Karnaugh Map (K-map)

- ✓ In the algebraic method of simplification, we need to write lengthy equations, find the common terms, manipulate the expressions etc., so it is time consuming work.

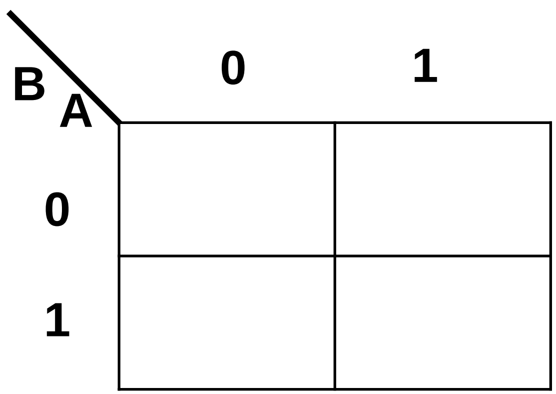
- ✓ Thus “K-map” is another simplification technique to reduce the Boolean equation.

Karnaugh Map (K-map)

- ✓ It overcomes all the disadvantages of algebraic simplification techniques.
- ✓ The information contained in a truth table or available in the SOP or POS form is represented on K-map.

Karnaugh Map (K-map)

- K-map Structure - 2 Variable
  - ✓ A & B are variables or inputs
  - ✓ 0 & 1 are values of A & B
  - ✓ 2 variable k-map consists of 4 boxes i.e. $2^2=4$



Karnaugh Map (K-map)

- K-map Structure - 2 Variable

  - ✓ Inside 4 boxes we have enter values of Y i.e. output

B \ A		$\bar{A}$ 0	A 1
$\bar{B}$	0	$\bar{A}\bar{B}$	$\bar{A}B$
B	1	$A\bar{B}$	AB

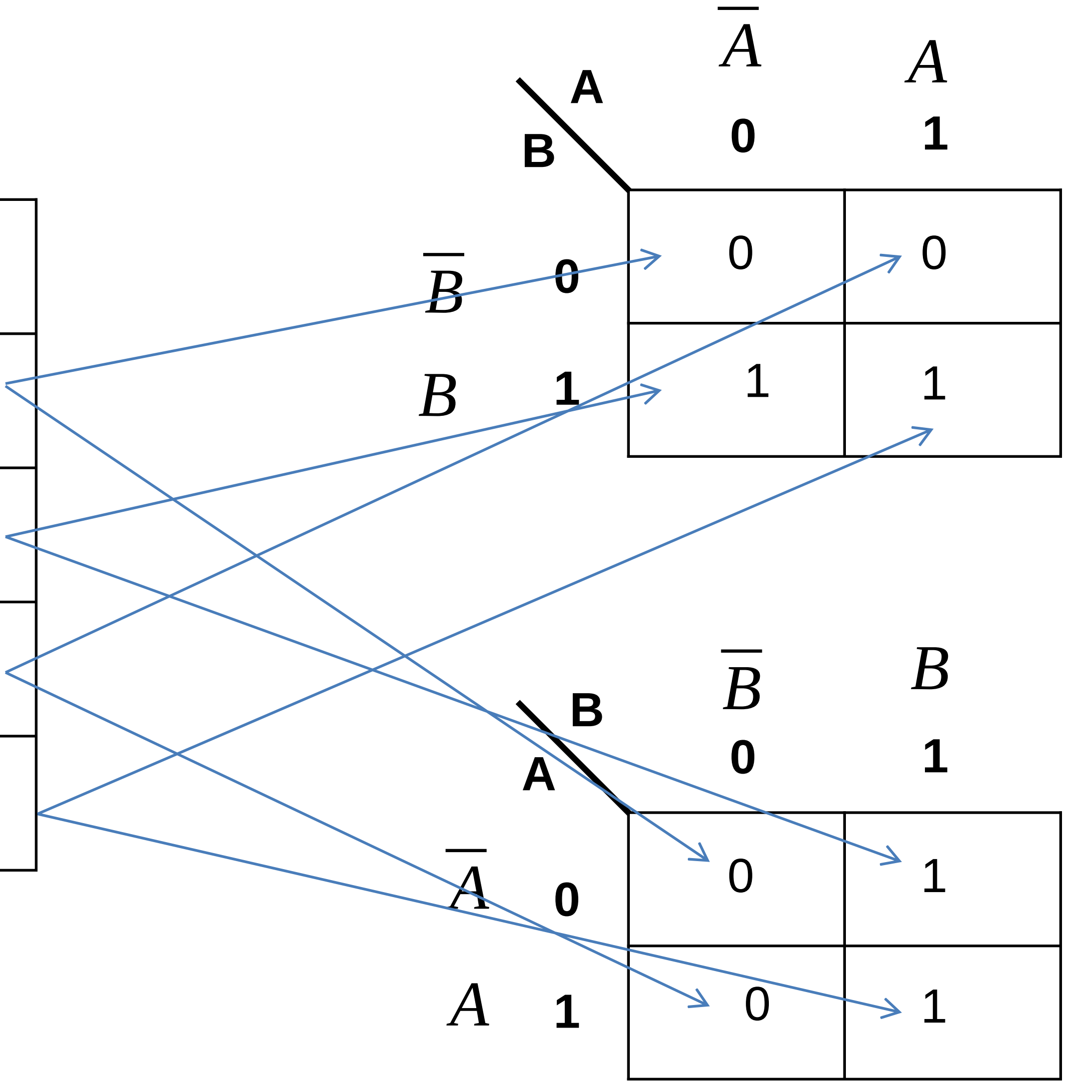
B \ A		$\bar{A}$ 0	A 1
$\bar{B}$	0	m_0	m_1
B		m_2	m_3

