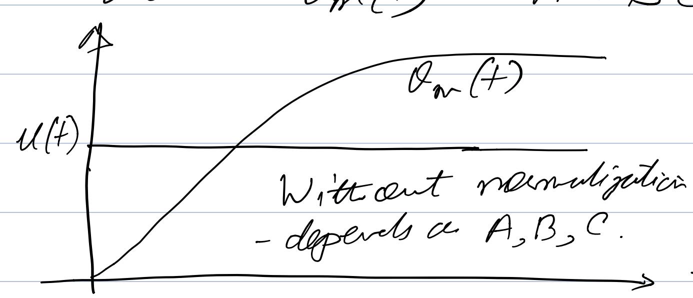
For $\theta_{ref} = u(t)$, solution depends on roots of $D^2 + \alpha D + KK_p = 0$

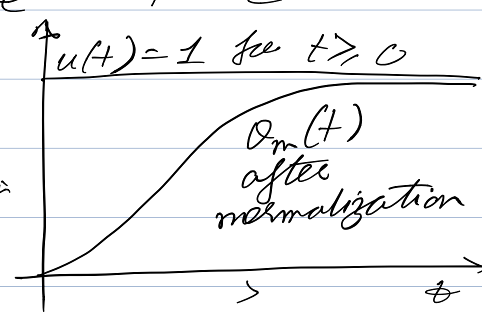
$$D_{1,2} = \frac{-\alpha \pm \sqrt{\alpha^2 - 4KK_p}}{2}$$

Depending on what K_p you choose, the roots $D_{1,2}$ can either be

1) Real & distinct (D_1, D_2) $\boxed{D_1, D_2 < 0}$

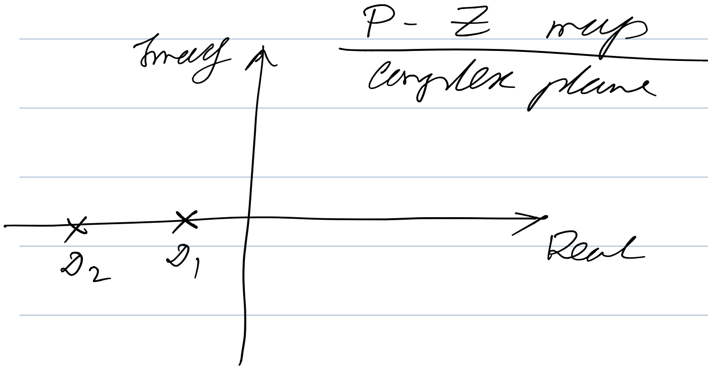
— then $\theta_m(t) = A + Be^{D_1 t} + Ce^{D_2 t}$

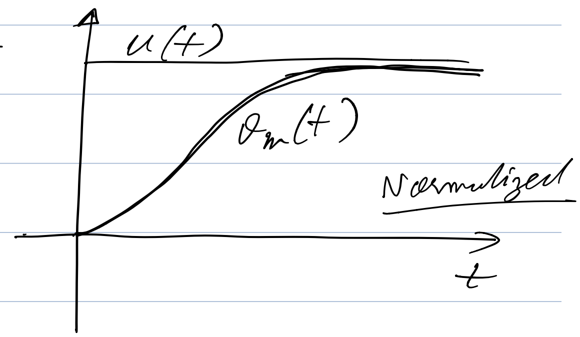




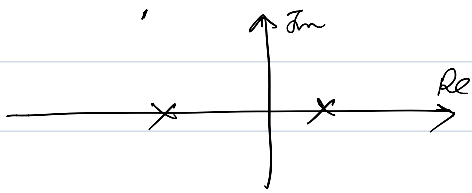
Such a response is called "OVERDAMPED"

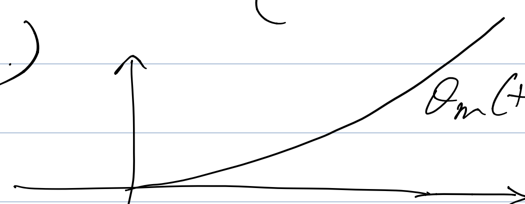
In controls, we try to make a direct correspondence between the location of D_1, D_2 & the nature of response.





