# 06-The-Definite-Integral

February 5, 2018

## 1 Using the Definition

$$\int_a^b f(x)dx = R(n) = \lim n \to \infty(\frac{b-a}{n})\sum_{i=1}^{n} f(a + i\frac{b-a}{n})$$

```
In [1]: import sympy as sy
```

```
In [2]: x, n = sy.symbols('x n')
```

```
In [3]: sy.pprint(sy.summation(x, (x, 1,n)))
```

```
 2
n    n
 +
2    2
```

```
In [4]: sy.pprint(sy.summation(x**2, (x, 1,n)))
```

```
 3    2
n    n    n
 +  +
3    2    6
```

```
In [5]: sy.pprint(sy.summation(x**3, (x, 1,n)))
```

```
 4    3    2
n    n    n
 +  +
4    2    4
```

```
In [6]: sy.pprint(sy.summation(x**4, (x, 1,n)))
```

```
 5    4    3
n    n    n    n
 +  +  -
5    2    3    30
```

### 1.0.1 Problem I

Use the definition and our summation formulas to evaluate the area under the given function and approximate what happens as $n \to \infty$ on given domain:

a. $f(x) = x$ on $[1, 4]$

b. $g(x) = x^2 - 2x$ on $[2, 4]$

c. $h(x) = x^3 - x + 1$ on $[1, 3]$

### 1.0.2 Problem II

We want to examine patterns on the interval $[0, b]$ for polynomial functions. Let's use the definition to prove the following:

a. $\int_0^b x = \frac{b^2}{2}$

b. $\int_0^b x^2 = \frac{b^3}{3}$

c. $\int_0^b x^3 = \frac{b^4}{4}$

### 1.0.3 Problem III

**Theorem**: Assume that $f(x)$ is continuous on $[a, b]$ and let $F(x)$ be an antiderivative of $f(x)$ on $[a, b]$. Then

$$\int_a^b f(x) = F(b) - F(a)$$

Use the attached table of integrals and the theorem above to evaluate the following definite integrals.

a. $\int_0^2 2x^2 - x\,dx$

b. $\int_{1/2}^2 \ln x\,dx$

c. $\int_0^{2\pi} \sin x\,dx$

### 1.0.4 Problem IV

**Interpreting the Integral as Total Change**:

Water flows into an empty bucket at a rate of $r(t)$ gallons per second. How much water is in the bucket after 5 seconds? If the rate is constant, we would have $0.3 \times 5 = 1.5$ gallons. If the rate is not constant, we can interpret the quantity of water as equal to the area under the graph of $r(t)$.

a. A survey shows that a mayoral candidate is gaining votes at a rate of $2000t + 150$ votes per day, where $t$ is the number of days since announcing her candidacy. How many supporters after 90 days?

b. A projectile is released with initial (vertical) velocity 100 m/s. Use the formula $v(t) = 100 - 98t$ for velocity to determine the distance traveled during the first 15 seconds.

c. The rate at which water drains from a tank is recorded at half-minute intervals. Use approximations to estimate the total amount of water drained during the first 3 minutes.

| liters | $t$(min) |
|---|---|
| 0 | 50 |
| 0.5 | 48 |
| 1 | 46 |
| 1.5 | 43 |
| 2 | 40 |
| 2.5 | 39 |
| 3 | 36 |

### 1.0.5 Problem V

**Area Between Curves**

Use the definite integral to find specified area between two curves.

1. Area between $y = x^3 - 2x^2 + 10$ and $y = 3x^2 + 4x - 10$.
2. Area between $y = 0.5x$ and $y = x\sqrt{1 - x^2}$
3. Area between $y = 4 - x^2$ and $y = x^2 - 4$