1 K-map & its associated minterms

Karnaugh Map (K-map)

- ✓ Relationship between Truth Table & K-map

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1



Karnaugh Map (K-map)

- K-map Structure - 3 Variable
  - ✓ A, B & C are variables or inputs
  - ✓ 3 variable k-map consists of 8 boxes i.e. $2^3=8$

C \ AB	00	01	11	10
0				
1				

A \ BC	00	01	11	10
0				
1				

BC \ A	0	1
00		
01		
11		
10		

Karnaugh Map (K-map)

✓ 3 Variable K-map & its associated product terms

C \ AB	00	01	11	10
0	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}B\overline{C}$	$AB\overline{C}$	$A\overline{B}\,\overline{C}$
1	$\overline{A}\,\overline{B}C$	$\overline{A}BC$	ABC	$A\overline{B}C$

A \ BC	00	01	11	10
0	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}C$	$\overline{A}BC$	$\overline{A}B\overline{C}$
1	$A\overline{B}\,\overline{C}$	$A\overline{B}C$	ABC	$AB\overline{C}$

BC \ A	0	1
00	$\overline{A}\,\overline{B}\,\overline{C}$	$A\overline{B}\,\overline{C}$
01	$\overline{A}\,\overline{B}C$	$A\overline{B}C$
11	$\overline{A}BC$	ABC
10	$\overline{A}B\overline{C}$	$AB\overline{C}$

Karnaugh Map (K-map)

✓ 3 Variable K-map & its associated minterms

C \ AB	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

BC \ A	0	1
00	m_0	m_4
01	m_1	m_5
11	m_3	m_7
10	m_2	m_6

Karnaugh Map (K-map)

- K-map Structure - 4 Variable
  - ✓ A, B, C & D are variables or inputs
  - ✓ 4 variable k-map consists of 16 boxes i.e. $2^4=16$

CD \ AB	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00				
01				
11				
10				

Karnaugh Map (K-map)

✓ 4 Variable K-map and its associated product terms

CD \ AB	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}\,\overline{D}$	$AB\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}\,\overline{D}$
01	$\overline{A}\,\overline{B}\,\overline{C}D$	$A\overline{B}\,\overline{C}D$	$AB\overline{C}D$	$A\overline{B}\,\overline{C}D$
11	$\overline{A}\,\overline{B}CD$	$A\overline{B}CD$	$ABCD$	$A\overline{B}CD$
10	$\overline{A}\,\overline{B}C\overline{D}$	$A\overline{B}C\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$

AB \ CD	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}D$	$\overline{A}\,\overline{B}CD$	$\overline{A}\,\overline{B}C\overline{D}$
01	$A\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$
11	$AB\overline{C}\,\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
10	$A\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

Karnaugh Map (K-map)

✓ 4 Variable K-map and its associated minterms

CD \ AB	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{11}	m_{10}

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Representation of Standard SOP form expression on K-map

For example, SOP equation is given as

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$

- ✓ The given expression is in the standard SOP form.
- ✓ Each term represents a minterm.
- ✓ We have to enter ‘1’ in the boxes corresponding to each minterm as below

$\overline{A}\,\overline{B}\,\overline{C}$ $\quad$ $\overline{A}\,\overline{B}C$

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	1	1	0	0
A 1	1	0	1	1

$AB\overline{C}$

$A\overline{B}\,\overline{C}$ $\quad$ ABC

Simplification of K-map

- ✓ Once we plot the logic function or truth table on K-map, we have to use the grouping technique for simplifying the logic function.
- ✓ Grouping means the combining the terms in adjacent cells.
- ✓ The grouping of either 1's or 0's results in the simplification of Boolean expression.