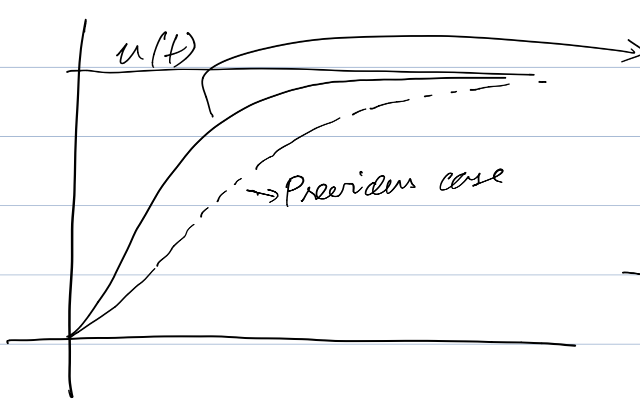
Q. Can any K_p make D_1/D_2 positive?
- Try it on the simulation (Have to remove saturation.)



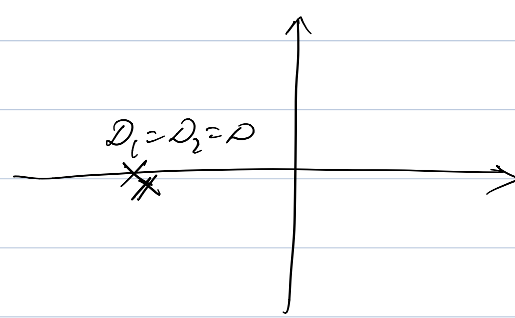


Calculation: Exercise

2> Real & repeated $D_1 = D_2 = D$
- then $\theta_m(t) = A + Be^{Dt} + Cte^{Dt}$

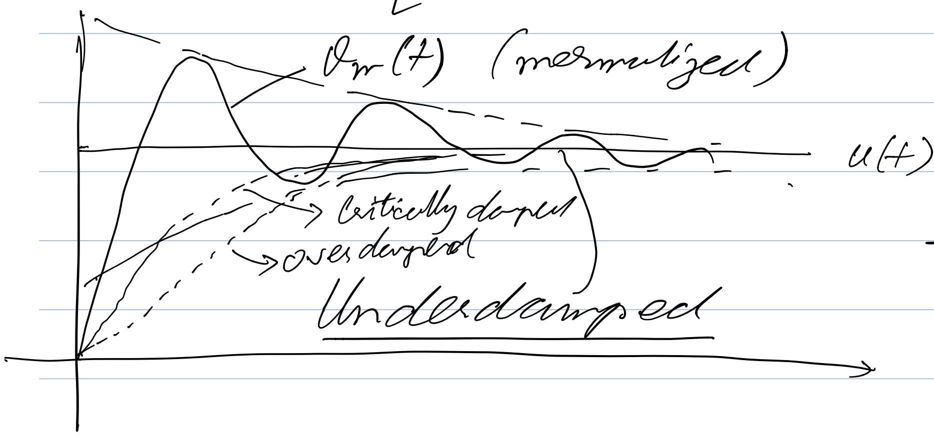


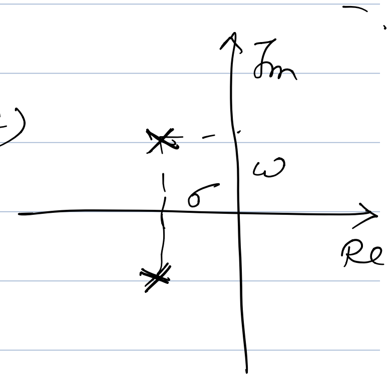
Critically damped



3) <u>Complex Poles</u>: $s_{1,2} = \sigma \pm j\omega \quad (\sigma < 0)$

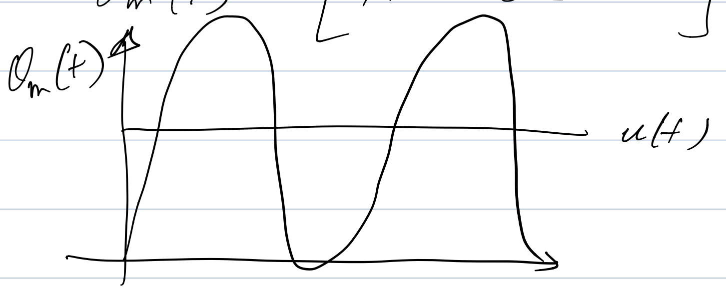
Then $\theta_m(t) = \left[A + B e^{\sigma t} \cos(\omega t - C) \right] u(t)$





