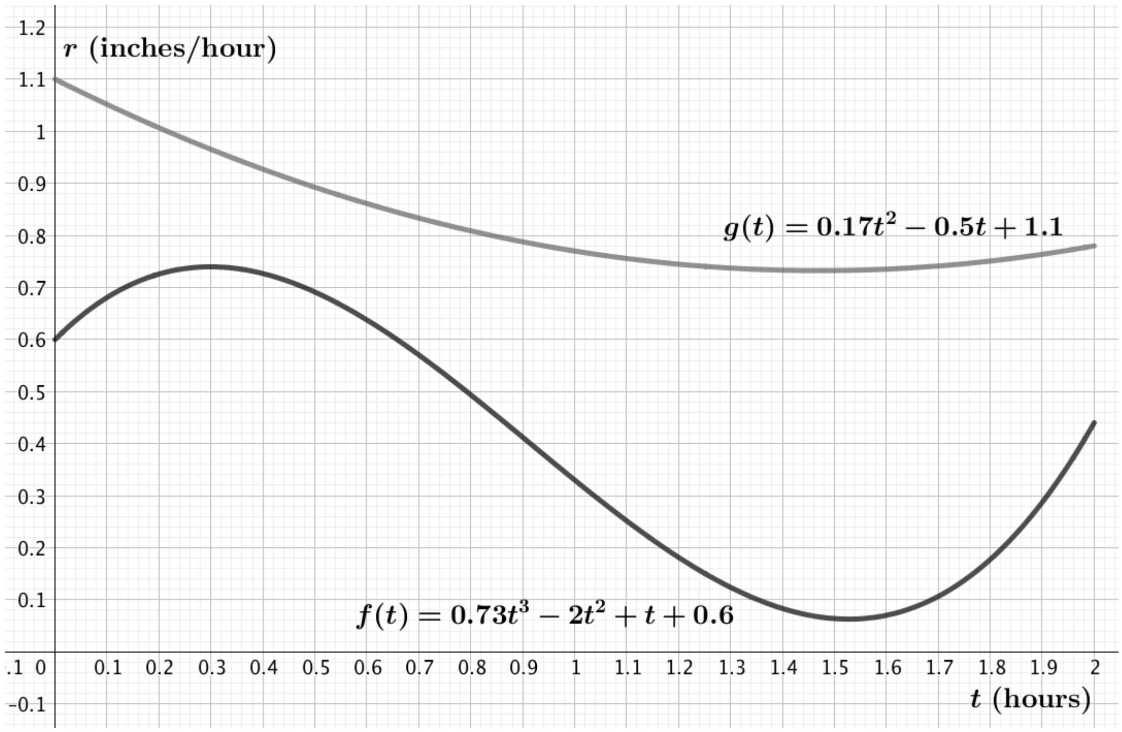
Exploration 5

Name________________________

In this exploration we will learn how to find the average of an infinite number of values; The average value of a continuous function defined over an interval.

1. How do we calculate the average of a finite number of values? Find the average of the numbers: 3, 3, 8, 2, 11, 9, 2, 10. What does this average value signify?

   The rates at which rain fell, in inches per hour, in two different locations (f and g) t hours after the start of a storm are given by $f(t) = 0.73t^3 - 2t^2 + t + 0.6$ and $g(t) = 0.17t^2 - 0.5t + 1.1$ (see graphs below).



2. Using the graph of f, find the approximate average rate of rain fall at location f by averaging 10 values of that function.

3. Using the graph of g, find the approximate average rate of rain fall at location g by averaging 10 values of that function.

4. How could we get better approximate averages?

5. Suppose we partition the interval $[0, 2]$ into n subintervals of equal width Δt. Write an expression for n in terms of Δt.

6. We get an estimate of the average rate of rain fall at location f by averaging the n values $f(t_1^*)$, $f(t_2^*) \ldots f(t_n^*)$, where t_i^* is a value in the ith subinterval. Write an expression for this average rate.

7. Using the relationship between n and Δt found above, rewrite the expression for our estimate of the average rate of rain fall at location f in terms of Δt. Simplify it if possible.

8. The more values of the function f we use to calculate the its average, the better the approximation becomes. We define *the average value* of f as the limit of the approximating averages as $n \to \infty$. This limit of Rieman sums can be written as a definite integral. Write a definite integral representing the average rate of rain fall at location f, then evaluate the integral.