Simplification of K-map

- ✓ If we group the adjacent 1's then the result of simplification is SOP form
- ✓ If we group the adjacent 0's then the result of simplification is POS form

Grouping

- ✓ While grouping, we should group most number of 1's.
- ✓ The grouping follows the binary rule i.e we can group 1,2,4,8,16,32,……..number of 1's.
- ✓ We cannot group 3,5,7,………number of 1's
- ✓ ***Pair***: A group of two adjacent 1's is called as Pair
- ✓ ***Quad***: A group of four adjacent 1's is called as Quad
- ✓ ***Octet***: A group of eight adjacent 1's is called as Octet

Grouping of Two Adjacent 1's : Pair

✓ <u>*A pair eliminates 1 variable*</u>

$\overline{A}BC$ $\overline{A}B\overline{C}$

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	0	0	1	1
A 1	0	0	0	0

$$Y = \overline{A}BC + \overline{A}B\overline{C}$$

$$Y = \overline{A}B(C + \overline{C})$$

$$Y = \overline{A}B \qquad (\because C + \overline{C} = 1)$$

Grouping of Two Adjacent 1's : Pair

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	0
A 1	1	0	0	1

AC \ B	$\overline{BC}$ 00	BC 01	BC 11	BC 10
$\bar{A}$ 0	0	1	1	1
A 1	0	0	1	0

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	0	0
A 1	0	1	0	0

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	1	1
A 1	1	0

Grouping of Two Adjacent 1's : Pair

AB \ CD		$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$	00	0	1	0	0
$\overline{A}B$	01	0	0	0	0
AB	11	0	0	0	0
$A\overline{B}$	10	0	1	0	0

Possible Grouping of Four Adjacent 1's : Quad

✓ ***A Quad eliminates 2 variable***

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	1	1	1

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	0	0
$\bar{A}B$ 01	0	1	0	0
AB 11	0	1	0	0
$A\bar{B}$ 10	0	1	0	0

Possible Grouping of Four Adjacent 1’s : Quad

✓ *A Quad eliminates 2 variable*

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	1	1	0	0
AB 11	1	1	0	0
$A\bar{B}$ 10	0	0	0	0

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	1	0
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	0	1	1	0

Possible Grouping of Four Adjacent 1's : Quad

✓ *A Quad eliminates 2 variable*

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	0	0	1

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	1	0	0	1
AB 11	1	0	0	1
$A\bar{B}$ 10	0	0	0	0

Possible Grouping of Four Adjacent 1's : Quad

✓ ***A Quad eliminates 2 variable***

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	1	1	1
AB 11	0	1	1	1
$A\bar{B}$ 10	0	0	0	0

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	0

Possible Grouping of Eight Adjacent 1's : Octet

✓ ***A Octet eliminates 3 variable***

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0	0	0	0
$\overline{A}B$ 01	0	0	0	0
AB 11	1	1	1	1
$A\overline{B}$ 10	1	1	1	1

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0	1	1	0
$\overline{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\overline{B}$ 10	0	1	1	0

Possible Grouping of Eight Adjacent 1's : Octet

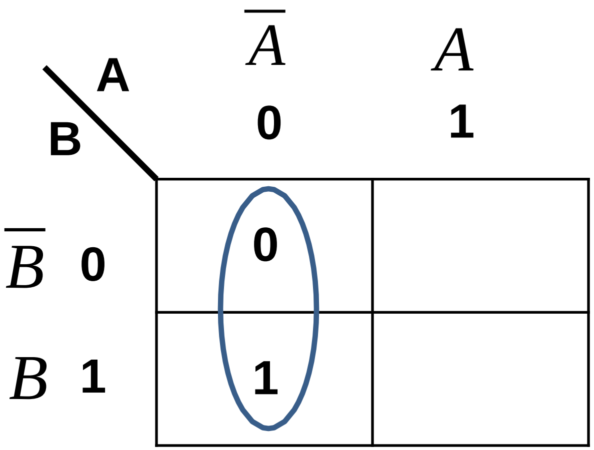
✓ <u>*A Octet eliminates 3 variable*</u>

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	1	1	1	1
$\overline{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\overline{B}$ 10	1	1	1	1

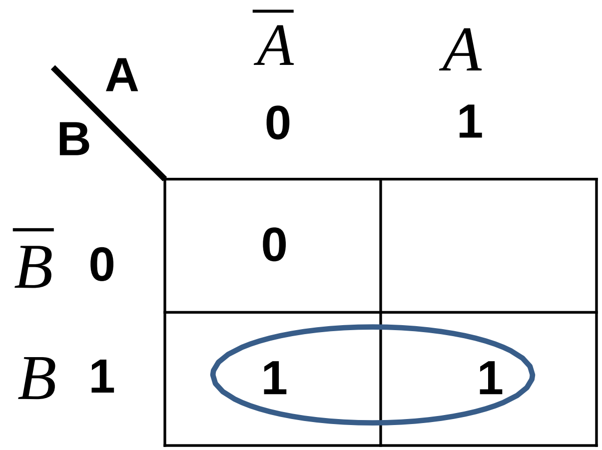
AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	1	0	0	1
$\overline{A}B$ 01	1	0	0	1
AB 11	1	0	0	1
$A\overline{B}$ 10	1	0	0	1

