Curve fitting using Least squares Approximation

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Outline

- Introduction
- Method of Least squares : overview
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- Curve fitting
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Least squares: Overview

- The method of Least squares is a standard approach to the approximate solution of overdetermined systems, i.e., sets of equations in which there are more equations than unknowns.
- "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation.
- The least-squares method is usually credited to Carl Friedrich Gauss (1795).

Least squares: Overview

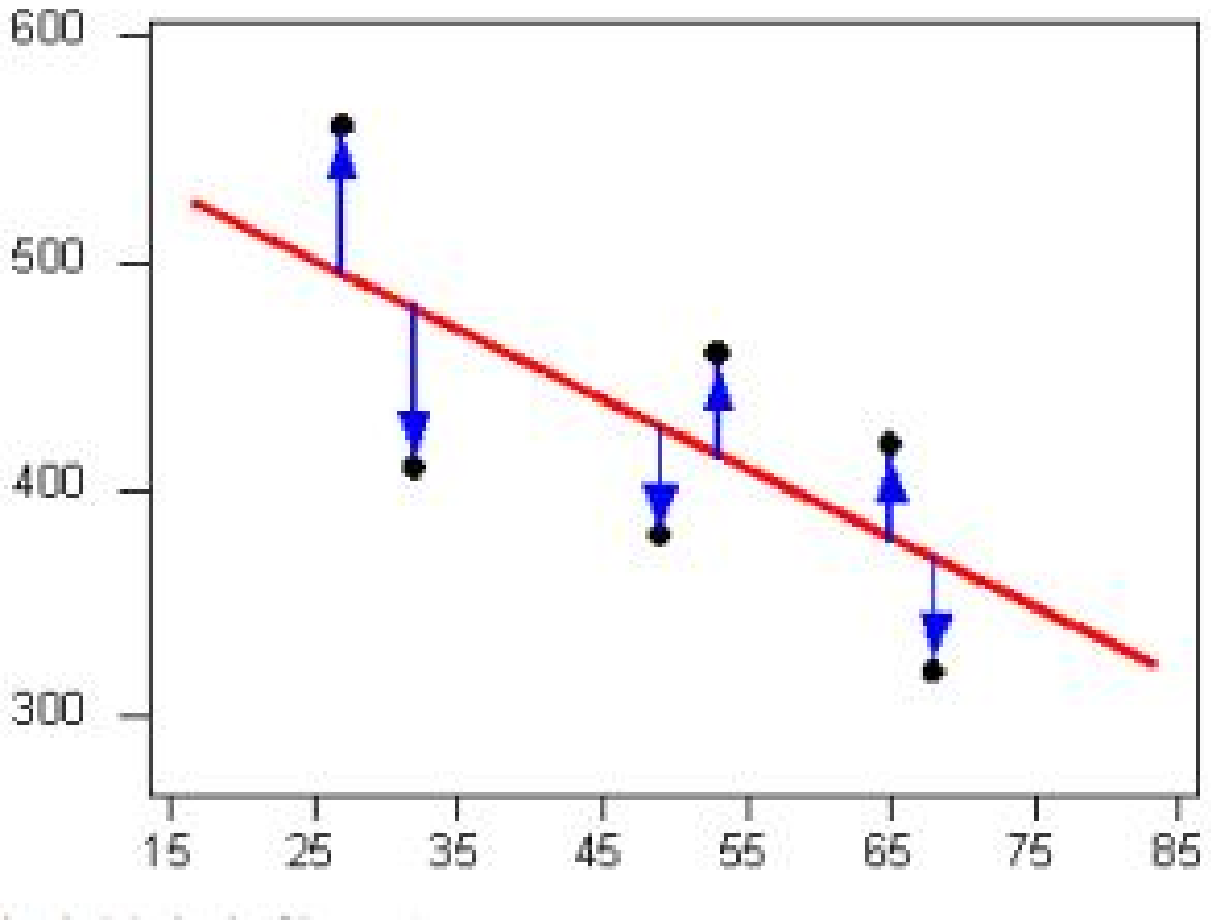


Figure 1: Method of Least Squares

Applications

- The most important application is in data fitting. The best fit in the least-squares sense minimizes the sum of squared residuals, a residual being the difference between an observed value and the fitted value provided by a model.

