Indian Institute of Science Education and Research, Mohali

Project Report

Course Code - PHY312

Monte Carlo simulation of polarised photon transport in scattering media

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1 Introduction

Monte Carlo methods are a broad class of algorithms that use randomness to simulate a model of interest. In this project, we use Monte Carlo methods to simulate light propagation through a scattering media.

2 Implementation of the model

In our model, photons are successively emitted, and their individual paths are computed and tracked one by one. In light propagation through the medium, the photon propagation is simultaneous. But, we consider successive emission of photons for simplicity and finally merge the individual trajectories to simulate the propagation of light through the medium and obtain the desired data. This doesn't affect the final data obtained, as the properties of the medium remain unchanged during photon propagation and photon-photon interactions are absent.

3 Mueller Matrix

Mueller matrices (a.k.a scattering matrices) are matrices that describe the relationship between the polarization of the incident and emerging light beam after passing through a medium. The Mueller matrices and Stokes vector is a compact and elegant formalism. Lenses, polarisers etc. can be represented as a mathematical entity and this simplifies calculation in optics significantly.

4 Geometry

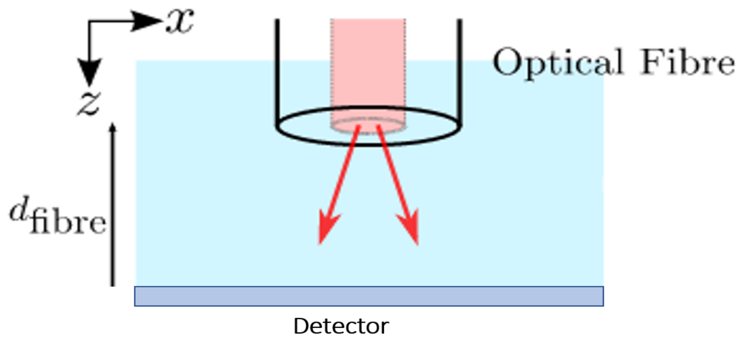


Figure 1: Geometry

The experimental setup simulated consists of a micro-sphere solution medium, with a detector at a distance d from an optical fibre. Light of different polarisations is incident on the medium, is scattered multiple times and collected at the detector. The polarisation of light at the detector is then analysed.

5 Euler Monte Carlo

Different methods have been developed to track the polarisation reference frame during the simulation. In Euler Monte Carlo method,a triplet of unit vectors that are rotated by an azimuth and scattering angle at each scattering step, is used. These unit vectors are rotated using Euler angles by general transformation matrices during scattering. This is the method we have implemented in our program.

6 Algorithm

6.1 Flow chart

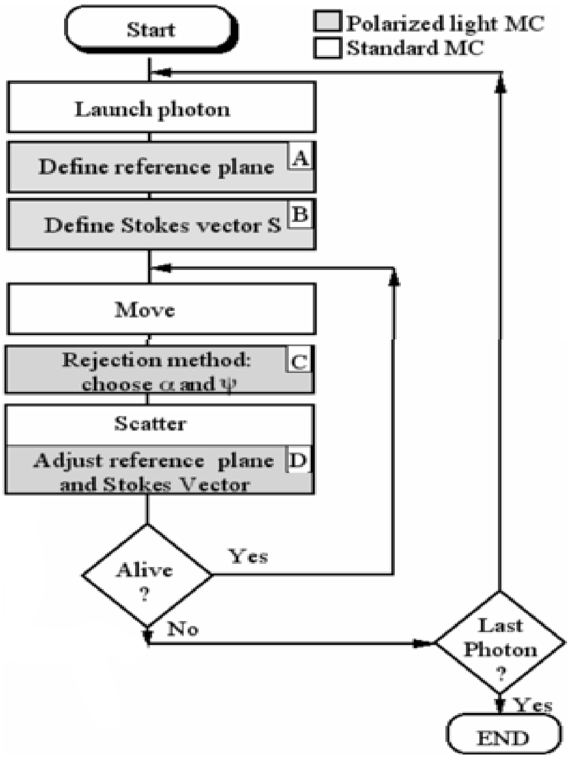


Figure 2: Flow chart

6.2 Launch photons

The position, direction cosines of the photons of a given polarisation are initialised. The photon is assigned a weight of 1. Weight of photon is used to account for absorption of light by the scattering media.

