Image Morphology

By

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Basic Morphological Methods: Erosion, Dilation, Opening, Closing, Hit-or-Miss Transformation.

Advanced Morphological Methods: Skeletonization, Thinning, Thickening, Pruning, Boundary Extraction.

Notation From Set Theory

Let A and B be sets.

To indicate a is an element of A write $a \in A$.

To indicate a is not an element of A write $a \notin A$.

A is a subset of B, written $A \subseteq B$, if for every $a \in A$, $a \in B$.

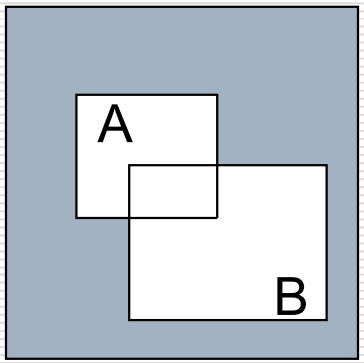
The union of A and B, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

The intersection of A and B, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

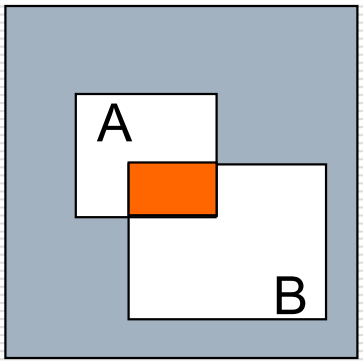
The complement of A, $A^c = \{x \mid x \notin A\}$.

The difference of A and B, $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

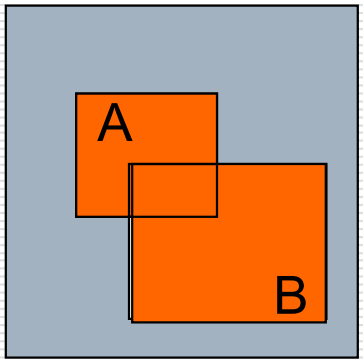
Examples



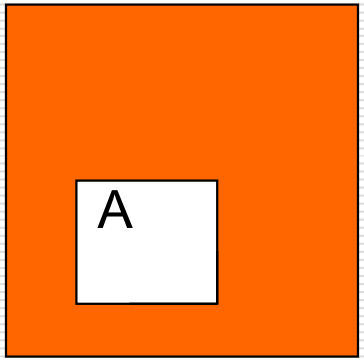
$A \cap B$



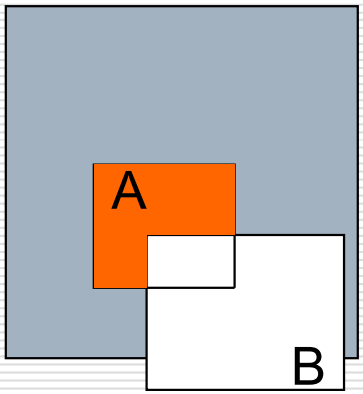
$A \cup B$



A^c



$A - B = A \cap B^c$



The Cross-product

Let A and B be sets.

The cross - product of A and B, written A x B, is given by

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

Example

If Z represents the set of integers, then $Z^2 = \{(z_1, z_2) \mid z_1, z_2 \in Z\}$.

Morphology

- Deals with forms and structures.
- Mathematical morphology is a tool for extracting image components useful in:
  - Representation and description of region shape
  - Pre or post processing(filtering, thinning etc).
- Based on set theory.
- Some morphological operations are:

  Dilation & Erosion, Opening & Closing, Hit-or-Miss Transform, Basic Algorithms

- Mathematical operations are used for the following purpose:

  - Image pre-processing(noise filtering, shape simplification)
  - Enhancing object structure (skeletonising, thinning, thickening, convex hull, object marking)
  - Segmenting objects from the background
  - Quantitaive description of object(area, perimeter etc).

Binary Dilation & Erosion

- Sets of black and white pixels constitute a description of a binary image.
- Assume that only black pixels are considered

Dilation

- Let A and B be 2 subsets in 2 dimensional space Z^2 representing two object regions containing elements {a} and {b}, where

  a = {a1, a2} & b= {b1, b2}

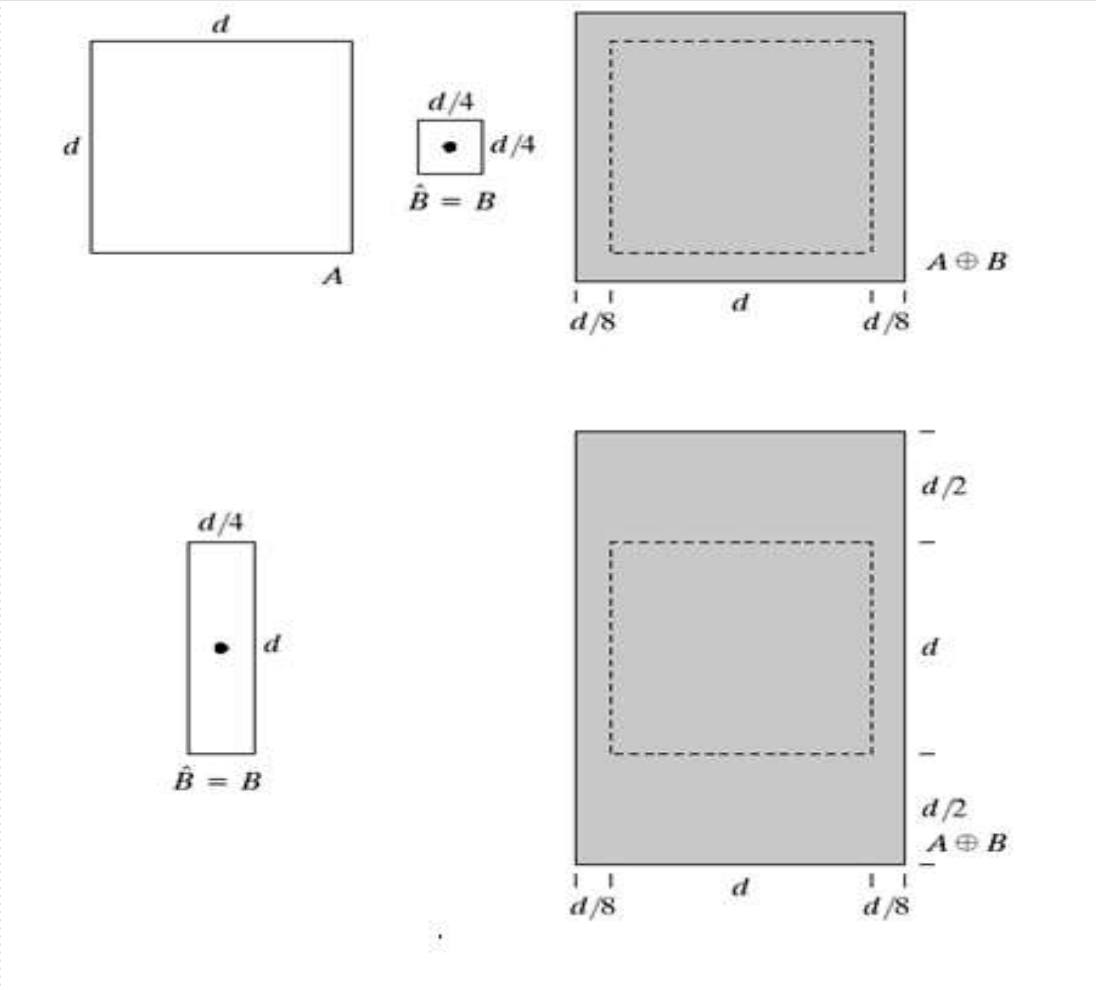
- The dilation of A by B is defined as

  $A \oplus B=\{p \in Z^2;\ p=a+b,\ a \in A,\ b \in B\}$

  where p= a+b={(a1+b1), (a2+b2)}

- Dilation operation expands an object to the closest pixels of the neighborhood.
- It has the effect of filling in the valleys between the spiky region edges.

