```
(define (for-each L pred?)
1
     (match L
       ([] true)
       ((list-of x xs) (&& (pred? x) (for-each xs pred?)))))
   (define (for-some L pred?)
     (match L
       ([] false)
       ((list-of x xs) (|| (pred? x) (for-some xs pred?)))))
10
11
   (define (negate t)
12
    (match t
      (true false)
13
14
      (false true)))
15
   (define (for-none L pred?)
16
    (for-each L (lambda (x) (negate (pred? x)))))
17
18
19 (define (member? x L)
    (for-some L (lambda (y) (equal? x y))))
20
2 # The following returns the index of the first (leftmost) occurrence of x in L.
^{23} # If x does not occur in L, then false is returned. Index-counting starts with 1.
24
25
  (define (member-index x L)
     (letrec ((search (lambda (L i)
26
                         (match L
27
                           ([] false)
29
                           ((list-of y rest) (check ((equal? x y) i)
30
                                                     (else (search rest (plus i 1))))))))
31
       (search L 1)))
32
   (define (member-eq? x L eq?)
    (match L
34
       ([] false)
35
36
       ((list-of y rest) (check ((eq? x y) true)
                                 (else (member-eq? x rest eq?))))))
37
  (define (subset? L1 L2)
39
    (for-each L1 (lambda (x) (member? x L2))))
41
  (define (subset-eq? L1 L2 eq)
42
43
    (for-each L1 (lambda (x) (member-eq? x L2 eq))))
44
   (define (equal-lists-as-sets L1 L2)
45
    (&& (subset? L1 L2)
46
         (subset? L2 L1)))
47
48
  (define (for-some' list pred?)
49
     (letrec ((loop (lambda (previous rest)
                        (match rest
51
52
                          ([] [])
53
                          ((list-of x more) (check ((pred? x) [(rev previous) x more])
                                                    (else (loop (add x previous) more))))))))
54
        (loop [] list)))
55
  (define (filter L f)
     (letrec ((loop (lambda (L results)
58
                       (match L
59
60
                        ([] (rev results))
                        ((list-of x rest) (check ((f x) (loop rest (add x results)))
61
                                           (else (loop rest results)))))))
       (loop L [])))
63
   (define (filter-out L f)
65
     (filter L (lambda (x) (negate (f x)))))
66
68 (define (filter-and-complement L f)
```

```
(letrec ((loop (lambda (L sat unsat)
                        (match L
                          ([] [sat unsat])
                          ((list-of x rest) (check ((f x) (loop rest (add x sat) unsat))
72
                                                     (else (loop rest sat (add x unsat)))))))))
73
74
        (loop L [] [])))
75
   (define (filter-and-complement-rev L f)
     (letrec ((loop (lambda (L sat unsat)
77
                        (match L
78
                          ([] [(rev sat) (rev unsat)])
79
                          ((list-of x rest) (check ((f x) (loop rest (add x sat) unsat))
80
                                                     (else (loop rest sat (add x unsat))))))))
        (loop L [] [])))
82
83
   (define (map-proc f 1)
84
     (match 1
85
        ([] ())
        ((list-of x rest) (seq (f x) (map-proc f rest)))))
87
88
   (define iter map-proc)
89
90
91
   (define (map-proc2 f 1)
     (letrec ((loop (lambda (L)
92
93
                        (match L
                          ([] ())
94
                          ((list-of x rest) (seq (f x) (loop rest)))))))
95
         (loop 1)))
97
   (define (list-diff L1 L2)
98
     (let ((len1 (length L1))
99
100
            (len2 (length L2)))
        (check ((greater? (plus len1 len2) 100)
101
                  (let ((T (table len2))
102
                         (_ (map-proc (lambda (x) (table-add T [x --> true])) L2))
103
                         (in-L2 (lambda (x) (try (table-lookup T x) false))))
104
                    (filter L1 (lambda (x) (negate (in-L2 x))))))
               (else (filter L1 (lambda (x) (negate (member? x L2)))))))
106
107
108
   (define (remove x L)
     (letrec ((patch (lambda (front back)
109
                         (match front
110
111
                           ([] back)
                           ((list-of y rest) (patch rest (add y back))))))
112
113
               (loop (lambda (L front)
114
                          ((list-of (val-of x) rest) (patch front (loop rest [])))
                          ((list-of y rest) (loop rest (add y front)))
116
117
                          (_ front)))))
        (loop L [])))
118
119
120
   (define (remove x L)
121
122
     (letrec ((patch (lambda (front back)
123
                         (match front
                           ([] back)
124
125
                           ((list-of y rest) (patch rest (add y back))))))
               (loop (lambda (L front)
126
127
                        ({\tt match}\ L
                          ((list-of (val-of x) rest) (patch (rev front) (loop rest [])))
128
                          ((list-of y rest) (loop rest (add y front)))
130
                          (_ (rev front)))))
131
        (loop L [])))
132
   (define (list-remove x L) (remove x L))
133
   ( define (remove-and-preserve-order a L)
135
136
     (letrec ((loop (lambda (rest already-seen)
137
                        (match rest
                          ((list-of x more)
138
```

```
(check ((equal? x a) (loop more already-seen))
                                      (else (loop more (add x already-seen)))))
140
                           (_ (rev already-seen))))))
141
        (loop L [])))
142
143
144
    (define list-remove remove-and-preserve-order)
145
    (define (rd L)
      (letrec ((loop (lambda (L res)
147
                        (match L
148
149
                           ((list-of x more) (check ((member? x res) (loop more res))
                                                      (else (loop more (add x res)))))
150
                           ([] res)))))
151
        (loop L [])))
152
153
    (define (rd-eq L equal?)
