Review for Quiz on Trig. Subst. and Integration of Rational Functions

Trigonometric Substitution

Use an appropriate trig. substitution to resolve each of the following definite or indefinite integrals. Your answers should *not* include the composition of any trig. and inverse trig. functions; For example, replace $\tan\left(\sin^{-1}\left(\frac{x}{4}\right)\right)$ with $\frac{x}{\sqrt{16-x^2}}$.

Problems

1. $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$

2. $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx$

3. $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx$

4. $\displaystyle\int_0^3 \frac{x^2}{\sqrt{36-x^2}}\,dx$

5. $\displaystyle\int_{\sqrt{2}}^{\sqrt{10}} \frac{t}{\sqrt{t^2-1}}\,dt$

6. $\displaystyle\int_0^a \frac{dx}{(a^2+x^2)^{3/2}}$

Rational Functions

Use polynomial division to express each improper rational function as the sum of a polynomial and proper rational function, then integrate.

Problems

7. $\displaystyle\int \frac{x^3-4x^2+4x+1}{x-1}\,dx$

8. $\displaystyle\int \frac{2x^3-x^2+2x+2}{x^2+1}\,dx$

Answers

1. $-\dfrac{\sqrt{4-x^2}}{4x}+C$

2. $-3\arccos\left(\dfrac{3}{x}\right)+\sqrt{x^2-9}+C$

3. $\dfrac{1}{3}(x^2-8)\sqrt{4+x^2}+C$

4. $3\pi-\dfrac{9\sqrt{3}}{2}$

5. 2

6. $\dfrac{1}{a^2\sqrt{2}}$

7. $\dfrac{x^3}{3}-\dfrac{3x^2}{2}+x+2\ln|x-1|+C$

8. $x^2-x+3\arctan(x)+C$