Examples of Dilation



A =

1	0	0
1	0	0
0	1	1
0	1	0
0	1	0

Let the structuring element B= | 1 | 1 |

A= {(0,0), (1,0), (2,1), (2,2), (3,1), (4,1)}

B={(0,0) , (0,1)}

$A \oplus B$={(0,0), (1,0), (2,1), (2,2), (3,1), (4,1),
(0,1), (1,1),(2,2),(2,3), (3,2), (4,2)}

A ⊕ B =

	0	1	2	3
0	1	1	0	0
1	1	1	0	0
2	0	1	1	1
3	0	1	1	0
4	0	1	1	0

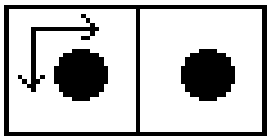
Original =

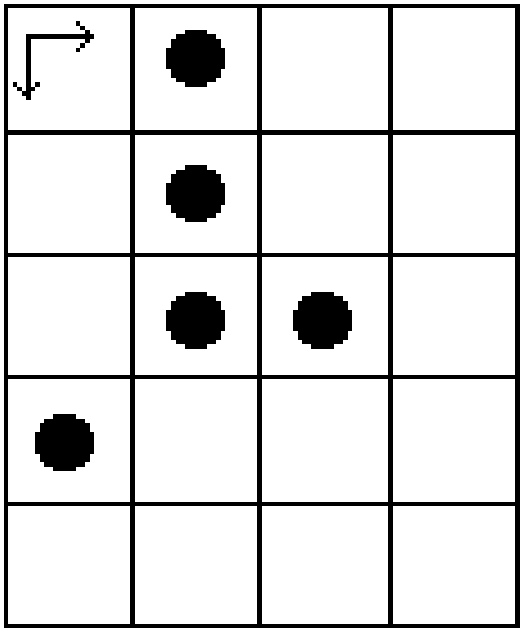
1	0	0
1	0	0
0	1	1
0	1	0
0	1	0

A = { (0,1), (1,1), (2,1), (2,2), (3,0) }

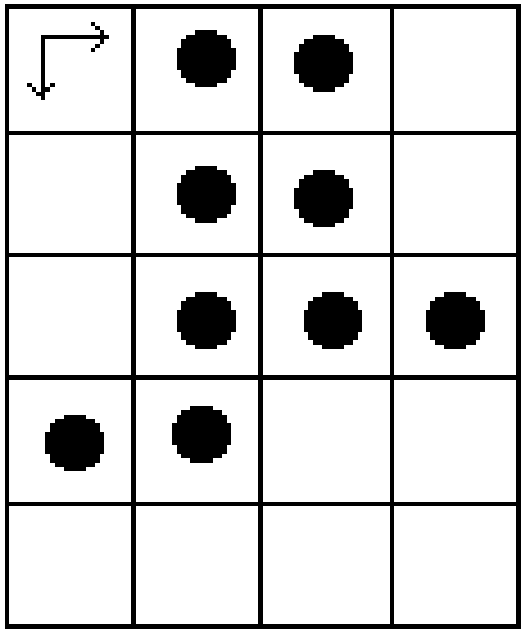
B = { (0,0), (0,1) }

B :

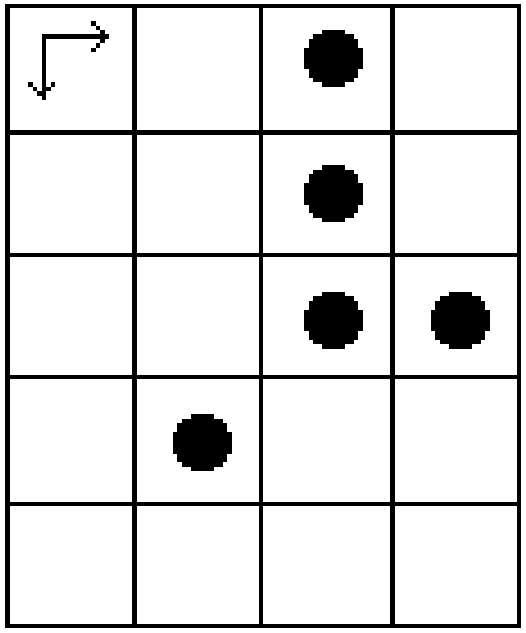




A



A ⊕ B



$A_{(0,1)}$

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

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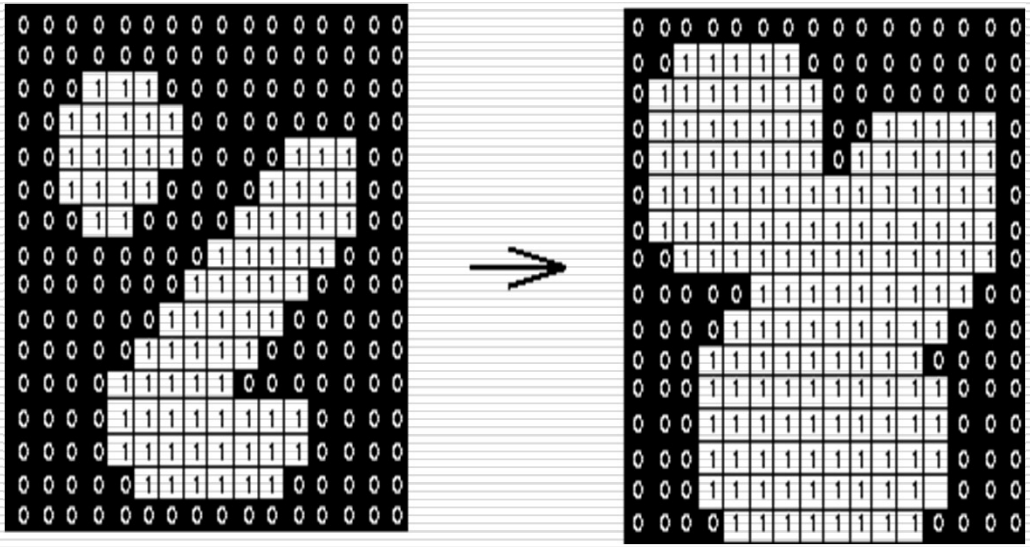
a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0



example : dilation using a 3 by 3 square structuring element for gap bridging.