154
      (letrec ((loop (lambda (L res)
155
                         (match L
                           ((list-of x more) (check ((member-eq? x res equal?) (loop more res))
157
                                                       (else (loop more (add x res)))))
158
                           ([] res)))))
159
160
        (loop L [])))
161
    (define (remove-duplicates L)
162
163
      (let ((T (table 50))
            (occurs-once (lambda (x)
164
                              (try (table-lookup T x)
165
                                    (seq (table-add T [x false])
166
                                         true)))))
167
        (filter L occurs-once)))
168
169
170
   (define rd remove-duplicates)
171
   (define remove-duplicates-eq rd-eq)
172
173
   (define (map-with-index f L)
174
      (letrec ((loop (lambda (L results i)
175
                         (match L
176
                           ([] (rev results))
177
                           ((list-of \times rest) (loop rest (add (f \times i) results) (plus i 1)))))))
178
        (loop L [] 1)))
179
180
    (define (app-with-index f L)
181
      (letrec ((loop (lambda (L i)
182
183
                         (match L
                           ([] ())
184
185
                           ((list-of x rest) (seq (f x i) (loop rest (plus i 1))))))))
        (loop L 1)))
186
187
   (define (map2 f L1 L2)
188
      (letrec ((loop (lambda (L1 L2 results)
189
                         (match [L1 L2]
190
                           ([[] _] (rev results))
191
192
                           ([_ []] (rev results))
                           ([(list-of x xs) (list-of y ys)]
193
                            (loop xs ys (add (f x y) results)))))))
194
        (loop L1 L2 [])))
195
196
   # (map2 add [1 2 3] [[4 5] [6 7] [8 9 10]]) = [[1 4 5] [2 6 7] [3 8 9 10]]
197
   # (map2 add [1] [[4 5] [6 7] [8 9 10]])
                                                = [[1 4 5]]
198
   # (map2 add [] [[4 5] [6 7] [8 9 10]])
                                                    = []
200
  # (map2 add [1 2 3] [])
   # To test the above:
201
   # (let ((a 'a) (b 'b) (c 'c) (d 'd) (e 'e) (f 'f))
202
203
        (seq
         (writeln-val (map2 add [1 2 3] [[4 5] [6 7] [8 9 10]]))
         (writeln-val\ (map 2\ add\ [1]\ [[4\ 5]\ [6\ 7]\ [8\ 9\ 10]]))
   #
205
         (writeln-val (map2 add [] [[4 5] [6 7] [8 9 10]]))
206
         (writeln-val (map2 add [1 2 3] []))))
207
   #
208
```

```
define (non-empty? L) := (match L ((list-of _ _) true) ([] false))
210
   (define (map-select f L pred)
211
212
     (letrec ((loop (lambda (L results)
                        (match L
213
214
                          ([] (rev results))
                          ((list-of x rest) (let ((res (f x)))
215
                                               (check ((pred res) (loop rest (add res results)))
217
                                                      (else (loop rest results))))))))
         (loop L [])))
218
219
   (define (map-select-2 f L pred)
220
     (letrec ((loop (lambda (L results)
221
222
                       (match L
223
                         ([] (rev results))
                          ((list-of x rest) (check ((pred x) (loop rest (add (f x) results)))
224
                                                    (else (loop rest results))))))))
225
         (loop L [])))
227
   (define map-list map)
228
229
230
   (define (zip L1 L2)
     (letrec ((f (lambda (L1 L2 res)
231
232
                    (match [L1 L2]
233
                       ([(list-of x1 rest1) (list-of x2 rest2)] (f rest1 rest2 (add [x1 x2] res)))
234
                       (_ (rev res))))))
        (f L1 L2 [])))
235
236
   (define list-zip zip)
237
238
   (define (unzip L)
239
     (letrec ((loop (lambda (L L1 L2)
241
                        (match L
                          ([] [(rev L1) (rev L2)])
242
243
                          ((list-of [x y] rest) (loop rest (add x L1) (add y L2)))))))
        (loop L [] [])))
