Extra Review Problems for Exam 2

Problems

1. Find a vector function for the line tangent to the curve $\boldsymbol{r}(t) = \langle \cos t, \sin t, \cos 4t \rangle$ at $t = \pi/3$.
2. An object moves with velocity vector $\boldsymbol{v} = \left\langle t, t^2, \cos t \right\rangle$, starting at $\langle 0, 0, 0 \rangle$ at $t = 0$. Find the position function $\boldsymbol{r}(t)$.
3. The position function of a particle is given by $\boldsymbol{r}(t) = \left\langle t^2, 5t, t^2 - 16 \right\rangle$, for $t \geq 0$. When (at what value of t) is the speed of the particle a minimum?
4. A curve is given by the position function $\boldsymbol{r}(t) = \left\langle t^2, 2, t^3 \right\rangle$. Find the length of the curve over the interval $0 \leq t \leq 1$.
5. Find the curvature of the curve $\boldsymbol{r}(t) = \left\langle t, t^2, t \right\rangle$ at time $t = 1$.

Answers

1. The tangent line to the curve $\boldsymbol{r}(t)$ at $t = \pi/3$ is given by $\boldsymbol{L}(t) = \left\langle \frac{1}{2} - \frac{t\sqrt{3}}{2}, \frac{\sqrt{3}}{2} + \frac{t}{2}, -\frac{1}{2} + 2t\sqrt{2} \right\rangle$
2. $\boldsymbol{r}(t) = \left\langle \frac{1}{2}t^2, \frac{1}{3}t^3, \sin t \right\rangle$
3. The speed function is $\sqrt{8t^2 + 25}$. Clearly, by inspection, its minimum value occurs at $t = 0$. The minumum speed is 5.
4. The length of the curve $\boldsymbol{r}(t)$ over the interval $[0, 1]$ is $s = \int_0^1 t\sqrt{4 + 9t^2}\, dt = \frac{13\sqrt{13} - 8}{27} \approx 1.44$
5. The curvature of $\boldsymbol{r}(t)$ at time $t = 1$ is $\kappa(1) = \frac{\sqrt{3}}{9} \approx 0.192$