- The basic effect of the operator on a binary image is to gradually enlarge the boundaries of regions of foreground pixels (i.e. white pixels, typically).
- Thus areas of foreground pixels grow in size while holes within those regions become smaller.

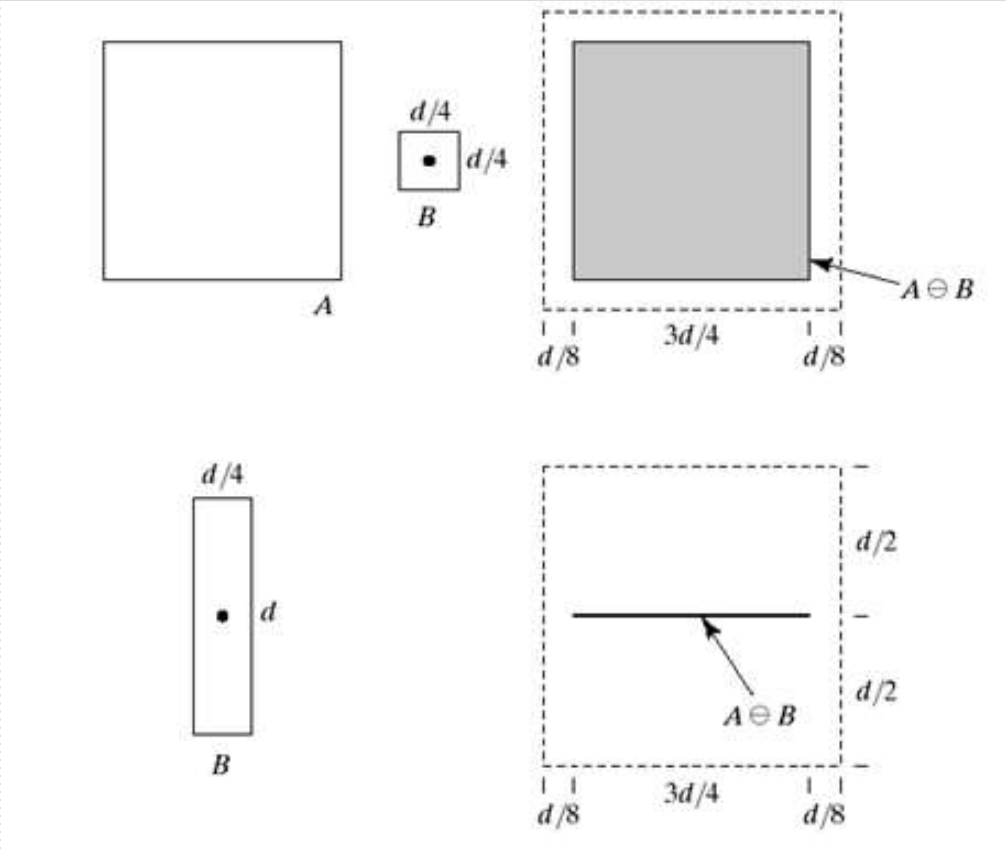
Erosion

- Let A and B be 2 subsets in 2 dimensional space Z^2 representing two object regions containing elements {a} and {b}.
- The erosion of A by B is defined as

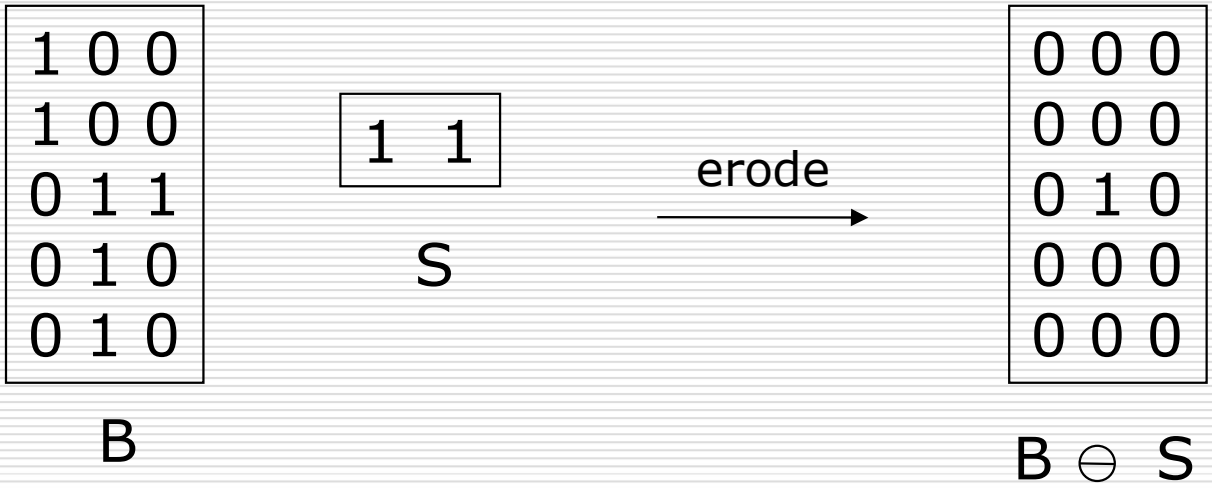
  $A \ominus B = \{P \in Z^2 : p+b \in A, b \in B\}$