244
## that takes an input x (usually a sentence) and produces
247
248
   ## a theorem. We apply M to all elements of the input list L,
   ## collecting the results along the way, ultimately passing them
249
   ## to the continuation K. Note that an error will result if
251
   ## the application of M to an element of the list L fails.
252
   (define (map-method M L K)
253
     (dletrec ((loop (method (L res)
254
255
                         (dmatch L
                          ([] (!K (rev res)))
256
257
                           ((list-of x rest) (dlet ((th (!M x))))
                                               (!loop rest (add th res))))))))
258
        (!loop L [])))
259
   ## Here the input method M is also a regular method
261
   \#\# that takes an input x (usually a sentence) and produces
   ## a theorem, but we don't fail if applying M to an
264 ## element of the list L produces an error. Conceivably,
265 ## M could fail on every element of L, in which case we
   ## will simply pass the empty list to the continuation K:
266
267
   (define (map-method-non-strictly M L K)
268
     (dletrec ((loop (method (L res)
270
                        (dmatch L
                           ([] (!K (rev res)))
271
272
                           ((list-of x rest) (dlet ((ok? (cell false))
                                                     (th (dtry (dlet ((res (!M x))
273
                                                                       (_ (set! ok? true)))
                                                                 (!claim res))
275
276
                                                               (!true-intro)))
                                                     (res' (check ((ref ok?) (add th res))
277
                                                                   (else res))))
278
```

```
(!loop rest res')))))))
         (!loop L [])))
280
281
282
   (define (map-method-non-strictly-2 M L K)
     (dletrec ((loop (method (L res productive)
283
284
                         (dmatch L
                           ([] (!K (rev res) productive))
285
                           ((list-of x rest) (dlet ((ok? (cell false))
                                                     (th (dtry (dlet ((res (!M x))
287
                                                                       (_ (set! ok? true)))
288
289
                                                                  (!claim res))
                                                                (!true-intro)))
290
                                                     ([res' productive'] (check ((ref ok?) [(add th res) (add x productive)]
291
                                                                                  (else [res productive]))))
292
                                                (!loop rest res' productive')))))))
293
         (!loop L [] [])))
294
295
   \#\# map-methods-non-strictly takes a list of methods [M\_1 \ldots M\_k] and (non-strictly) applies
   ## each M_i to the input list L, keeping track of the results along the way. All results
297
   ## thereby obtained are ultimately passed to the continuation K:
298
299
300
   (define (map-methods-non-strictly method-list L K)
     (dletrec ((loop (method (methods all-results)
301
                         (dmatch methods
302
303
                           ([] (!K all-results))
                           (({f list-of} M rest) (!map-method-non-strictly M L
304
                                                 (method (new-results)
305
                                                   (!loop rest (join new-results all-results)))))))))
306
         (!loop method-list [])))
307
308
309
310
   (define (map-methods-non-strictly-2 method-list L K)
311
     (dletrec ((loop (method (methods all-results productive)
312
                         (dmatch methods
313
                           ([] (!K all-results (remove-duplicates productive)))
314
                           ((list-of M rest) (!map-method-non-strictly-2 M L
                                                 (method (new-results new-productive)
316
                                                   (!loop rest (join new-results all-results)
317
                                                                (join new-productive productive)))))))))
318
         (!loop method-list [] [])))
319
320
321
   ## map-methods-non-strictly* is similar, but iterates this process up to a fixed
   ## point, or until the number of iterations exceeds the limit specified by max-iterations,
322
323
   ## whichever occurs first. (The parameter max-iterations is optional. The unit value,
324 ## or any other non-numeric value can be given to it, in which case the method will
325 ## iterate up to a fixed point.) Specifically, first we apply all methods to the input
   ## list L, obtaining the first generation of results. Then we apply the methods to
326
   ## those results, obtaining a second generation of results, and so on. A fixed point
   ## is reached when the latest yield of results does not give us anything we haven't
328
   ## already obtained in some previous generation. Finally, all the results are
   ## cleaned up (removing duplicates, etc.) and passed to the continuation K:
331
332
   (define (map-methods-non-strictly* method-list L K max-iterations)
     (dlet ((test (match max-iterations
333
334
                     (x:Int (lambda (j) (greater? j max-iterations)))
335
                     (_ (lambda (_) false)))))
        (dletrec ((loop (method (latest-results all-results i)
336
                           (!map-methods-non-strictly method-list latest-results
337
                              (method (new-results)
338
