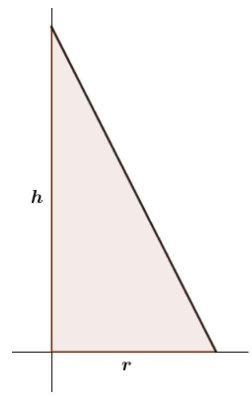
Volumes of Revolution, part 2

In this exploration we will determine the volume of a cone or base radius r and height h as the volume obtained by revolving a triangular region of base length r and height h about the y axis. We will do this by partitioning the triangular region into thin rectangular strips (either vertical or horizontal) and summing the volumes of the shapes obtained by rotating these strips about the y axis.

Hint: Make use of similar triangles.

1. First we will partition the triangular region along the x axis into thin vertical strips of width dx. When such strips are rotated about the y axis, thin cylindrical shells are generated.

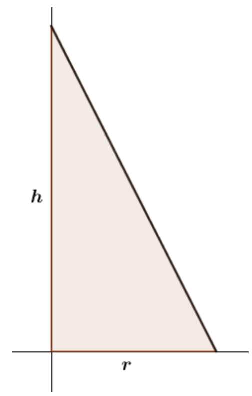


   (a) Find the volume of the shell at a given location x with $0 \leq x \leq r$. Hint: Imagine cutting the cylinder vertically and unrolling it (flattening it out). The volume of the cylinder equals the volume of the flattened out shape.

   (b) Write a definite integral representing the sum of the volumes of all the cylindrical shells (the volume of the sphere) for all $0 \leq x \leq r$.

   (c) Evaluate the definite integral found in the previous problem.

2. Next we will partition the semicircle along the y axis into thin horizontal strips of width dy. When such strips are rotated about the y axis, thin circular disks are generated.



(a) Find the volume of the disk at a given location y with $0 \leq y \leq h$.

(b) Write a definite integral representing the sum of the volumes of all the circular disks (the volume of the sphere) for all $0 \leq y \leq h$.

(c) Evaluate the definite integral found in the previous problem.