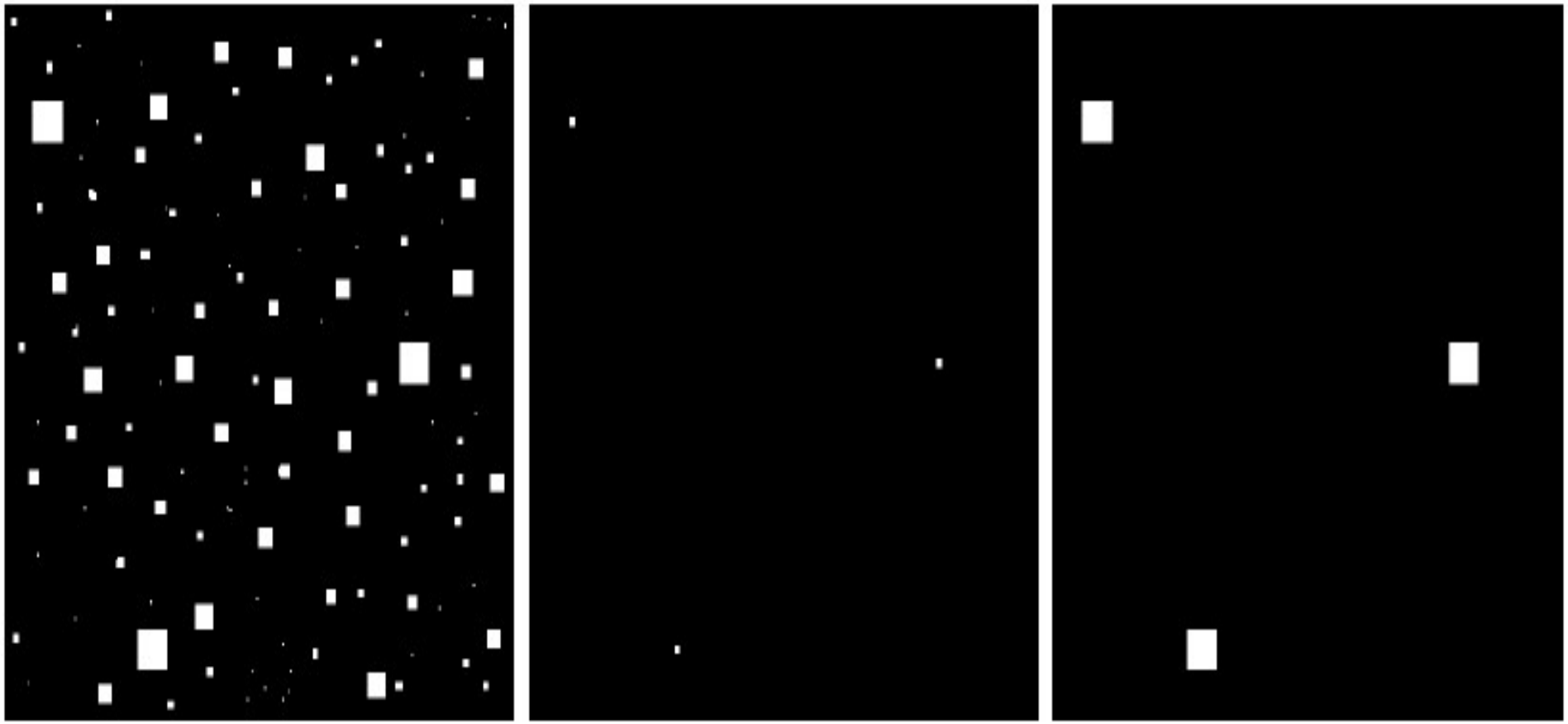
  i.e. every pt. p from the image is tested and the result of erosion is given by those point p for which all possible p+b are in A.

Examples of Erosion



Erosion





a b c

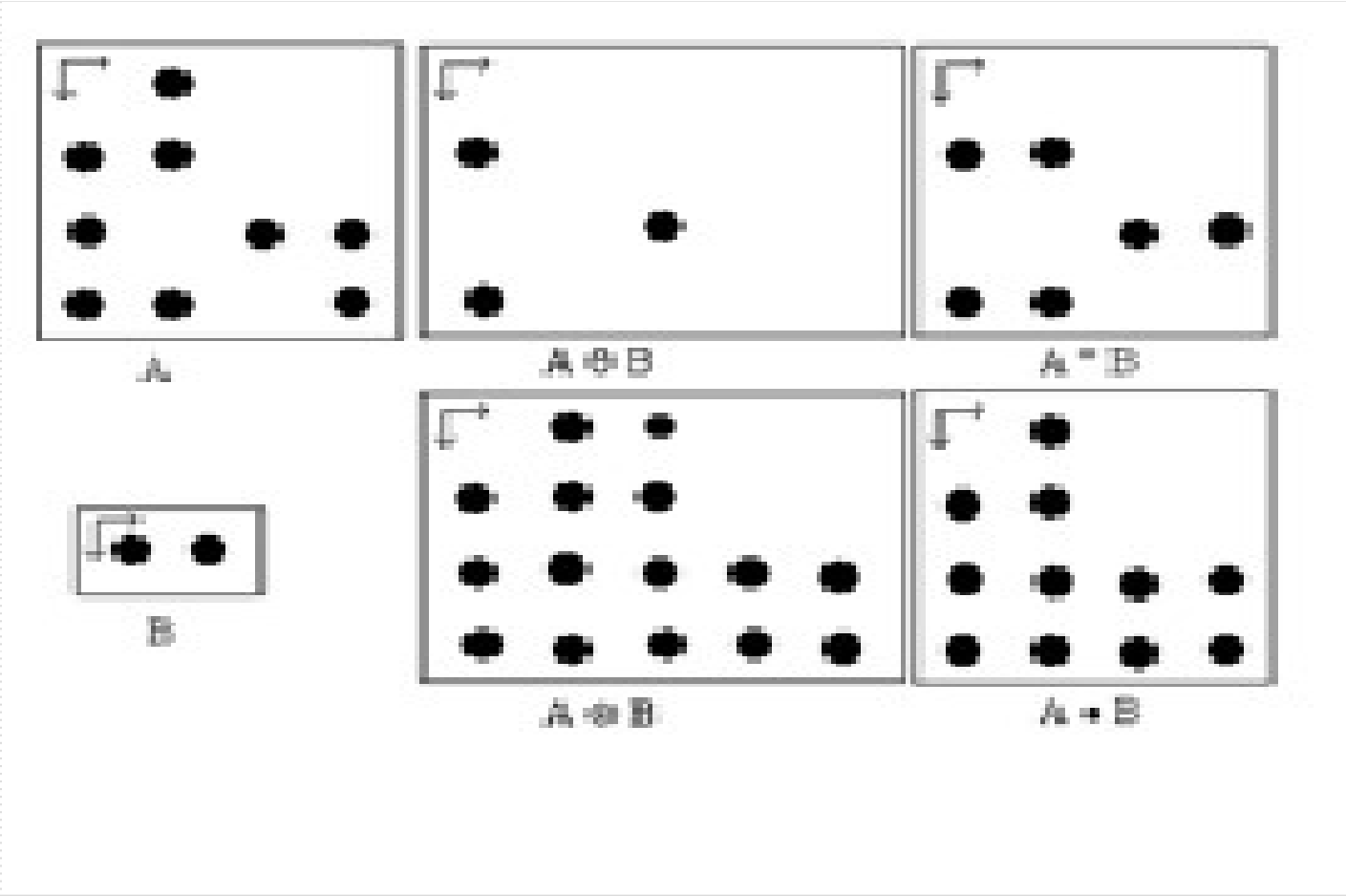
FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Closing and opening