Rules for K-map simplification

1. Groups may not include any cell containing a zero.



Not Accepted



Accepted

Rules for K-map simplification

2. Groups may be horizontal or vertical, but may not be diagonal

B \ A	$\overline{A}$ 0	A 1
$\overline{B}$ 0	0	1
B 1	1	0

Not Accepted

B \ A	$\overline{A}$ 0	A 1
$\overline{B}$ 0	0	1
B 1	1	1

Accepted

Rules for K-map simplification

3. Groups must contain 1,2,4,8 or in general 2^n cells

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	0	1	1	1
A 1	0	0	0	0

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	0	1	1	1
A 1	0	0	0	0

B \ A	$\overline{A}$ 0	A 1
$\overline{B}$ 0	1	1
B 1	0	1

Not Accepted

B \ A	$\overline{A}$ 0	A 1
$\overline{B}$ 0	1	1
B 1	0	1

Accepted

Rules for K-map simplification

4. Each group should be as large as possible

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	1	1	1	1
A 1	0	0	1	1

Not Accepted

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	1	1	1	1
A 1	0	0	1	1

Accepted

Rules for K-map simplification

5. Each cell containing a one must be in at least one group

BC / A	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 0	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	1
A 1	0	0	1	0

Rules for K-map simplification

6. Groups may be overlap

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 0	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	1	1	1
A 1	0	0	1	1

Rules for K-map simplification

7. Groups may wrap around the table. The leftmost cell in a row may be grouped with rightmost cell and the top cell in a column may be grouped with bottom cell

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	1	1	1

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	0	0	1
A 1	1	0	0	1

Rules for K-map simplification

8. There should be as few groups as possible, as long as this does not contradict any of the previous rules.

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 0	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	1	1	1
A 1	0	0	1	1

Not Accepted

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 0	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	1	1	1
A 1	0	0	1	1

Accepted

Rules for K-map simplification

9. ***A pair eliminates one variable.***

10. ***A Quad eliminates two variables.***

11. ***A octet eliminates three variables***

Example 1

For the given K-map write simplified Boolean expression

AB / C	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	0	1	1	1
C 1	0	0	1	0

Example 1

continue…..

A B \ C	$\overline{AB}$ 00	AB 01	AB 11	AB 10
$\overline{C}$ 0	0	1	1	1
C 1	0	0	1	0

AC

$\overline{B}C$

AB

Simplified Boolean expression

$$Y = B\overline{C} + AB + A\overline{C}$$

Example 2

For the given K-map write simplified Boolean expression

C \ AB		$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$	0	1	1	0	1
C	1	1	0	0	1

Example 2

continue…..

AB / C	$\overline{A}\overline{B}$ 00	$\overline{A}B$ 01	AB 11	$A\overline{B}$ 10
$\overline{C}$ 0	1	1	0	1
C 1	1	0	0	1

$\overline{A}\overline{C}$

$\overline{B}$

Simplified Boolean expression

$$Y = \overline{B} + AC$$

Example 3

A logical expression in the standard SOP form is as follows;

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,C$$

Minimize it with using the K-map technique

Example 3

continue……

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,C$$

A \ BC	$\overline{BC}$ 00	BC 01	BC 11	BC 10
$\overline{A}$ 0	1	0	1	1
A 1	0	1	0	0

AB

$\overline{A}\,\overline{C}$

$A\overline{B}C$

Simplified Boolean expression

$$Y = \overline{A}\,\overline{C} + \overline{A}B + A\overline{B}C$$

Example 4

A logical expression representing a logic circuit is;

$$Y = \Sigma m(0, 1, 2, 5, 13, 15)$$

Draw the K-map and find the minimized logical expression

Example 4

continue…..

$$Y = \Sigma m(0,1,2,5,13,15)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0: **1**	1: **1**	3: **0**	2: **1**
$\bar{A}B$ 01	4: **0**	5: **1**	7: **0**	6: **0**
AB 11	12: **0**	13: **1**	15: **1**	14: **0**
$A\bar{B}$ 10	8: **0**	9: **0**	11: **0**	10: **0**

$\overline{ABD}$

$\bar{A}\bar{C}D$

ABD

Simplified Boolean expression

$$Y = \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}D + ABD$$

Example 5

Minimize the following Boolean expression using K-map ;

$$f(A,B,C,D) = \Sigma m(1,3,5,9,11,13)$$

Example 5

continue…..