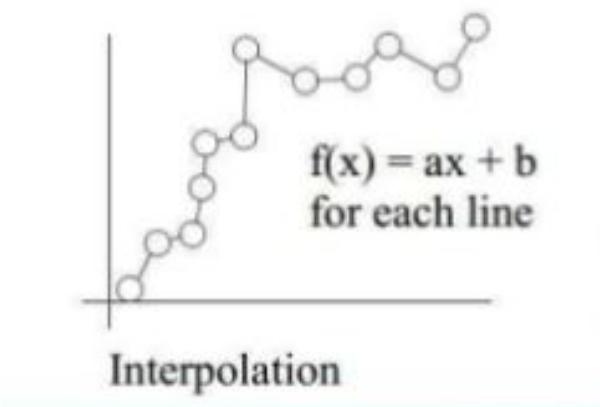
Curve Fitting

- **Definition :** Curve fitting is the process of constructing a curve, or mathematical function, that has the best fit to a series of data points, possibly subject to constraints.

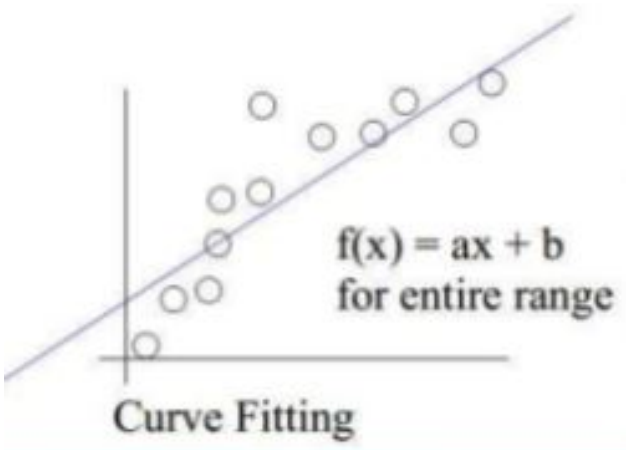
Curve Fitting

There are two general approaches for curve fitting:

- **Least Squares regression:** Data exhibit a significant degree of scatter. The strategy is to derive a single curve that represents the general trend of the data.
- **Interpolation:** Data is very precise. The strategy is to pass a curve or a series of curves through each of the points.



Interpolation
(Source:)



Curve fitting
(Source:)

Proposed Methodology. . .

- We know, the general form of a straight line :

$$f(x) = ax + b$$

$$err = \sum(d_i^2) = (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 + (y_3 - f(x_3))^2 + (y_4 - f(x_4))^2$$

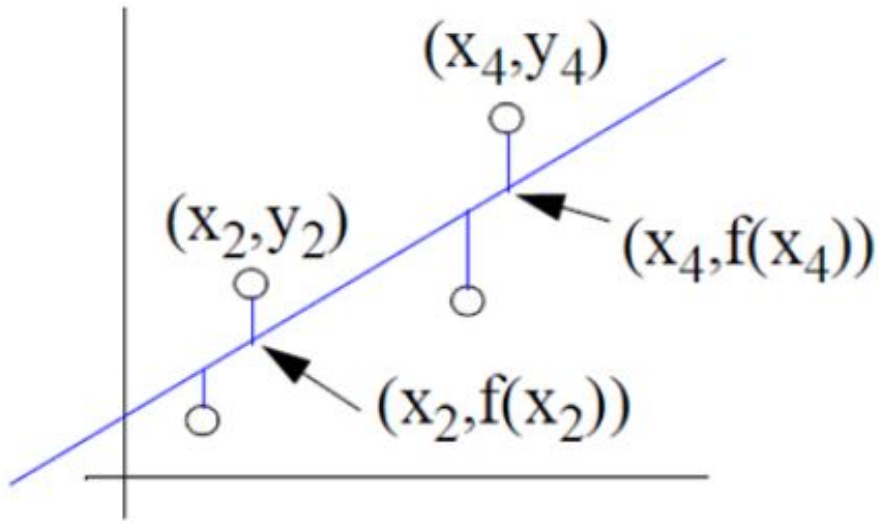


Figure 2: Procrustes Problem [5]

Proposed Methodology. . .