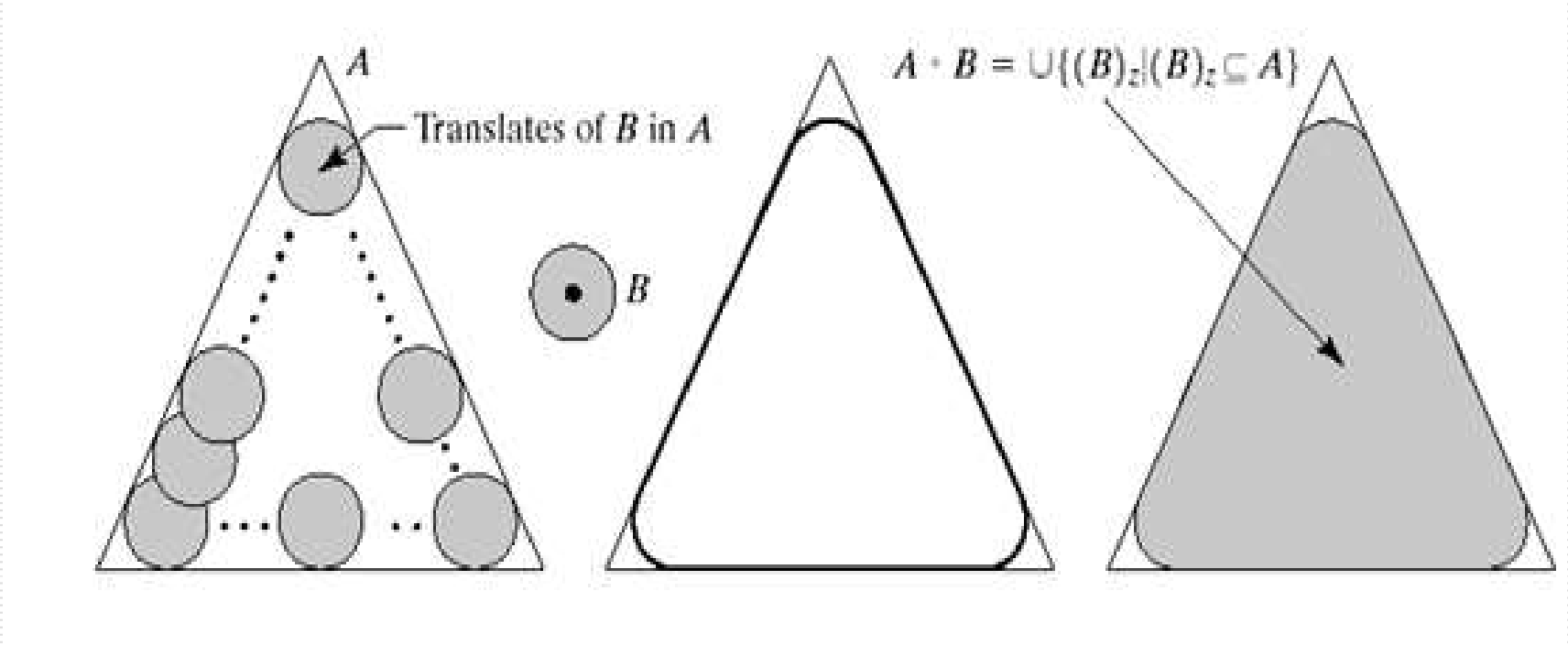
- Closing is a dilation followed by an erosion
- Smoothes sections of contours.
- Fuses narrow breaks
- Fills gaps in the contours
- Eliminates small holes.

- Opening is an erosion followed by dilation.
- Opening smoothes the contours of objects
- Breaks narrow isthmuses
- Eliminates thin protrusions.

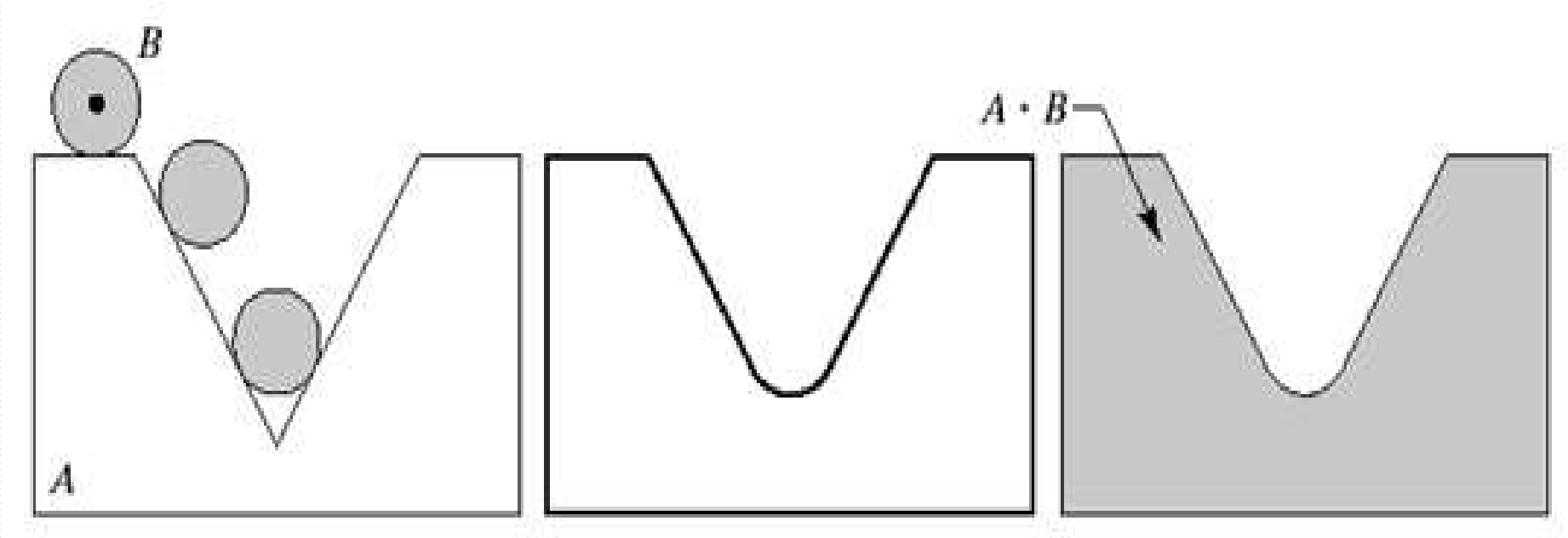
Opening & Closing



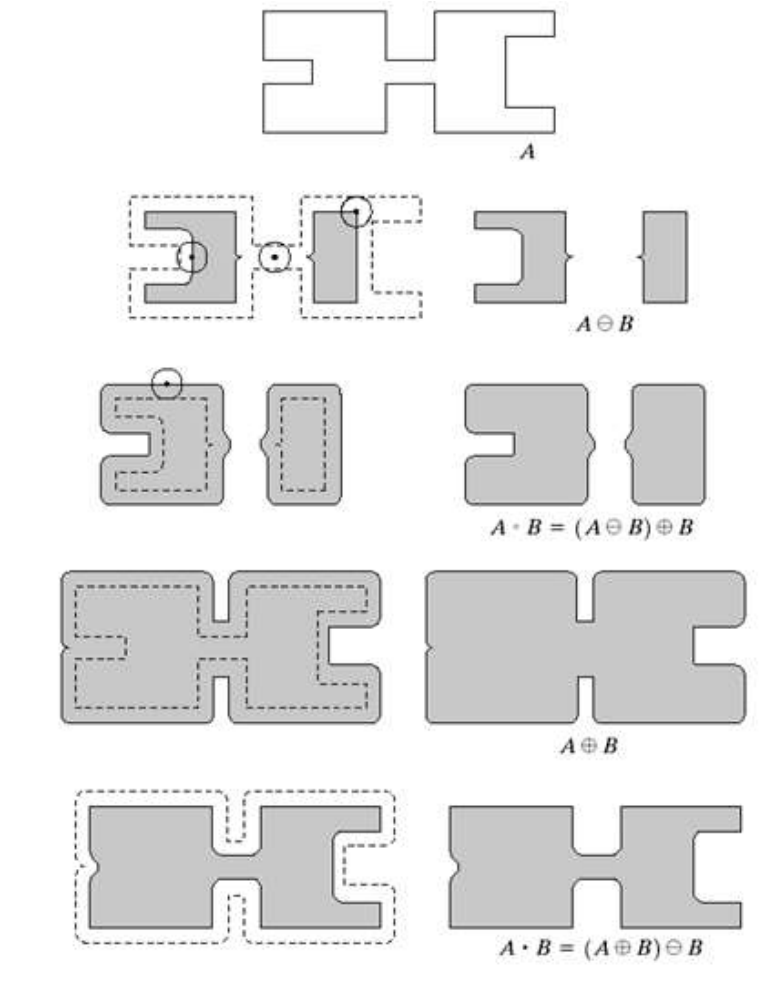
Opening [A∘B]



