```
(load-file "msr.ath")
2 (load-file "rewriting.ath")
  (define ars rewrite-search)
5 (define gr get-all-rewrites)
  (define R1 (commutative +))
8 (define R2 (associative +))
9 (define R3 (commutative *))
10 (define R4 (associative *))
12 (define rules [R1 R2 R3 R4])
13 (assert rules)
14
15 (define (test-df s t)
    (let ((t1 (time))
16
           (_ (!drs-df s t rules rewrite-one-redex))
17
           (t2 (time))
18
           (time-spent (minus t2 t1)))
19
       (println (join "Time: " (val->string time-spent) " seconds"))))
20
22 (define (test-rf s t)
    (let ((t1 (time))
24
           (_ (!drs-rf s t rules rewrite-one-redex))
           (t2 (time))
25
           (time-spent (minus t2 t1)))
      (println (join "Time: " (val->string time-spent) " seconds"))))
27
29 (define (test-bf s t)
    (let ((t1 (time))
30
31
           (_ (!drs-bf s t rules rewrite-one-redex))
           (t.2 (time))
32
           (time-spent (minus t2 t1)))
       (println (join "Time: " (val->string time-spent) " seconds"))))
34
35
  (define (test-pairs L)
37
     (let ((failures (cell 0))
          (sum (cell 0))
39
           (pair-num (length L))
           (_ (print "\nHave " pair-num " equations to test...\n"))
41
           (index (cell 0))
42
43
           (test-bf (lambda (s t)
                       (let ((t1 (time))
44
                             (_ (set! index (plus (ref index) 1)))
45
                             (res (dtry (!drs-bf s t rules rewrite-one-redex) (!true-intro)))
46
                             (t2 (time))
48
                             (time-spent (minus t2 t1)))
                        (match res
49
                          (true (seq (print "\nFAILED test #" (ref index) "\n")
                                     (set! failures (plus (ref failures) 1))))
51
                          (_ (seq (print "\nFinished test #" (ref index) " in " time-spent " seconds...\n")
                                 (set! sum (plus (ref sum) time-spent))))))))
53
           (_ (map-proc (lambda (pair) (match pair ([s t] (test-bf s t)))) L))
54
           (successes (minus pair-num (ref failures)))
           (av-time (div (ref sum) successes)))
       (print "\nManaged all except for " (ref failures) " tests.\nAverage proof time: " av-time "\n")))
58
59
  (define (dtest-pairs L)
    (let ((failures (cell 0))
61
           (sum (cell 0))
           (pair-num (length L))
63
           (_ (print "\nHave " pair-num " equations to test...\n"))
64
           (index (cell 0))
65
           (test-bf (lambda (s t)
66
                       (let ((t1 (time))
                             (_ (set! index (plus (ref index) 1)))
68
```

```
(res (dtry (!drs-df s t rules rewrite-one-redex) (!true-intro)))
                           (t.2 (time))
70
                           (time-spent (minus t2 t1)))
72
                       (match res
                        (true (seq (print "\nFAILED test #" (ref index) "\n")
73
74
                                   (set! failures (plus (ref failures) 1))))
                         (_ (seq (print "\nFinished test #" (ref index) " in " time-spent " seconds...\n")
75
                                (set! sum (plus (ref sum) time-spent))))))))