                                (dcheck ((|| (test i) (subset? new-results all-results))
339
340
                                           (!K (list-diff (remove-duplicates all-results) L)))
                                        (else (!loop new-results (join new-results all-results) (plus i 1)))))))))
341
342
           (!map-methods-non-strictly method-list L
             (method (first-results)
343
               (!loop first-results first-results 1))))))
345
346
   (define (map-methods-non-strictly-2* method-list L K max-iterations)
347
     (dlet ((test (match max-iterations
                     (x:Int (lambda (j) (greater? j max-iterations)))
348
```

```
(_ (lambda (_) false)))))
        (dletrec ((loop (method (latest-results all-results productive i)
350
                           (!map-methods-non-strictly-2 method-list latest-results
352
                              (method (new-results new-productive)
                                (dcheck ((|| (test i) (subset? new-results all-results))
353
                                          (!K (list-diff (remove-duplicates all-results) (join L productive))))
354
                                        (else (!loop new-results (join new-results all-results)
355
                                                                   (join new-productive productive) (plus i 1))))))))
           (!map-methods-non-strictly-2 method-list L
357
             (method (first-results productive)
358
359
               (!loop first-results first-results productive 1))))))
360
361
   ## A "multi-method" is one that takes an input x and produces a *list*
362
   ## of theorems. For that reason, a multi-method must be binary: it takes
363
   \#\# a continuation K as a second argument, and passes the list of its results
365 ## to it when it is done. A typical example of a multi-method is a conjunction
   ## elimination method that returns both the left and the right component
   ## of a given conjunction:
367
368
   ##
   ##
369
370
   ## (define (conj-elim p K)
371 ##
        (dmatch p
           ((and _ _) (dlet ((p1 (!left-and p))
   ##
372
373
   ##
                               (p2 (!right-and p)))
   ##
                          (!K [p1 p2])))
374
375
            (_ (!K []))))
376
   ##
   ## Note that multi-methods should be written to be fail-safe, so that
377
   ## if an error occurs, the empty list of theorems is passed to the given
   ## continuation (instead of halting execution with an error). This is
379
   ## the reason why conj-elim was written as above with pattern matching,
381 ## rather than as follows:
   ##
382
383
   ## (define (conj-elim p K)
        (dlet ((pl (!left-and p))
   ##
384
   ##
                (p2 (!right-and p)))
385
           (!K [p1 p2])))
   ##
386
   ##
387
   ## Of course, the first way of writing the method is not 100% fail-safe
388
   ## either. E.g., if it's applied to a conjunction that is not in the a.b.,
389
   ## we'll get an error. We can make a multi-method completely safe
   ## as follows:
391
392
   ##
   ## (define (make-multi-method-safe M)
393
   ##
       (method (L K)
394
395
   ##
           (dtry (!M L K)
   ##
                 (!K []))))
396
   ##
   ## However, use of make-multi-method-safe is inefficient and not recommended.
398
399
   ## "map-multi-method" below must be used to map a multi-method M to a list:
400
   ##
401
402
   (define (map-multi-method M L K)
403
     (dletrec ((loop (method (L results)
404
405
                        (dmatch L
                           ([] (!K (rev results)))
406
                           ((list-of x rest) (!M x (method (new-results)
407
408
                                                      (!loop rest (join new-results results)))))))))
         (!loop L [])))
410
   ## A list method is one that can be applied to a list of inputs
411
   ## and return a list of theorems. Thus a list method must be binary,
412
   ## taking a continuation K as its second argument. The list of resulting
413
414 ## theorems is eventually passed to the continuation. Any regular method
   ## (taking an input x and producing a theorem) can be turned into
415
416
   ## a list method as follows:
417
   (define (make-list-method M)
418
```

```
419
      (method (L K)
        (!map-method-non-strictly M L K)))
420
421
422
    (define (map-multi-method* M L K max)
423
424
      (dletrec ((loop (method (latest-results all-results i)
                           (!map-multi-method M latest-results
425
                               (method (new-results)
                                (dcheck ((|| (subset? new-results all-results)
427
                                               (less? max i)
428
429
                                               (equal? max i))
                                            (!K (remove-duplicates all-results)))
430
                                          (else (!loop new-results (join new-results all-results) (plus i 1)))))))))
431
        (!map-multi-method M L
432
          (method (results)
433
            (!loop results results 1)))))
434
435
437
   (define (fold f lst id)
438
      (match 1st
439
440
        ([] id)
441
        ([x] (f x id))
        ((list-of x (list-of y more))
442
443
           (fold f (add (f x y) more) id))))
444
   (define (foldl f e l)
445
446
      (match 1
        ([] e)
447
448
        ((list-of x xs) (foldl f (f e x) xs))))
449
   (define (foldr f e l)
      (match 1
451
        ([] e)
452
        ((list-of x rest) (f x (foldr f e rest)))))
453
454
   define (flatten L) := (foldl join [] L)
456
457
   (define (nth i 1)
458
      (match [i 1]
459
460
        ([1 (list-of x _)] x)
461
        (_ (nth (minus i 1) (tail 1)))))
