

Image Segmentation

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Image Segmentation

- ❑ An image processing method where input is an image and the output are attributes extracted from the image.

Definitions

- Partitions the image into connected subsets that maximize some “uniformity” criteria
- Identify possibly overlapping but maximal connected subsets that satisfy some uniformity criterion

Features

- ❑ Sub-divides images into its constituent regions or objects.
- ❑ Should stop when the objects of interest have been isolated.
- ❑ Determines the eventual success or failure of computerized analysis procedures.

Properties of Intensity Values

□ Discontinuity

Partitions an image based on abrupt changes in intensity; eg. edges.

□ Similarity

Partitions an image into regions that are similar according to a set of predefined criterion; eg. thresholding, region-growing etc.

Detection of Discontinuities

□ 3 basic type of discontinuities:

-Points

-Lines

-Edges

□ Run a mask

□ Response of the mask $R = \sum_{i=1}^9 w_i z_i$

Point Detection

- A point is detected at the location on which mask is centered if,

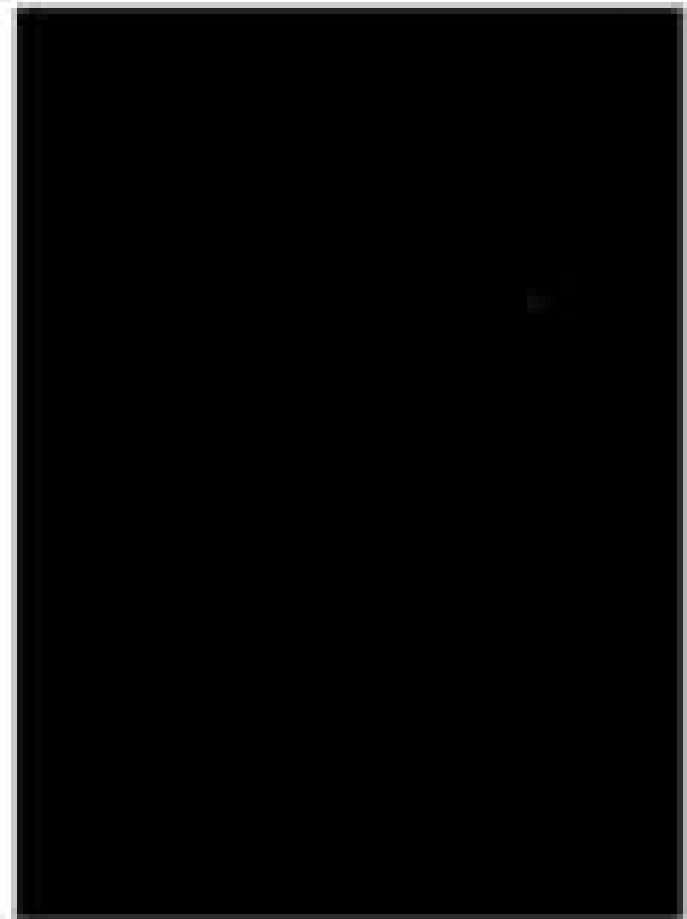
$$|R| \geq T$$

where T = non-negative threshold

- Weighted difference between the center point and its neighbours is measured.

-1	-1	-1
-1	8	-1
-1	-1	-1

Result of Point Detection



Line Detection

-1	-1	-1
2	2	2
-1	-1	-1

Horizontal

-1	-1	2
-1	2	-1
2	-1	-1

+45

-1	2	-1
-1	2	-1
-1	2	-1

Vertical

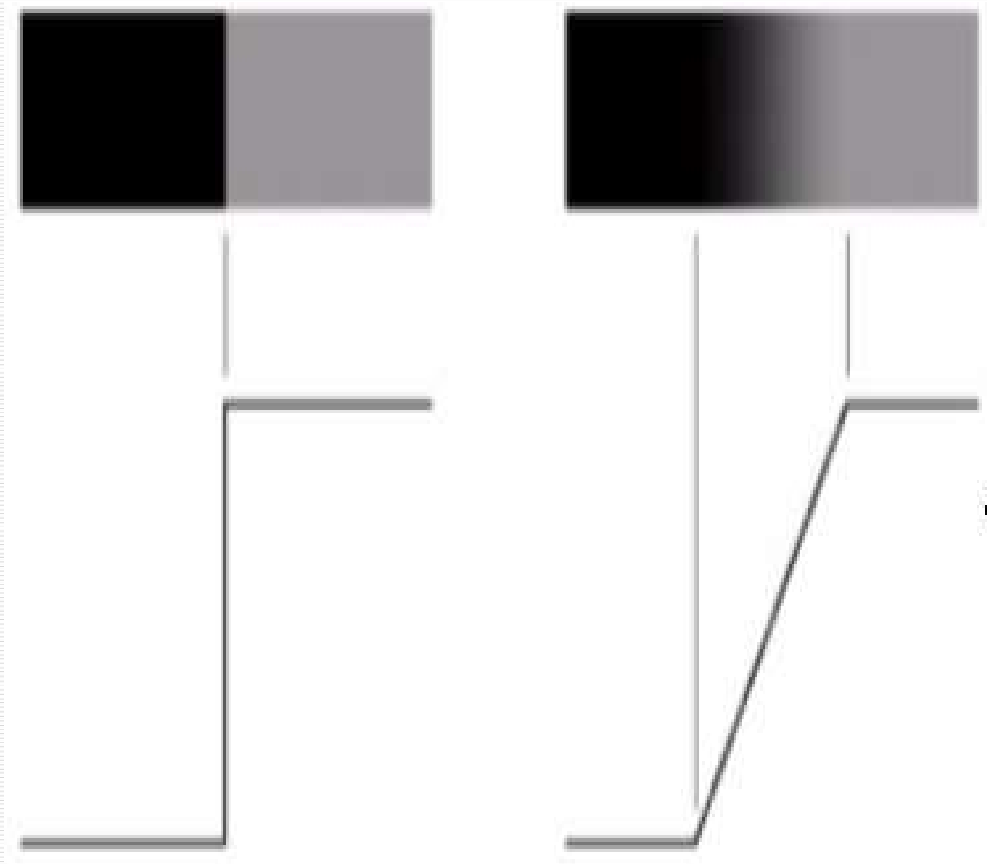
2	-1	-1
-1	2	-1
-1	-1	2

-45



Edge Detection

- Ability to measure gray level transitions.
- **Ideal Edge**: 1 pixel thick path
- **Ramp Edge**:
 - Blurred
 - Slope of ramp is proportional to the degree of blurring in the edge
 - Edge point is any point contained in the ramp
 - Thickness of the edge = length of the ramp(determined by the slope)



