# 1 Incorporation of plasticity in the context of data-driven solid mechanics

In order to render the data-driven algorithm suitable for materials showing plasticity in their stress response, the penalty function $F_e(\epsilon_e, \sigma_e)$ remains unchanged, however the distance function (that is the norm) is changed to

$$||(\epsilon_e, \sigma_e)||_e = \left(W_e^{\xi}(\xi_e - \xi_e^{'}) + W_e^{\sigma}(\sigma_e - \sigma_e^{'}) + W_e^{\rho}(\rho_e - \rho_e^{'}) + W_e^{\tau}(\tau_e - \tau_e^{'}) + W_e^{\varphi}(\varphi_e - \varphi_e^{'})\right)^{1/2},$$

with the *energy densities*

$$W_e^{\xi}(\xi_e) = \frac{1}{2} k_e^{\xi} \xi^2, \tag{1}$$

$$W_e^{\sigma}(\sigma_e) = \frac{1}{2} k_e^{\sigma} \sigma^2, \tag{2}$$

$$W_e^{\rho}(\rho_e) = \frac{1}{2} k_e^{\rho} \rho^2, \tag{3}$$

$$W_e^{\tau}(\tau_e) = \frac{1}{2} k_e^{\tau} \tau^2 \qquad \text{and} \tag{4}$$

$$W_e^{\varphi}(\varphi_e) = \frac{1}{2} k_e^{\varphi} \varphi^2. \tag{5}$$

The first and second energy density are identical to the *reference strain and complementary energy densities* from the linear elasticity framework, however presented in a slightly different notation. In the context of plasticity, $\xi$ denotes the *ending* or *last strain value* of the load path and $\sigma$ the corresponding stress state. $\tau$ and $\varphi$ then denote the *maximum nominal strain and stress value* of the load path and $\rho$ encodes the sign of this stress-strain state, in detail $\rho = 1$ denotes tension and $\rho = -1$ compression.
Introducing these additional distances may not increase the dimension of the phase space (which is still two-dimensional for a one-dimensional material), however it increases the data-set size. The phase-space can not be sampled as before, that is for example by prescribing the strain equidistantly over the phase space. It now needs to be sampled by prescribing multiple load paths. For the example at hand, theses load paths are characterized by two quantities, $\epsilon^{\max}$ and $\epsilon^{\text{end}}$, denoting the maximum prescribed strain and the last prescribed strain. Thus the sampling is done by prescribing $\epsilon^{\max}$, which leads to a point on the envelope of the functions, red line in Figure 1, and then prescribing multiple $\epsilon^{\text{end}}$ for this $\epsilon^{\max}$. Exemplary, the blue line in Figure 1 shows a randomly picked $\epsilon^{\min}$ for a random $\epsilon^{\max}$. For this load path, the stress-strain response is solved using the material parameters depicted in Table 1 and the aforementioned stress and strain quantities are saved into the data-set. For the example at hand, which is depicted in Figure 1, $\epsilon^{\max}$ was prescribed as

$$\epsilon_i^{\max} = i\Delta\epsilon \quad i = 1, ..., 50 \qquad \Delta\epsilon = 0.0001, \tag{6}$$

and for each $\epsilon_i^{\max}$ the ending strain $\epsilon^{\text{end}}$ was prescribed as

$$\epsilon^{\text{end}} = j\Delta\epsilon \quad j = 1, ..., i \tag{7}$$

resulting in 1275 data points.