462
   ## The above gets into an infinite loop for negative indices.
463
   ## ith below fixes that. It also takes the list as the first
464
465
   ## argument, the index
466
467
   (define (ith L i)
      (check ((greater? i 1) (ith (tail L) (minus i 1)))
468
              ((equal? i 1) (head L))))
469
470
   (define at ith)
471
472
   (set-precedence at 107)
473
474
   (define (nth i L) (ith L i))
475
476
477
    (define (nth-tail 1 n)
      (match n
478
479
        (0 \ 1)
        (_ (nth-tail (tail 1) (minus n 1)))))
480
481
482
    (define (decompose-nth L n)
      (letrec ((loop (lambda (L L' i ith)
483
484
                         (match [i L]
                           ([1 (list-of x rest)] (loop rest L' (minus i 1) x))
485
                           ([_ (list-of x rest)] (loop rest (add x L') (minus i 1) ith))
486
                           ([_ []] [ith L'])))))
487
        (loop L [] n ())))
488
```

```
(define (prefix? s1 s2)
490
      (match [s1 s2]
491
492
        ([[] _] true)
        ([(list-of x rest1) (list-of x rest2)] (prefix? rest1 rest2))
493
494
        (_ false)))
495
   (define (find-min-rest 1 fun)
      (letrec ((f (lambda (rem rest min i)
497
                     (match rem
498
499
                       ([] [min rest])
                       ((list-of x more) (check
500
                                             ((greater? i 0) (check ((less? (fun x) (fun min)) (f more (add min rest)
501
                                                                 x (plus i 1)))
502
                                                                       (else (f more (add x rest) min (plus i 1)))))
503
                                             (else (f more [] x (plus i 1))))))))
504
        (check ((null? 1) ()) (else (f 1 [] () 0)))))
505
   (define (non-null L) (negate (null? L)))
507
508
   (define (take 1 n)
509
510
      (letrec ((f (lambda (l n res)
                     (match [l n]
511
                       ([_ 0] (rev res))
512
513
                       ([[] _] (rev res))
                        (_ (f (tail 1) (minus n 1) (add (head 1) res)))))))
514
        (f l n [])))
515
516
   (define (drop L n)
517
      (check ((less? n 1) L)
518
             (else (drop (tail L) (minus n 1)))))
519
520
   (define (split-list l n)
521
      (letrec ((f (lambda (l n res)
522
523
                     (match [l n]
                       ([_ 0] [(rev res) 1])
524
                       ([[] _] [(rev res) 1])
525
                        (_ (f (tail 1) (minus n 1) (add (head 1) res)))))))
526
        (f l n [])))
527
528
   (define (cut-in-half L)
529
530
      (split-list L (div (length L) 2)))
531
   (define (from-to i j)
532
533
      (letrec ((loop (lambda (k res)
                         (check ((less? k i) res)
534
                                 (else (loop (minus k 1) (add k res)))))))
         (check ((less? j i) [])
536
537
                 (else (loop j [])))))
538
   (define to from-to)
539
540
   (define (quant* Q var-list P)
541
542
     (match var-list
      ([] P)
543
       ((list-of v more-vars) (Q v (quant* Q more-vars P)))))
544
545
   (define (forall* vl P) (quant* forall vl P))
546
547
   (define (exists* vl P) (quant* exists vl P))
548
549
550
   (define (close x)
551
      (match x
552
        ((some-sentence p) (forall* (free-vars p) p))
        ((some-list L) (map close L))
553
554
        (_ x)))
555
556
   (define (make-selector-axioms S)
557
      (let ((constructors (constructors-of S))
            (selectors-of (lambda (con) (map-select-2 string->symbol
558
```

```
(selector-names con)
                                                            (lambda (str) (negate (null? str))))))
560
            (make-straight-axioms
561
562
                (lambda (con)
                  (let ((range (from-to 1 (arity-of con)))
563
                         (vars (map (lambda (_) (fresh-var)) range))
564
                         (con-term (make-term con vars))
565
                         (sel-vars (zip (selectors-of con) vars)))
567
                    (map (lambda (sel-var-pair)
                            (match sel-var-pair
568
                              ([sel v] (close (= (sel con-term) v)))))
569
                          sel-vars))))
570
            (make-option-axioms
571
                (lambda (con)
572
                  (let ((range (from-to 1 (arity-of con)))
573
574
                         (vars (map (lambda (_) (fresh-var)) range))
                         (con-term (make-term con vars))
575
                         (selectors (selectors-of con))
                         (sel-vars (zip selectors vars))
577
                         (other-constructors (filter constructors (lambda (c) (negate (equal? c con)))))
578
                         (main-axioms (map (lambda (sel-var-pair)
579
                                               (match sel-var-pair
580
                                                ([sel v] (close (= (sel con-term) ((string->symbol "SOME") v))))))
581
                                             sel-vars))
582
583
                         (option-axioms (lambda (sel)
584
                                             (map (lambda (c)
                                                      (let ((con-term (make-term c (map (lambda (_) (fresh-var)) (from-to 1 (ar
585
                                                        (close (= (sel con-term) (string->symbol "NONE")))))
586
                                                  other-constructors)))
587
                        (all-option-axioms (flatten (map option-axioms selectors))))
588
                   (join main-axioms all-option-axioms)))))
589
       (match (struc-with-option-valued-selectors? S)
590
591
         (true (flatten (map make-option-axioms constructors)))
         (_ (flatten (map make-straight-axioms constructors))))))
592