4) <u>Purely Imaginary Poles</u> : $s_{1,2} = \pm j\omega$

$\theta_m(t) = \left[A + \cos \omega t \right] u(t)$



<u>Undamped</u>

Q. Can you get this response in the simulation?
→ Try to justify.

<u>IMPORTANT QUESTION</u> : You could do (see) all these by trial & error. Why do you need the math?
→ Cannot tune (in most cases) on the real device

→ We will see systems where manual tuning is too hard.

Q. What can K_p do?

Recall : $s_{1,2} = -\frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\alpha^2 - 4kK_p}$

$(\alpha, k, K_p > 0)$

– $K_p < \frac{\alpha^2}{4k}$ → real distinct roots (Overdamped)

→ Over damped – Very slow response

– $K_p = \frac{\alpha^2}{4k}$ → real repeated roots
still slow – (Critically damped)

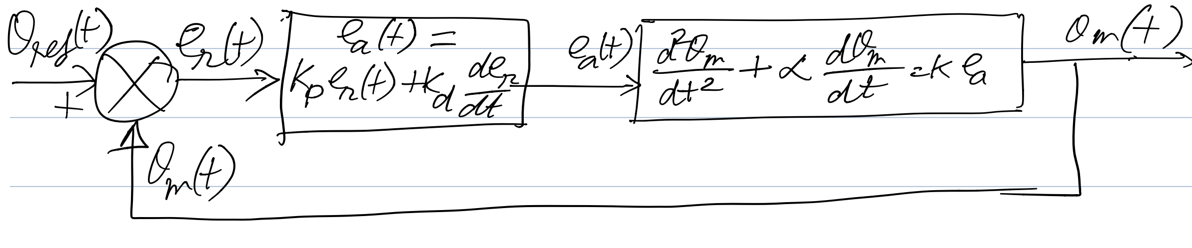
→ $K_p > \frac{\alpha^2}{4k}$ → complex conjugate – faster but oscillatory – oscillations not good.

$\boxed{\text{Show Antenna animation}}$

We want fast but no oscillation

– Can we add damping? – Remember that system cannot be changed!

P – D Controllers



$$e_r(t) = \theta_{ref}(t) - \theta_m(t) \quad \text{———— (1)}$$

$$e_a(t) = K_p\, e_r(t) + K_d \frac{de_r(t)}{dt} \quad \text{———— (2)}$$

Using (1) in (2), $e_a(t) = K_p\left(\theta_{ref}(t) - \theta_m(t)\right) + K_d \frac{d}{dt}\left(\theta_{ref}(t) - \theta_m(t)\right)$

Then

$$e_a(t) = K_p\theta_{ref} - K_p\theta_m(t) - K_d \frac{d\theta_m(t)}{dt} + K_d \frac{d\theta_{ref}(t)}{dt}$$

Using (3) in the system eqn:

$$\frac{d^2\theta_m}{dt^2} + \alpha \frac{d\theta_m}{dt} = K\left[K_p\theta_{ref} - K_p\theta_m(t) - K_d\frac{d\theta_m(t)}{dt} + K_d\frac{d\theta_{ref}(t)}{dt}\right]$$

$$\Rightarrow \frac{d^2\theta_m}{dt^2} + \underline{(\alpha + KK_d)}\frac{d\theta_m(t)}{dt} + KK_p\,\theta_m(t) = KK_p\,\theta_{ref} + KK_d\frac{d\theta_{ref}}{dt}$$

Q. How to analyse such diff. eqns? → Will learn later

For the moment assume $\theta_{ref}(t) =$ constant (Step input — sorry about the lack of sigma)

$$\Rightarrow \frac{d^2\theta_m}{dt^2} + \underline{(\alpha + KK_d)}\frac{d\theta_m(t)}{dt} + KK_p\,\theta_m(t) = KK_p\,\theta_{ref}$$