6.2.1 Define reference plane

Two unit vectors u,v are defined as reference vectors. In Euler Monte Carlo method, a triplet of unit vectors (u,v,w) are used to track the polarisation reference plane, where u denotes the direction of photon propagation. W is defined as the cross product of u and v, and hence isn't explicitly used in the program.

6.2.2 Define Stokes vector

The Stokes vector $S = [I, Q, U, V]$ for each photon is defined based on its initial polarisation.

6.3 Move

6.3.1 Compute stepsize

The step size of a photon denotes the distance traversed by a photon before scattering, and is determined as: $s = -ln(rnd)/(\mu_s + \mu_a)$, where rnd is random number between 0 and 1.

6.3.2 Record trajectory

At each timestep, the position of the photon is recorded. This is used to trace the path of the photon through the medium. The precision of the reconstructed path depends on the time intervals at which the photon positions are recorded.

6.4 Rejection Method

The angle of scattering (α) and the angle of rotation into the scattering plane (β) are selected by a rejection method, based on the phase function.

$$P(\alpha, \beta) = s_{11}(\alpha)I_0 + s_{12}(\alpha)[Q_0 cos(2\beta) + U_0 sin(2\beta)] \tag{1}$$

where, $[I_0, Q_0, U_0, V_0] = S_0$, the stokes vector of the incident photon
$s_{11}(\alpha)$ and $s_{12}(\alpha)$ are elements of scattering matrix, and are obtained from the scattering amplitudes $S_1(\alpha)$ and $S_2(\alpha)$. S_1 and S_2 depend on the size parameter x, the complex index of refraction of the particle $n_{particle}$,and are obtained using miepython module.

6.4.1 Miepython

`miepython` is a Python module which provides functions for calculating the extinction efficiency, scattering efficiency, backscattering, scattering asymmetry etc. It is used to calculate light scattering by non-absorbing, partially-absorbing, or perfectly conducting spheres. This package is based on Mie theory. The results of this package has been validated by experimental research.

6.5 Scatter

The new position and direction cosines of the photon are updated.

6.5.1 Adjust reference plane and Stokes vector

The reference unit vectors u,v are updated.
V is rotated about u by angle θ.
$v_{new} = R_{euler}(u, \beta)v$
u is the rotated about v by an angle α, and is updated as $u_{new} = R_{euler}(v, \alpha)u$
The Stokes vector is updated as : $S_{new} = M(\alpha)R(\beta)S$, where $M(\alpha)$ is the scattering matrix and $R(\beta)$ is the rotational matrix.

6.6 Checking if the photon reached the detector

If the photon reaches the detector before scattering, the co-ordinates and polarisation of the photon (Stokes vector) at the detector is recorded.

6.7 Checking photon status

The photon weight is compared to a threshold, and the photon is terminated if the photon weight ¡ threshold. If photon weight ¿ threshold, a new step size is determined and the above steps are repeated.
The above steps are 7 times, with a different polarisation being assigned to a photon each time.

