Utrecht University

Simulation of Ocean, Atmosphere and Climate

Daisyworld: A simple climate-biosphere feedback model

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Abstract

In this project we investigate the interactions between biota and environment and the properties of this coupled system through the Daisyworld model. Daisyworld is an imaginary planet similar to Earth with a simple biosphere of black and white daisies and the environment represented by a single climate variable, the temperature. In this study we computed two models: a 1D model and a latitude dependent model using the 4th order Runge-Kutta method to solve the differential equations explaining our models. Moreover we investigated different parametetrizations of the models: simple greenhouse effect, heat transport and orbital forcings. We show that the interactions between the variables of the system effectively regulate the planetary temperature by keeping it constant, even for a large range of solar luminosity values and different model parametrizations.

1 Introduction

Daisyworld, presented by James Lovelock and Andrew Watson in 1983 [1], is a planetary model which aims at demonstrating the long-term effect of connections between biota and its environment. This model is presented in defence of the Gaia theory which suggests that the living organisms and nonliving parts of Earth form a complex interacting system which behaves as a single organism. It evolves through feedback systems operated unconsciously by the biosphere which helps to maintain and perpetuate the conditions for life on the planet.

This modelled world is made of two species of daisies: black and white daisies. The environment of Daisyworld is reduced to a meteorological or climate variable: temperature. Its surface temperature is determined by the solar luminosity and the daisies albedo, which is, in turn, influenced by the coverage of the two daisy types. In the following study, we will see that this temperature and vegetation interplay produces a nonlinear system with interesting self-regulating properties.

The primary goals of this study is to simulate and analyse variations of Daisyworld. Firstly, we focus on both a "warming" (increasing solar luminosity) and "cooling" (decreasing solar luminosity) flat disk planet with two species of daisies, black and white. We investigate the population density of the white and black daisies and the equilibrium surface temperature that is estimated when the whole planetary system achieves steady state for each incremental or decremental level of solar luminosity. Afterwards, the Daisyworld model is expanded by adding a third species of daisies. We also investigate the sensitivity of the model to changes in the value of parameters b, the relaxation constant, and β, the heat transport between the species and the environment. In order to apply the model with more realistic conditions, we then add a latitudinal distribution of the solar insolation, as this is the case for our planet Earth. Taking benefit from this latitude dependent Daisyworld model we thus investigate the effect of changing the meridional distribution of solar radiation as the result of changing Earth orbital configurations.

2 Models' descriptions

Daisyworld is an Earth like cloudless planet with no oceans, but with plentiful resources for life. The planet orbits an artificial sun that is warming slowly over time resulting in an increase in planetary temperature. It consists of only two different types of daisy, black and white, which may be considered distinct species or as distinct phenotypes of the same species. The growth rates of the two daisy populations depend on temperature and determine the planetary albedo which affects the global temperature [2]. In the following subsections, we present the equations that control the dynamics and feedback mechanisms of Daisyworld, and how they are applied to create our model.

Here we focus on the steady state solution of Daisyworld. In system's theory, if the variables that determine a system's/process' behavior do not change over time or, in more complex systems, when the behaviour varies over a stable range of possibilities, the system/process is in a steady state. Therefore, steady states of the system as a whole were found for each incremental level of solar luminosity by numerical integration using the 4^{th} order of Runge-Kutta methods. In order to ensure the model is not too sensitive to the solving method used, we additionally investigated the performance of Matsuno's and Heun's numerical schemes but we don't include the results here. However, we decided to use the RK4 method since it offers a good balance between order of accuracy and cost of computation. The methods are extensively discussed in the Appendix section 5.

2.1 The Original Daisyworld model

In Daisyworld, the two different daisy colors imply different albedo properties for each species. Albedo defines the proportion of incident light, reflected back from a surface. Black daisies have a small albedo and so absorb more solar radiation than bare ground, whereas white daisies have a larger albedo and thus reflect more solar radiation than bare ground. For this model we consider black daisies with an albedo of 0.15, the white daisies with an albedo of 0.75, and the bare ground albedo is 0.25. The mean planetary albedo a_p is defined as:

$$a_p = a_g A_g + a_b A_b + a_w A_w \tag{1}$$

where A_g is the area, and a_g the albedo of bare ground. A_b (resp. a_b) is the area (resp. albedo) of ground covered by black daisies, and A_w (resp. a_w) is the area (resp. albedo) covered by white daisies. All areas are measured as fractions of the total planetary area.

The area of land covered by daisies is governed by daisy growth. Daisy growth is modelled according to the following equation:

$$\frac{dA_i}{dt} = A_i(A_g\beta_i - \gamma), \qquad i \in \{w, b\} \tag{2}$$

where β_i is the growth rate of the daisies per unit of time, and γ_i is the death rate equal to 0.3 per unit of time. For steady state, we assume that:

$$\frac{dA_i}{dt} = A_i(A_g\beta_i - \gamma) = 0, \qquad i \in \{w, b\} \tag{3}$$

The area of fertile ground which is uncolonized by daisies is:

$$A_g = P - A_b - A_w \tag{4}$$

where P is the proportion of the planet area which is fertile ground.

The growth rate of the daisy species, β_i, is assumed to be a parabolic function of local temperature, T_i:

$$\beta_i = 1 - 4\frac{(T_{opt} - T_i)^2}{(T_{max} - T_{min})^2}, \qquad i \in \{w, b\} \tag{5}$$

It has a maximum value of 1 at the optimal temperature, T_{opt}, for daisy growth which is equal to 22.5°C. Growth may only occur in a limited range of temperatures (Fig. 1).

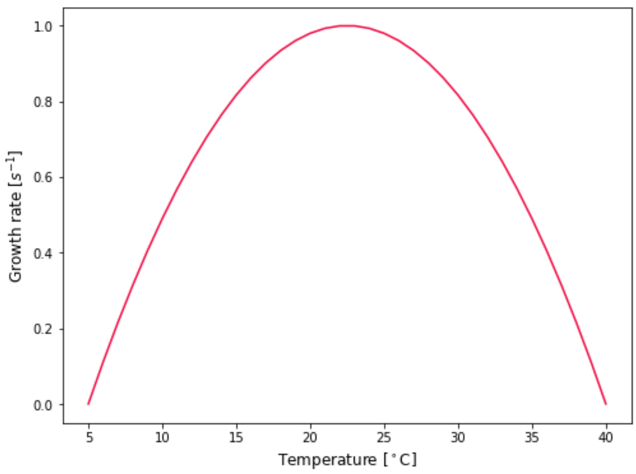

Fig. 1. Growth rate of the daisy species as a function of local temperature.