$$f(A,B,C,D) = \Sigma m(1,3,5,9,11,13)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	**0** (0)	**1** (1)	**1** (3)	**0** (2)
$\bar{A}B$ 01	**0** (4)	**1** (5)	**0** (7)	**0** (6)
AB 11	**0** (12)	**1** (13)	**0** (15)	**0** (14)
$A\bar{B}$ 10	**0** (8)	**1** (9)	**1** (11)	**0** (10)

$\bar{B}D$

$\bar{C}D$

Simplified Boolean expression

$$f = \bar{B}D + \bar{C}D$$

$$f = D(\bar{B} + \bar{C})$$

Example 6

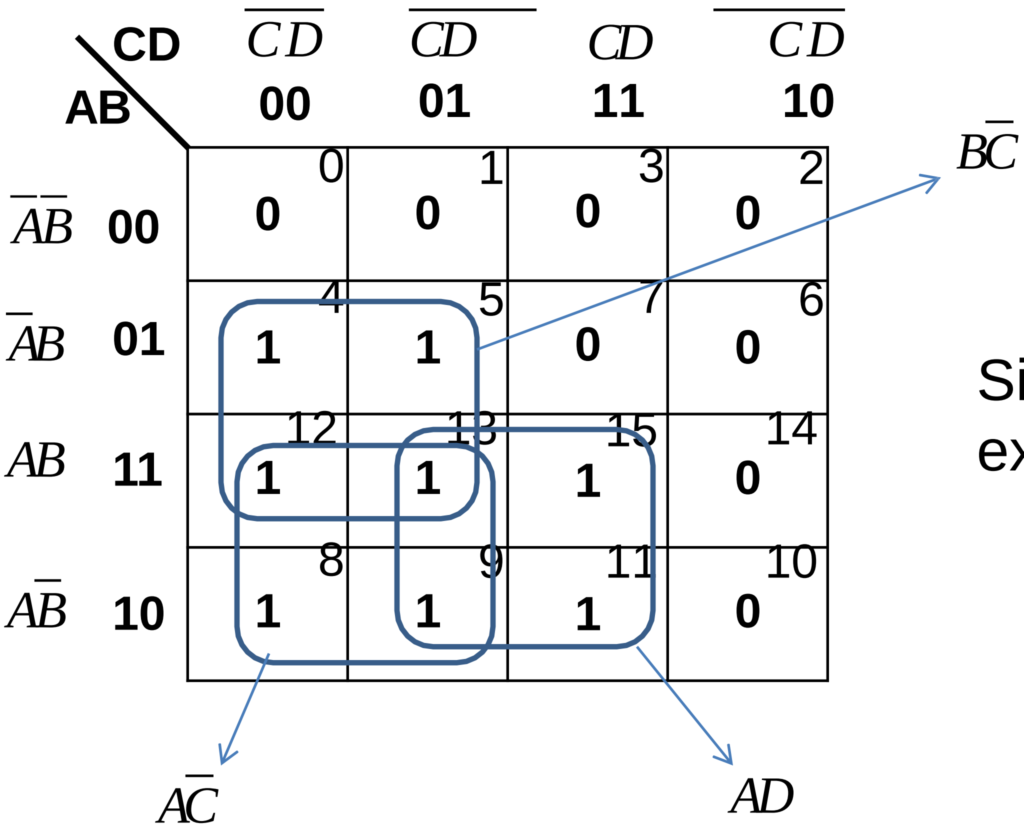
Minimize the following Boolean expression using K-map ;

$$f(A,B,C,D) = \Sigma m(4,5,8,9,11,12,13,15)$$

Example 6

continue…..

$$f(A,B,C,D)=\Sigma m(4,5,8,9,11,12,13,15)$$



Simplified Boolean expression

$$f = B\overline{C} + A\overline{C} + AD$$

Example 7

Minimize the following Boolean expression using K-map ;

$$f_2(A, B, C, D) = \Sigma m(0,1,2,3,11,12,14,15)$$

Example 7

continue…..

$$f_2(A,B,C,D) = \Sigma m(0,1,2,3,11,12,14,15)$$

AB \ CD	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	1 (0)	1 (1)	1 (3)	1 (2)
$\overline{A}B$ 01	0 (4)	0 (5)	0 (7)	0 (6)
AB 11	1 (12)	0 (13)	1 (15)	1 (14)
$A\overline{B}$ 10	0 (8)	0 (9)	1 (11)	0 (10)