Blurred edges tend to be **thick!!**

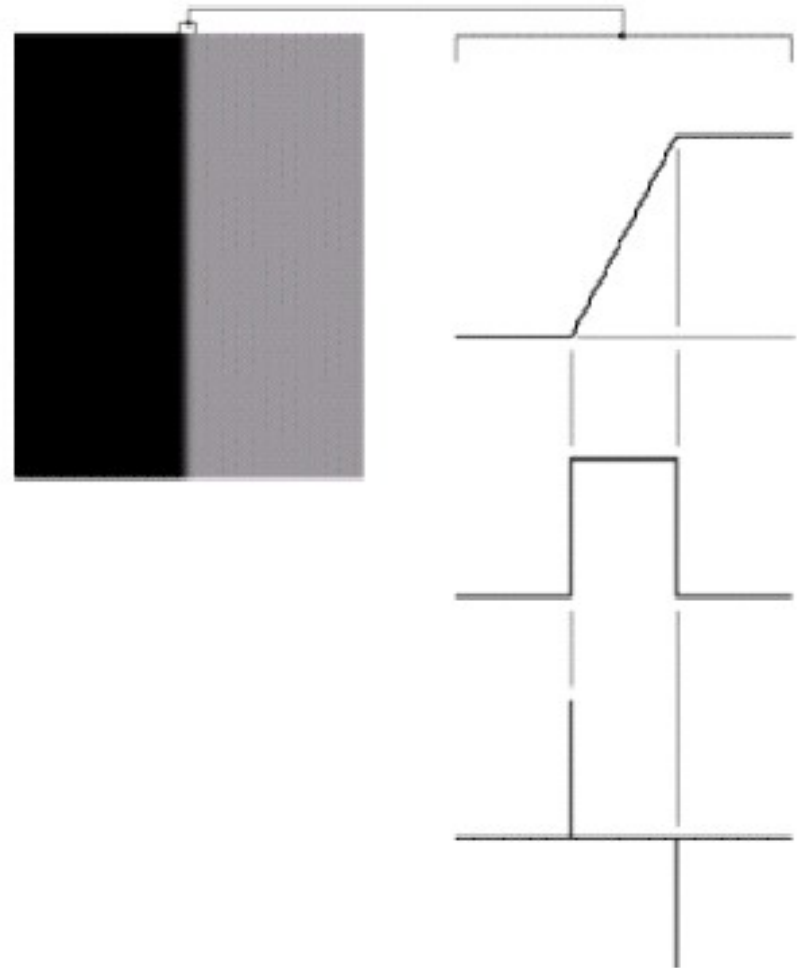
Ramp Profile

□ 1st Derivative:

- Positive at points of transition into and out of ramp.
- Constant for points in ramp
- Zero in constant gray area

□ 2nd Derivative

- Positive with transition associated with dark side of edge
- Zero along constant gray and ramp
- Negative at light side



Signs of derivative would reverse for an edge transiting from light to dark!!

Gradient Operators

- Gradient vector points in direction of maximum rate of change of “f” at (x,y).
- Magnitude = $(G_x^2 + G_y^2)^{1/2}$
- Direction = $\tan^{-1}(G_y/G_x)$
- Direction of an edge is perpendicular to the direction of the gradient vector

Gradient Operators

- ☐ Roberts
- ☐ Prewitt
- ☐ Sobel

\mathcal{E}_1	\mathcal{E}_2	\mathcal{E}_3
\mathcal{E}_4	\mathcal{E}_5	\mathcal{E}_6
\mathcal{E}_7	\mathcal{E}_{10}	\mathcal{E}_9

-1	0	0	-1
0	1	1	0

Roberts

-1	-1	-1	-1	0	1
0	0	0	-1	0	1
1	1	1	-1	0	1

Prewitt

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

Sobel

Robert's

- $G_x = (Z_9 - Z_5)$ $G_y = (Z_8 - Z_6)$
- 2X2 masks are awkward to implement.

Prewitt's

- $G_x = (Z_7 + Z_8 + Z_9) - (Z_1 + Z_2 + Z_3)$
- $G_y = (Z_3 + Z_6 + Z_9) - (Z_1 + Z_4 + Z_7)$
- Much simpler to implement.

Sobel's

- $G_x = (Z_7 + 2Z_8 + Z_9) - (Z_1 + 2Z_2 + Z_3)$
- $G_y = (Z_3 + 2Z_6 + Z_9) - (Z_1 + 2Z_4 + Z_7)$
- Superior noise suppression.

□ All gradients give zero response in constant gray-level areas.

□ $\nabla f = |G_x| + |G_y|$

Illustration of the Gradient and its Components



Original Image



Gx component of
Gradient



Gy component of
gradient



Gradient image
 $|G_x| + |G_y|$

The Laplacian

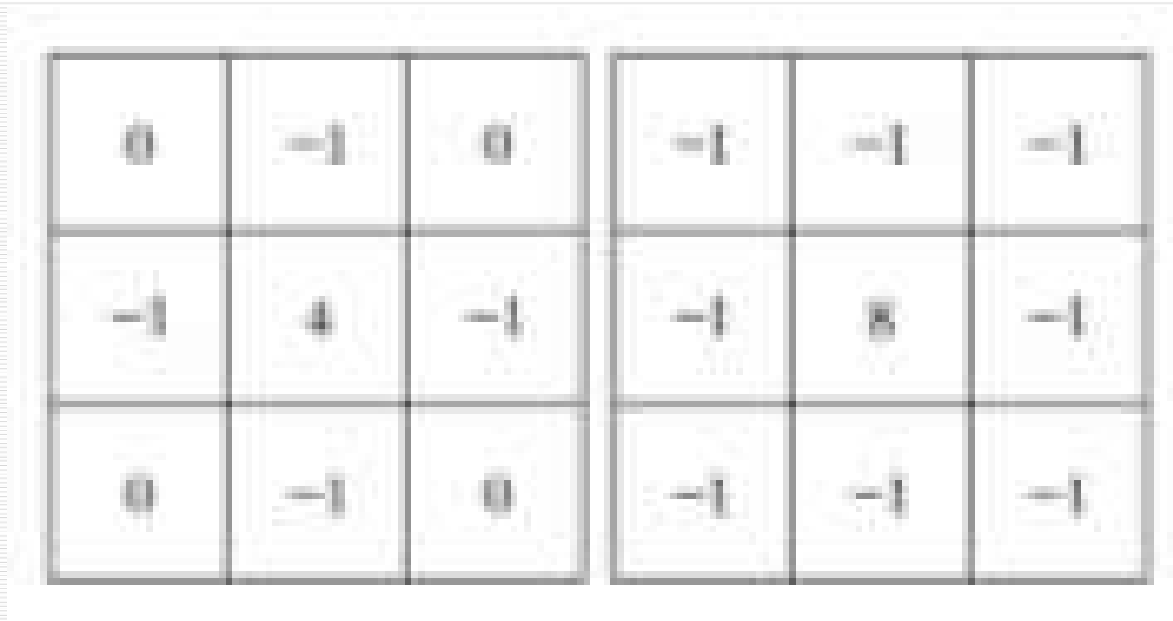
□ Defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

□ Laplacian is generally not used in its original form.

Laplacian Masks

Digital approximation to the Laplacian:



0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

Not used in its Original Form

- ❑ Highly sensitive to noise (2nd order derivative)
- ❑ Magnitude of Laplacian produces double edges (complicates segmentation)
- ❑ Unable to detect edge direction

Role of Laplacian

- ❑ Zero crossing property used for edge location.
- ❑ To establish if a pixel is on dark or light side of an edge.
- ❑ Magnitude of the first derivative can be used to detect the presence of an edge at a point in an image.
- ❑ Sign of 2nd derivative used to determine whether edge pixel is on light or dark side.

Additional Properties of 2nd Derivative

- Produces 2 values for every edge.
- An imaginary straight line joining extreme positive and negative values, crosses "0" at mid-points of an edge.
- Zero crossing property used for detecting centre of thick edges.