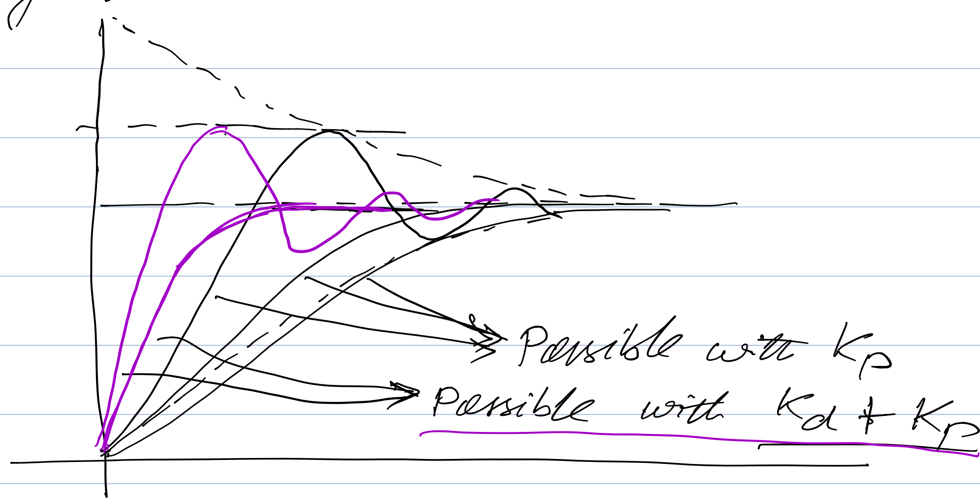
We successfully added damping without changing the original system.

The solution will depend on the roots of

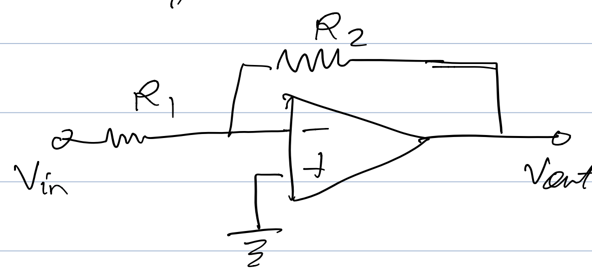
$$D^2 + (\alpha + KK_d)D + KK_p = 0$$

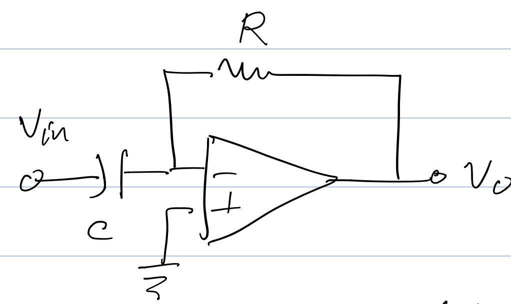
$$D_{1,2} = -\frac{(\alpha + KK_d)}{2} \pm \frac{1}{2}\sqrt{(\alpha + KK_d)^2 - 4KK_p}$$

Note that we can change the real part of the roots



<u>Analog circuit implementation</u> (Ideal)

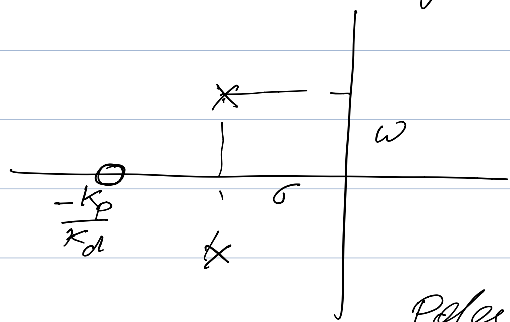




$$V_o = -\frac{R_2}{R_1} V_{in} \qquad V_o = -RC\frac{dV_{in}}{dt}$$

Very rarely used — usually digitally implemented in micro-controllers (?).