           (_ (map-proc (lambda (pair) (match pair ([s t] (test-bf s t)))) L))
77
           (successes (minus pair-num (ref failures)))
78
79
           (av-time (div (ref sum) successes)))
       (print "\nManaged all except for " (ref failures) " tests.\nAverage proof time: " av-time "\n")))
80
82 (define (btest-pairs L)
    (let ((failures (cell 0))
83
84
           (sum (cell 0))
           (pair-num (length L))
85
           (_ (print "\nHave " pair-num " equations to test...\n"))
           (index (cell 0))
87
           (test-bf (lambda (s t)
88
                     (let ((t1 (time))
89
90
                           (_ (set! index (plus (ref index) 1)))
                           (res (dtry (!drs-rf s t rules rewrite-one-redex) (!true-intro)))
91
                           (t2 (time))
92
93
                           (time-spent (minus t2 t1)))
                       (match res
94
                        (true (seq (print "\nFAILED test #" (ref index) "\n")
95
                                   (set! failures (plus (ref failures) 1))))
                        (_ (seq (print "\nFinished test #" (ref index) " in " time-spent " seconds...\n")
97
                                (set! sum (plus (ref sum) time-spent))))))))
           (_ (map-proc (lambda (pair) (match pair ([s t] (test-bf s t)))) L))
99
           (successes (minus pair-num (ref failures)))
101
           (av-time (div (ref sum) successes)))
       (print "\nManaged all except for " (\mathbf{ref} failures) " tests.\nAverage proof time: " av-time "\n")))
102
103
104
   (define (test s t)
     (let ((res (ars s t [R1 R2] "depth-first" 50000 "silent")))
106
107
108
  109
                                       Some tests:
110 #
112
113 (define s1 (+ ?a (+ ?b ?c)))
114 (define t1 (+ ?a (+ ?c ?b)))
115 (define t2 (+ (+ ?b ?c) ?a))
116 (define t3 (+ (+ ?a ?b) ?c))
117
   (define t4 (+ (+ ?b ?a) ?c))
118 (define t5 (+ ?c (+ ?b ?a)))
119
   (rewrite-search s1 t1 rules 'best-first)
121
122
   (define t6 (+ (+ ?c ?a) ?b))
123
124
125 # (gr s1 rules)
126
   # (load-file "msr-tests.ath")
127
128
130
  (test-bf s1 t1)
131
132
   (test-bf s1 t1)
133
134 (test-bf s1 t2)
135
136
137 (test-bf s1 t3)
```

138

```
(test-bf s1 t4)
140
   (test-bf s1 t5)
142
143
144
   (test-bf s1 t5)
145
147 (define A (+ ?g (+ ?e (+ (+ ?a ?b) (+ ?d ?c)))))
   (define B (+ (+ ?c (+ ?d (+ ?e (+ ?b ?a)))) ?g))
148
   (define C (+ ?a (+ ?b (+ ?c (+ ?d (+ ?e ?g))))))
149
   (define D
150
      (+ ?g
151
        (+ ?a
152
            (+ ?e
153
               (+ (+ ?c ?b)
154
                  ?d)))))
155
157 (define E (+ (+ (+ ?g (+ ?c ?e)) (+ ?b ?a)) ?d))
158
159
160 (define (test-bf a b) ())
161
162 (test-bf A B)
163
   (test-bf B A)
164
165 (test-bf B C)
166 (test-bf C B)
167
168
   (test-bf A C)
169
170 (test-bf C A)
171
172 (test-bf A D)
173
   (test-bf D A)
174
175 (test-bf B D)
176 (test-bf D B)
177
178 (test-bf C D)
179 (test-bf D C)
180
181
   (define A1 (+ ?e (+ (+ ?a ?b) (+ ?d ?c))))
182
   (define B1 (+ ?c (+ ?d (+ ?e (+ ?b ?a)))))
183
184
185 (test-bf A1 B1)
186 (test-bf B1 A1)
188
189 # Best-first seach with some randomness succeeds in interconverting any two of
190 # the terms A, B, C, and D below (on the basis of the comm/assoc properties of + and \star).