Edge Linking & Boundary Detection

□ Local Processing

□ Global Processing

Local Processing

Establish similarity of edge pixels by analysing:

- Strength of the response of the gradient operator
- Direction of the gradient operator

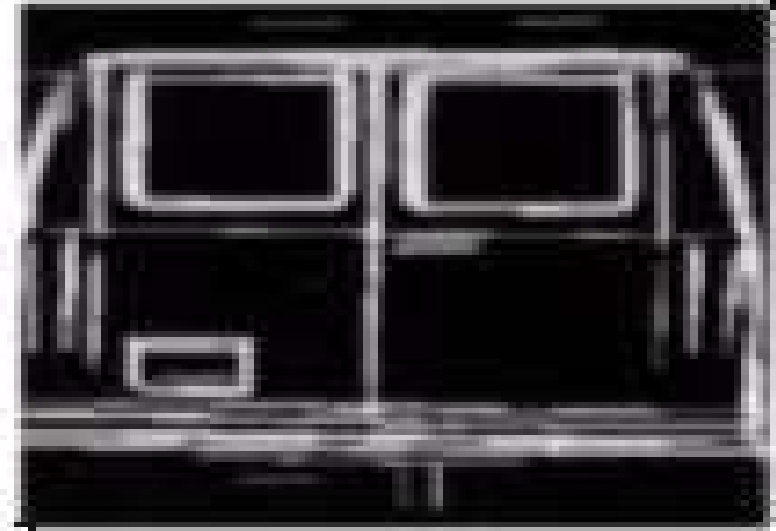
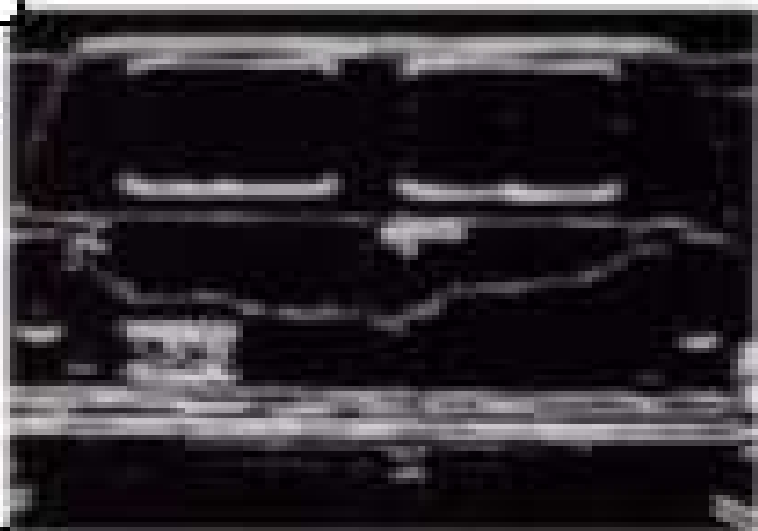
$$\left| \underset{\text{threshold}}{\nabla f(x,y)} - \nabla f(x_0, y_0) \right| \leq E, \quad E = \text{non-negative}$$

$$\left| \underset{\text{threshold}}{a(x,y)} - a(x_0, y_0) \right| \leq A, \quad A = \text{non-negative}$$

Local Processing.....

- A point in the predefined neighbourhood of (x,y) is linked to the pixel if both magnitude and direction criteria are specified.
- Process is repeated at every location of the image

Edge Linking

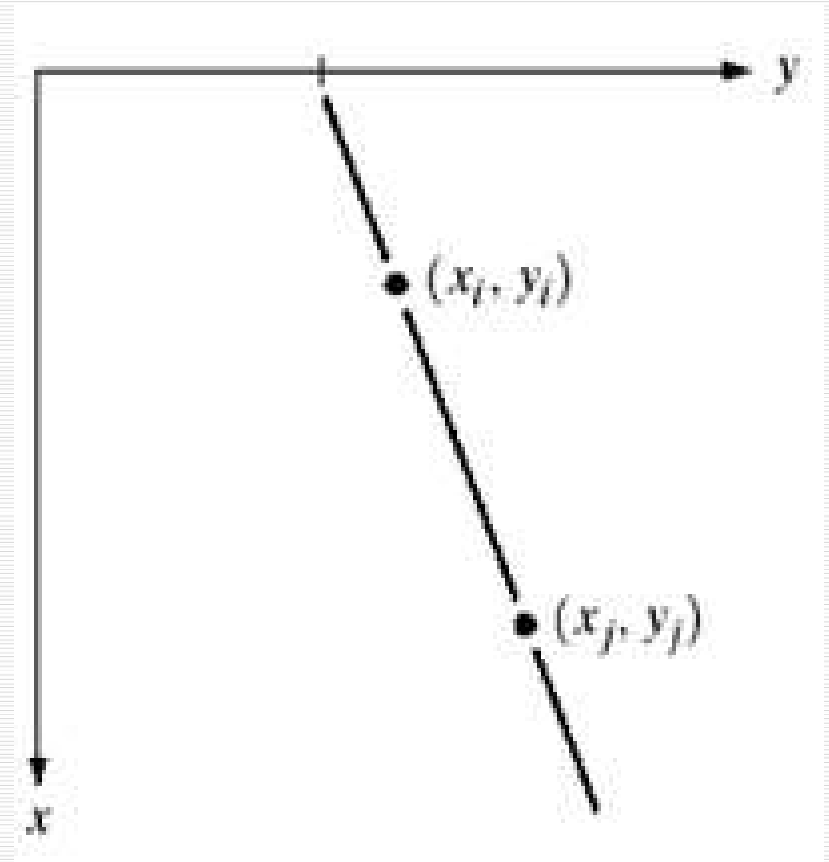


Global Processing

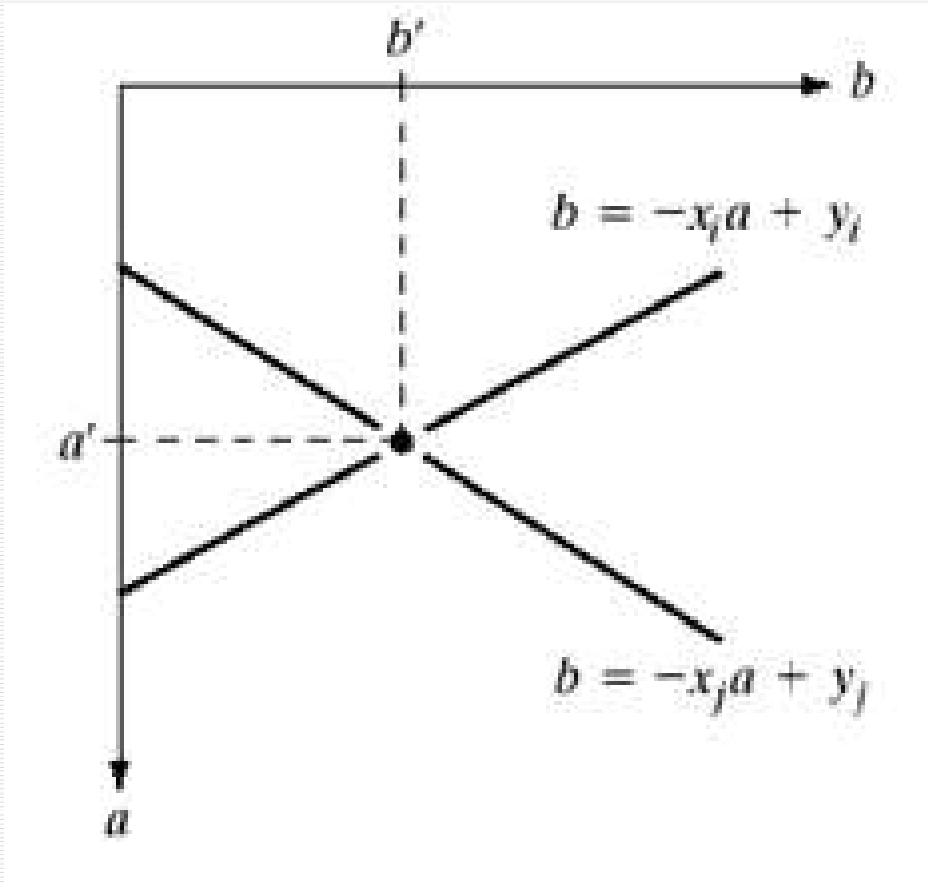
- ❑ Points are linked by determining first if they lie on a curve of specified shape.
- ❑ Global relationship between pixels is considered
- ❑ Hough Transform, Graph Theoretic Technique