- Our fit is a straight line, so now substitute : f(x) = ax + b.

$$err = \sum_{i=1}^{datapoints} (y_i - f(x_i))^2 = \sum_{i=1}^{datapoints} (y_i - (ax_i + b)^2$$

- The 'best' line has minimum error between line and data points. This is called the least squares approach, since we minimize the square of the error.
- So, we will minimise these least squares using calculus to get our best fit line.
- Similarly, we can proceed to any Curve Fitting.

Least-Squares as an Orthogonal Projection. . .

- In Least squares method, the errors are parallel to the y axis.
- since the line we're fitting is of the form $y = ax + b$, we can calculate the entire vector of predicted y's for each data point just by multiplying: $\hat{y} = ax + b$. We can also compute all the errors in the same form: $err = \bar{y} - y$.

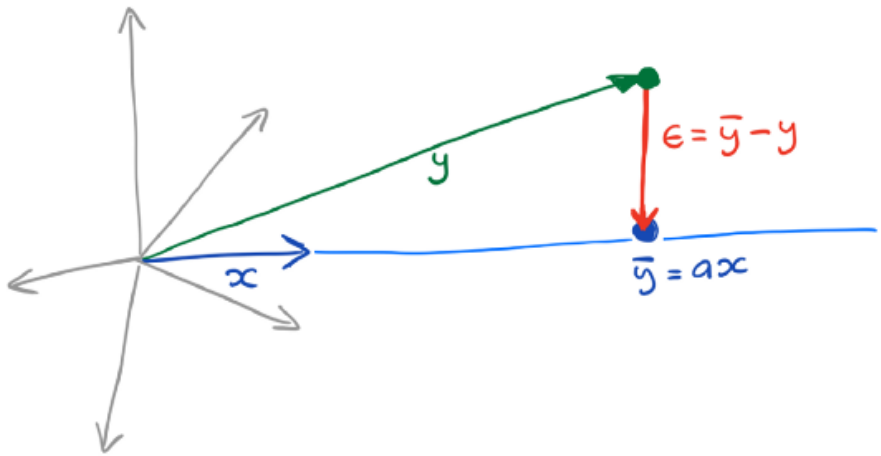


Figure 3: Least-Squares as an Orthogonal Projection

Results. . .