The local temperature of each species of daisy and the planetary temperature are given by the energy balance equation:

$$C\frac{dT_i}{dt} = \frac{1}{4}S_0L(1 - \alpha_i) - (I_0 + bT_i) - \beta(T_i - T_p), \qquad i \in \{w, b, p\} \tag{6}$$

All parameters used in equations are listed in Tab. 1. The first term on the right hand side of eq. (6) describes the shortwave incoming radiation absorbed by the Earth. The last term on the right hand side of eq. (6) represents redistribution of energy from warm to cold or in other words the relaxation term. Where β is the heat transport between daisies and the environment. The β parameter should not be confused with the growth rate of the daisies, β_i. Finally, the second term on the right hand side of eq. (6) describes the Outgoing Longwave Radiation (OLR) from the Earth. It is an empirical estimation of the OLR seen from the satellites at the top of the atmosphere. The parameter b is the OLR per degree difference and is a measure of the greenhouse effect.

By assuming steady state solution for eq. (6), we find the two following equations for the local temperatures, T_i, and the effective temperature at which the planet radiates, T_p.

$$T_i = \frac{\frac{1}{4}S_0 L(1-a_i) - I_0 - T_p}{b+\beta} \qquad i \in \{w, b\} \tag{7}$$

$$T_p = \frac{\frac{1}{4}S_0 L(1-a_p) - I_0}{b} \tag{8}$$

2.2 Radiative forcing and the heat transport in the original Daisyworld model

Greenhouse gasses alter planetary temperatures by trapping the OLR, which is known as the greenhouse effect. We can simulate the greenhouse effect by altering the value of b in eqs. (7) and (8). This value is deduced from satellite measurements relating the OLR, of which the transmission is influenced by the green house gas containing atmosphere between the satellite and Earth's surface and the ground temperature at Earth's surface [3].

We use a default value of b equal to $2.2W/m^2K$ [4]. When increasing the greenhouse effect, we decrease the value of b and thus reduce the OLR. More OLR is trapped between the Earths' surface and the atmosphere, increasing the planetary temperature. The opposite is true when decreasing b.

The change of heat transport between the species and the environment is described by the relaxation constant, β. Lower values of β simulate lower heat transport between the daisies and the environment while higher values of β simulate increased heat transport.

2.3 Latitude Dependent model

In the previous model (section 2.1) we used $S = S_0$, a solar flux independent of latitude. However, in reality the incoming solar flux for a given area on Earth does depend on latitude. Therefore, it motivates us to develop and simulate Daisyworld within a latitude dependent model framework.

The incoming solar flux per unit surface area can be defined as Q in eq. (9).

$$Q = S_0(\frac{\bar{d}}{d})^2 \cos(\theta_s) \tag{9}$$

with $\bar{d}$ the mean distance at which the solar constant (S_0) is measured, d the actual distance from the sun, and θ_s the solar zenith angle (the angle between the normal to Earth's surface and the solar flux). The fraction ($\bar{d}/d$) is equal to 1 since we assume that the eccentricity of the Daisyworld's orbit around the sun is zero (perfect circle). The zenith angle depends on the latitude, season and hour of the day and the cosine of this angle is given by eq. (10).

$$\cos(\theta_s) = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(h) \tag{10}$$

with ϕ the latitude and δ the declination angle. When $\cos(\theta_s) < 0$, the sun is under the horizon and we should take $Q = 0$ (no insolation). The declination angle is the angle between the sun rays and the equator measured at noon and indicates the season [5]. Lastly, h is the hour angle; the longitude of the subsolar point relative to its position at noon, progressing from $h = \pi$ to $h = -\pi$. The hour angle, h_t, which separates the illuminated side from the dark side of the planet, can be calculated by [6]:

$$\cos(h_t) = -\tan(\phi)\tan(\delta) \tag{11}$$

In our model, we do not include seasonality, thus δ is assumed to be zero. Therefore, solving eq. (11) for $\delta = 0$, gives us $h_t = \pi/2$.

By combining eqs. (9) and (10), and afterwards integrating over all the hour angles, from $h = h_t$ to $h = -h_t$, we get an average solar flux per unit surface area of that day.

$$Q = \frac{1}{2\pi} \int_{-h_t}^{h_t} Q dh \tag{12}$$

$$Q = \frac{S_0}{\pi} \left[h_t \sin(\phi) \sin(\delta) + \cos(\phi) \cos(\delta) \sin(h_t)\right] \tag{13}$$

According to Budyko (1969) [4], an additional term is added in the energy balance equation that approximates the relationship between temperature distribution and horizontal heat transfer in the atmosphere, to estimate the radiation variation influence on the temperature of each latitude. We call this term a relaxation term and is estimated as:

$$A = \beta(T_{lat} - T_p) \tag{14}$$

where T_{lat} is the mean temperature at a given latitude and T_p in this case, is defined as the average temperature of the planet. Based on the above, the latitude dependent temperature we use is given by eq. (15).

$$T_{lat} = \frac{QL(1 - \alpha_p) - I_0 + \beta T_p}{b + \beta} \tag{15}$$

The latitude dependent temperature can now be used to compute the local temperature of each species as shown in eq. (16):

$$T_i = \frac{1}{4} \frac{QL(a_p - a_i)}{b + \beta} + T_{lat}, \qquad i \in \{w, b\} \tag{16}$$

2.4 Orbital parameters

As an additional functionality for the model presented in section 2.3, we added a varying latitudinal distribution of solar luminosity, using Earth orbital parameters as a model input. The three orbital parameters we need to take into account are e, the eccentricity, Φ, the obliquity and Λ, the longitude of perihelion.

Eccentricity is a measurement of how non-circular an orbit is. If eccentricity equals to 0, the orbit of the planet around the sun is a perfect circle.

The obliquity represents the tilt of the Earth. This tilt is the angle between the normal of the orbital plane and the Earth's axis which varieties from $22.1°$ and $24.5°$ [7].