Hough Transform

- Consider a point (x_i, y_i) and the general equation of a line in slope-intercept form,
$$y_i = ax_i + b$$
- All lines passing through this (x_i, y_i) satisfy $y_i = ax_i + b$, for varying values of a and b



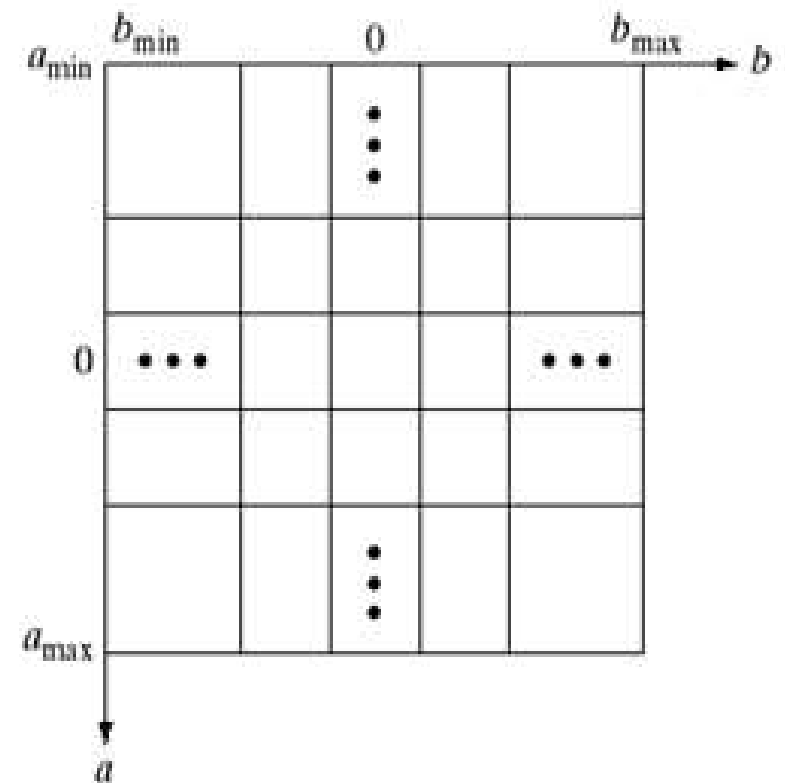
Parameter Space



- Writing the eqn as
$$b = -x_i a + y_i$$
and considering a-b plane, yields eqn of a single line for a fixed pair of (x_i, y_i)
- (x_j, y_j) also has a line in parameter space.
- The 2 lines intersect at (a', b')

Accumulator

- Subdivide the parameter space into accumulator cells
- (a_{\max}, a_{\min}) & (b_{\max}, b_{\min}) are expected range of slope and intercept values



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- The cell at co-ordinate (i,j) with accumulator value $A_{(i,j)}$ corresponds to the square associated with parameter space co-ordinates (a_i, b_j)
 - Initially, set cells to zero
 - At every point (x_k, y_k) in the image let parameter "a" equal each of the allowed sub-division values on the a-axis and solve for corresponding "b", using $b = -x_k a + y_k$

-
- Round off resulting b 's
 - If a_p leads to b_q , we let $A_{(p,q)} = A_{(p,q)} + 1$
 - At the end of the procedure, value of Q in $A_{(i,j)}$ corresponds to Q points in the x - y plane lying on the line
$$y = a_i x + b_j$$
 - No. of sub-divisions in the a - b plane determines the accuracy of collinearity of these points

Problem with using $y = ax + b$

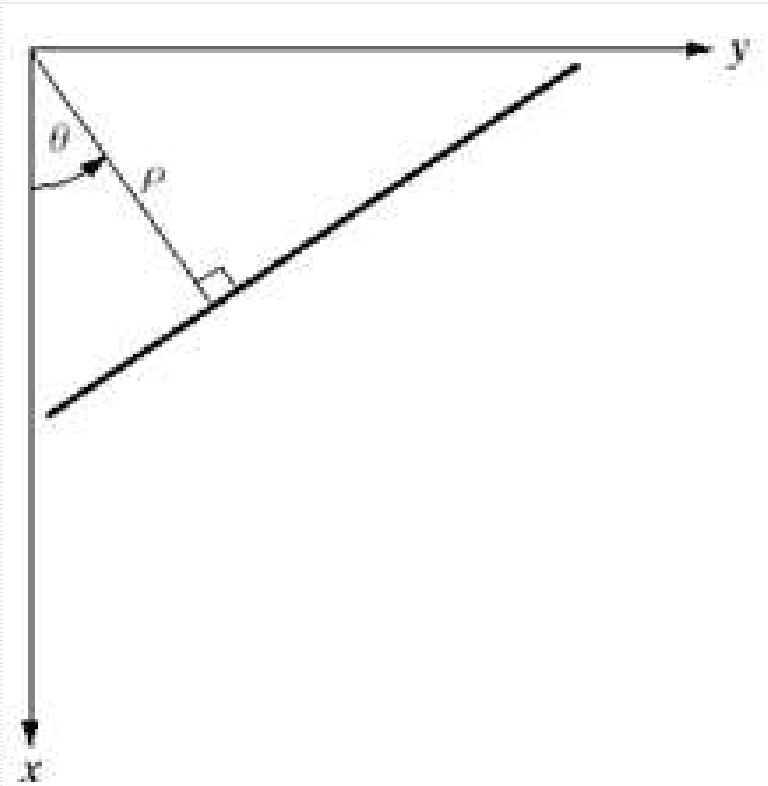
□ Slope approaches infinity as the line approaches vertical

□ Solution:

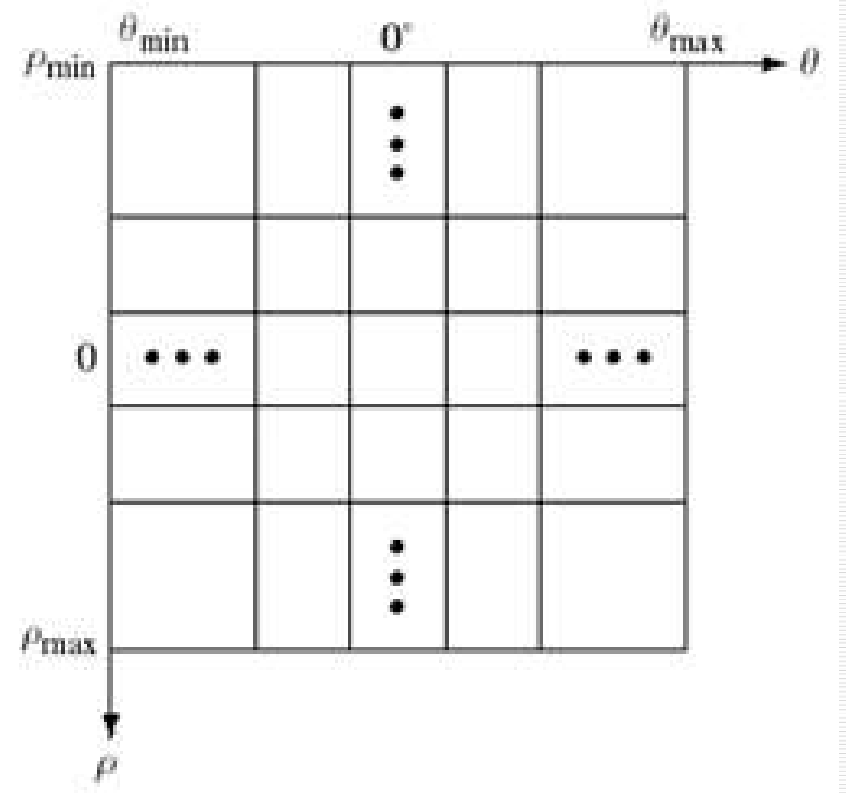
Use normal representation of a line:

