

Midterm Part 1 - Nonlinear Dynamics and Chaos

NAME: _____ SCORE:

Deadline: Wednesday 26 November 2025 (by 9pm)

Credits: 12 points (6 questions) Type of evaluation: MT Paper

In your own words, provide concise answers to the following statements:

1. **(2 points) Phase portraits in 1D**

Explain: (i) what a phase portrait represents for a 1D dynamical system $\dot{x} = f(x)$, and (ii) what information can be extracted from it about long-term behaviour.

2. **(2 points) Linear stability**

(i) What does it mean for a fixed point of a non-linear system to be linearly stable? (ii) Why is the sign of $f'(x^*)$ important?

3. **(2 points) Bifurcations in 1D**

Briefly describe: (i) what a bifurcation is, and (ii) two common types of local bifurcations encountered in 1D systems.

4. **(2 points) Nullclines in 2D systems**

Explain (i) what nullclines are and (ii) how they help determine the behaviour of a 2D dynamical system.

5. **(2 points) Limit cycles**

Explain: (i) what a limit cycle is, and (ii) how it differs from a stable spiral or a center.

6. **(2 points) Sketching a saddle-node bifurcation**

Consider the system $\dot{x} = r + x^2$. Sketch the bifurcation diagram showing fixed points vs. the parameter r , and clearly indicate: (i) stable and unstable branches, (ii) the location of the bifurcation point/s.