Simpson's Rule

Simpson's rule is a formula which allows us to approximate the value of a definite integral for those situations when it is either difficult or impossible to ust the Fundamental Theorem of Calculus. To approximate the value of the definite integral

$$\int_a^b f(x)\,dx$$

we partition the interval $[a, b]$ into an *even* number n of subintervals of equal width Δx with partition points $a = x_0, x_1, \ldots, x_n = b$. Simpson's rule tells us that

$$\int_a^b f(x)\,dx \approx \frac{1}{3}\Delta x\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right)$$

where the error E_S (the difference between the exact value of the integral and the approximation) can be estimated by

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

with K a constant such that $\left|f^{(4)}(x)\right| \leq K, \forall x \in [a, b]$.

Problems

1. A sky diver jumps out of a plane at time $t = 0$ seconds, opens his parachute two minutes later and lands five minutes after his jump. The table below shows his speed $v(t)$ (in meters per second) at different times t (in seconds) during his jump. The distance he fell is given by the integral $\int_0^{300} v(t)\,dt$. Use Simpson's rule to estimate this distance.

t	0	25	50	75	100	125	150	175	200	225	250	275	300
$v(t)$	0	176	264	307	329	294	159	100	74	63	59	57	56

2. Let $f(x) = \sin(x^2)$.

   (a) Use Simpson's rule with $n = 8$ subintervals to approximate the value of $\int_0^{\pi} f(x)\, dx$.

   (b) Use the error estimation formula for Simpson's rule to estimate the error between the exact value of the integral and the approximation. Use the graph of the fourth derivative of f below, to determine a value for K in the formula that gives the best error bound.

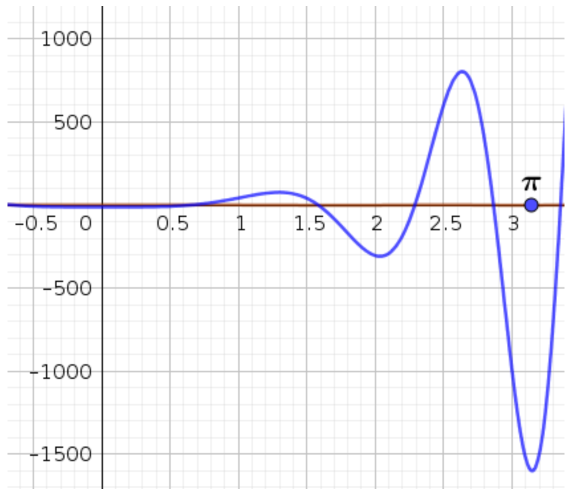

Figure 1: Graph of $f^{(4)}(x)$