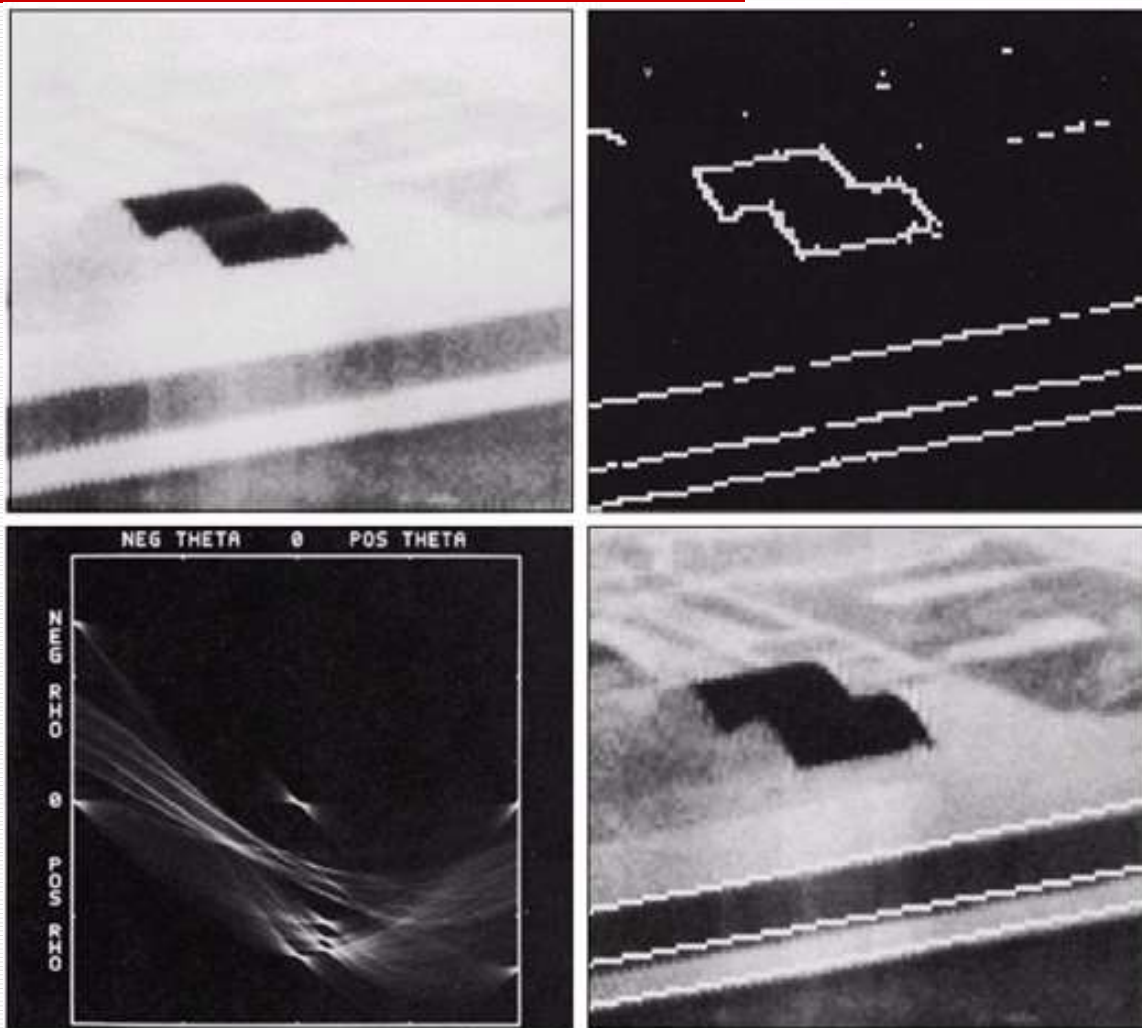
$$x \cos \theta + y \sin \theta = \rho$$

Representation of a line in ρ - θ plane



Subdivision of ρ - θ plane into cells



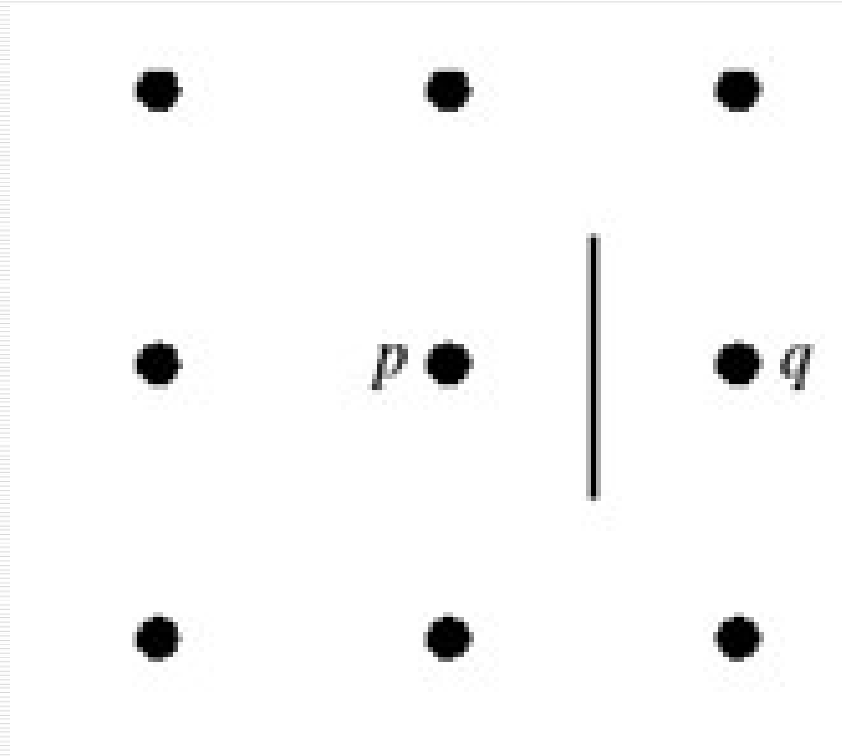


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Graph Theoretic Technique

- ❑ Global approach for edge detection and linking
 - ❑ Based on representing edge segments in the form of a graph
 - ❑ Searching the graph for low cost paths
 - ❑ Rugged approach-performs well in the presence of noise
 - ❑ Complicated procedure
 - ❑ Increased processing time
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Edge Element between Pixels p and q



