# ME212 Project

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**Type:**

Type 1 (Small problem of Solid Mechanics solved using any software package or writing a code in any platform)

### Language and Technologies Used

Python, Jupyter Notebook, Matlplotlib library (for plotting graphs)

### GitHub URL for the project

Atharvat/ME212_Project (github.com) (https://github.com/Atharvat/ME212_Project)

**Important Note: To run the code for this project, Python need to be installed on the PC. If Jupyter Notebooks is installed, run the 'index.ipynb' file, and if Jupyter Notebooks is not installed, run the 'index.py' file.**

## Question

The cantilever beam AB is of uniform cross section and carries a load P at its free end A (Fig. 1). Determine the equation of the elastic curve and the deflection and slope at A and plot the graphs for the deflection and slope of the beam.

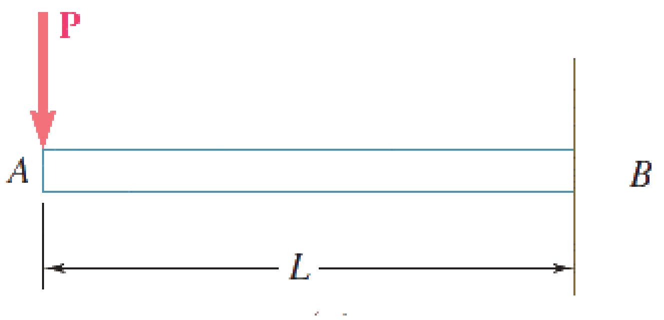



Figure 1

## Solution

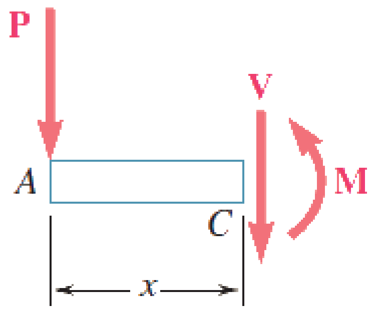



Figure 2

Using the free-body diagram of the portion AC of the beam (Fig. 2), where $C$ is located at a distance $x$ from end $A$,

$$M = -Px \tag{1}$$

We know, the differential equation for the elastic curve is given by

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} \tag{2}$$

Substituting for $M$ from $Eq.$ $(1)$ into $Eq.$ $(2)$ and multiplying both members by the constant $EI$ gives

$$EI\frac{d^2y}{dx^2} = -Px \tag{3}$$

Integrating in $x$,

$$EI\frac{dy}{dx} = -\frac{1}{2}Px^2 + C_1 \tag{4}$$

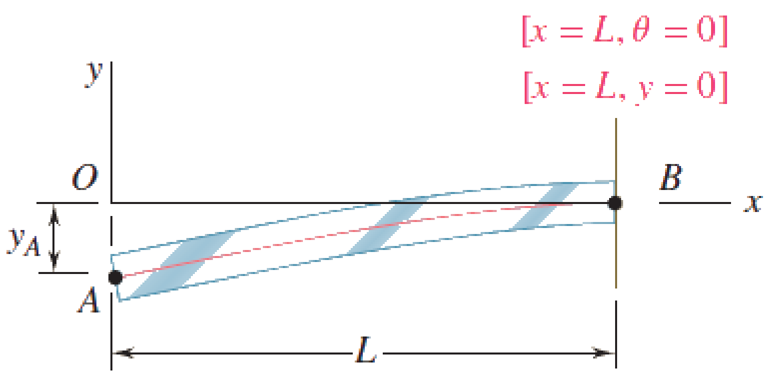



Figure 3

Now at the fixed end $B,\ x = L$ and $\theta = \frac{dy}{dx} = 0$ (Fig. 3).
Substituting these values into $Eq.\,(4)$ and solving for $C_1$ gives

$$C_1 = \frac{1}{2}PL^2 \tag{5}$$

which we carry back into $Eq.\,(4)$:

$$EI\frac{dy}{dx} = -\frac{1}{2}Px^2 + \frac{1}{2}PL^2 \tag{6}$$

Integrating both members,

$$EIy = -\frac{1}{6}Px^3 + \frac{1}{2}PL^2x + C_2 \tag{7}$$

But at B, $x = L,\ y = 0$. Substituting into $Eq.\,(7)$,

$$0 = -\frac{1}{6}Px^3 + \frac{1}{2}PL^2x + C_2 \tag{8}$$

$$C_2 = -\frac{1}{3}PL^3 \tag{9}$$

Carrying the value of $C_2$ back into $Eq.\,(7)$, the equation of the elastic curve is

$$EIy = -\frac{1}{6}Px^3 + \frac{1}{2}PL^2x - \frac{1}{3}PL^3 \tag{10}$$

or

$$y = \frac{P}{6EI}(-x^3 + 3L^2x - 2L^3) \tag{11}$$

The deflection and slope at $A$ are obtained by letting $x = 0$ in $Eqs.\,(11)$ and $(6)$

$$y_A = -\frac{PL^3}{3EI} \tag{12}$$

and

$$\theta_A = \left(\frac{dy}{dx}\right)_A = \frac{PL^2}{2EI} \tag{13}$$

## Code

## Importing libraries

In [ ]:

```
import matplotlib.pyplot as plt
import numpy as np
```

## Taking inputs

In [ ]:

```
P = int(input('Enter the value of load at in N(Newtons)'))
L = int(input('Enter the length of the beam AB in m(metres)'))
E = int(input('Enter the modulus of Elasticity of the beam in Pa(Pascals)'))
I = int(input('Enter I (the moment of inertia of the cross section of the beam about its neutral axis) in kg*m^2'))
print('You have entered: P = ',P,'N, L = ',L,'m, E = ',E,'Pa, I = ',I, 'kg*m^2')
```

```
You have entered: P =  100 N, L =  10 m, E =  20000 Pa, I =  10 kg*m^2
```