Closing [A•B]



Application of Opening & Closing



Hit or Miss Transform

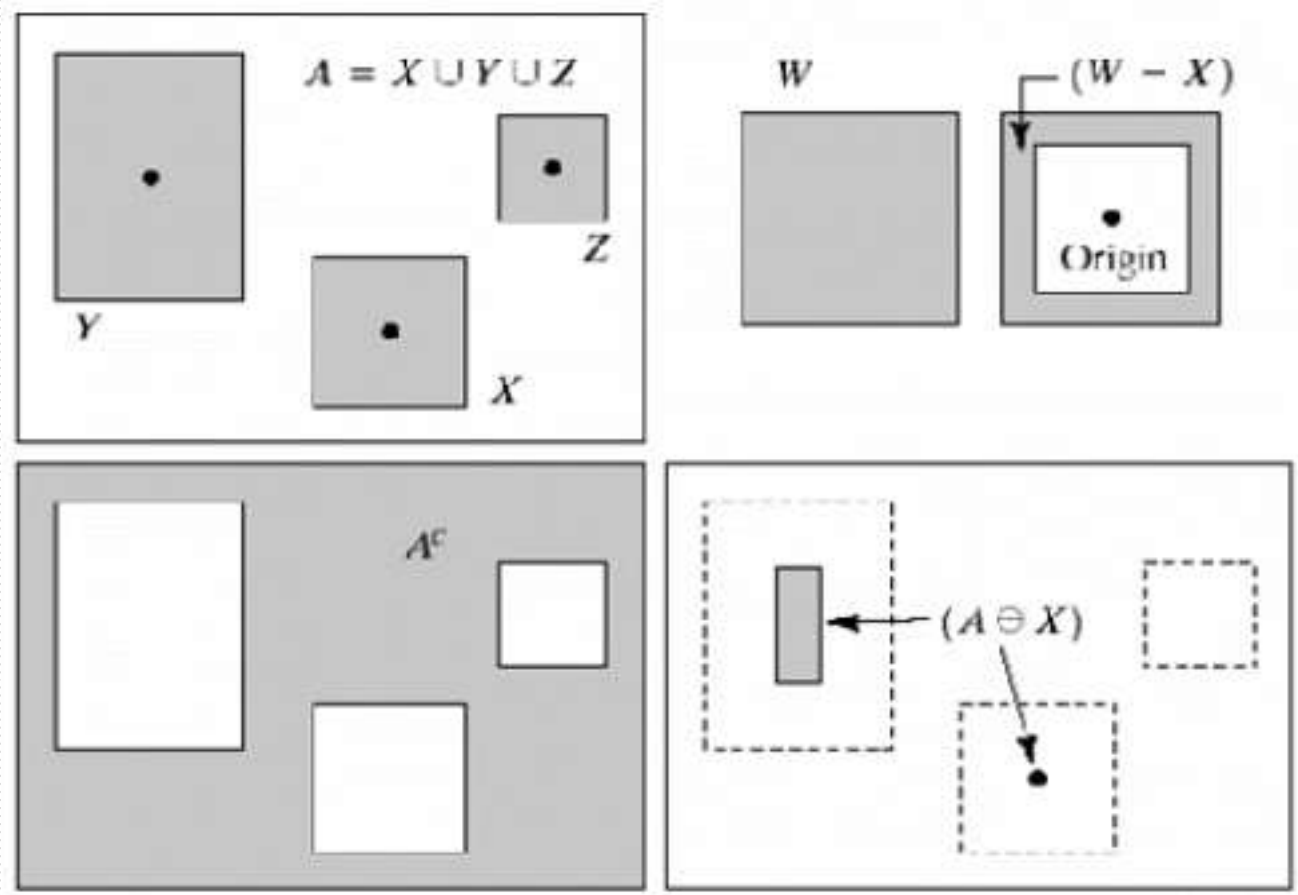
- Basic tool for shape detection.
- Used to look for particular patterns of foreground and background pixels in an image.

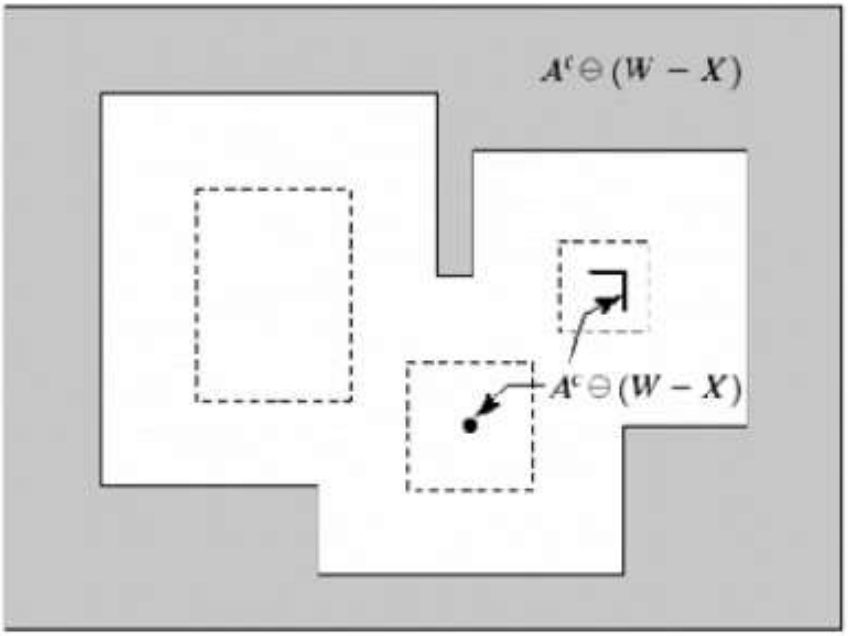
- Uses 2 structuring elements to identify structures which are specific to the fore ground and background.