Q. Where is the pole zero map?



$$\frac{d^2\theta_m}{dt^2} + (\alpha + KK_d)\frac{d\theta_m}{dt} + KK_p$$
$$= KK_d \frac{d\theta_{ref}(t)}{dt} + KK_p\,\theta_{ref}(t)$$

Poles → Roots of the LHS = 0

Zeros → Roots of the RHS = 0

$$D^2 + (\alpha + KK_d)D + KK_p = 0 \rightarrow D_{1,2} = \sigma \pm j\omega$$

$$K_d D + K_p = 0 \longrightarrow D_1 = -\frac{K_p}{K_d}$$

<u>Fundamental question</u>: How to "<u>place</u>" poles & zeros to have "good" response?

⇔ How to choose K_p & K_d to have "good" response.

Q. What is good response?

— Control specifications

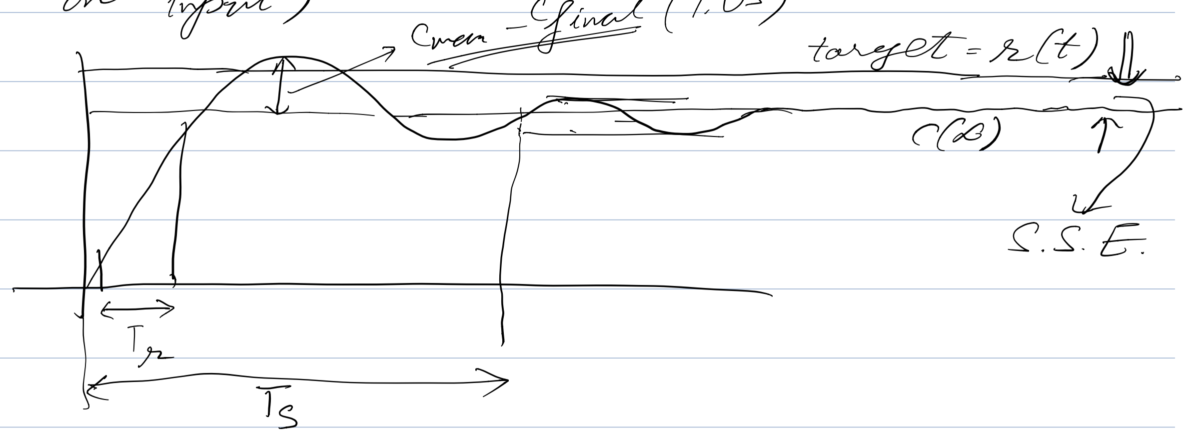
1> <u>Rise time</u> → time for the response to rise from 10% to 90% of its final value

2> <u>Settling Time</u> → time taken by response to reach & stay within 2% of final value

3> <u>Percentage Overshoot</u>: $\%OS = \frac{C_{max} - C_{final}}{C_{final}} \times 100\%$

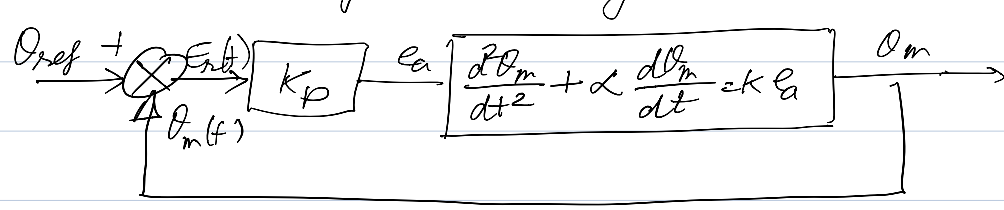
(1, 2, 3 are for step input)

4) <u>Steady State error</u> : SSE is the difference between output & input as $t \to \infty$ (depends on input)



Q. Did we have S.S.E in our example?

→ Proportional feedback case



$E_r(t) = \theta_{ref} - \theta_m$

$e_a(t) = K_p E_r(t)$

$$\frac{d^2\theta_m}{dt^2} + \alpha \frac{d\theta_m}{dt} = (k K_p) e_r$$

$$\frac{d^2(\theta_{ref} - e_r)}{dt^2} + \alpha \frac{d(\theta_{ref} - e_r)}{dt} = k K_p e_r$$