Finally, the longitude of perihelion, Λ, is defined as the angle between the Earth-Sun line at vernal equinox and the line from the Sun to perihelion.

For the present day, the values of these parameters are: $e = 0.017236$, $\Phi = 23.446°$ and $\Lambda = 281.37°$ [8].

Parameters	Value	Units	Description
α_g	0.25	-	Albedo of bare ground
α_w	0.75	-	Albedo of white daisies
α_b	0.15	-	Albedo of black daisies
γ	0.3	s^{-1}	Death rate
P	1	-	Proportion of area that is fertile ground
S_0	1366	Wm^{-2}	Average Solar Energy incident on the planet's surface
b	2.2	$Wm^{-2}K^{-1}$	Outgoing Longwave Radiation (OLR) per degree difference
C	-	-	Heat capacity
L	-	-	Adjustable parameter for solar luminosity
β	16	$Wm^{-2}K^{-1}$	Heat transport between daisies and the environment
I_0	220	Wm^{-2}	Outgoing Longwave Radiation (OLR) at 0 Celcius

Tab. 1. List of parameters used in the equations

3 Results

3.1 The Original Daisyworld model

Here we present the results for both a warming planet and cooling planet for the cases without daisies, with only one of the two species and with both species existing. In the absence of daisies, the planetary temperature will steadily increase as the luminosity increases, or decrease if the luminosity decreases (Fig. 2b,d,f).

In the second case, only white daisies may grow. For a warming planet (starting the simulation in a cold Daisyworld with increasing luminosity), we observe from Fig. 2a,b, that for low values of luminosity, the white daisies grow up slowly because of their large albedo. When they appear, they cool the planet and make it less favorable for themselves. However, as the luminosity increases, the area which is covered by white daisies increases and the planetary temperature stays constant, below what it would be if the planet was bare due to their large albedo compared to ground's albedo. Therefore, the planet is cooling, contributing to a negative feedback, and an equilibrium is made between white daisies and climate for values of L varying between $L = 1.1$ and $L = 2.1$. As the luminosity increases further, the growth of the white daisies is limited due to the limited space in the planet. The planet warms, but since the area covered by daisies is too small to keep the average albedo needed for the suitable temperature. Their growth rate thus decreases as temperatures becomes too high, and eventually, they die.

In the third case, only black daisies can grow. For a warming planet we observe from Fig. 2c,d that the black daisies rapidly grow up for $L = 0.90$ to a maximal area coverage of about 70% for $L \simeq 0.95$. Since their albedo is lower than ground's albedo, the planet is warming, and therefore the planetary temperature is higher compared to what it would be if the planet was bare (positive feedback). As the luminosity increases further, the growth of the black daisies is limited due to the limited space in the planet. The planet becomes warmer and warmer but daisies cannot hold the temperature to a suitable level and eventually, they die similar to the second case.

Figs. 2e,f illustrate the complete model (both daisies exist) on a warming planet. For low values of luminosity ($L < 0.90$), no daisies can grow. At $L = 0.90$, we observe a drastic growth of black daisies in a small range of luminosity values rapidly covering 70% of the planet, similarly to the black daisies only case. They appear first because they absorb more of the sun's energy and reach a growth temperature more quickly than white daisies. Of note, this is the maximum area they can cover because of their death rate of 0.3, similarly to the case of only black daisies as no white daisies have grown yet and conditions are similar. As luminosity increases, black daisies can remain for a longer luminosity range then in the case of only black daisies thanks to the albedo of the white daisies that appear. As the temperature of the world continues to increase, the white daisies gradually replace the black ones, and for $1.0 < L < 1.60$ the two species coexist. When all the black daisies die, the white daisies continue slowly growing till they occupy 70% of the planet for $L = 2.1$. If luminosity increases further, the white daisies disappear as well. Planetary temperature remains approximately constant at around 23°C.