$\overline{AB}$

$AB\overline{D}$

ACD

Simplified Boolean expression

$$f_2 = \overline{A}\,\overline{B} + AB\overline{D} + ACD$$

Example 8

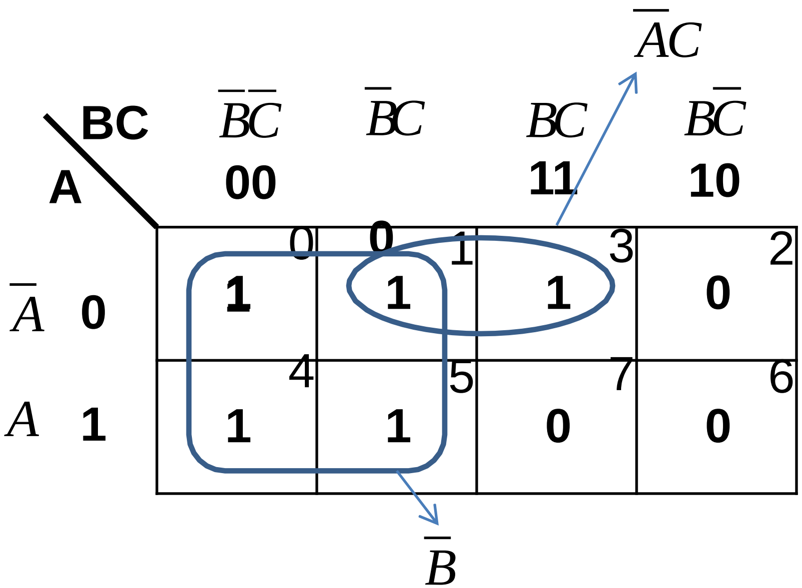
Solve the following expression with K-maps;

1. $f_1(A, B, C) = \Sigma m(0,1,3,4,5)$
2. $f_2(A, B, C) = \Sigma m(0,1,2,3,6,7)$

Example 8

continue……

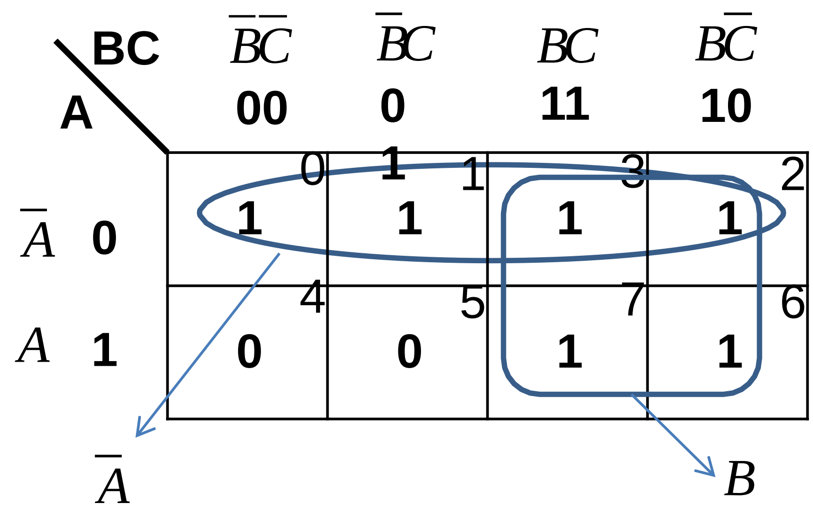
$f_1(A,B,C) = \Sigma m(0,1,3,4,5)$



Simplified Boolean expression

$f_1 = \overline{A}C + \overline{B}$

$f_2(A,B,C) = \Sigma m(0,1,2,3,6,7)$



Simplified Boolean expression

$f_2 = \overline{A} + B$

Example 9

Simplify ;

$$f(A, B, C, D) = \Sigma m(0,1,4,5,7,8,9,12,13,15)$$

Example 9

continue…..

$$f\,(A, B, C, D) = \Sigma m(0,1,4,5,7,8,9,12,13,15)$$

CD / AB	$\overline{C}\,\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\,\overline{B}$ 00	0: **1**	1: **1**	3: **0**	2: **0**
$\overline{A}B$ 01	4: **1**	5: **1**	7: **1**	6: **0**
AB 11	12: **1**	13: **1**	15: **1**	14: **0**
$A\overline{B}$ 10	8: **1**	9: **1**	11: **0**	10: **0**

$\overline{C}$

BD

Simplified Boolean expression

$$f = \overline{C} + BD$$

Example 10

Solve the following expression with K-maps;

1. $f_1(A, B, C, D) = \Sigma m(0,1,3,4,5,7)$
2. $f_2(A, B, C) = \Sigma m(0,1,3,4,5,7)$