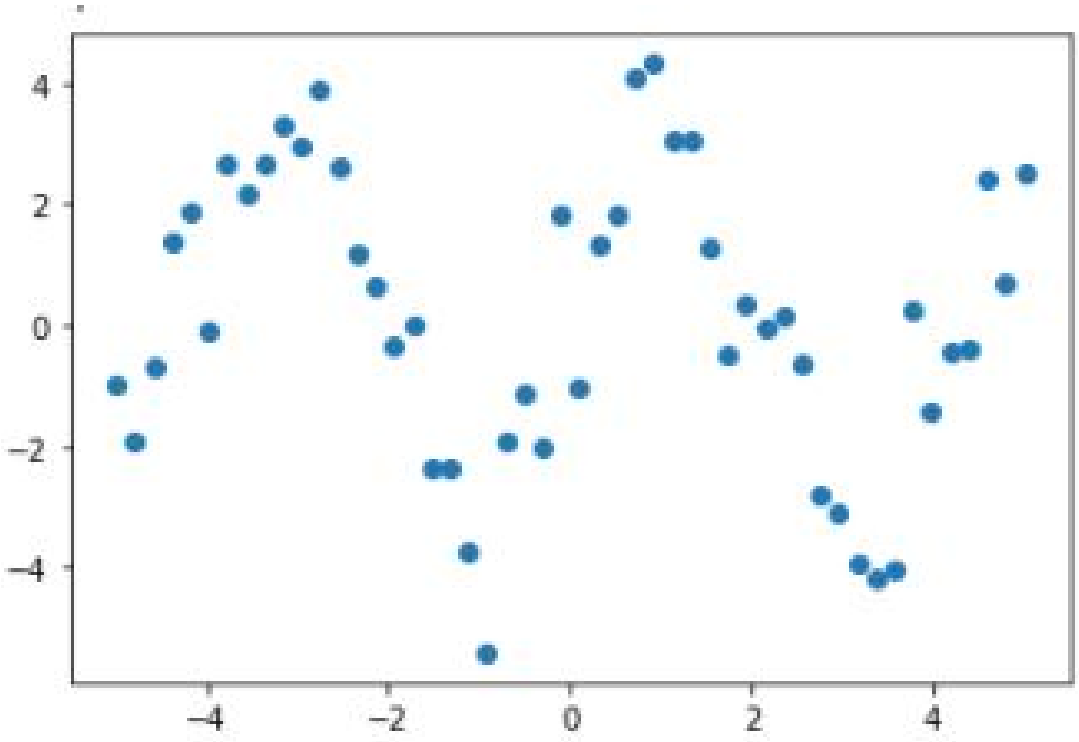


Figure 4: Data points plotting

Results. . .

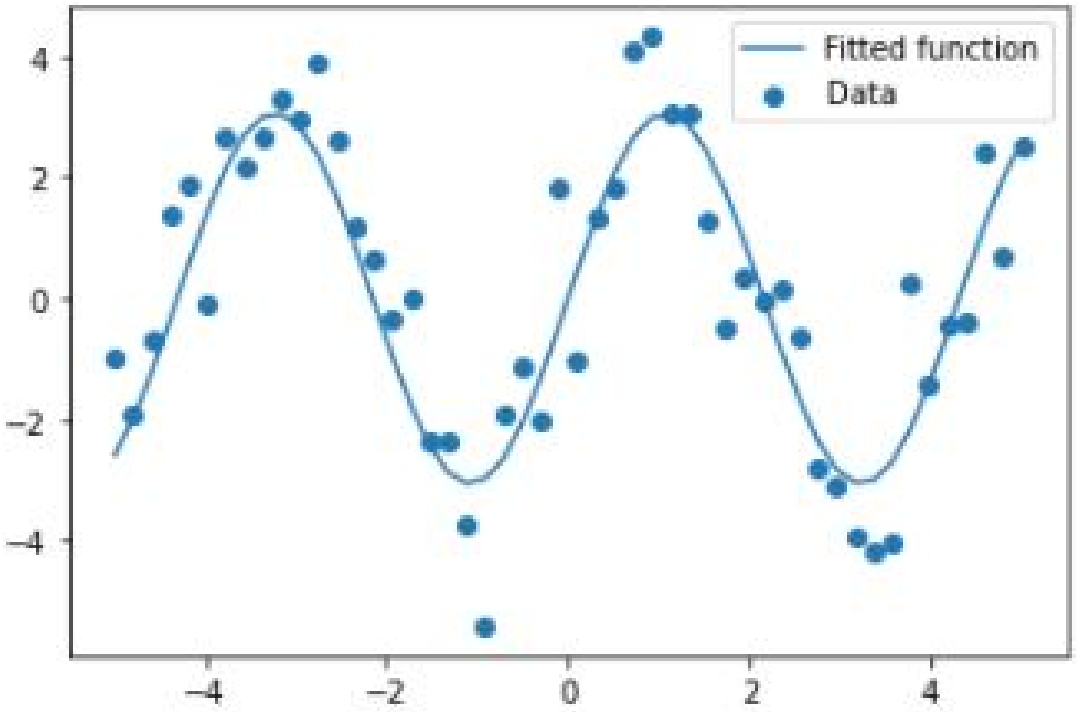


Figure 5: Best fit curve

Applications

The curve fitting models has two general application in engineering:

- Trend Analysis : Predicting values of dependent variables.
- Hypothesis Testing : We can perform comparison of existing mathematical models with measured data.

Conclusion

- In this study we deomnstrated how to solve a curve fitting problem in data modelling by using method of Least square approximations.
- Fitting a curve onto onto some data points is clearly a crucial application of Method of Least squares.
- Experiment results exhibits that the proposed approach obtains best fit curve for Curve fitting to a sinusoidal function wave form data points.

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Thank
You!