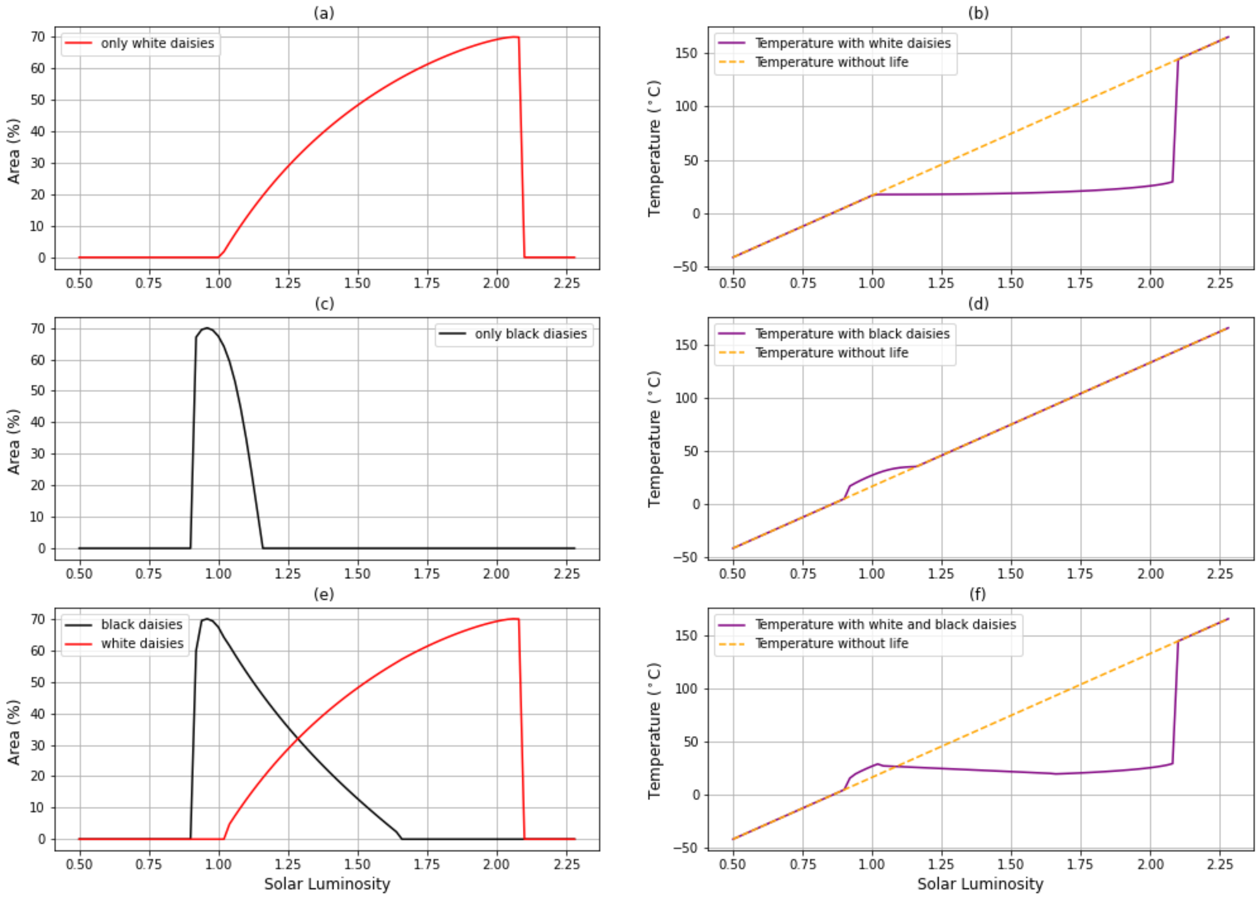

Fig. 2. Steady state responses of Daisyworld. Areas of black and white daisies and planetary temperature are plotted against increasing values of the luminosity parameter L. Dashed yellow line indicates the temperature of the planet without life. Subplots (a) and (b) for a population only of white daisies, subplots (c) and (d) for a population only of black daisies, subplots (e) and (f) for a population of both black and white daisies.

Finally, Fig. 3a,b, shows the complete model for a cooling planet (starting the simulation in a warm Daisyworld and decreasing luminosity). In Fig. 3a, we see that white daisies are the first ones to appear. This makes the temperature drop due to the higher than ground albedo of these emerging white daisies (Fig. 3b). For a little lower luminosity, the black daisies appear and cover a bit more area than the white flowers. This causes the average albedo to be slightly higher than the ground albedo, roughly increasing the temperature. Now the temperature increases slowly with decreasing luminosity because the white daisies population decrease linearly with luminosity lowering the average albedo and black daisies increase linearly, also lowering the average albedo. Due to this decrease in albedo, more light is absorbed and the planet is warmed. However, this inhibits black daisies to grow till they drop sharply, since the temperature becomes too low for them to survive. Here, also the planetary temperature starts to drop till it reaches the temperature as without daisies on the planet (i.e. 5°C) (Fig. 3b).

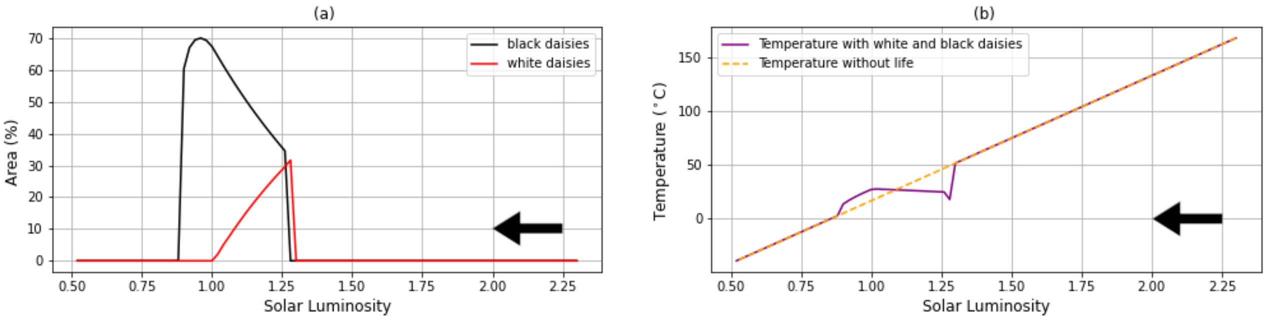

Fig. 3. Steady state responses of Daisyworld. Areas of black and white daisies (a) and planetary temperature (b) are plotted against decreasing values of the luminosity parameter L when reading from right to left. Dashed yellow line indicates the temperature of the planet without life.

Screen shots of the Daisyworld are shown in Fig. 4 for different solar luminosity. Due to space constraints, only solar luminosity from 1.0 to 1.7 are illustrated. For $L = 1.0$, most of the planet's area is covered by black daisies. At $L = 1.1$, the first white daisies appear. For $1.2 < L < 1.3$, the white and black daisies occupy almost equal amount of the surface area of the planet. For $L > 1.4$, the black daisies start disappearing and eventually die, while the white daisies start occupying most of the planet's area [9].

