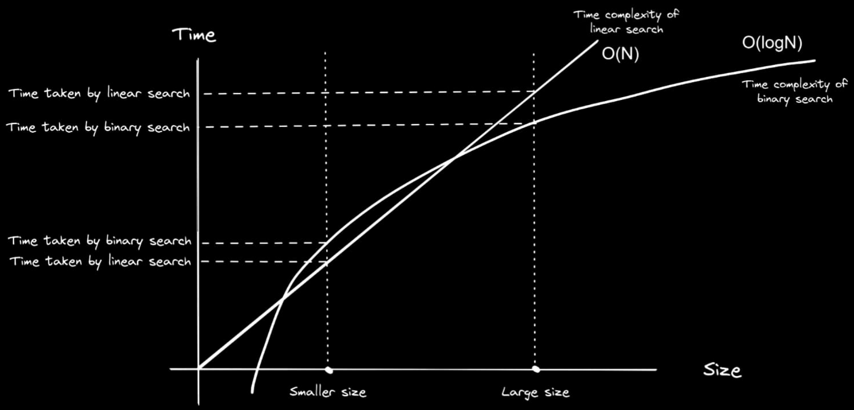
Time Complexity

Time Complexity is a function which gives us the relationship about how the time will grow as the input size grows.

Observe the graph below :-



In the above example, we are comparing linear search time complexity with the time complexity of binary search.

Here are some key observations from the above graph :-

1. For smaller input sizes, the time taken by linear search is less compared to the time taken by binary search.

2. For larger input sizes, the time taken by linear search is more as compared to the time taken by binary search. And it keeps on increasing as compared to the time taken by binary search.

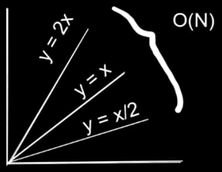
3. We only consider the case having large number of input sizes i.e. the worst case because it allows us to know whether the algorithm is efficient in the long term or not. Worst case analysis helps us to determine real world scenario.

4. Thus, we can say that binary search is more efficient as compared to linear search.

Key points to consider when dealing with time complexity :-

1. Always look for worst case scenario.

2. Always take large input sizes / infinity into consideration.

3. 

Here, the actual time is different but in all cases the time is growing linearly as the input grows!

- Don't care about the actual time.
- Ignore constants.

4. Ignore less dominating terms.

Example :- O (N^3 + log(N))

for N = 1x10^6 = 1 million,

O ((10^6)^3 + log(10^6))

= O ((10^6)^3 + 6) → Here, 6 is very small as compared to 1 million cube! That's why we ignored it.

~ O ((10^6)^3)

~ O (N^3)

Big O Notation

A mathematical notation which describes the upper limit of the function depicting the relationship between the time and input size.

For example, for

O(N^3)

The time complexity cannot exceed O(n^3). It can be O(n^2), O(n), O(log n), but it can't be greater than O(n^3).

Let's dive into the mathematical part

If

$$f(n) = O(g(n))$$

then,

$$\boxed{\lim_{n \to \infty} \frac{f(n)}{g(n)} < \infty}$$

f(n) O(g(n))

$$6(n^3) + 3n + 5 = O(n^3)$$

$$\lim_{n \to \infty} \frac{6(n^3) + 3n + 5}{n^3}$$

$$= \lim_{n \to \infty} 6 + \frac{3}{n^2} + \frac{5}{n^3}$$

Substituting n = infinity

$$= 6 + \frac{3}{\infty} + \frac{5}{\infty}$$

$$= 6 + 0 + 0$$

$$= 6$$

$$< \infty$$

And that's why we ignore constants as well as less dominating terms!