593
594
   (define selector-axioms make-selector-axioms)
595
596
   (define (upper-case-alpha-char? c)
597
      (&& (member? (compare-chars c 'A) ['greater 'equal])
598
          (member? (compare-chars c 'Z) ['less 'equal])))
599
600
601
   (define (downcase c)
      (check ((upper-case-alpha-char? c) (char (plus (char-ord c) 32)))
602
603
              (else c)))
604
   (define (downcase-string str)
      (map downcase str))
606
607
   (define (separate L token)
608
      (match L
609
       ([] "")
610
        ([s] s)
611
612
        ((list-of s1 (bind rest (list-of _ _)))
613
          (join s1 token (separate rest token)))))
614
615
   (define (auto-induction-definition structure-name)
       (\textbf{let} \ ( (\texttt{prequel} \ (\texttt{join} \ \texttt{"} (\texttt{define} \ (\texttt{"} \ ( \texttt{downcase-string} \ \texttt{structure-name}) \ \texttt{"-induction} \ \texttt{goal-property}) \\ \\ \texttt{`n"})) 
616
            (make-constructor-line
617
               (lambda (c)
618
                  (let ((arity (arity-of c))
620
                         (pattern (check ((less? arity 1)
                                              (join "(bind state " (symbol->string c) ")"))
621
                                           (else (join "(bind state (" (symbol->string c) " " (separate (map (lambda (_) "_")
622
                         (body (join "(!prove (goal-property state))")))
623
                    (join "(" pattern " " body ")"))))
            (lines (separate (map make-constructor-line (constructors-of structure-name)) "\n"))
625
           (cmd (join prequel "\n (by-induction (forall ?x (goal-property ?x))\n" lines "))")))
626
627
        (process-input-from-string cmd)))
628
```

```
630
631
   (make-private "auto-induction-definition")
632
633
   # New additions, July 05 2015:
634
635
   # (datatype (List-Of T)
      Ni 7
637
        (Cons hd:T tl:(List-Of T)))
638
639
   #datatype (List T) := nil | (cons hd:T tl:(List T))
640
641
   datatype (List T) := nil | (:: hd:T tl:(List T))
642
643
   define [Nil Cons] := try { [nil ::] | [nil cons] }
644
645
    (define (makeList L)
      (match L
647
       ([] Nil)
648
       ((list-of x rest) (Cons x (makeList rest)))))
649
650
   (define (alist->list L)
652
653
      (match L
        ([] (Nil))
654
        ((list-of x rest) (Cons x (alist->list rest)))
655
656
        (_ L)))
657
658
   (define (alist->list inner)
659
660
      (letrec ((loop (lambda (L)
661
                         (match L
                           ([] (Nil))
662
                           ((list-of x rest) (Cons (inner x) (loop rest)))
663
                            (_ L)))))
664
         loop))
666
   (define (list->alist L)
667
668
      (match L
        (Nil [])
669
        ((Cons x rest) (add x (list->alist L)))
670
671
        (_ L)))
672
   (define (list->alist inner)
673
      (letrec ((loop (lambda (L)
674
675
                         (match L
                           (Nil [])
676
677
                           ((Cons x rest) (add (inner x) (loop rest)))
678
                            (_ L)))))
         loop))
679
680
681
682
   (define
      (even-positions L)
683
        (match L
684
          ((list-of _ rest) (odd-positions rest))
685
686
          (_ []))
687
      (odd-positions L)
688
        (match L
          ((list-of x rest) (add x (even-positions rest)))
690
          (_ [])))
691
   (define (merge L1 L2 less?)
692
      (match [L1 L2]
693
        ([(list-of x rest1) (list-of y rest2)]
             (\mathbf{check}\ ((less?\ x\ y)\ (add\ x\ (merge\ rest1\ L2\ less?)))
695
696
                     (else (add y (merge rest2 L1 less?)))))
        ([[] _] L2)
697
        (_ L1)))
698
```

```
(define (merge-sort' L less?)
700
      (match L
701
        ((list-of _ (list-of _ rest)) (merge (merge-sort' (odd-positions L) less?)
702
                                                (merge-sort' (even-positions L) less?))
703
704
705
   (define (merge-sort L less?)
      (letrec ((sort (lambda (L)
707
708
709
                          ((list-of _ (list-of _ _))
                             (let (([L1 L2] (cut-in-half L)))
710
                                (merge (sort L1) (sort L2) less?)))
711
712
                          (_ L)))))
        (sort L)))
713
714
   (define (skip-until list pred)
715
716
      (match list
        ([] [])
717
        ((list-of x rest) (check ((pred x) list)
718
                                   (else (skip-until rest pred))))))
719
720
721
   (define (skip-until' list pred)
722
723
      (letrec ((loop (lambda (L so-far)
                        (match T
724
                           ([] [(rev so-far) []])
725
726
                           ((list-of c more)
                                (check ((pred c) [(rev so-far) L])
727
                                       (else (loop more (add c so-far)))))))))
728
        (loop list [])))
729
731
732
733
   (define (tokenize L delims)
     (let ((pred (lambda (c)
734
                     (member? c delims)))
            (pred' (lambda (c) (negate (member? c delims))))
736
            (add' (lambda (str tokens)
737
738
                      (check ((null? str) tokens)
                             (else (add str tokens))))))
739
        (letrec ((loop (lambda (L tokens)
740
741
                           (match L
742
                               ([] (rev tokens))
                               (_ (let (([_ rest] (skip-until' L pred'))
743
                                        ([token more] (skip-until' rest pred)))
744
745
                                    (loop more (add' token tokens))))))))
          (loop L []))))
746
   (define tokenize-string tokenize)
748
749
   \# (first-image-that-works L f) returns (f x) for the first (leftmost) element x
   \# of L such that (f x) is defined (its computation doesn't lead to an error).