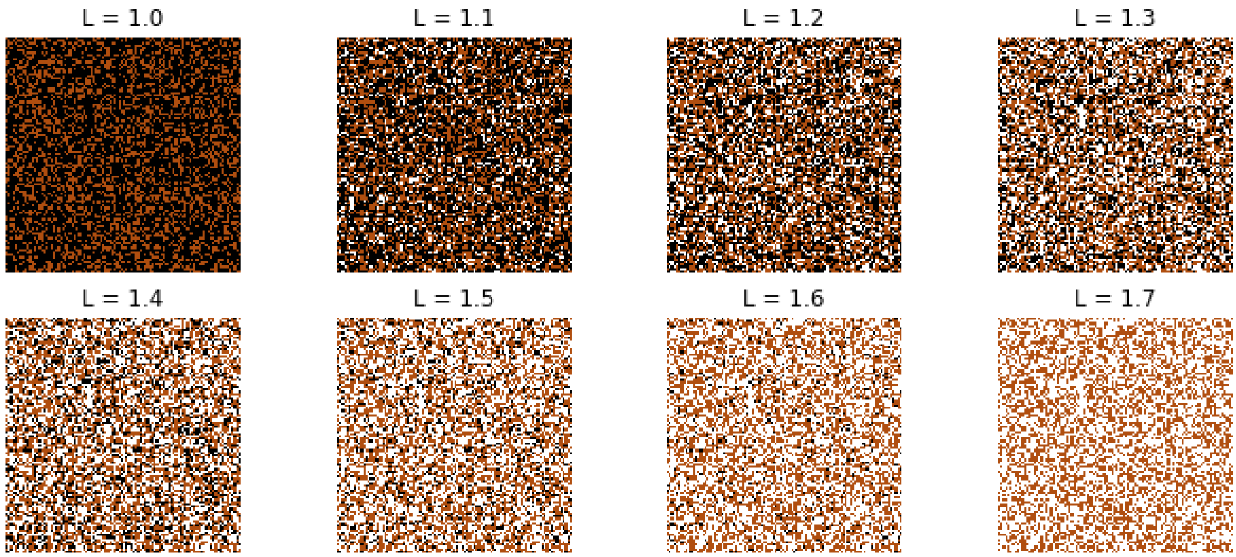

Fig. 4. Population map for different luminosity for the original model. The brown color corresponds to the area that is not covered by any daisy, the white color corresponds to the area covered by white daisies and the black color represents the area covered by black daisies.

Now we introduce another species of daisy: grey daisies with albedo equal to 0.45, and similar growth and death rate as the other daisies, illustrated in Fig. 5a,b. The black daisies still grow first at approximately $L = 0.90$ and raise the planetary temperature above the temperature of a planet without daisies whereas as the grey and white daisies grow they keep the planet's temperature below this reference temperature. Fig. 5a shows a gradual change in dominance from the black daisies to the grey ones and eventually the white ones. The presence of grey species makes little difference to the planetary temperature regulation (Fig. 5b).

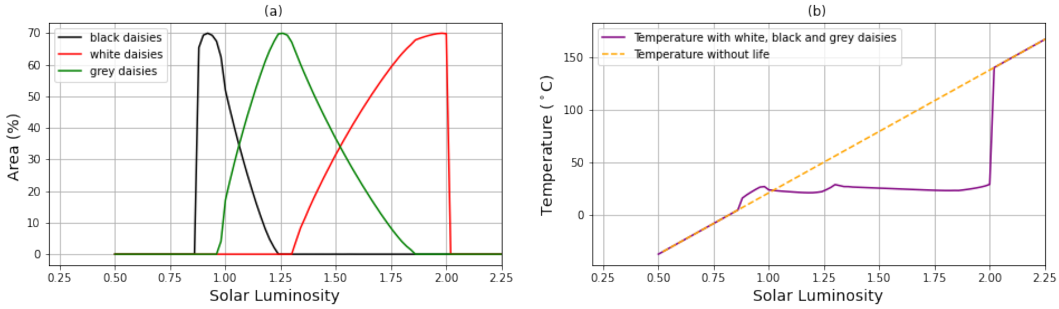

Fig. 5. Steady state responses of Daisyworld. Areas of black, grey and white daisies (a) and planetary temperature (b) are plotted against increasing values of the luminosity parameter L. Dashed yellow line indicates the temperature of the planet without life.

3.2 Radiative forcing and the heat transport in the original Daisyworld model

Fig. 6 shows the effect of alternating the value of the OLR parameter, b, representing the strength of the greenhouse effect. By looking Fig. 6a,b, when increasing the value of b ($= 4\ Wm^{-2}K^{-1}$), reducing the greenhouse effect, luminosity must increase more than normally to reach the minimum temperature for the species to grow. Both species grow at higher luminosity due to the decreased trapped heat. The regulation of the temperature lasts for a larger range of luminosity.

By looking Fig. 6c,d, when simulating Daisyworld with a lower value of b ($= 1Wm^{-2}K^{-1}$), representing a higher greenhouse effect, the black daisies start growing at almost the same luminosity but have a lower maximum and start disappearing earlier than with a lower greenhouse effect. White daisies exhibit a similar behaviour by growing and dying also for lower values of luminosity. This is expected since the temperature at which the white flowers start growing is reached for lower values of luminosity due to the increased trapped heat. Notice from Fig. 6d, that the stabilization of the planetary temperature lasts for a shorter range of luminosity, since this increased greenhouse effect, warms the planet even more and even faster.

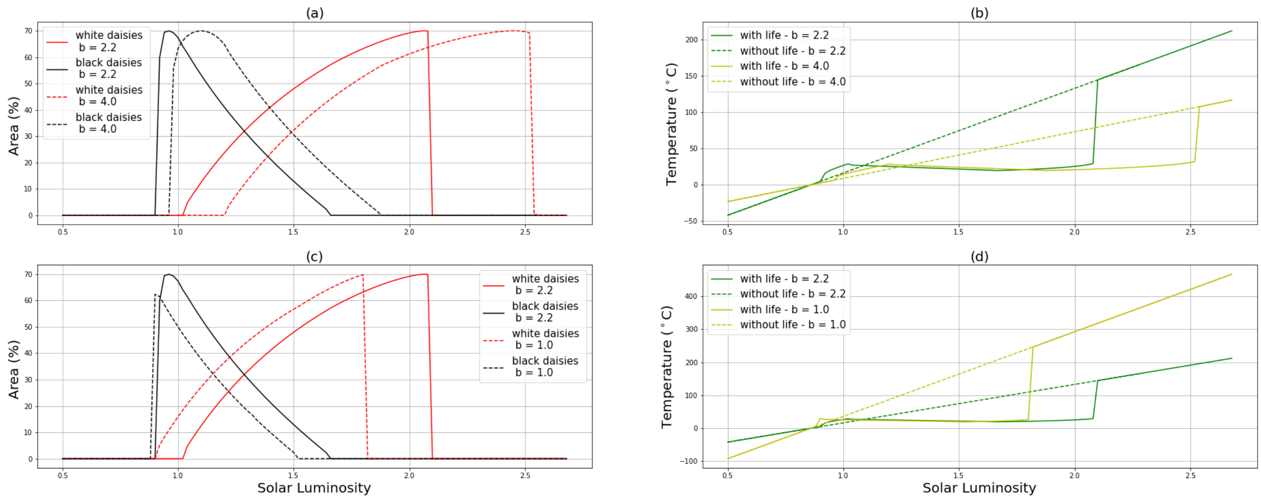

Fig. 6. Steady state responses of Daisyworld by varying the greenhouse effect. Areas of black and white daisies (subplots (a) and (c)) and planetary temperature (subplots (b) and (d)) are plotted against increasing values of the luminosity parameter L. In subplots (a) and (b), the straight and dashed lines corresponds to $b = 2.2$ and $b = 4.0$ respectively while in subplots (c) and (d), the straight and dashed lines corresponds to $b = 2.2$ and $b = 1.0$ respectively. Dashed lines in subplots (b) and (d) indicate the temperature of the planet without life.