6.8 Serialisation of detector data

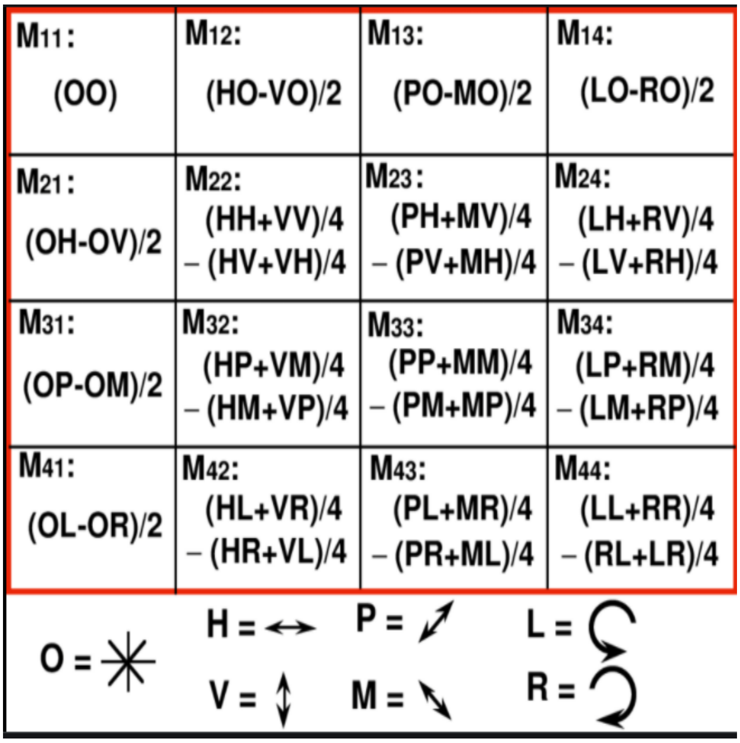
Serialisation is the process of converting an object into a stream of bytes to store the object. We use the module pickle to serialise the detector data, which comprises the spatial co-ordinates and polarisation data (Stokes vector) of each photon.

6.9 Parallel computing

In computation intensive programs, it is customary to trade time for memory and the other way around to manage resources and runtime. This can be achieved using parallel computing techniques like multithreading and multiprocessing. Whenever parts of code can function independently and without chronology constraints, we can run the parts simultaneously.
Here, we can exploit the fact that the photons are non-interactive and run more than one photon at a time using multiple CPU threads and CPU cores. On top of that, some out of: the binning, the matrix algebra, the rejection sampling, calculating the Mueller matrix elements, animating trajectories etc., can be done simultaneously. Since multithreading requires prerequisite technical knowledge, we have made use of a primitive version of multiprocessing to obtain our results.

6.10 Determining Mueller Matrix elements

The combined data is then used to analyse the polarisation of photons at the detector. The elements of the Mueller matrix for each spatial bin are then constructed as:



M_{11}: (OO)	M_{12}: (HO-VO)/2	M_{13}: (PO-MO)/2	M_{14}: (LO-RO)/2
M_{21}: (OH-OV)/2	M_{22}: (HH+VV)/4 − (HV+VH)/4	M_{23}: (PH+MV)/4 − (PV+MH)/4	M_{24}: (LH+RV)/4 − (LV+RH)/4
M_{31}: (OP-OM)/2	M_{32}: (HP+VM)/4 − (HM+VP)/4	M_{33}: (PP+MM)/4 − (PM+MP)/4	M_{34}: (LP+RM)/4 − (LM+RP)/4
M_{41}: (OL-OR)/2	M_{42}: (HL+VR)/4 − (HR+VL)/4	M_{43}: (PL+MR)/4 − (PR+ML)/4	M_{44}: (LL+RR)/4 − (RL+LR)/4

Figure 3: Mueller Matrix

In each of the elements, the first and second letter denotes the polarisation of incident photon, and photon at the detector respectively. For example, OO represents the unpolarised light detected, for unpolarised incident light.

7 Parameters

- Wavelength of light in vacuum: 543 nm
- Concentration of medium: $0.05 spheres/microns^3$
- Scattering coefficient of medium: $11.88cm^{-1}$
- Absorption coefficient of medium: 0
- Refractive index of medium: 1.33
- Refractive index of micro spheres: 1.59
- Sphere diameter: 2.02 microns
- Core diameter of optical fibre: 1.2 mm

The simulation was run for 5537 photons per polarisation.