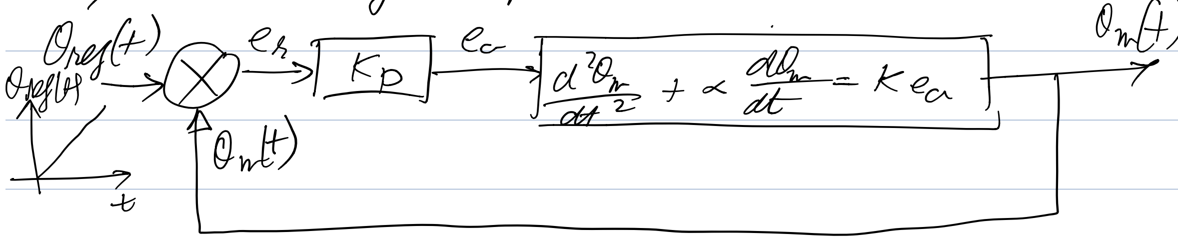
$$\frac{d^2 e_r}{dt^2} + \alpha \frac{d e_r}{dt} + k K_p e_r = 0$$

$\Rightarrow e_r(t) \to 0$ as $t \to \infty$

<u>So no error at steady state.</u>

Speed Control — Need for PI

A) With only K_p: θ



$$\theta_{ref}(t) = \omega_{ref}\, t$$

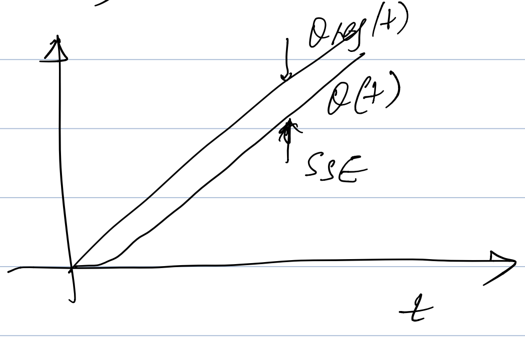
Same steps as above: $\frac{d^2(\theta_{ref} - e_r)}{dt^2} + \alpha \frac{d(\theta_{ref} - e_r)}{dt} = K K_p e_r$

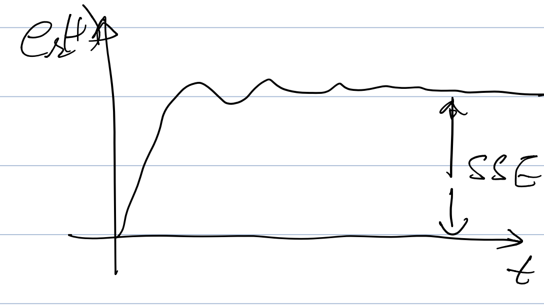
$$\Leftrightarrow \quad -\frac{d^2 e_r}{dt^2} + \alpha \omega_{ref} - \alpha \frac{d e_r}{dt} = K K_p e_r$$

$$\Leftrightarrow \quad \frac{d^2 e_r}{dt^2} + \alpha \frac{d e_r}{dt} + K K_p e_r = \alpha \omega_{ref}$$

Solution:
$$e_r(t) = \begin{cases} A + B e^{p_1 t} + C e^{p_2 t} & (p_1 \neq p_2 \text{ real}) \\ A + B e^{pt} + C t e^{pt} & (p_1 = p_2 = p \text{ real}) \\ A + B e^{\sigma t} \cos(\omega t + C) & (\sigma \pm j\omega) \end{cases}$$

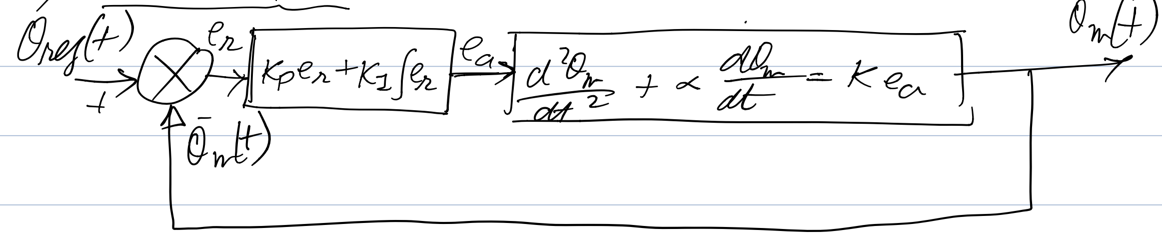
$e_r(t) \to A$ as $t \to \infty$ $\Leftrightarrow$ S.S.E $\neq 0$.