In Fig. 7 we see the effect of the β parameter, representing the heat transport between the species and the environment. For low heat transport ($\beta = 6Wm^{-2}K^{-1}$), we see in Fig. 7a that the black daisies vanish more rapidly because they interchange less heat with the environment and the other daisies and warm up faster to a level they cannot bare anymore. The stable temperature is higher in this simulation since the luminosity has increased further until white daisies appear. The white flowers have a shifted growth behaviour towards higher luminosity. They warm up less fast now and need more luminosity to arrive at the minimum temperature they need to start growing. Due to the behaviour of the flowers the stabilizing feedback mechanisms holds over longer luminosity values.

For higher heat transport ($\beta = 26Wm^{-2}K^{-1}$), we see in Fig. 7c that white daisies die at lower luminosity while the black ones at higher luminosity. So, the range of luminosity at which daisies live and stabilize the temperature is smaller due to the increased redistribution of energy between biota and environment.

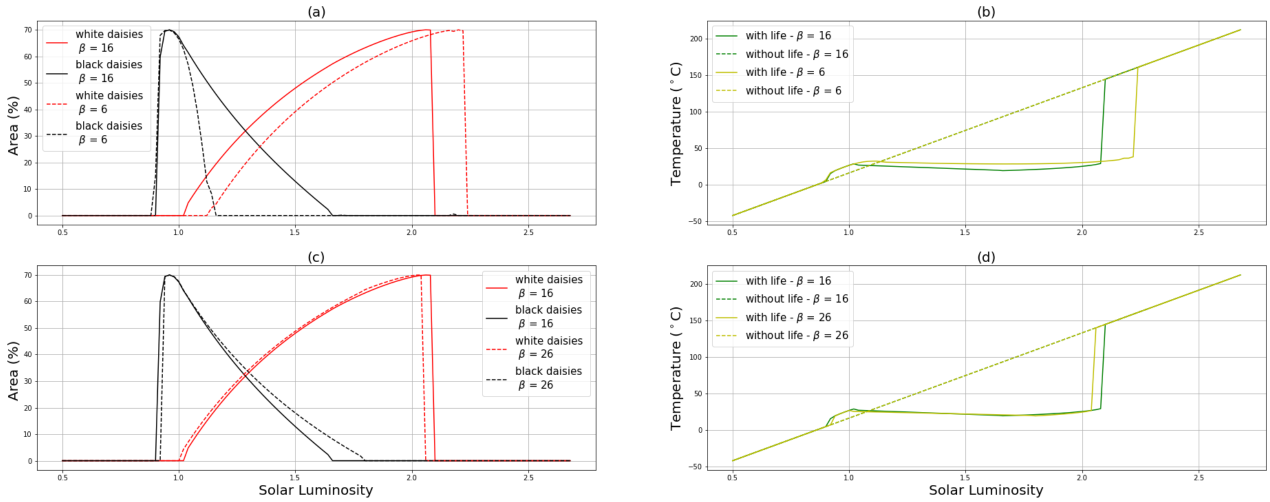

Fig. 7. Steady state responses of Daisyworld by varying the relaxation parameter (β). Areas of black and white daisies (subplots (a) and (c)) and planetary temperature (subplots (b) and (d)) are plotted against increasing values of the luminosity parameter L. In subplots (a) and (b), the straight and dashed lines corresponds to $\beta = 16$ and $\beta = 6$ respectively while in subplots (c) and (d), the straight and dashed lines corresponds to $\beta = 16$ and $\beta = 26$ respectively. Dashed lines in subplots (b) and (d) indicate the temperature of the planet without life.

3.3 Latitude Dependent model

In the latitude dependent model, we incorporate a distribution of incoming solar radiation based on latitude in the original model with both daisy species. Fig. 8 illustrates the zonal latitudinal planetary temperature, as well as the white and black daisies fraction coverage as a function of solar luminosity and latitude, using the constants $b = 2.2Wm^{-2}K^{-1}$ and $\beta = 16Wm^{-2}K^{-1}$.

For $L = 0.9$, black daisies start growing around the equator where their growth temperature is reached faster than in the rest of the planet, decreasing the planetary albedo, and hence increasing planetary temperature. As luminosity increases, the black daisies migrate towards the poles since the equator becomes too hot for them to survive. Meanwhile, white daisies take their place around the equator and begin to also spread outwards. As the temperature of the world continues to increase, the white daisies gradually replace the black ones, increasing the planetary albedo and hence cooling the planet down. As the luminosity increases further, the black daisies disappear and white daisies cover most of the planet. For $L > 2$, the white daisies die as well.

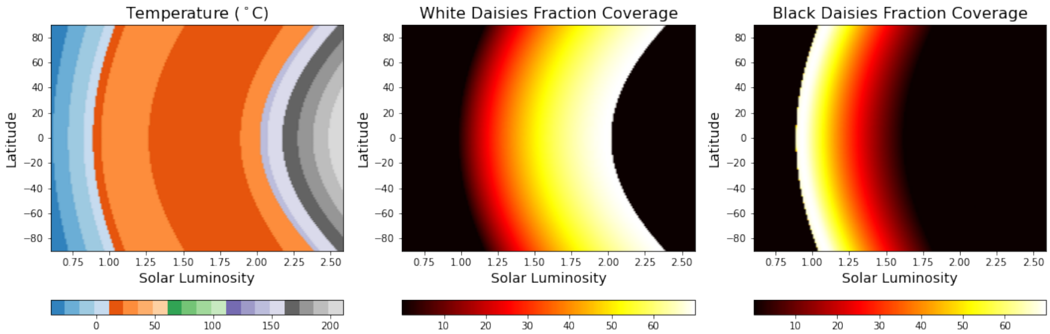

Fig. 8. Steady state responses of Daisyworld model which includes spatial dependency. The zonal temperature and areas of black and white daisies are plotted against increasing values of the luminosity parameter L for each latitude.