## Calculating Deflection and Slope at A:

We have derived the formulas: $y_A = -\frac{PL^3}{3EI}$ and $\theta_A = \frac{PL^2}{2EI}$

In [ ]:

```
y_a = -1 * P * L**3 / (3 * E * I)
theta_a =  P * L**2 / (2 * E * I)
print('The deflection of the beam at A is ', y_a, 'm')
print('The angle of the beam at A is ', theta_a, 'radians')
```

```
The deflection of the beam at A is  -0.16666666666666666 m
The angle of the beam at A is  0.025 radians
```

## Equations for the deflection and slope of the beam

$y = \frac{P}{6EI}(-x^3 + 3L^2x - 2L^3)$ and $\frac{dy}{dx} = \frac{1}{EI}(-\frac{1}{2}Px^2 + \frac{1}{2}PL^2)$

In [ ]:

```
def y(x):
    return P * (-x**3 + 3*x*L**2 - 2*L**3) / (6*E*I)
def theta(x):
    return (-(P*x**2)/2 + (P*L**2)/2) / (E*I)
```

In [ ]:

```
print(theta(0))
```

```
0.025
```

## Plotting the graphs for Deflection and Slope

In [ ]:

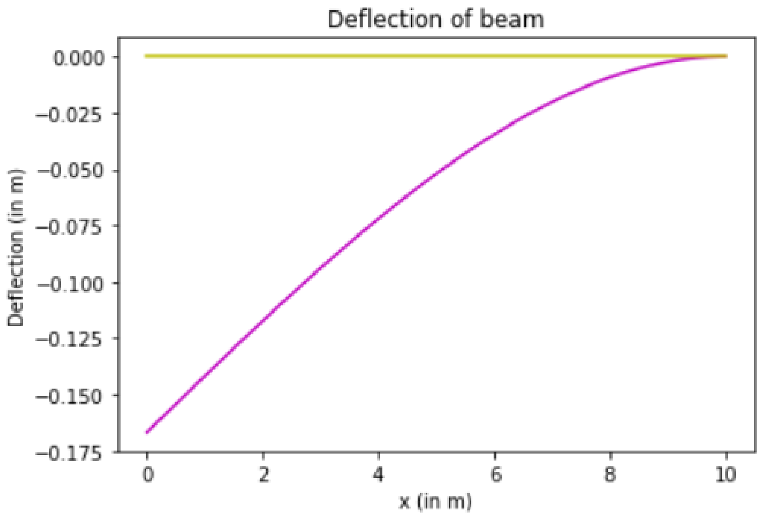
```
l=np.linspace(0,L,1000)
X=[]
deflections_list=[]
slopes_list=[]

for x in l:
  X.append(x)
  deflections_list.append(y(x))
  slopes_list.append(theta(x))
```

## Deflection plot

In [ ]:

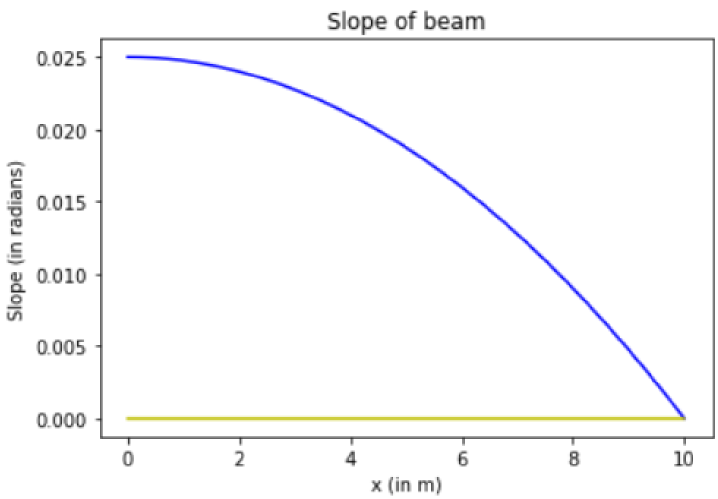
```
plt.subplot(1,1,1)
plt.plot(X,deflections_list,color='m')
plt.plot([0,L],[0,0],color='y')
plt.title("Deflection of beam")
plt.xlabel("x (in m)")
plt.ylabel("Deflection (in m)")
plt.show()
```




## Slope Plot

In [ ]:

```
plt.subplot(1,1,1)
plt.plot(X,slopes_list,color='b')
plt.plot([0,L],[0,0],color='y')
plt.title("Slope of beam")
plt.xlabel("x (in m)")
plt.ylabel("Slope (in radians)")
plt.show()
```




So, for $P = 100N, L = 10m, E = 20000Pa, I = 10kg * m^2$: The value of deflection at $A$ is -0.167m and slope at $A$ is 0.0025 radians.

# Thank You!