191 # Depth-first search fails on all of them with a max depth of 4000:
193 (test-bf B C)
194 (test-bf C B)
195
196 (test-bf A C)
197
   (test-bf A C)
198
   (test-bf A D)
200 (test-bf D A)
201
   (test-bf B D)
202
203 (test-bf D B)
205 (test-bf C D)
206 (test-bf D C)
207
208 (test-bf D A)
```

```
(test-bf D A)
210
   (test-bf B D)
212 (test-bf D B)
213
214
   (test-bf D C)
215
   (test-bf A E)
   (test-bf E A)
217
218
   (test-bf B E)
219
220 (test-bf E B)
221 (test-bf C E)
222 (test-bf E C)
223
224
   (define E1 (+ ?c (+ ?b (+ ?a (+ ?d (+ ?g ?e))))))
225
227 (test-bf E E1)
   (test-bf E1 E)
228
   (test-bf E1 A)
229
230 (test-bf E1 C)
231 (test-bf D E1)
232
   (distance D C)
234
   (distance E1 C)
235
236
237 (define a (+ (+ ?a ?b) (+ ?c ?d)))
   (define b1 (+ ?d (+ ?c (+ ?b ?a))))
   (define b2 (+ ?c (+ ?d (+ ?b ?a))))
239
   (define b3 (+ ?b (+ ?d (+ ?c ?a))))
241 (define b4 (+ (+ (+ ?d ?c) ?b) ?a))
   (define b5 (+ (+ (+ ?d ?c) ?a) ?a))
242
243
   (define start1 (+ (+ (+ ?a ?b) ?c) (+ ?d ?e)))
244 (define finish1 (+ ?e (+ ?d (+ ?c (+ ?b ?a)))))
245 (define start2 (+ ?foo (+ (+ (+ ?a ?b) ?c) (+ ?d ?e))))
246 (define finish2 (+ (+ ?e (+ ?d (+ ?c (+ ?b ?a)))) ?foo))
   (define finish2a (+ ?e (+ ?d (+ ?c (+ ?b (+ ?foo ?a))))))
247
248
   (define start3 (+ (+ ?foo ?goo) (+ (+ (+ ?a ?b) ?c) (+ ?d ?e))))
249
   (define finish3 (+ (+ ?e (+ ?d (+ ?c (+ ?b ?a)))) (+ ?goo ?foo)))
   (define finish4 (+ ?e (+ ?d (+ ?c (+ ?b (+ ?goo (+ ?a ?foo)))))))
251
252
253 (test-bf a b1)
254 (test-bf a b1)
255 (test-bf a b2)
   (test-bf a b3)
256
   (test-bf a b4)
258
259 (test-bf start1 finish1)
260 (test-bf finish1 start1)
261
   (test-bf start2 finish2)
   (test-bf finish2 start2)
263
265 (test-bf start2 finish2a)
   (test-bf finish2a start2)
266
268 (test-bf start3 finish3)
   (test-bf finish3 start3)
270
   (test-bf start3 finish4)
271
272
   (test-bf finish4 start3)
273
275 (define X1 (+ finish4 start3))
276
   (define Y1 (+ start3 finish3))
   (define X2 (+ start2 start3))
278 (define Y2 (+ finish3 finish2))
```

```
(test-bf X1 Y1)
280
   (test-bf X2 Y2)
282
283
284
285
   (define s (+ (+ finish4 start3) (+ start2 start3)))
   (define f (+ (+ start3 finish3) (+ finish3 finish2)))
287
   (test-bf s f)
288
289
   (define S1 (+ (* ?a ?b)
290
                   (+ (+ ?A (* ?B ?C))
291
                      (* ?e (* ?d ?f)))))
292
293
   (define S2 (+ (+ (* (* ?f ?d) ?e) (+ ?A (* ?C ?B)))
294
                   (* ?b ?a)))
295
   (define S3 (+ (+ (+ (* ?C ?B) ?A)
297
                     (* (* ?d ?e) ?f))
298
                   (* ?b ?a)))
299
300
301
   (define pairs [[A B] [B A] [B C] [C B] [A C] [C A] [A D] [D A] [B D] [D B] [C D] [D C] [A E] [E A] [B E] [E B] [C E]
302
303
                    [E E1] [E1 E] [E1 A] [E1 C] [D E1]
                   [a b1] [a b2] [a b3] [a b4] [start1 finish1] [finish1 start1] [start2 finish2] [finish2 start2]
304
                   [start2 finish2a] [finish2a start2] [start3 finish3] [finish3 start3] [start3 finish4] [finish4 start3]
305
306
                   [X1 Y1] [X2 Y2] [S1 S2] [S2 S1] [S1 S3] [S3 S1] [S2 S3] [S3 S2]])
307
   # Can't do [s f]
308
309
310
311
   (define (t) (test-pairs pairs))
312
313
   (load-file "msr-tests.ath")
