```
# Infinite streams here are represented as pairs of the form
  \# [x thunk], where x is the first element of the stream and
  # thunk is a nullary closure able to generate the rest of
  # the stream. An empty stream is represented by [].
   (define empty-stream [])
  (define stream-head first)
(define (stream-tail e) ((second e)))
12 (define empty-stream? null?)
13
  (define (stream-cons x s)
14
    [x (lambda () s)])
15
16
   (define (list->stream L)
17
18
     (match L
19
      ([] empty-stream)
       ((list-of x (some-list rest)) [x (lambda () (list->stream rest))])))
20
   (define (stream-nth stream i)
22
    (check ((less? i 2) (stream-head stream))
23
24
            (else (stream-nth (stream-tail stream) (minus i 1)))))
25
   (define (stream-map f s)
    (check ((empty-stream? s) s)
27
            (else [(f (stream-head s))
29
                   (lambda () (stream-map f (stream-tail s)))])))
30
   (define (stream-for-each s pred N)
31
     (letrec ((loop (lambda (s i)
32
                        (check ((|| (greater? i N) (empty-stream? s)) true)
                               ((pred (stream-head s)) (loop (stream-tail s) (plus i 1)))
34
35
                               (else false)))))
36
       (loop s 1)))
37
  (define (stream-for-some s pred N)
     (letrec ((loop (lambda (s i)
39
                        (check ((|| (greater? i N) (empty-stream? s)) false)
                               ((pred (stream-head s)) true)
41
                               (else (loop (stream-tail s) (plus i 1)))))))
42
43
       (loop s 1)))
44
45
   (define (stream-filter s pred)
46
47
     (check ((empty-stream? s) s)
48
            (else (let ((x (stream-head s)))
                    (check ((pred (stream-head s)) [x (lambda () (stream-filter (stream-tail s) pred))])
49
                            (else (stream-filter (stream-tail s) pred)))))))
51
  # This is a friendly version of take: If the input stream is finite and
  # has no more elements than n, then the entire stream is returned (as a list):
54
   (define (stream-take stream n)
55
     (letrec ((loop (lambda (S i res)
56
                       (check ((leq? i 1) (check ((empty-stream? S) (rev res))
                                                  (else (rev (add (stream-head S) res)))))
58
                               (else (check ((empty-stream? S) (rev res))
59
                                             (else (loop (stream-tail S) (minus i 1) (add (stream-head S) res)))))))))
       (check ((less? n 1) [])
61
              (else (loop stream n [])))))
63
   # Interleaving two streams:
64
65
   (define (weave-streams s1 s2)
66
     (check ((empty-stream? s1) s2)
67
68
            (else [(stream-head s1) (lambda () (weave-streams s2 (stream-tail s1)))])))
```

```
# Interleaving infinite streams raises questions of fairness and
70
71 # element distribution. The above version of weave always
n # swaps orders on each call. The version below prefers drawing elements
  # from the first stream with probability p, where 0 \le p \le 1.
   # Thus, in the long run, (weave-streams-with-probability s1 s2 0.5)
   # should give the same results as (weave-streams s1 s2):
   (define (weave-streams-with-probability s1 s2 p)
77
     (check ((empty-stream? s1) s2)
78
79
             (else (let ((x (random-int 100)))
                     (check ((leq? x (times p 100)) [(stream-head s1) (lambda () (weave-streams-with-probability (stream-
80
                            (else [(stream-head s2) (lambda () (weave-streams-with-probability s1 (stream-tail s2) p))])))
82
83
   # The procedure zip generates a stream representing the Cartesian product
   # of input streams S1 and S2, either of which may be infinite. Moreover,
85 # it does this in a way that is fair and gives priority to left elements
86 # from both input streams. It walks through the (potentially infinite)
\mbox{\it 87} \mbox{\it \#} two-dimensional matrix of all values from S1 and S2 in the style
   # of Cantor's encoding of the rational numbers: by sweeping the matrix
89 # starting from the left upper-hand corners, then moving to the right,
90 # then down and left, and then back up and to the right again.