Cost










- Each edge element defined by pixel p and q has an associated cost:

$$C(p,q) = H - [f(p) - f(q)]$$

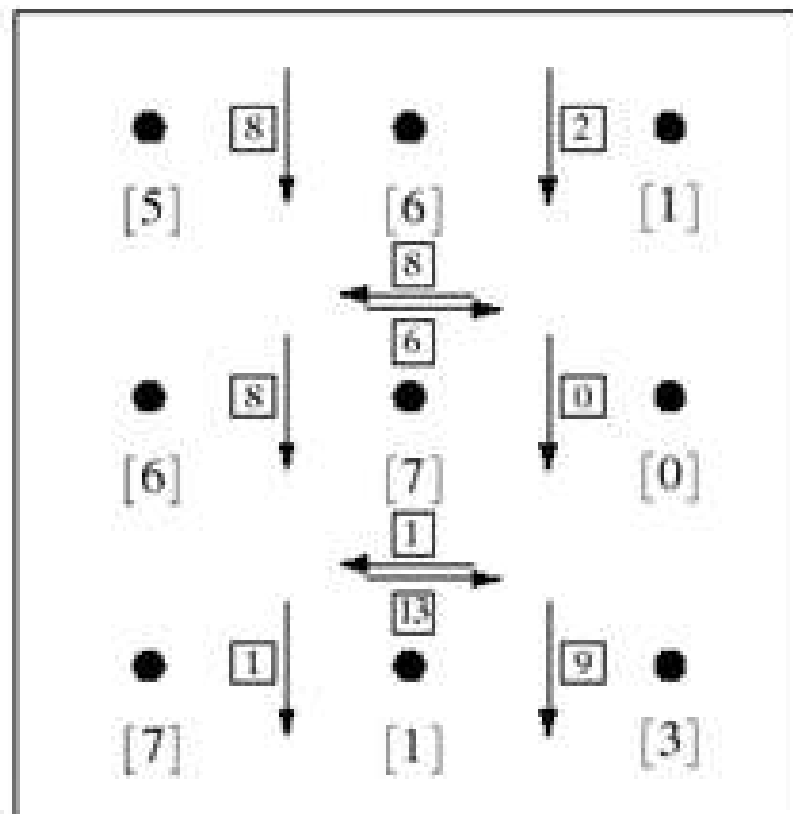
H= Highest gray level value in the image

f(p), f(q)= gray level values of p & q

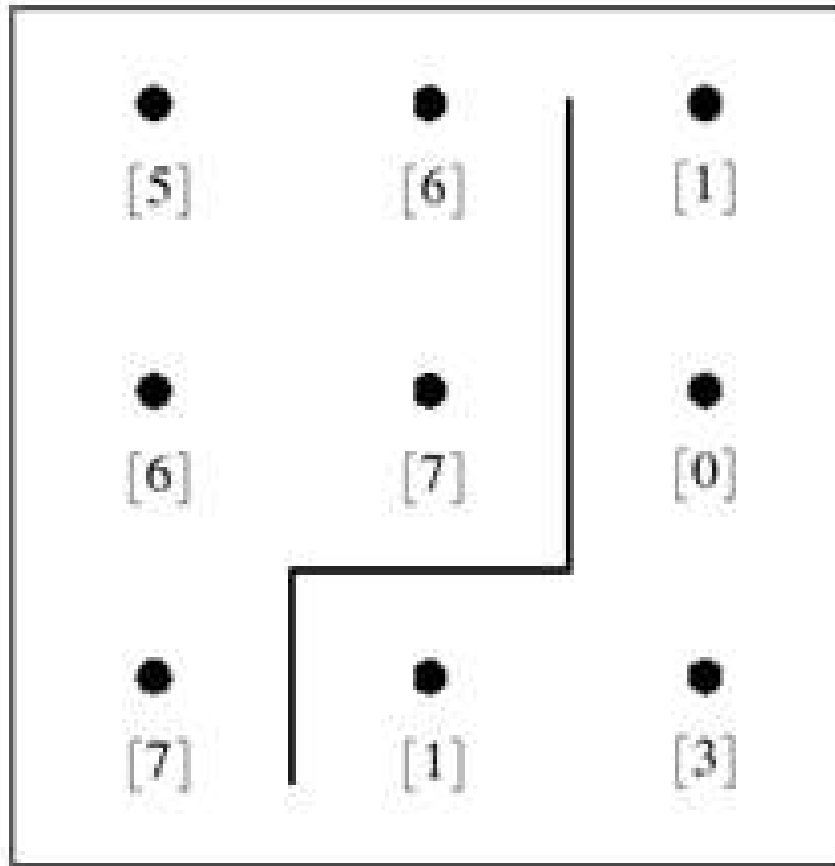
- p and q are 4 neighbours
- Point p is on the right hand side of the direction of travel along edge elements
- Edge starts in the top row and terminates in the last row (supposedly)

	1	2	3
1	 [5]	 [6]	 [1]
2	 [6]	 [7]	 [0]
3	 [7]	 [1]	 [3]