Example 10

continue……

$$f_1(A,B,C,D)=\Sigma m(0,1,3,4,5,7)$$

AB \ CD	$\bar C\bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A\bar B$ 00	0 **1**	1 **1**	3 **0**	2 **0**
$\bar A B$ 01	4 **1**	5 **1**	7 **1**	6 **0**
AB 11	12 **0**	13 **0**	15 **0**	14 **0**
$A\bar B$ 10	8 **0**	9 **0**	11 **0**	10 **0**

$\bar A\bar C$

$\bar A D$

Simplified Boolean expression

$$f_1=\bar A\bar C+\bar A D$$

$$f_2(A,B,C)=\Sigma m(0,1,3,4,5,7)$$

A \ BC	$\bar B\bar C$ 00	$\bar B C$ 0 1	BC 11	$B\bar C$ 10
$\bar A$ 0	0 **1**	1 **1**	3 **1**	2 **0**
A 1	4 **1**	5 **1**	7 **1**	6 **0**

$\bar B$

C

Simplified Boolean expression

$$f_2=\bar B+C$$

K-map for Product of Sum Form (POS Expressions)

- ✓ Karnaughmap can also be used for Boolean expression in the Product of sum form (POS).
- ✓ The procedure for simplification of expression by grouping of cells is also similar

K-map for Product of Sum Form (POS Expressions)

- ✓ The letters with bars (NOT) represent 1 and unbarred letters represent 0 of Binary.
- ✓ A zero is put in the cell for which there is a term in the Boolean expression
- ✓ Grouping is done for adjacent cells containing zeros.

Example 11

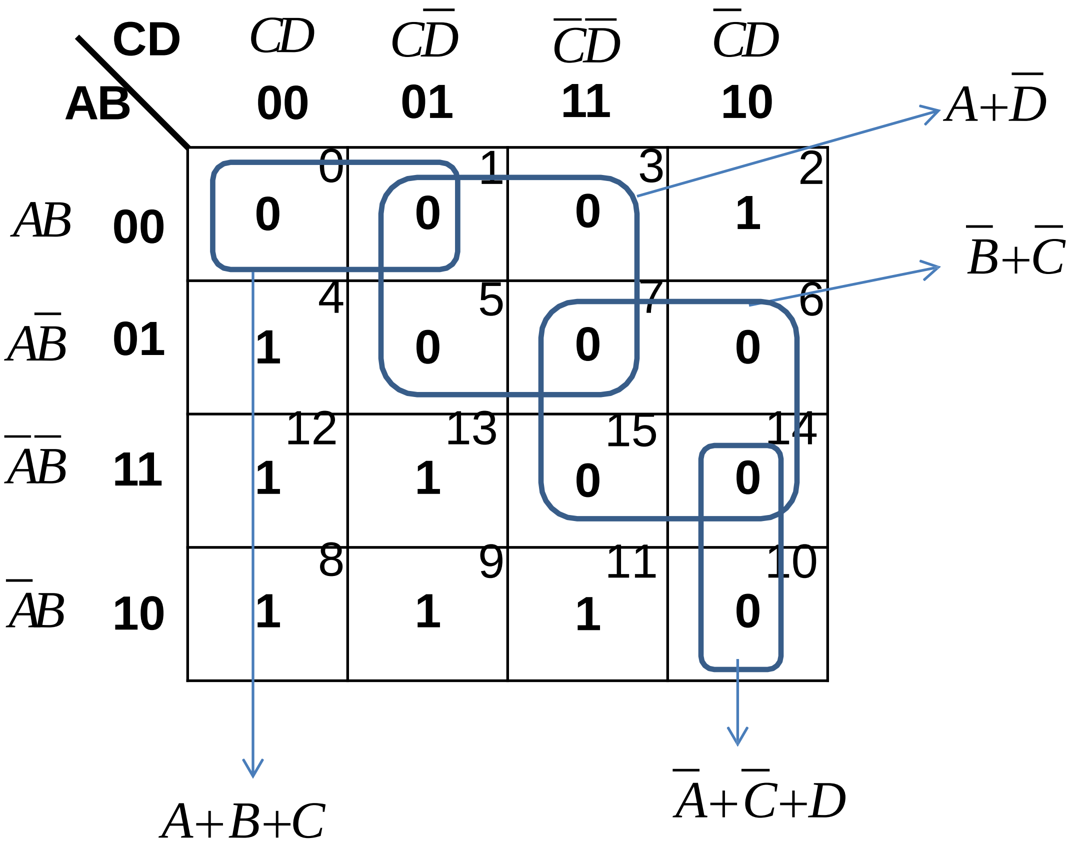
Simplify ;

$$f(A,B,C,D)=\Pi M(0,1,3,5,6,7,10,14,15)$$

Example 11

continue…..

$$f(A,B,C,D)=\Pi M(0,1,3,5,6,7,10,14,15)$$



Simplified Boolean expression

$$f=(A+\overline{D})(\overline{B}+\overline{C})(\overline{A}+\overline{C}+D)(A+B+C)$$