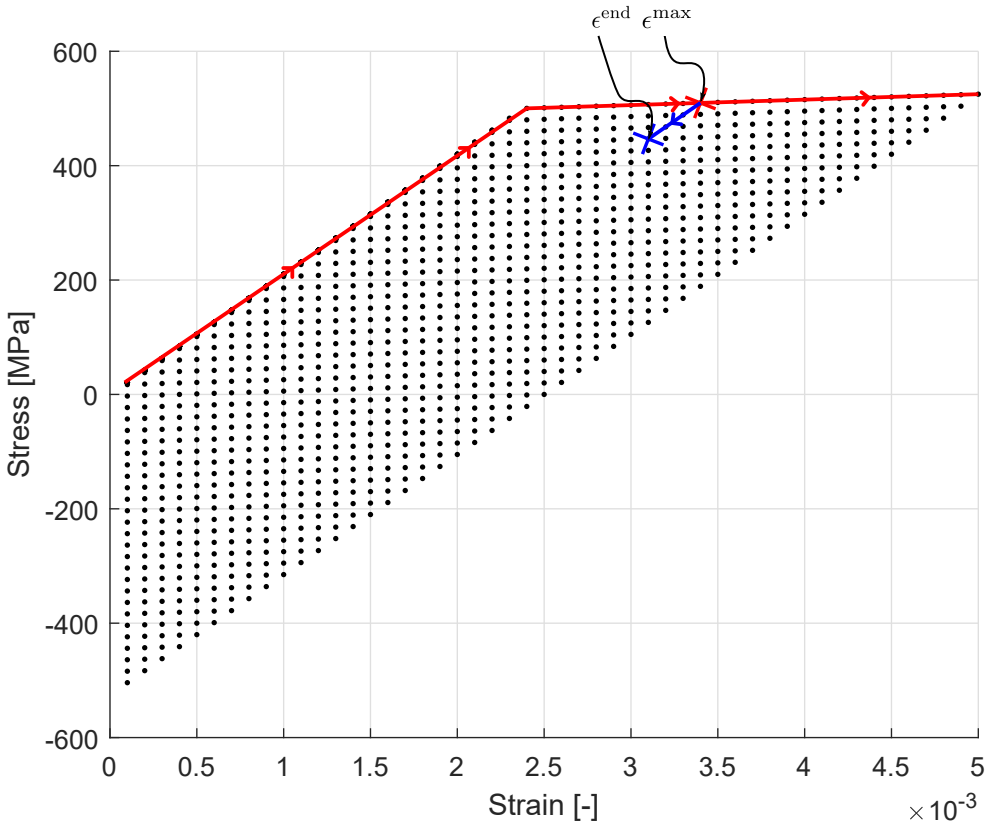


Figure 1: Schematic visualization of the global and local coordinate system of each truss.

Table 1: Material parameters used for the data-set.

| Material parameter | Value |
|---|---|
| Young's modulus | 210000 [MPa] |
| Hardening modulus | 10000 [MPa] |
| Yield limit | 500 [MPa] |

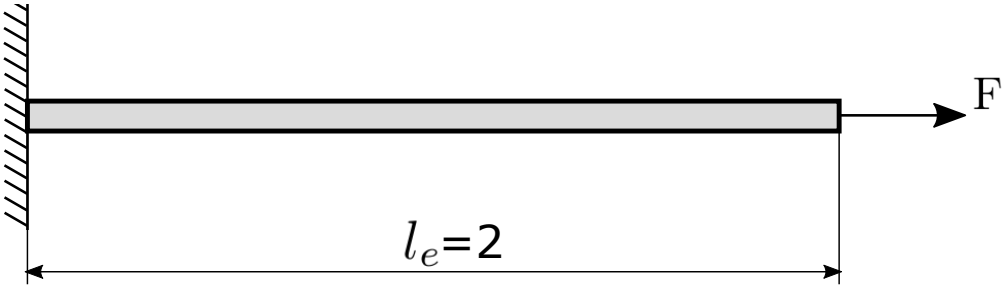


Figure 2: Geometry of the single truss.

In order to validate this new distance function, the geometry depicted in Figure 2 is used, showing a single truss being clamped at the left end and being subjected to a prescribed force at the right end. The load path is prescribed via the prescribe force, which is 5.1 in maximum and then released to 4.59. Since the cross-section of the bar is prescribed with $A = 0.01$, the force corresponds to a stress of 510 in maximum and 459 in minimum, which is exactly the stress-strain path depicted in Figure 1 by the red and blue line. The geometry is discretized in 100 elements and given two different initial starting points, being the first data-set entry, depicted in Figure 3a, and random data-set entries, depicted in Figure 3b. The calculations were performed with the weighting parameters shown in Table 2. With the same starting point for every element, one can see that the algorithm does find the correct stress state and the best data-set point, which is data-set entry 592 in is this example, for every element. However, when given random starting points, the algorithm is not able to find the exact same solution in every element. The algorithm either finds data-set entry 592 when the respective element starts *on the left* of the correct solution, or data-set entry 593 when the respective element started *on the right*. When taking a closer look at these data-set entries

$$D(592): \quad \begin{aligned} \xi &= 0.0031 \\ \sigma &= 446.7228 \\ \tau &= 0.0034 \\ \rho &= 1 \\ \varphi &= 509.7249 \end{aligned} \tag{8}$$

$$D(593): \quad \begin{aligned} \xi &= 0.0032 \\ \sigma &= 467.7228 \\ \tau &= 0.0034 \\ \rho &= 1 \\ \varphi &= 509.7249 \end{aligned} \tag{9}$$

one can see that both data-set points are coming from the same maximum strain and stress ($\tau$ and $\varphi$) but differ in the minimal stress and strain ($\xi$ and $\sigma$). When

additionally taking a look at the exact solution

$$\text{Exact solution}: \quad \begin{array}{l} \xi = 0.003157 \\ \sigma = 458.7 \\ \tau = 0.0034 \\ \rho = 1 \\ \varphi = 509.7 \end{array} \tag{10}$$

on can see that both solution are close to the exact solution, which is confirmed by the distance to the exact soltuion which is 0.0373 for $D(592)$ and 0.0284 for $D(593)$. However, since the distance to the corrected stresses is same for both, the algorithm is not able to find the "better" one. This should be able to be changed when adapting the search parameters.