8 Results

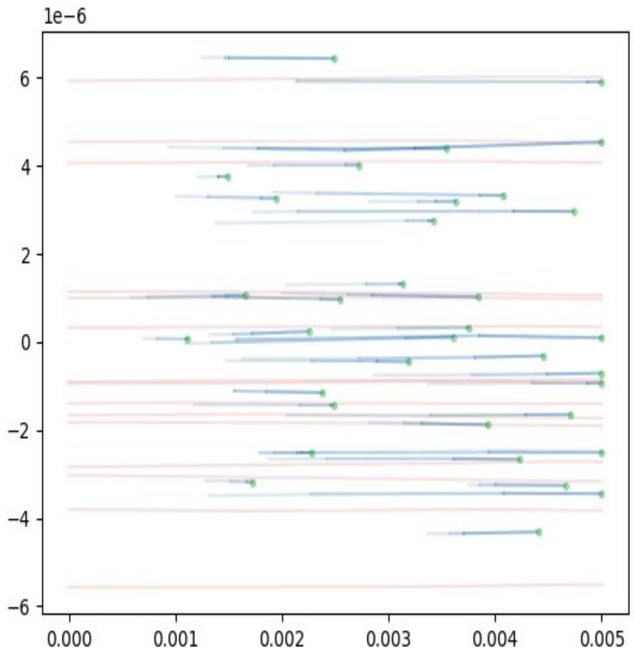


Figure 4: Animation

The animation of photon trajectory in the medium was obtained for different spatial resolutions, and the results have been attached.

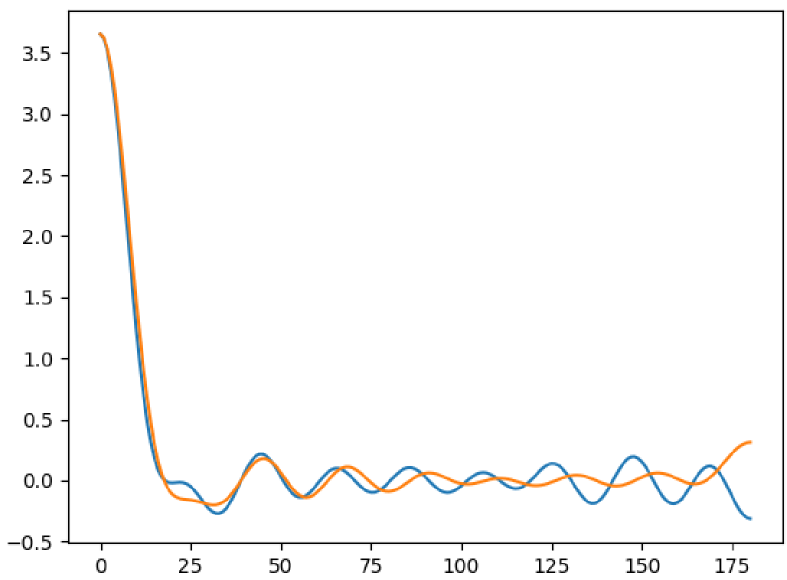


Figure 5: Scattering Phase Function for the given set of parameters

The scattering phase function is obtained for a scattering coefficient $11.88 cm^{-1}$, absorption coefficient 0, concentration $0.05 spheres/microns^3$, and sphere diameter 2.02 microns. The refractive index of medium and micro spheres used is 1.33 and 1.59 respectively.