Example 12

Simplify ;

$$f(A,B,C,D)=\Pi M(4,6,10,12,13,15)$$

Example 12 continue…..

$$f\,(A,B,C,D)=\Pi M\,(4,6,10,12,13,15)$$

CD \ AB	CD 00	$C\overline{D}$ 01	$\overline{C}\overline{D}$ 11	$\overline{C}D$ 10
AB 00	1 (0)	1 (1)	1 (3)	1 (2)
$A\overline{B}$ 01	0 (4)	1 (5)	1 (7)	0 (6)
$\overline{A}\overline{B}$ 11	0 (12)	0 (13)	0 (15)	1 (14)
$\overline{A}B$ 10	1 (8)	1 (9)	1 (11)	0 (10)

$A+\overline{B}+D$

$\overline{A}+\overline{B}+C$

$\overline{A}+B+\overline{C}+D$

$\overline{A}+\overline{B}+\overline{D}$

Simplified Boolean expression

$$f\;=(\overline{A}+B+\overline{C}+D)(A+\overline{B}+D)(\overline{A}+\overline{B}+\overline{D})(\overline{A}+\overline{B}+C)$$

K-map and don’t care conditions

- ✓ For SOP form we enter 1’s corresponding to the combinations of input variables which produce a high output and we enter 0’s in the remaining cells of the K-map.
- ✓ For POS form we enter 0’s corresponding to the combinations of input variables which produce a high output and we enter 1’s in the remaining cells of the K-map.

K-map and don't care conditions

- ✓ But it is not always true that the cells not containing 1's (in SOP) will contain 0's, because some combinations of input variable do not occur.
- ✓ Also for some functions the outputs corresponding to certain combinations of input variables do not matter.

K-map and don’t care conditions

- ✓ In such situations we have a freedom to assume a 0 or 1 as output for each of these combinations.
- ✓ These conditions are known as the “Don’t Care Conditions” and in the K-map it is represented as ‘X’, in the corresponding cell.
- ✓ The don’t care conditions may be assumed to be 0 or 1 as per the need for simplification

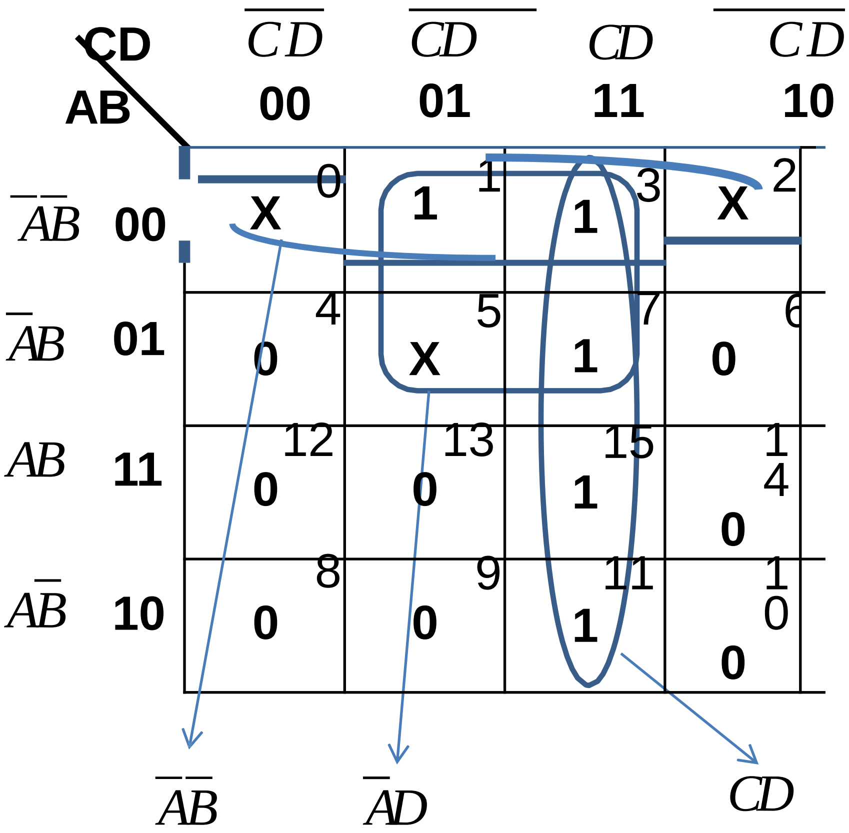
K-map and don’t care conditions - Example

Simplify ;

$$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$$

K-map and don’t care conditions - Example

$$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$$



Simplified Boolean expression

$$f = CD + \overline{A}\,\overline{B} + \overline{A}D$$