In Fig. 9, we can visualise the gradual change in dominance from the black daisies to the white ones for five different values of luminosity. For $L = 1.05$, the black daisies dominate the planet while the white daisies start to thrive only near the equator. For $L = 1.1$, white daisies start spreading out towards the poles. For $L = 1.2$, white daisies occupy further area (around 28%) while the black daisies still dominate the planet by approximately 37%. For $L = 1.4$, black daisies start dying around the equator, whereas the white daisies sharply grow, occupying 45% of the planet's surface. Finally, for $L = 1.6$, the white daisies dominate the planet and the black daisies are restricted to the poles.

3.4 Orbital parameters

Finally we implement orbital parameters with the latitude-variable Daisyworld modification into our model. In order to initialise and run our model, we used the current values of Earth's orbit (section 2.4), and we integrated the latitudinal distribution of solar luminosity over the year (Fig. 10).

By comparing Fig. 8 and Fig. 10, we observe some differences caused by the planet's tilt. The species move polewards at lower luminosity with respect to the simulation without orbital parametrisation. We also observe that at latitudes 60°-90°N and 60°-90°S there is a brief stabilisation of both temperature and the species fraction coverage for each level of luminosity. It is interesting that the regulation of the temperature lasts for wider range of luminosity as well.

Moreover, we expected an asymmetry between the north and south hemispheres since solar radiation is unevenly distributed between the hemispheres due to the planet's tilt. However, this asymmetry is not that evident. One possible explanation is that we calculated the annually average of the solar flux in order to run our model and therefore, the seasonal variations could be canceled.

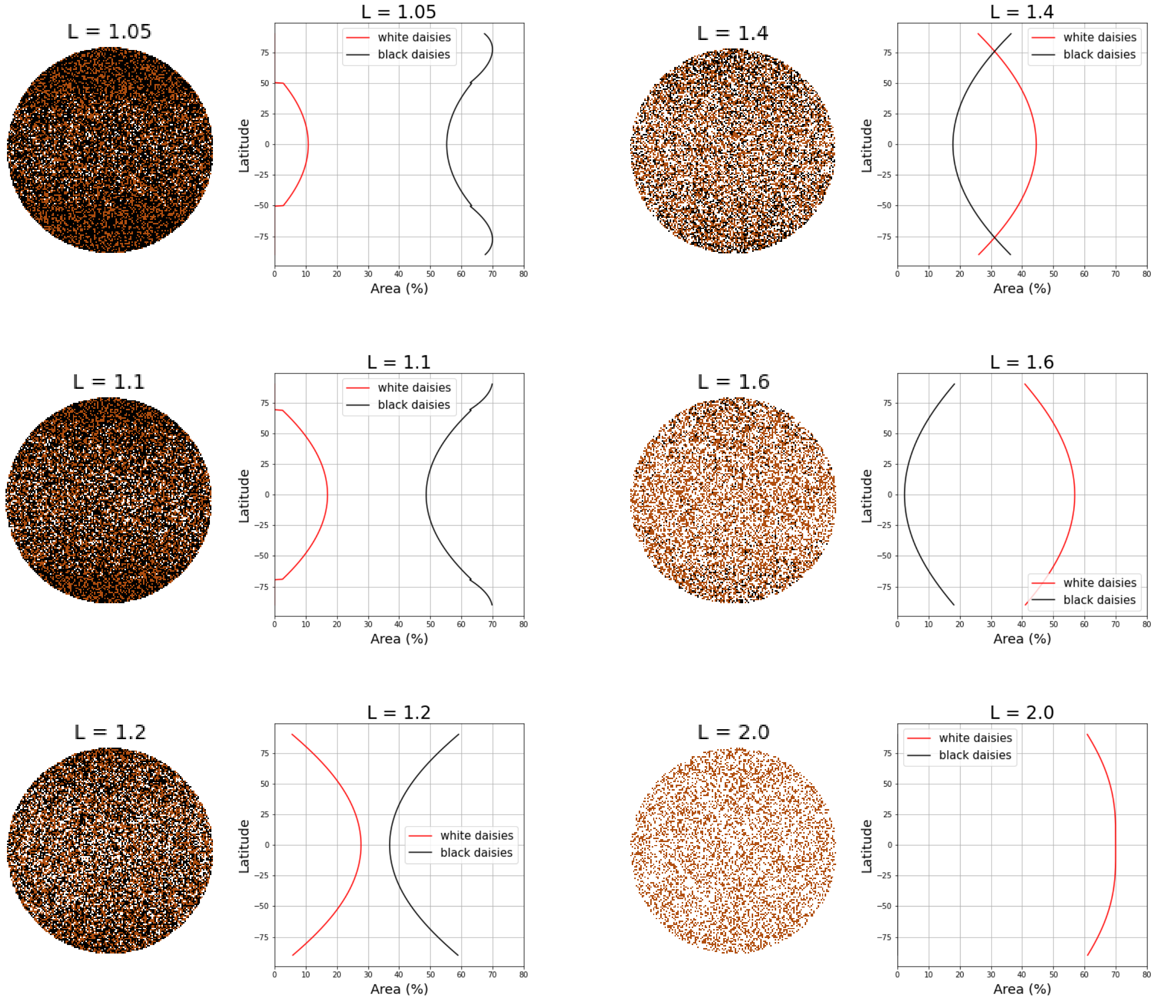

Fig. 9. Subpanels with population maps (on the left of each subpanel) and the area covered by each species of daisy for six different luminosity levels for the latitude dependent model. For the populations maps, the brown color corresponds to the area that is not covered by any daisy, the white color corresponds to the area covered by white daisies and the black color represents the area covered by black daisies.

