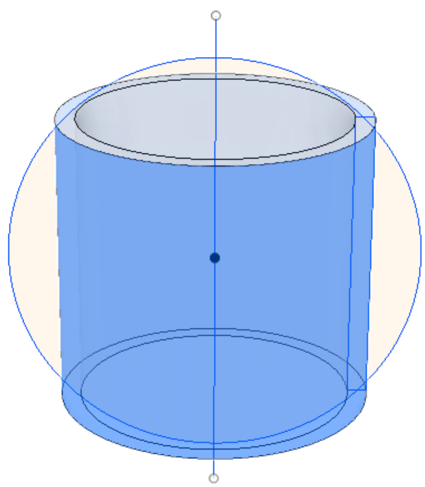
Volumes of Revolution

In this exploration we will determine the volume of a sphere or radius R as the volume obtained by revolving the right half semicircle of radius R about the y axis. We will do this by partitioning the semicircle into thin rectangular strips (either vertical or horizontal) and summing the volumes of the shapes obtained by rotating these strips about the y axis.

1. First we will partition the semicircle along the x axis into thin vertical strips of width dx. When such strips are rotated about the y axis, thin cylindrical shells are generated (see diagram below).

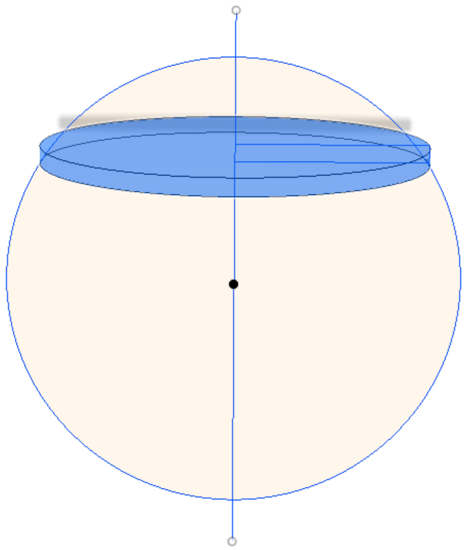


   (a) Find the volume of the shell at a given location x with $0 \le x \le R$. Hint: Imagine cutting the cylinder vertically and unrolling it (flattening it out). The volume of the cylinder equals the volume of the flattened out shape.

   (b) Write a definite integral representing the sum of the volumes of all the cylindrical shells (the volume of the sphere) for all $0 \le x \le R$.

   (c) Evaluate the definite integral found in the previous problem.

2. Next we will partition the semicircle along the y axis into thin horizontal strips of width dy. When such strips are rotated about the y axis, thin circular disks are generated (see diagram below).



(a) Find the volume of the disk at a given location y with $-R \leq y \leq R$.

(b) Write a definite integral representing the sum of the volumes of all the circular disks (the volume of the sphere) for all $-R \leq y \leq R$.

(c) Evaluate the definite integral found in the previous problem.