Exam 2 Review

1. An object's velocity $\mathbf{v}$ (in meters per second) at time t seconds is given by $\mathbf{v}(t) = 2t\hat{\imath} + 2\hat{\jmath} + \frac{1}{t}\hat{\mathbf{k}}$, for $t > 0$ seconds. Using this function do the problems below.
   (a) Find the position function $\mathbf{r}(t)$, given that $\mathbf{r}(1) = \mathbf{0}$.
   (b) Find the objects speed at $t = 1$ second.
   (c) Find the acceleration function $\mathbf{a}(t)$, and the acceleration vector at $t = 1$ second.
   (d) How far along the curve does the object travel between 1 and 5 seconds; i.e. what is the arc-length of the curve over $1 \le t \le 5$?
   (e) Find the unit tangent vector function $\mathbf{T}(t)$, then find the unit tangent vector at $t = 1$.
   (f) Find the curvature κ of the curve at $t = 1$ second.
2. Find the unit tangent vector $\mathbf{T}$ for the vector position function $\mathbf{r}(t) = \langle t, 2\sin(t), 3\cos(t)\rangle$ at $t = \pi/6$.
3. Evaluate the integral $\displaystyle\int_0^{\pi/4} \cos(2t)\hat{\imath} + \sin(2t)\hat{\jmath} + t\hat{\mathbf{k}}\, dt$.
4. Find the length of the curve $\mathbf{r}(t) = \langle t\sqrt{2}, e^t, e^{-t}\rangle$, $0 \le t \le 1$.
5. Find the unit tangent vector $\mathbf{T}(t)$ and the unit normal vector $\mathbf{N}(t)$ for the curve $\mathbf{r}(t) = \langle \frac{1}{3}t^3, t^2, 2t\rangle$.
6. Find the curvature of the curve $\mathbf{r}(t) = \langle \sin(t), \cos(t), \sin(t)\rangle$.
7. Find the velocity, acceleration and speed of the particle with position function $\mathbf{r}(t) = \langle t^2, t, t^3\rangle$ at $t = 1$.
8. Find the tangential and normal components of the acceleration vector if $\mathbf{r}(t) = \cos(t)\hat{\imath} + \sin(t)\hat{\jmath} + t\hat{\mathbf{k}}$.
9. Let $\mathbf{r}(t) = \langle 3\sin(t), 4t, 3\cos(t)\rangle$ be a vector valued function which describes a curve.
   (a) Reparametrize $\mathbf{r}$ in terms of arclength s.
   (b) Find $\mathbf{T}$ and $\mathbf{N}$ (the unit tangent and normal vectors respectively) to the curve at the point $(0, 0, 3)$.
   (c) Find the equation of the normal plane at the point $(0, 0, 3)$.
10. Given the accelleration of a particle $\mathbf{a}(t) = \langle t, t^2, \cos(2t)\rangle$, with initial velocity $\mathbf{v}(0) = \langle 1, 0, 1\rangle$ and initial position $\mathbf{r}(0) = \langle 0, 1, 0\rangle$
    (a) Find the velocity $\mathbf{v}(t)$.
    (b) Find the postion $\mathbf{r}(t)$.

Exam 2 Rev Answers

1. (a) $\mathbf{r}(t) = \left\langle t^2 - 1, 2t - 2, \ln(t) \right\rangle$.
   (b) $|\mathbf{r}(1)| = 3$.
   (c) $\mathbf{a}(t) = \left\langle 2, 0, -1/t^2 \right\rangle$, $\mathbf{a}(1) = \langle 2, 0, -1 \rangle$.
   (d) The length of the curve is $24 + \ln(5)$.
   (e) $\mathbf{T}(t) = \left\langle \frac{2t^2}{1+2t^2}, \frac{2t}{1+2t^2}, \frac{1}{1+2t^2} \right\rangle$, $\mathbf{T}(1) = \langle 2/3, 2/3, 1/3 \rangle$.
   (f) $\kappa(1) = 2/9$.

2. $\mathbf{T}(\pi/6) = \left\langle 2/5, 2\sqrt{3}/5, -3/5 \right\rangle$

3. $\displaystyle\int_0^{\pi/4} \cos(2t)\hat{\boldsymbol{\imath}} + \sin(2t)\hat{\boldsymbol{\jmath}} + t\hat{\mathbf{k}}\, dt = \frac{1}{2}\hat{\boldsymbol{\imath}} + \frac{1}{2}\hat{\boldsymbol{\jmath}} + \frac{\pi^2}{32}\hat{\mathbf{k}}.$

4. The length of the curve $\mathbf{r}(t)$ is $e - \frac{1}{e}$.

5. $\mathbf{T}(t) = \left\langle \frac{t^2}{t^2+2}, \frac{2t}{t^2+2}, \frac{2}{t^2+2} \right\rangle$, $\mathbf{N}(t) = \left\langle \frac{2t}{t^2+2}, \frac{2-t^2}{t^2+2}, \frac{-2t}{t^2+2} \right\rangle$.

6. $\kappa(t) = \dfrac{\sqrt{2}}{(1 + \cos^2(t))^{3/2}}$.

7. $\mathbf{v}(1) = \langle 2, 1, 3 \rangle$, $\mathbf{a}(1) = \langle 2, 0, 6 \rangle$, speed $= \sqrt{14}$.

8. $a_T = 0$, $a_N = 1$.

9. $\mathbf{r}(t) = \langle 3\sin(t), 4t, 3\cos(t) \rangle$
   (a) $\mathbf{r}(s) = \langle 3\sin(s/5), 4s/5, 3\cos(s/5) \rangle$.
   (b) $\mathbf{T}(0) = \langle 3/5, 4/5, 0 \rangle$ and $\mathbf{N}(0) = \langle 0, 0, -1 \rangle$
   (c) The equation of the normal plane at the point $(0, 0, 3)$ is $3x + 4y = 0$.

10. $\mathbf{a}(t) = \left\langle t, t^2, \cos(2t) \right\rangle$, with initial velocity $\mathbf{v}(0) = \langle 1, 0, 1 \rangle$ and initial position $\mathbf{r}(0) = \langle 0, 1, 0 \rangle$
    (a) $\mathbf{v}(t) = \left\langle 1 + \frac{t^2}{2}, \frac{t^3}{3}, 1 + \frac{\sin(2t)}{2} \right\rangle$.
    (b) $\mathbf{r}(t) = \left\langle t + \frac{t^3}{6}, 1 + \frac{t^4}{12}, \frac{1}{2} + t - \frac{\cos^2(t)}{2} \right\rangle$.