□ A 3X3 image region



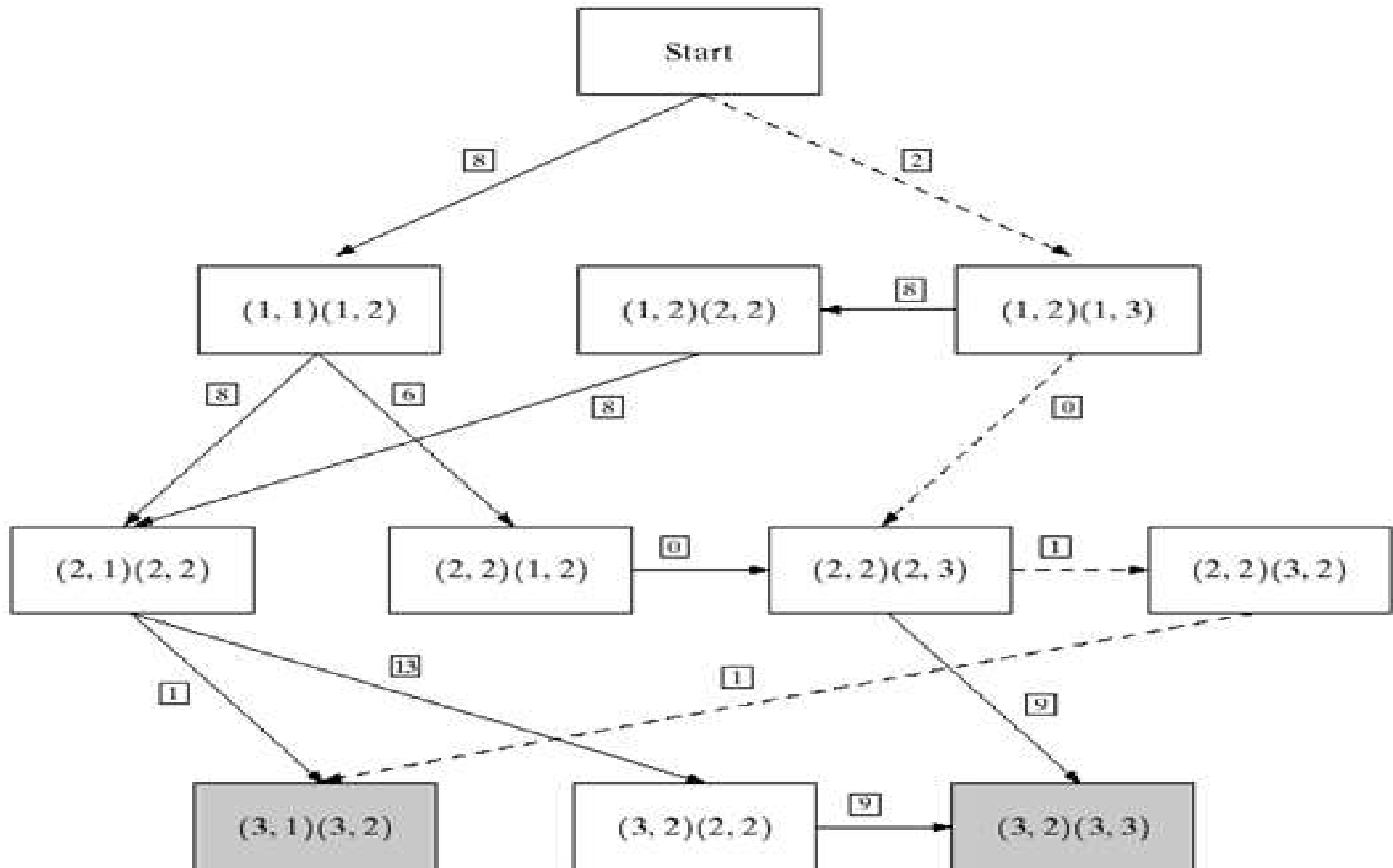
□ Edge segments and their costs



□ Edge corresponding to the lowest cost path

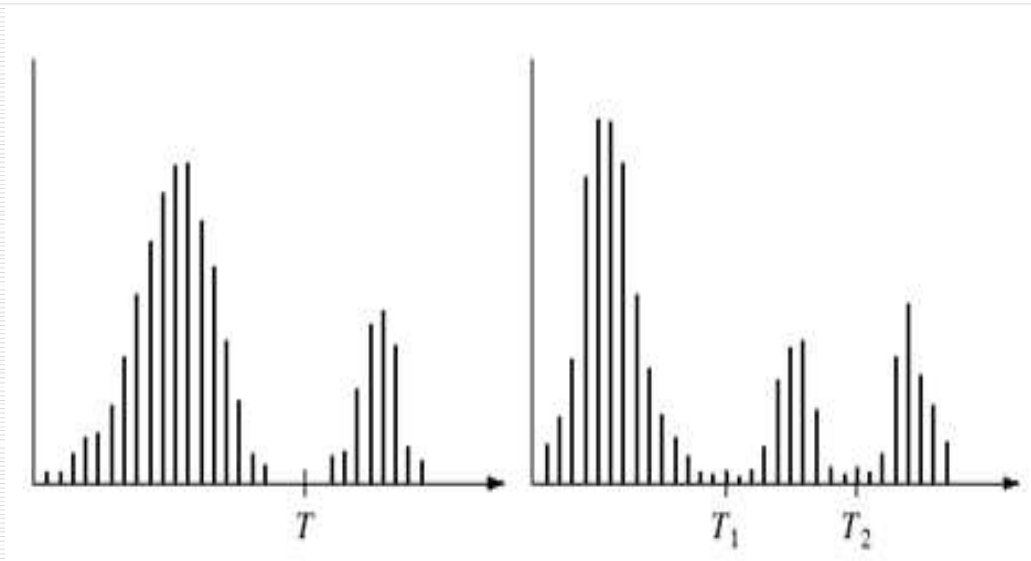
GRAPH:

Lowest cost path is shown dashed.....



Thresholding

- ❑ Objects and background have gray levels grouped into two dominant modes
- ❑ Preferred because of simplicity of implementation



Single Threshold

Multiple Threshold

□ Single

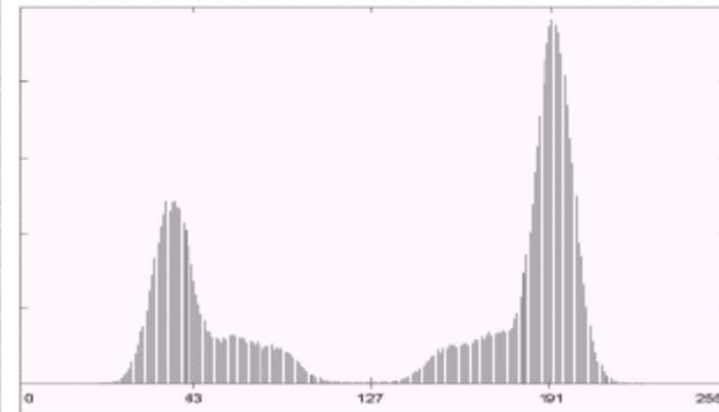
$f(x,y) > T$ Object
else Background

□ Multiple

$T_1 < f(x,y) \leq T_2$ _{object1}

$F(x,y) > T_2$ _{object2}

$F(x,y) \leq T_1$ _{background}



(a) Original image. (b) Image histogram. (c) Result of segmentation with the threshold estimated by iteration. (Original courtesy of the National Institute of Standards and Technology.)



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- Thresholding is viewed as an operation that involves test against a function T of the form,

$$T = T [x, y, p(x, y), f(x, y)]$$

- $f(x, y)$ = gray level of pt. (x, y)
- $P(x, y)$ = some local property of this point
- A threshold image $g(x, y)$ is defined as,

$$\begin{aligned} g(x, y) &= 1 && \text{if } f(x, y) > T \\ &= 0 && \text{if } f(x, y) \leq T \end{aligned}$$

Types

□ Global Threshold:

T depends only on $f(x,y)$

□ Local Threshold:

T depends on both $f(x,y)$ & $p(x,y)$

□ Dynamic or Adaptive Threshold:

T depends on above factors plus spatial co-ordinates.