751
   # If L has no such x, (first-image-that-works L f) fails in error:
753
   define (first-image L f) :=
754
755
     match L {
        (list-of x rest) => try { (f x) | (first-image rest f) }
756
757
758
   (define (find-element L pred success failure)
     (letrec ((search (lambda (list)
760
                          (match list
761
762
                             ([] (failure))
                            ((list-of x rest) (check ((pred x) (success x))
763
                                                        (else (search rest))))))))
         (search L)))
765
766
       (find-element' L pred f success failure) finds the first element
767
   \#\# x of L such that (pred (f x)) holds, if such x exists in L.
```

```
## If so, then (success (f x)) is called, otherwise (failure) is called.
770
   (define (find-element' L pred f success failure)
     (letrec ((search (lambda (list)
772
                          (match list
773
774
                             ([] (failure))
                            ((list-of x rest)
775
                                (let ((res (f x)))
                                  (check ((pred res) (success res))
777
                                         (else (search rest)))))))))
778
779
         (search L)))
780
   (define find-some-element find-element')
782
   \#\# (find-first L f) finds the first (leftmost) member x of list L such that
783
   \#\# (f x) returns something - call it y - other than false. That result, y,
784
   ## is the result of (find-first L pred). If L contains no such x, then an error occurs:
785
   (define (find-first L f)
787
     (match L
788
       ((list-of x rest) (match (f x)
789
790
                             (false (find-first rest f))
                              (res res)))))
791
792
793
   # find-first' works like find-first, except that if there is no element x in L
   # such that (f x) is non-false, then the nullary continuation K is invoked:
794
795
796
   (define (find-first' L f K)
     (match L
797
        ((list-of x rest) (match (f x)
798
                             (false (find-first' rest f K))
799
                             (res res)))
        (_ (K))))
801
802
803
804
   (define (find-list-member L pred? success failure)
     (dmatch L
806
        ([] (!failure))
807
        ((list-of x rest) (dcheck ((pred? x) (!success x))
808
                                    (else (!find-list-member rest pred? success failure))))))
809
811
   (define (find-some L M K)
812
813
     (dmatch L
       ([] (!K))
814
        ((list-of x rest) (dtry (!M x)
                                  (!find-some rest M K)))))
816
818
819
  # (find-in-list L P) takes finds the first element y in L such
820 # that y has the unary property P. In addition, the lists to the
   # left and right of y (in L) are also returned. E.g.,
821
   # (find-in-list [0 6 3 4 10] odd?) returns [[0 6] 3 [4 10]].
   # If no such y is found, the unit () is returned.
823
825
   (define (find-in-list L P)
     (letrec ((loop (lambda (L prefix)
826
                        (\mathtt{match}\ \mathtt{L}
827
828
                          ([] ())
                          ((list-of x more) (check ((P x) [(rev prefix) x more])
830
                                                     (else (loop more (add x prefix)))))))))
831
        (loop L [])))
832
   (define (find-decomposition L P)
833
      (letrec ((loop (lambda (L prefix)
                        (match L
835
836
                          ([] ())
                          ((list-of x more) (check ((P prefix x more) [prefix x more])
837
                                                     (else (loop more (join prefix [x]))))))))
838
```

```
(loop L [])))
840
   (define (max i j)
842
     (check ((less? i j) j)
843
844
             (else i)))
845
   (define (min i j)
    (check ((less? i j) i)
847
             (else j)))
848
849
   (define (min-or-max L seed f)
850
      (\textbf{match} \ \bot
851
       ([] seed)
852
        ((list-of x rest) (min-or-max rest (f seed x) f))))
853
854
   (define (generic-list-min L generic-min)
855
      (match L
       ([] ())
857
        (_ (min-or-max L (head L) generic-min))))
858
859
   (define (list-max L)
860
861
     (match L
       ([] ())
862
863
        (_ (min-or-max L (head L) max))))
864
   (define (list-min L)
865
866
      (match L
       ([] ())
867
        (_ (min-or-max L (head L) min))))
868
869
   (define (max* L)
871
     (match L
872
        ([] 0)
873
        (_ (list-max L))))
874
   (define (intersection A B)
876
877
     (letrec ((loop (lambda (A B result)
878
                         (match A
                           ([] result)
879
880
                           ((list-of x Amore)
                            (check ((&& (member? x B) (negate (member? x result)))
881
                                     (loop Amore B (add x result)))
882
883
                                    (else (loop Amore B result))))))))
        (loop A B [])))
884
886
   # define (fast-intersection A B) :=
   # let {T := (list->table A) }
888
         (filter B
889
                 lambda (x) (x HashTable.in T))
890
   #
891
   # (define (fast-intersection* lists)
892
       (match lists
893
         ([] [])
894
895
          ([L] L)
          ((list-of L more)
896
897
           (fast-intersection L (fast-intersection* more)))))
898
   (define list-intersection intersection)
900
   (define (intersection* lists)
901
902
     (match lists
       ([] [])
903
        ([L] L)
       ((list-of L more)
905
906
         (intersection L (intersection* more)))))
907
908 (define (fast-intersection L1 L2)
```

```
(let ((T (table 10))