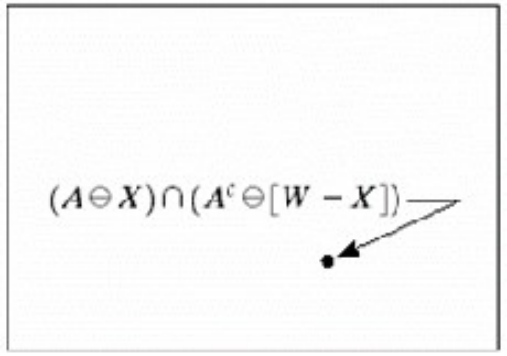
- Basic operation of binary morphology
- Other binary morphological operators can be derived from it.
- Input and output both are binary images.

Steps:

- We have a binary image that contains certain shapes (circles, squares, lines, etc.) called image A.
- We use another image or matrix to search image A for a particular pattern of bits - call this pattern 'shape B'.
- We then search image A for shape B.
- Whenever there is a 'hit', we indicate where the center of shape B was on image A.


A = X ∪ Y ∪ Z
Y
Z
X
W
(W − X)
Origin
A^c
(A ⊖ X)


$A^c \ominus (W - X)$
$A^c \ominus (W - X)$


$(A \ominus X) \cap (A^c \ominus [W - X])$

Objective

- To find a disjoint region (set) in an image

- B denotes the set composed of X and its background

- the match/hit (or set of matches/hits) of B in A, is

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

- Generalized notation:

  B = (B1,B2)

B1:Set formed from elements of B associated with an object

B2:Set formed from elements of B associated with the corresponding background

- B1 = X and B2 = (W − X)
- More general definition:

$$A \circledast B = (A \ominus B_1) \cap [A^c \ominus B_2]$$

- $A \circledast B$ contains all the origin points at which, simultaneously, B1 found a hit in A and B2 found a hit in A^c

- A background is necessary to detect disjoint sets
- When we only aim to detect certain patterns within a set, a background is not required, and simple erosion is sufficient

Some Basic Morphological Algorithms

Skeletonization

- To reduce all objects in an image to lines
- Essential structure of the image retained

Method used in image processing to extract a simplified representation of an object while preserving its structural properties.

Stages of Skeletonization

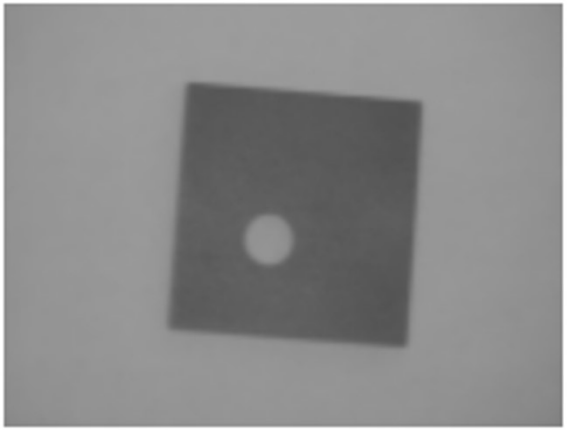
1. **Thinning**:

- Pimary step
- Iteratively erodes the foreground pixels until the object is reduced to a one-pixel-wide skeleton
- Process continues until no further erosion can occur without breaking the connectivity of the object.

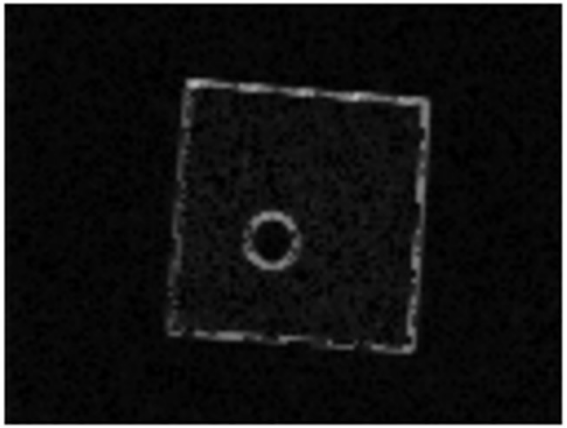
2. **Pruning**:

- Removes extraneous branches or noise.
- Obtains a cleaner and more accurate representation of the object's structure