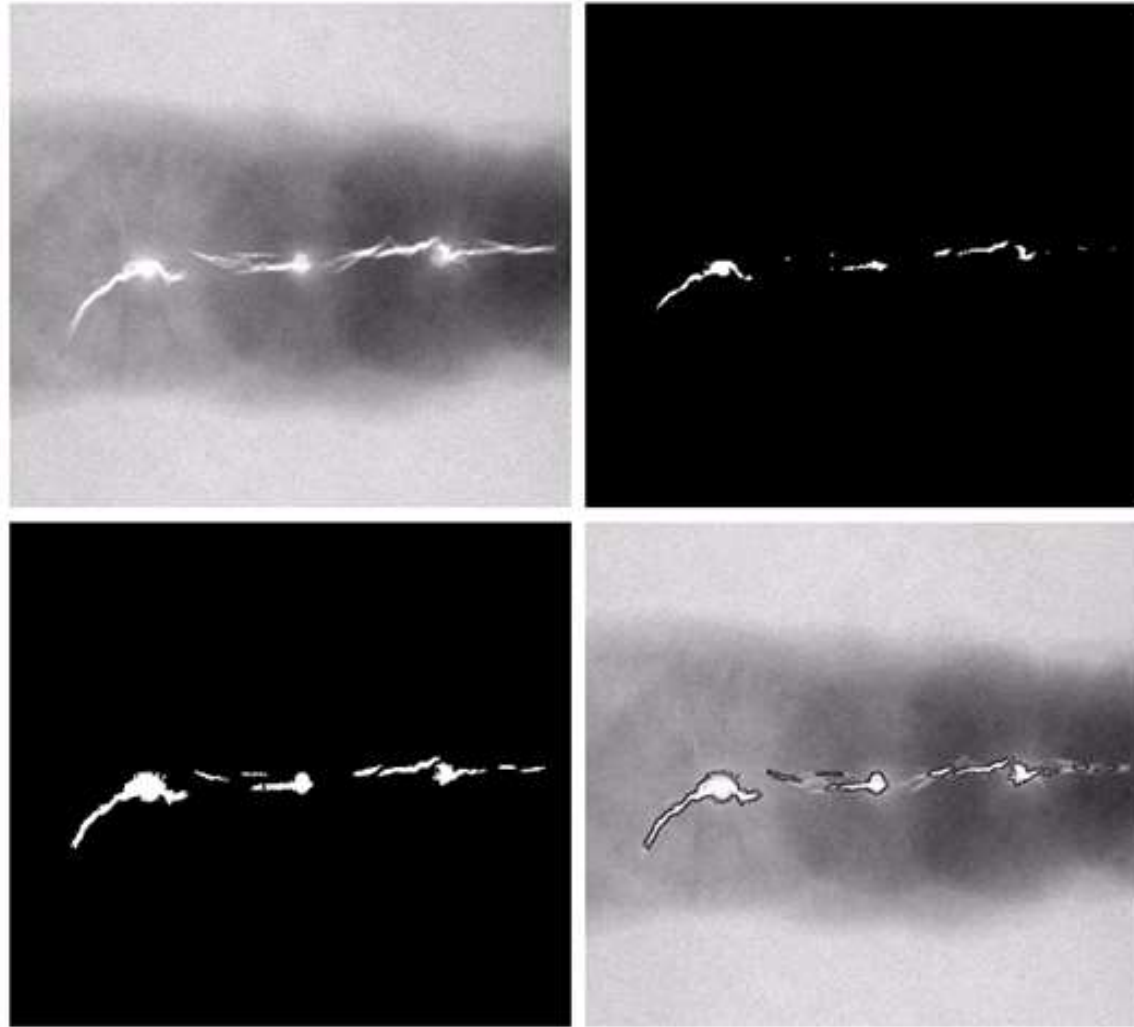
Region-Based Segmentation

- Region Growing
- Region Splitting & Merging

Region Growing

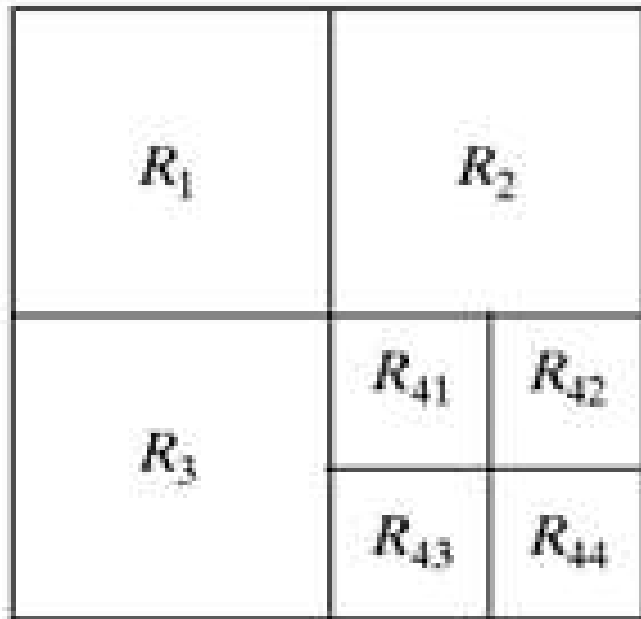
- ❑ Groups pixels or sub-regions into larger regions based on pre-defined criteria
- ❑ Start with a set of “seed” points and grow regions
- ❑ Selection of similarity criteria depends on problem under consideration and type of image data available.
- ❑ Stop when no more pixels satisfy the criteria

Eg. of Region Growing

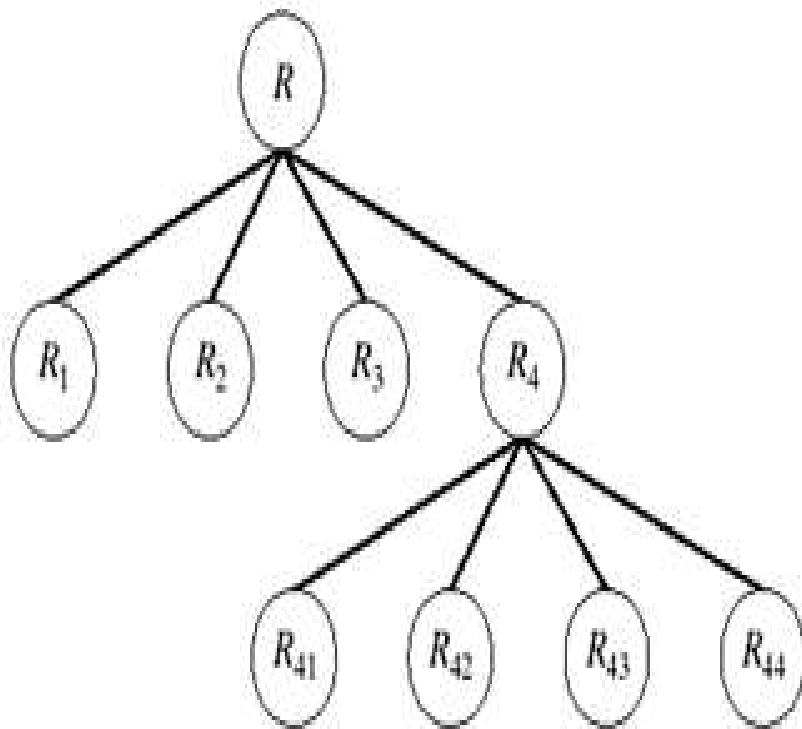


Region Splitting & Merging

- Sub-divide an image into a set of arbitrary disjointed regions and then merge or split the regions to satisfy conditions.



- R = Entire image region
- P = Predicate
- For any region R_i ,
 $P(R_i) = \text{TRUE}$
- If $P(R_i) = \text{FALSE}$, divide the image into quadrants.
- If P is false for any quadrant, divide the quadrant into subquadrants, and so on.
- Quadtree (root of the tree is the entire image).



- ❑ Root corresponds to entire image.
- ❑ Each node corresponds to a subdivision.
- ❑ Two adjacent regions R_j and R_k are merged only if $P(R_j \cup R_k) = \text{TRUE}$

Quad-Tree Segmentation

- Split R_i into 4 disjoint quadrants for which $P(R_i) = \text{FALSE}$.
- Merge R_j and R_k for which $P(R_j \cup R_k) = \text{TRUE}$
- Stop when no further splitting or merging is possible.

Questions

- ❑ Explain with suitable example region splitting and merging technique for image segmentation. (10 marks)
 - ❑ Develop an algorithm for converting one pixel thick 8 connected path to 4-connected path. (10 marks)
 - ❑ Write a short note on: Hough Transform. (4/10 marks)
 - ❑ Justify Hough transform used to detect the curves. (5 marks)
 - ❑ Write a short note on Edge Linking and Boundary detection via graph theoretic Technique. (5 marks)
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- Assume that the edge starts in the first row and ends in the last row for the following gray image :

5	6	1
6	7	0
7	1	3

- Sketch all possible paths and determine the edge corresponding to minimum cost path. (10 marks)
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- ☐ Write a short note on: Split and merge. (5 marks)
 - ☐ Explain basic principles of detecting following in the images. (i) Points (ii) Lines (iii) Edges. Give 3×3 mask for each of them and explain their operation. (10 marks)
 - ☐ Explain the method of segmentation of images by Region Splitting and merging. (10 marks)
 - ☐ Justify/contradict following statement: Image resulting from poor illumination cannot be segmented easily. (4 marks)
 - ☐ Justify/contradict the following statement: Segmentation algorithm for monochrome image generally based on 2 basic properties of grey level values.
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- What is an edge? Give typical edge profiles and convolution masks for Roberts, Laplace and Prewitt edge detectors. (10 marks)
- Assuming that edge starts in the first row and ends in the last row. For following gray level image, sketch all possible paths and determine edge corresponding to minimum cost path

2	2	7
2	7	5
0	1	5