

# Midterm Part 1 - Nonlinear Dynamics and Chaos

NAME: \_\_\_\_\_ SCORE: \_\_\_\_\_

Deadline: Wednesday 26 November 2025 (by 9pm)

Credits: 12 points (6 questions) Type of evaluation: MT Paper

In your own words, provide concise answers to the following statements:

1. (2 points) Phase portraits in 1D

Explain: (i) what a phase portrait represents for a 1D dynamical system  $\dot{x} = f(x)$ , and (ii) what information can be extracted from it about long-term behaviour.

2. (2 points) Linear stability

(i) What does it mean for a fixed point of a non-linear system to be linearly stable? (ii) Why is the sign of  $f'(x^*)$  important?

3. (2 points) Bifurcations in 1D

Briefly describe: (i) what a bifurcation is, and (ii) two common types of local bifurcations encountered in 1D systems.

**4. (2 points) Nullclines in 2D systems**

Explain (i) what nullclines are and (ii) how they help determine the behaviour of a 2D dynamical system.

**5. (2 points) Limit cycles**

Explain: (i) what a limit cycle is, and (ii) how it differs from a stable spiral or a center.

**6. (2 points) Sketching a saddle-node bifurcation**

Consider the system  $\dot{x} = r + x^2$ . Sketch the bifurcation diagram showing fixed points vs. the parameter  $r$ , and clearly indicate: (i) stable and unstable branches, (ii) the location of the bifurcation point/s.