Q. How to get SSE to zero?
→ Use PI.

B) With PI



$$\frac{d^2\theta_m}{dt^2} + \alpha \frac{d\theta_m}{dt} = K\left(K_p e_r + K_I \int_0^t e_r\,dt\right)$$

$$\frac{d^2(\theta_{ref}(t) - e_r)}{dt^2} + \alpha \frac{d(\theta_{ref}(t) - e_r)}{dt} = K\left(K_p e_r + K_I \int e_r\,dt\right)$$

For $\theta_{ref}(t) = \omega_{ref}\, t$,

$$-\frac{d^2 e_r}{dt^2} + \alpha\,\omega_{ref} - \alpha \frac{de_r}{dt} = KK_p e_r + KK_I \int_0^t e_r\,dt$$

Diff. again:

$$\frac{d^3 e_r}{dt^3} + \alpha \frac{d^2 e_r}{dt^2} + \overset{\beta}{(KK_p)} \frac{de_r}{dt} + \overset{\gamma}{(KK_I)}\, e_r = 0$$

$\rightarrow$ 3rd order diff. eqn.: But no forcing function.

We guess that a solution is of the form:

$$e_r(t) = e^{Dt}$$

Then $\quad D^3 e^{Dt} + \alpha D^2 e^{Dt} + \beta D e^{Dt} + \gamma e^{Dt} = 0$

$\Leftrightarrow \quad D^3 + \alpha D^2 + \beta D + \gamma = 0$

Let the roots be D_1, D_2, D_3.

$e_r(t) = A e^{D_1 t} + B e^{D_2 t} + C e^{D_3 t}$ (real distinct)

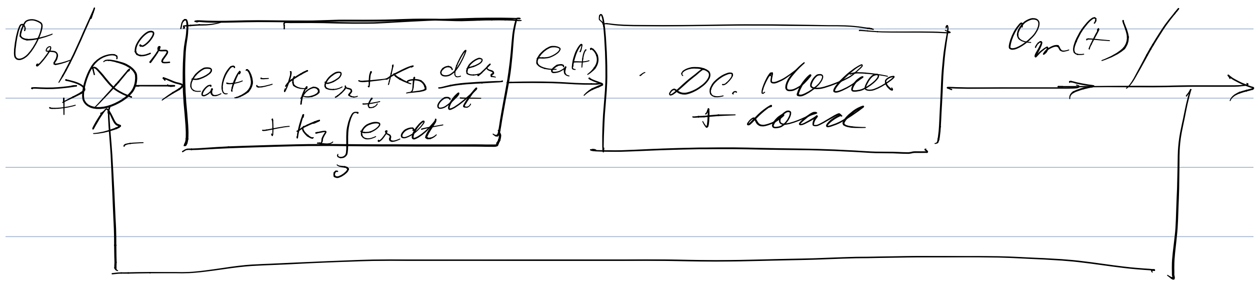
$e_r(t) = A e^{D_1 t} + B e^{Dt} + C t e^{Dt}$ (real, $D_1 \neq D_2 = D_3$)

$e_r(t) = A e^{Dt} + B t e^{Dt} + C t^2 e^{Dt}$ ($D_1 = D_2 = D_3$ real)

$$e_r(t) = Ae^{\lambda_1 t} + Be^{\sigma t}\cos(\omega t + C) \quad \left(\begin{matrix}\lambda_1 \text{ real} \\ \sigma \pm j\omega\end{matrix}\right)$$

In each case, $e_r(t) \to 0$ as $t \to \infty$.

So: final feedback controller:



PID Controllers are used in most Control Systems

Q. How to tune? 3 parameters → many possibilities.

Q. How to ensure that your design will work in the real world? → for inaccurate models & arbitrary disturbances.

→ Better answers in 3rd year - EE302/EE324.

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