Table 2: Distance parameters used for the calculations.

| Distance parameter | Value | |
|---|---|---|
| $k_e^{\xi}$ | E | |
| $k_e^{\sigma}$ | 1/E | E= 210 000 |
| $k_e^{\tau}$ | E | |
| $k_e^{\rho}$ | 1 | |
| $k_e^{\varphi}$ | 1/21; | |

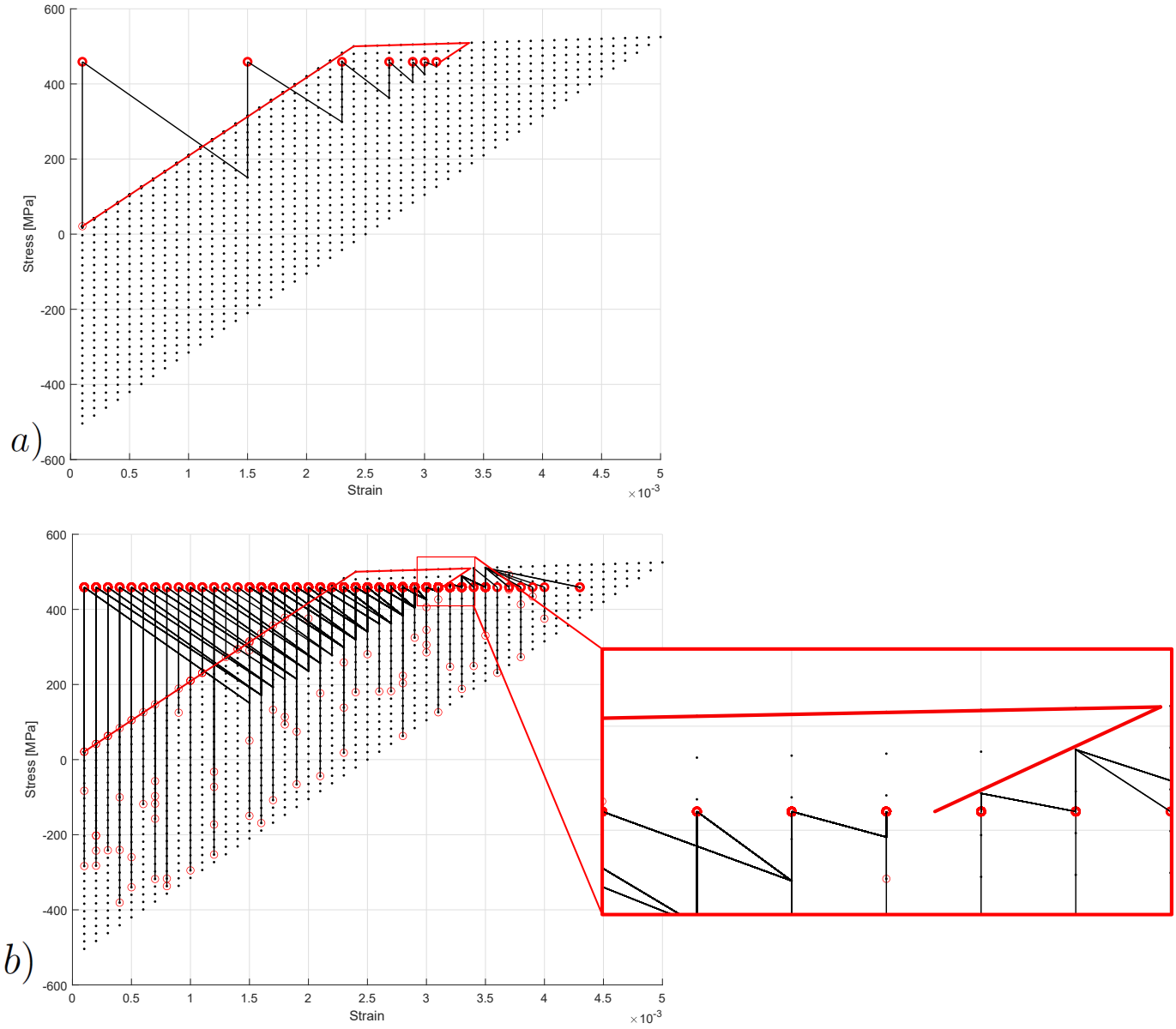


Figure 3: Iteration path for identical starting points for every element in a) or random starting points in b).