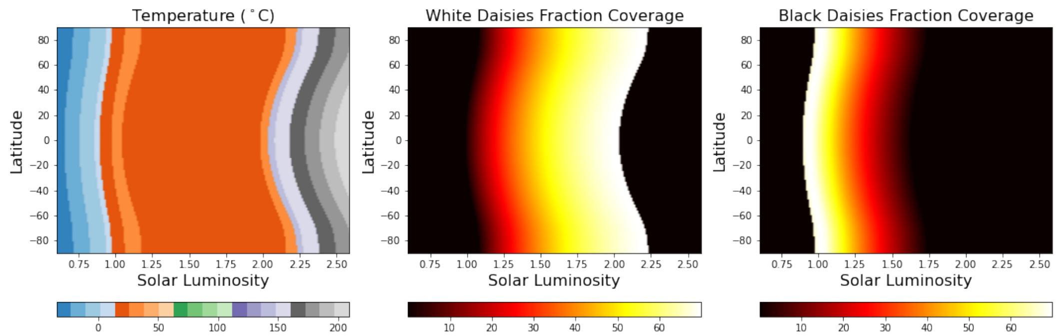

Fig. 10. Steady state responses of Daisyworld model which includes spatial dependency and orbital parameters. The zonal temperature and areas of black and white daisies are plotted against increasing values of the luminosity parameter L for each latitude.

4 Conclusion and Discussion

In our study, we simulated and analysed Daisyworld. The model demonstrates that the interaction feedbacks between daisies and environment are effectively regulating the planetary temperature by keeping it constant. At low and high radiation range, there is an overall positive feedback. For $1.1 < L < 2.1$, since white daisies start dominating the planet, there is a negative feedback cooling the planet and keeping the temperature constant at around 23°C. Moreover, by varying the strength of the greenhouse effect or the heat transport between life and environment, the planetary temperature is kept constant again at around the optimal growth temperature of the daisies.

In the latitude dependent simulation, the equator will always receive more heat than the polar regions. Therefore, conditions should in general be more hospitable to white daisies around the equator, and black daisies should have an advantage in the polar regions.

The implementation of orbital parameters shows that daisies move polewards at lower luminosity compared to the model without orbital parameters and temperature is stabilised over a larger luminosity range. However, this extension of Daisyworld needs further improvement

Overall, we conclude that Daisyworld shows that biota and environment can form a coupled system. Nevertheless, the real world is far more complex. The actual mechanisms of climate regulation in Daisyworld were not intended to explicitly represent those on the Earth. Many more extensions could be interesting to be implemented on our models that could have improved the results and created more realistic conditions of Earth, and thus further show that self-regulation might also occur in more complex systems. Further studies are, for instance, adding oceans, clouds and seasonal variations.

5 Appendix

5.1 Numerical Schemes

For the simulation of Daisyworld we investigated the performance of three different numerical schemes: Matsuno scheme, Heun scheme and the 4^{th} order of Runge-Kutta methods.

5.1.1 Matsuno's and Heun's scheme

Matsuno's and Heun's scheme is a multi-step scheme which means it uses the information of the previous steps to calculate the next one. In general the multi-step scheme can be represented by eq. (17) with f a ordinary function.

$$\frac{dx}{dt} = f(t, x, y) \tag{17}$$

Multi-step scheme contains two steps of computation; the first, predictor step, (18) and the second, corrector step, (19). With $*$ representing the first guess and n the time index.

$$x^*_{n+1} - x_n = \Delta t f_n \tag{18}$$

$$x_{n+1} - x_n = \Delta t \left(\alpha f_n + c f^*_{n+1}\right) \tag{19}$$

where the sum of α and c must be 1. When filling in eqs. (18) and (19) for Daisyworld we get to the following equations:

$$(A^*_i)_{n+1} = (A_i)_n + f_n(t, A_w, A_b)\, dt \tag{20}$$

$$(A_i)_{n+1} = (A_i)_n + [\alpha f_n(t, A_w, A_b) + c f^*_{n+1}(t, A_w, A_b)]\, dt \tag{21}$$

where A_i is the area of the ground which is covered by white or black daisies. The ordinary function f is given by eq. (2). In this case the function f is independent of t.

For Matsuno's scheme the values of α is 0 and c is 1. This makes the scheme "semi-implicit" and of first order with respect to Δt. For Heun's scheme both values α and c are 0.5, which makes the scheme of second order with respect to Δt.

5.1.2 4^{th} order Runge-Kutta method

Runge–Kutta method can be used to construct high order accurate numerical method. In this model, we used the two-variable Runge-Kutta algorithm. Suppose a system with two first-order differential equations as are shown below: [10][11]

$$\frac{dx}{dt} = f(t, x, y), \qquad x(t_0) = x_0 \tag{22}$$

$$\frac{dy}{dt} = g(t, x, y), \qquad y(t_0) = y_0 \tag{23}$$

The formulas for the Runge-Kutta algorithm are:

$$t_{n+1} = t_n + h \tag{24}$$

$$x_{n+1} = x_n + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \tag{25}$$

$$y_{n+1} = y_n + \frac{1}{6}h(l_1 + 2l_2 + 2l_3 + l_4) \tag{26}$$

The next value x_{n+1} and y_{n+1} is determined by the present value x_n and y_n, respectively, plus the weighted average of four increments. The step-size h corresponds to the time-step. The four increments for each differential equation are defined as:

$$
\begin{aligned}
k_1 &= f(t_n,\ x_n,\ y_n) & l_1 &= g(t_n,\ x_n,\ y_n)\\
k_2 &= f(t_n + \frac{h}{2},\ x_n + h\frac{k_1}{2},\ y_n + h\frac{l_1}{2}) & l_2 &= g(t_n + \frac{h}{2},\ x_n + h\frac{k_1}{2},\ y_n + h\frac{l_1}{2})\\
k_3 &= f(t_n + \frac{h}{2},\ x_n + h\frac{k_2}{2},\ y_n + h\frac{l_2}{2}) & l_3 &= g(t_n + \frac{h}{2},\ x_n + h\frac{k_2}{2},\ y_n + h\frac{l_2}{2})\\
k_4 &= f(t_n + h,\ x_n + hk_3,\ y_n + hl_3) & l_4 &= g(t_n + h,\ x_n + hk_3,\ y_n + hl_3)
\end{aligned}
\tag{27}
$$

By applying this numerical method in the Daisyworld model, the eqs. (22) and (23) take the following form:

$$\frac{dA_w}{dt} = f(t, A_w, A_b), \qquad A_w(t_0) = 0.01 \tag{28}$$

$$\frac{dA_b}{dt} = g(t, A_w, A_b), \qquad A_b(t_0) = 0.01 \tag{29}$$

where f and g are given by eq. (2). Both functions f and g are independent of t.

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