314
   (test-pairs pairs)
316
317
318
   (dtest-pairs pairs)
319 (btest-pairs pairs)
320
321 (test-bf S1 S2)
322
323 (test-bf S2 S1)
   (test-bf S2 S1)
324
326 (test-bf S1 S3)
327
   (test-bf S3 S1)
328
   (test-bf S2 S3)
329
   (test-bf S3 S2)
331
   (define S4 (+ (+ (+ (+ ?A1 (+ ?A2 (* ?C ?B))) ?A)
332
                     (* (* ?d ?e) ?f))
333
                   (* ?b ?a)))
334
335
336
   (define S5 (+ (+ (* (* ?f ?d) ?e) (+ (+ ?A1 (+ ?A2 (* ?C ?B))) ?A))
337
                  (* ?b ?a)))
338
340 (test-bf S4 S5)
   (test-bf S5 S4)
341
342
   (define S6 (+ S4 S5))
343
344 (define S7 (+ S5 S4))
345
346
   (test-bf S6 S7)
347 (test-bf S7 S6)
```

348

```
(define S8 (+ S6 (+ S5 S4)))
   (define S9 (+ (+ S4 S5) S6))
350
   (test-bf S8 S9)
352
353
   354
355
   (load-file "rewriting.ath")
357
   (domain Z)
358
   (datatype Nat zero (succ Nat))
359
360 (declare Minus (-> (Nat Nat) Nat))
361 (declare Zzero Z)
362 (declare neg (-> (Nat) Z))
363
   (define pl (forall ?x:Nat (= (Minus ?x ?x) zero)))
364
  (define p2 (= Zzero (neg zero)))
365
366 (assert p1 p2)
367
   (define a ?a:Nat)
368
   (define s (neg (Minus a a)))
369
370 (define t Zzero)
371
   (define rules [p1 p2])
372
373
   (!drs-bf s t rules rewrite-one-redex)
374
375
## D: Added 1/21/2009 to document an apparent bug:
377
   # Uncomment the following if starting here:
379
381
   (load-file "rewriting.ath")
382
383
   (datatype Nat zero (succ Nat))
384
   (domain E)
386
   (declare T (-> (E) Nat))
387
388
389 (declare (s1 s2) E)
390
391 (declare t Nat)
392
   (define p1 ((T s1) = t))
393
394
395
  (define p2 ((T s2) = (succ t)))
396
397
   (assert p1 p2)
398
   # This fails
399
400
   (!drs-bf (T s2) (succ (T s1)) [p1 p2] rewrite-one-redex)
401
   # This works
403
   (!drs-bf (succ (T s1)) (T s2) [p1 p2] rewrite-one-redex)
404
405
406 # This works:
  (!chain [(T s2)
407
           = (succ t)
                                [p2]
408
           = (succ (T s1))
                                [p1]])
410
   # So does this, since chain tries both calls of drs-bf:
411
412
413 (!chain [(T s2)
           = (succ (T s1))
                               [p1 p2]])
415
416 \# But if we do essentially the same thing with a binary symbol T1
   # instead of T, it fails:
417
418
```

```
(domain C)
420
   (declare T1 (-> (C E) Nat))
421
422
   (declare C1 C)
423
424
425
   (declare (x1 x2) E)
427 (declare t1 Nat)
428
429 (define p1 ((T1 C1 \times1) = t1))
430
431
   (define p2 ((T1 C1 x2) = (succ t1)))
432
   (assert p1 p2)
433
434
435 # This fails:
436 (!drs-bf (T1 C1 x2) (succ (T1 C1 x1)) [p1 p2] rewrite-one-redex)
437
438 # and so does this:
439 (!drs-bf (succ (T1 C1 x1)) (T1 C1 x2) [p1 p2] rewrite-one-redex)
440
441 # So, although this works:
442 (!chain [(T1 C1 x2)
443
            = (succ t1)
            = (succ (T1 C1 x1)) [p1]])
444
445
446 \# the following fails (calls external theorem prover)
447 # since both drs-bf calls fail:
449 (!chain [(T1 C1 x2)
           = (succ (T1 C1 x1)) [p1 p2]])
451
452 # This works:
453
454 (define p3 (!sym p1))
456 (!drs-bf (T1 C1 x2) (succ (T1 C1 x1)) [p3 p2] rewrite-one-redex)
457
458 # hence this does too:
459 (!chain [(T1 C1 x2)
            = (succ (T1 C1 x1)) [p3 p2]])
```