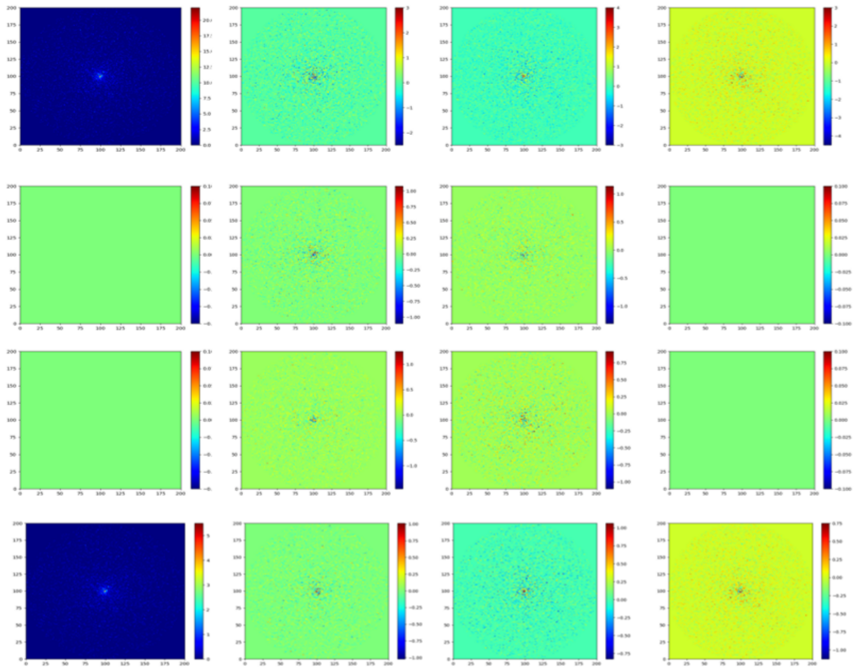


Figure 6: Mueller Matrix

The Mueller matrix elements for each spatial bin are computed and is visualised in the form of the plots shown above.

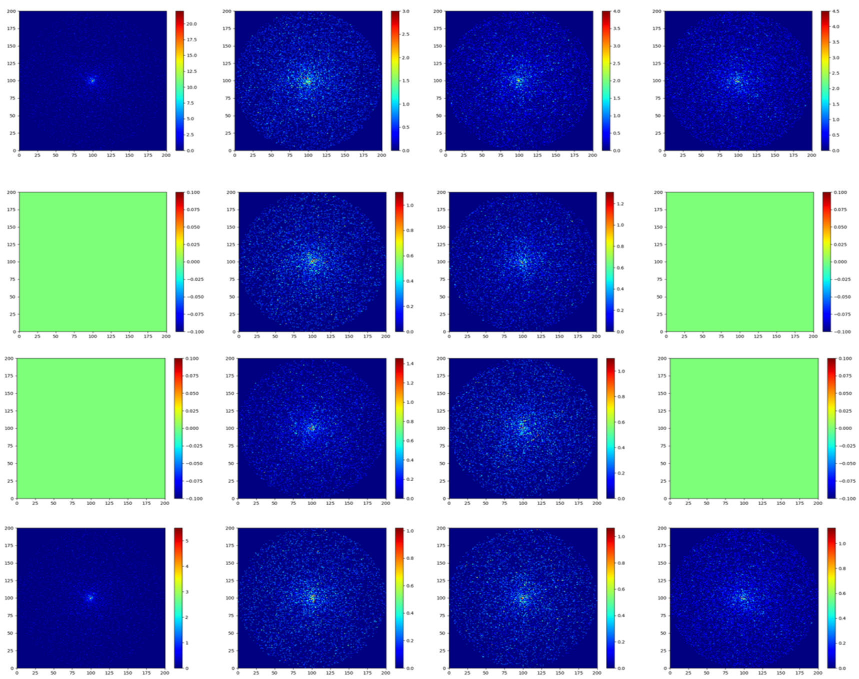


Figure 7: contrast enhanced Mueller matrix

In the second plot, the absolute value of the matrix elements are plotted in order to obtain a contrast-enhanced plot.

9 Limitations

- The number of photons per polarisation used in this simulation is 5537 only. A larger number of photons can provide a better insight and accurate data and plots of Mueller Matrix. But, this couldn't be accomplished due to the limitations in computational power available.
- Due to the longer runtime, the detector had to be placed nearer in order to reduce the number of scattering events.
- Experimenting with different values of parameters for optimisation couldn't be done, as the runtime for a single set of parameters was very high.

References

[1] B. Kaplan, G. Ledanois, and B. Villon *Mueller Matrix of Dense Polystyrene Latex Sphere Suspensions: Measurements and Monte Carlo Simulation," Appl. Opt. 40, 2769–2777, (2001)*

[2] Jessica C. Ramella-Roman, Scott A. Prahl, Steve L. Jacques *Three Monte Carlo programs of polarized light transport into scattering media: part I*

[3] Jessica C. Ramella-Roman, Scott A. Prahl, Steven L. Jacques4 *Three Monte Carlo programs of polarized light transport into scattering media: part II*

[4] `https://omlc.org/calc/mie_calc.html`

[5] `http://omlc.ogi.edu/software/polarization/`

[6] `https://miepython.readthedocs.io/en/latest/`