            (_ (map-proc (lambda (x) (table-add T [x --> true])) L1)))
910
        (filter L2 (lambda (x) (try (table-lookup T x) false)))))
911
912
   (define (all-but-last L)
913
914
      (match L
        ([] [])
915
        ([x] [])
916
        ((list-of x rest) (add x (all-but-last rest)))))
917
918
   (define (find L pred success-cont failure-cont)
919
      (letrec ((loop (lambda (L)
920
                         (match L
921
                           ([] (failure-cont))
922
                           ((list-of x rest) (check ((pred x) (success-cont x))
923
924
                                                       (else (loop rest))))))))
         (loop L)))
925
   (define (find-list-element L pred success-cont failure-cont)
927
      (letrec ((loop (lambda (L)
928
                         (match To
929
930
                           ([] (failure-cont))
                           ((list-of x rest) (check ((pred x) (success-cont x))
931
                                                       (else (loop rest))))))))
932
933
         (loop L)))
934
   (define (list->string L sep)
935
936
      (\mathtt{match}\ \ \mathsf{L}
        ([] "")
937
        ([x] (val->string x))
938
        ((list-of x (bind rest (list-of _ _)))
939
          (join (val->string x)
941
                 sep
                 (list->string rest sep)))))
942
943
   (define (weave x L)
944
      (letrec ((loop (lambda (front back results)
945
                         (match back
946
                           ([] (rev (add (join front [x]) results)))
947
                           ((list-of y rest) (loop (join front [y])
948
                                                      rest
949
950
                                                      (add (join front [x] back) results))))))
951
         (loop [] L [])))
952
953
   (define (cprod L1 L2)
954
955
      (let ((f (lambda (x)
                  (map (lambda (y) [x y]) L2))))
956
957
       (flatten (map f L1))))
958
   (define (all-list-elements L)
959
960
       (letrec ((loop (lambda (L results)
                           (match L
961
962
                             ([] (rev results))
                             ((list-of (some-list L') rest)
963
                                  (let ((values (all-list-elements L')))
964
965
                                     (loop rest (join (rev values) results))))
                             ((list-of x rest)
966
967
                                 (loop rest (add x results)))))))
          (loop L [])))
968
969
970
   (define (cprod* lists)
      (letrec ((loop (lambda (lists)
971
972
                         (match lists
                           ([L] L)
973
974
                           ((list-of L rest) (cprod L (loop rest)))))))
        (match lists
975
976
           ([L] (map (lambda (x) [x]) L))
           (_ (map all-list-elements (loop lists))))))
977
978
```

```
(define X cprod)
980
    (define (cprod-k L1 L2 k)
981
      (check ((less? k 3) (cprod L1 L2))
982
              (else (let ((f (lambda (x)
983
                                   (map (lambda (y) (add x y))
984
                                        (cprod-k L1 L2 (minus k 1))))))
985
                       (flatten (map f L1))))))
987
    (define (iterate f x k)
988
989
      (check ((less? k 1) x)
              (else (iterate f (f x) (minus k 1)))))
990
991
992
    (define (cprods L)
993
      (letrec ((loop (lambda (lists)
994
                           (match lists
995
                              ((list-of L more) (cprod L (loop more)))))))
997
         (let ((k (length L))
998
                (f (lambda (cp)
999
                       (map (lambda (L)
1000
1001
                                (flatten (map (lambda (x)
                                                   (match x
1002
1003
                                                     ((some-list _) x)
1004
                                                     (_ [x])))
                                                 L)))
1005
1006
                             cp))))
           (match L
1007
             ([only-list] [only-list])
1008
             (_ (iterate f (loop L) k)))))
1009
1010
    (define (all-distinct-pairs L)
1011
      (let ((all-pairs (cprod L L)))
1012
1013
         (letrec ((loop (lambda (L res)
                             (match L
1014
                               ([] res)
                               ((list-of (as p [x y]) more)
1016
                                   ( check ((&& (negate (equal? x y)) (negate (member? [y x] res)))
1017
1018
                                             (loop more (add p res)))
                                           (else (loop more res))))))))
1019
            (loop all-pairs []))))
1020
1021
1022
    (define (occ-num x L)
1023
      (letrec ((loop (lambda (L res)
1024
1025
                          (match L
                            ([] res)
1026
1027
                            (((list-of y more) where (equal? x y)) (loop more (plus res 1)))
                            ((list-of y more) (loop more res))))))
1028
        (loop L 0)))
1029
1030
1031
1032
    define (find-first-element M list) :=
      match list {
1033
         (list-of first rest) =>
1034
          try { (!M first) | (!find-first-element M rest) }
1035
1036
1037
    define (list->counts L) :=
1038
1039
      letrec {loop := lambda (L M)
1040
                          match L {
                             [] => M
1041
                           | (list-of x rest) => (loop rest (Map.add M [[x try { (1 plus (M x)) | 1}]]))
1042
                           } }
1043
        (loop L (Map.make []))
```