Skeletonization



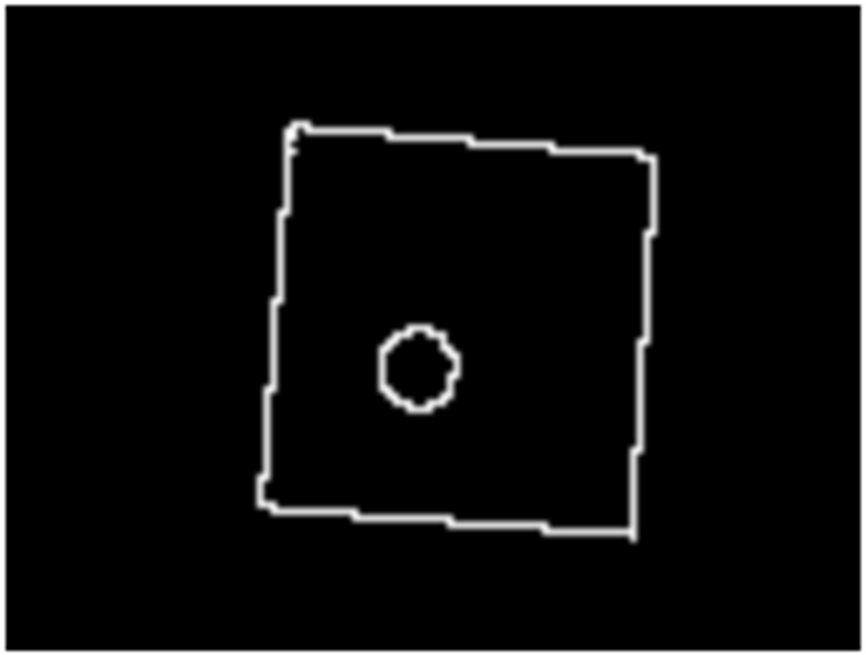
Original Image

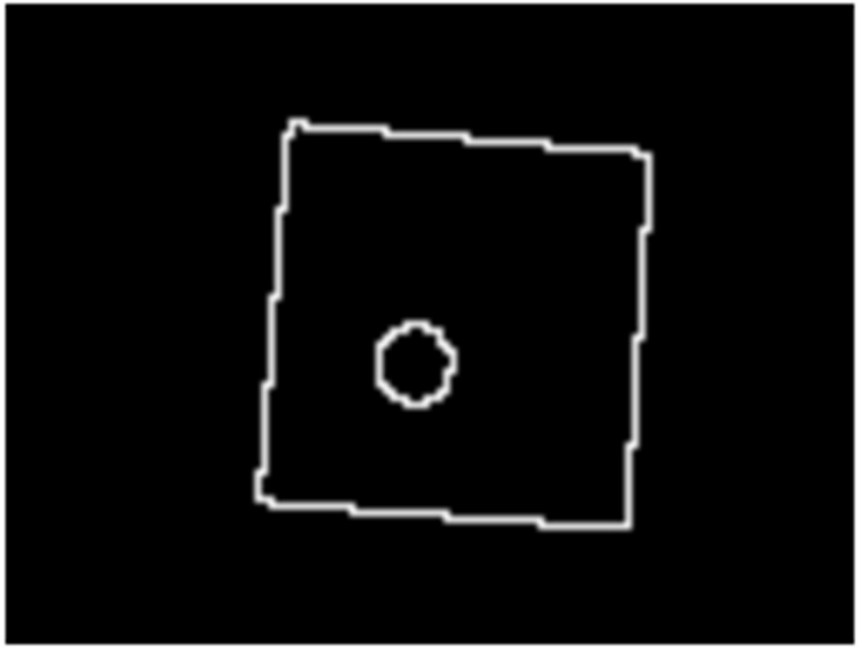


Edge Detection Output

Thresholding

Thinning & Pruning



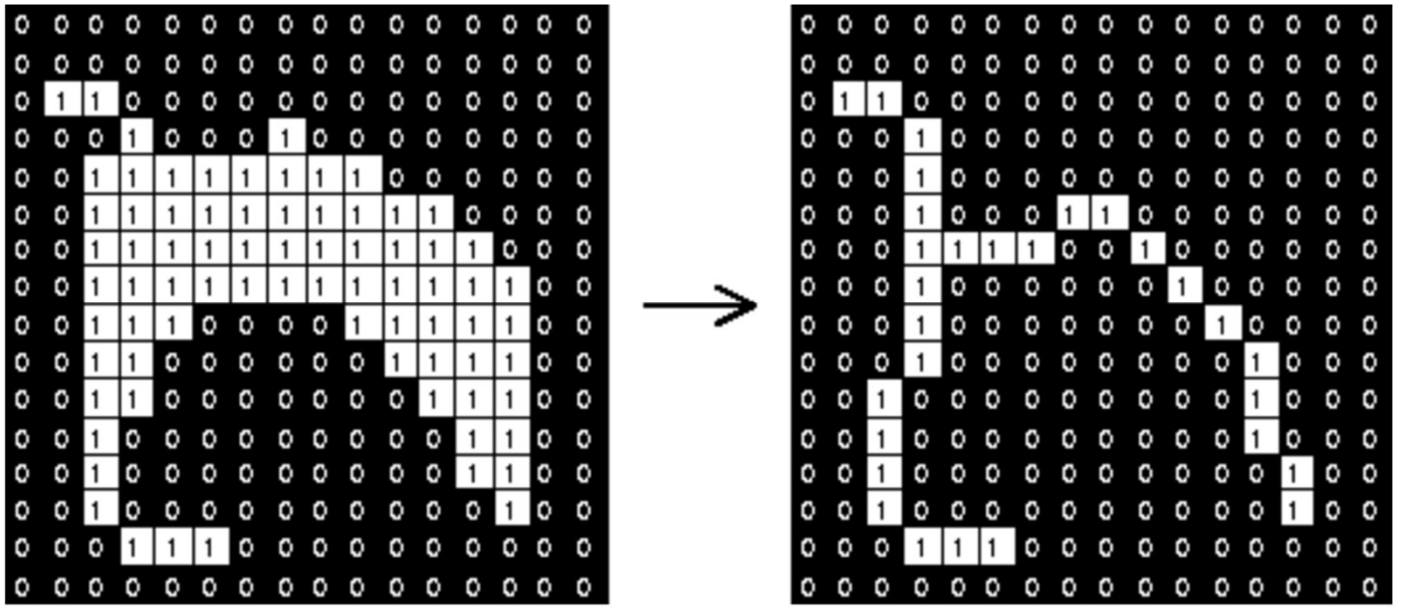


Thinning (spurs visible)

Pruning

Thinning

- Transform into a simplified representation
- Objects are reduced to one-pixel-wide lines.



Iterative Erosion

1. Thinning is achieved through an iterative erosion process
2. In each iteration, the foreground pixels of the image are eroded using a structuring element (often a 3x3 or 5x5 kernel)
3. Repeated until no further erosion can occur without breaking the connectivity of the objects

4. The structuring element moves over the image, and if it matches the shape of the object in the neighborhood, the central pixel is marked for deletion

Thickening

- Inverse of thinning.
- Expands the thickness of foreground pixels while maintaining connectivity.
- Useful in applications uch image restoration, boundary correction, or pattern completion tasks.

Pruning

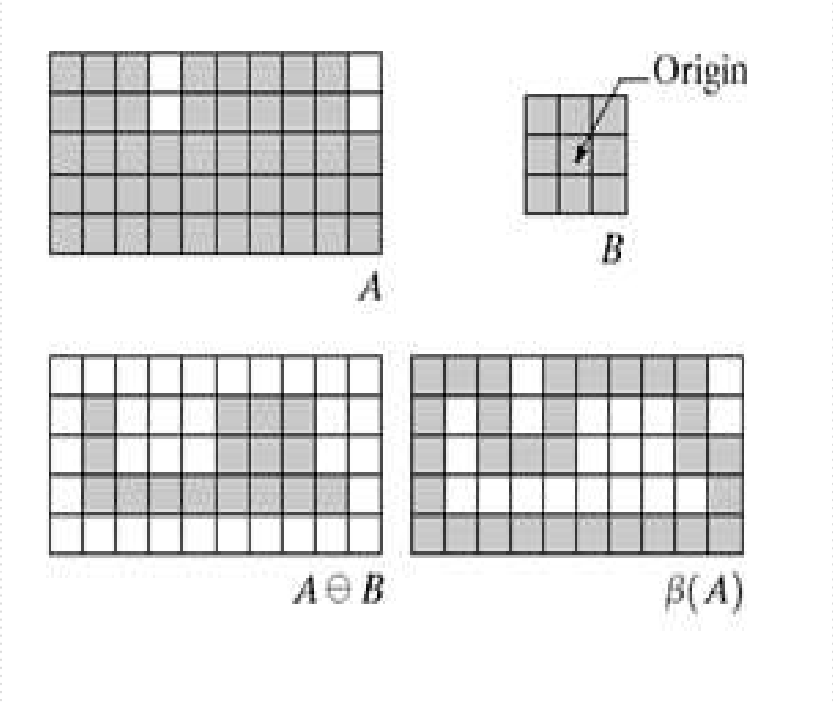
- Used to refine the results of other operations

  (such as skeletonization or thinning)

- Removes unnecessary or spurious branches or components from the extracted structures

- Particularly useful in applications where the extracted structures contain extraneous details, noise, or artifacts that may interfere with subsequent analysis or interpretation.

Boundary Extraction



- Perform erosion of A by B.
- Perform set diff between A and its erosion.

Thinning, Thickening, Pruning using Hit-or-Miss Transform