91 # The procedure does not need to keep track of positions (i,j) in
  # the matrix: instead, it passes (and consumes) the matrix rows
   # (each of which represents an infinite stream) dynamically as
   # arguments. This leads to a quite efficient implementation.
94
   (define (stream-zip S1 S2)
     (letrec ((getNext (lambda (front-streams back-streams first-stream)
97
                          (match front-streams
98
                            ([] (check ((empty-stream? first-stream)
99
100
                                           (match back-streams
101
                                             ([] empty-stream)
                                             (_ (getNext (rev back-streams) [] first-stream))))
102
                                        (else (let ((x (stream-head first-stream))
103
                                                    (new-stream (stream-map (lambda (y) [x y]) S2)))
104
                                                (getNext (rev (add new-stream back-streams)) [] (stream-tail first-stream))
                            ((list-of stream-of-pairs more-streams)
106
                                 (check ((empty-stream? stream-of-pairs)
107
108
                                           (getNext more-streams back-streams first-stream))
                                        (else (let ((pair (stream-head stream-of-pairs)))
109
                                                [pair (lambda ()
111
                                                         (getNext more-streams
                                                                  (add (stream-tail stream-of-pairs) back-streams)
112
113
                                                                   first-stream))])))))))
        (check ((|| (empty-stream? S1) (empty-stream? S2)) empty-stream)
114
                (else (getNext [] [] S1)))))
116
117
   (define (fair-weave stream-list)
     (letrec ((getNext (lambda (front-streams back-streams)
118
                          (match front-streams
119
                            ([] (match back-streams
120
                                   ([] empty-stream)
121
122
                                   (_ (getNext back-streams []))))
                            ((list-of (some-list S) (some-list more)) (check ((empty-stream? S) (getNext more back-streams
123
                                                       (else (let ((x (stream-head S)))
124
                                                               [x (lambda () (getNext more (add (stream-tail S) back-streams
125
         (match stream-list
126
            ([] empty-stream)
127
            (_ (getNext stream-list [])))))
128
130
   (define (flatten-tuple L)
131
     (match L
132
        ((list-of (as L' (list-of _ _)) more) (join (flatten-tuple L') (flatten-tuple more)))
133
        ((list-of x more) (add x (flatten-tuple more)))))
135
  # The following is a generalization of stream-zip that can take any number
136
137
   # of streams as input, packaged in a list:
138
```

```
(define (stream-zip* streams)
      (letrec ((loop (lambda (streams res)
140
                        (match streams
141
142
                          ([] res)
                          ((list-of (some-list S) (some-list more)) (loop more (stream-zip res S))))))))
143
         (match streams
144
           ([] empt.v-st.ream)
145
           ((list-of (some-list S) (some-list more))
              (stream-map flatten-tuple (loop more S))))))
147
148
149
   (define (weave-streams* L)
                        (match L
150
151
                           ([] empty-stream)
152
                           ([(some-list s)] s)
                           (_ (let (([L1 L2] [(even-positions L) (odd-positions L)]))
153
                                 (weave-streams (weave-streams* L1) (weave-streams* L2))))))
154
155
   # Split an infinite stream in two roughly equal parts:
157
158
   (define
159
160
     (stream-even-positions S)
         (check ((empty-stream? S) empty-stream)
161
                (else (stream-odd-positions (stream-tail S))))
162
163
      (stream-odd-positions S)
         (check ((empty-stream? S) empty-stream)
164
                (else [(stream-head S) (lambda () (stream-even-positions (stream-tail S)))])))
165
166
   # Here L is an infinite stream of infinite streams:
167
168
   (define (weave-streams** L)
169
170
      (check ((empty-stream? L) empty-stream)
171
             ((empty-stream? (stream-tail L)) (stream-head L))
             (else (let (([L1 L2] [(stream-even-positions L) (stream-odd-positions L)]))
172
173
                      (weave-streams (weave-streams** L1) (weave-streams** L2))))))
174
   (define (all-from i) [i (lambda () (all-from (plus i 1)))])
176
177
   (define (all-negative-integers-from i) [i (lambda () (all-negative-integers-from (minus i 1)))])
178
179
   (define all-non-negative-integers (all-from 0))
180
181
   (define all-positive-integers (all-from 1))
   (define all-integers-less-than-or-equal-to-zero (all-negative-integers-from 0))
182
183
   (define all-integers (weave-streams-with-probability all-positive-integers all-integers-less-than-or-equal-to-zero 0.8
184
185
   (define all-identifiers (stream-map (lambda (i)
186
187
                                            (string->id (join "x" (val->string i))))
                                          all-positive-integers))
188
189
   (define non-negative-integers (all-from 0))
190
191
192
   (define all-reals
      (stream-map (lambda (pair)
193
                      (match pair
194
                        ([(some-term i1) (some-term i2)] (string->num (join (val->string i1) "." (val->string i2))))))
195
                    (stream-zip non-negative-integers non-negative-integers)))
196
197
198
   (define all-numbers non-negative-integers)
199
   (define (make-var n) (string->var (join "a" (symbol->string n))))
200
201
202
   (define all-variables (stream-map make-var non-negative-integers))
203
   (define (stream-append s1 s2)
      (check ((empty-stream? s1) s2)
205
             (else [(stream-head s1)
206
                     (lambda () (stream-append (stream-tail s1) s2))])))
207
208
```

```
(define (stream-append* stream-list)
      (letrec ((loop (lambda (streams res)
210
                         (match streams
211
212
                          ([] res)
                           ((list-of (some-list stream) (some-list more)) (loop more (stream-append res stream)))))))
213
214
         (loop stream-list empty-stream)))
215
   #(define s (stream-append* [all-numbers all-variables]))
217
   (define st stream-take)
218
219
   (define (stream-tail-k S k)
220
     (letrec ((loop (lambda (S i)
221
                       (check ((leq? i 0) S)
222
                               (else (check ((empty-stream? S) empty-stream)
223
                                             (else (loop (stream-tail S) (minus i 1))))))))
224
        (loop S k)))
225
   (define (stream-shuffle S k)
227
     (let ((chunk (st S k))
228
           (rest (stream-tail-k S k)))
229
       (stream-append (list->stream (rev chunk))
230
231
                       (check ((empty-stream? rest) empty-stream)
                               (else [(stream-head rest) (lambda () (stream-shuffle (stream-tail rest) k))])))))
232
233
   ## Flatten an infinite stream of (finite or infinite) streams into one infinite stream:
234
235
236
   (define (stream-flatten* S)
     (check ((empty-stream? S) empty-stream)
237
238
             (else (let ((S1 (stream-head S)))
                      (check ((empty-stream? S1) (stream-flatten* (stream-tail S)))
239
                              (else [(stream-head S1) (lambda () (stream-flatten* (stream-cons (stream-tail S1) (stream-tai
```