Math 253 # Exploration 19 Name______________________________

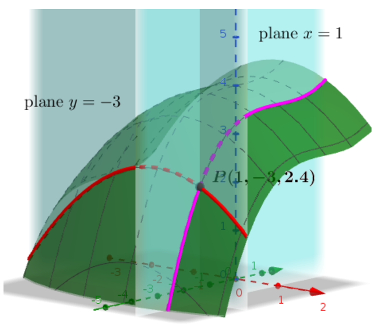
Partial Derivatives

Let $w = f(x_1, \ldots, x_n)$ be a function of the n independent variables $x_1, \ldots, x_n$. The n (first) partial derivatives (f_{x_i} for $i = 1, \ldots, n$)[1] of f are obtained by differentiating f with respect to one of its inputs, while treating all the other inputs as constants; The n partial derivatives are the derivatives of n single variable functions.

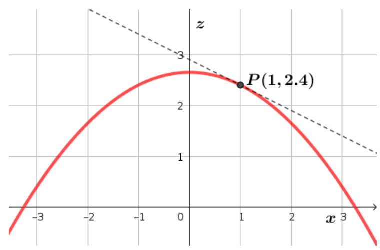
For example, the three (first) partial derivatives of the function $f(x, y, z) = 3x^2y - 2xy^3z^2 + y^2z^5$ are: $f_x = 6xy - 2y^3z^2$, $f_y = 3x^2 - 6xy^2z^2 + 2yz^5$, and $f_z = -4xy^3z + 5y^2z^4$.

We can gain a geometric understanding of the (first) partial derivatives of a function $f(x, y)$ of two input variables as the slopes of tangent lines to the traces of the graph of f in the planes $x = a$ and $y = b$, where a and b are constants.

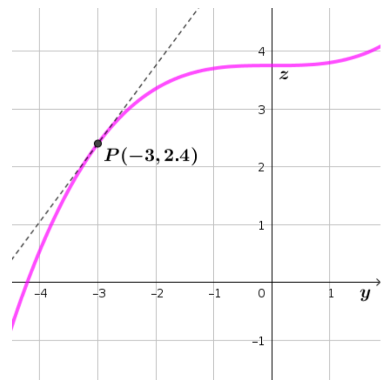
Consider the graph of $z = f(x, y) = 4 - \frac{1}{4}\left(x^2 - \frac{1}{5}y^3\right)$ and its traces in the planes $y = -3$ and $x = 1$ below.



(a) Graph of $f(x, y)$



(b) Trace of f in the plane $y = -3$



(c) Trace of f in the plane $x = 1$

Problems

1. What is the equation of the trace depicted in figure (b)?
2. What is the equation of the trace depicted in figure (c)?
3. Use figures (b) and (c) to estimate the values of $z_x(1, -3)$ and $z_y(1, -3)$.
4. Use the derivative of the function from problem 1 to find the exact value of $z_x(1, -3)$. How does the exact value compare to the approximation you found in problem 3?
5. Use the derivative of the function from problem 2 to find the exact value of $z_y(1, -3)$. How does the exact value compare to the approximation you found in problem 3?

A Physical Example

Imagine a quantity of gas confined to a cylinder with a piston that can be used to compress or expand the gas[2]. If the gas is an ideal gas, the relationship between its pressure P (in "kilo Pascals" kPa), volume V (in liters L) and temperature T (in Kelvin K) obey the ideal gas law

$$PV = nRT$$

[1] Other notations for these (first) derivatives are w_{x_i}, $\partial f/\partial x_i$, $\partial w/\partial x_i$ and $D_{x_i} f$

[2] Similar to the cylinders and pistons in the engines of a gasoline powered car.

where n is the number of moles[3] of gas, and R is the ideal gas constant. This equation relates the three parameters P, V and T. We can solve the equation for any one of the parameters in terms of the other two, giving us three different (but related) functions of two variables: $P(T, V)$, $V(T, P)$ and $T(V, P)$.

Problems

Supposing that $nR = 10$ L kPa/K

1. Calculate $P_T(300, 2)$ and interpret its meaning.
2. Calculate $P_V(300, 2)$ and interpret its meaning.
3. Calculate $V_T(300, 5)$ and interpret its meaning.
4. Calculate $V_P(300, 5)$ and interpret its meaning.

[3] A measure of the number of molecules in the cylinder