Statistical Signal Processing I
MATLAB Homework

Objective

This homework assignment aims to deepen your knowledge and skills in linear algebra, probabilities, multivariate densities, and matrices as they apply to statistical signal processing. Please complete all exercises and submit your MATLAB code for evaluation.

Instructions (Please read carefully)

Return your solution to Moodle before 11.10.2023 16:15. Late returning will lead to deduction of points.

Upload your solutions to Moodle as ZIPPED file containing all files needed to run your codes. Your codes must be ready to run without any additional user configuration.

Use filename `MATLAB_Group.zip`. In addition to the well-commented program, make sure the report includes the following:

- The full name and the student number for each group member.
- Clear answers to the problems posted (including for example figures obtained by running the simulation program).
- Plagiarism is strictly forbidden and will lead to immediate rejection!
- You will get separate grade for each MATLAB exercise and you must need passing grade in order to pass the whole course.

Exercise 1

1. Create a 5x5 random matrix $\boldsymbol{A}$ with entries drawn from a Gaussian distribution $\mathcal{N}(0, 1)$.
2. Compute the singular value decomposition (SVD) of $\boldsymbol{A}$.
3. Reconstruct $\boldsymbol{A}$ using the SVD components, and name it $\boldsymbol{A}_{\mathbf{svd}}$.
4. Verify the reconstruction quality by computing the Frobenius norm $\|\boldsymbol{A} - \boldsymbol{A}_{\mathbf{svd}}\|_F^2$.

Hints: use command *svd*, *randn*, and *norm*$(\cdot)$.

Exercise 2

Consider the following probability density function of the continuous uniform distribution:

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases}$$

1. Generate 1000 random samples, where each sample is drawn from the uniform distribution with mean $\mu = 2$ and variance $\sigma^2 = 3$.

2. Plot a histogram of the generated samples.

   Hints: For the first question, you have to find the values of a and b. Use MATLAB functions *rand* for uniform distribution and *histogram* for plotting.

Exercise 3

Suppose you have a 2D matrix $\boldsymbol{D}$ with dimensions 500×2.

1. Generate the first column of $\boldsymbol{D}$, namely $\boldsymbol{x}$, that contains 500 samples drawn from a Gaussian distribution with mean $\mu = .5$ and variance $\sigma^2 = .5$.

2. Generate the second column of $\boldsymbol{D}$, namely $\boldsymbol{y}$, that contains 500 samples drawn from a Gaussian distribution with mean $\mu = 0$ and variance $\sigma^2 = 2$.

3. compute the sample mean and the sample variance of $\boldsymbol{z} = \boldsymbol{x} + \boldsymbol{y}$, and compare them with the theoretical mean and the theoretical variance of $\boldsymbol{z}$.

Exercise 4

1. Create a 2x2 covariance matrix $\boldsymbol{\Sigma}$ with non-zero off-diagonal elements.

2. Generate a white Gaussian random vector $\boldsymbol{w}$ with dimension 2×1000, where each column has a zero mean and an identity covariance matrix.

3. Perform the Cholesky decomposition on $\boldsymbol{\Sigma}$ and obtain its lower triangular matrix $\boldsymbol{L}$.

4. Use the matrix $\boldsymbol{L}$ to map $\boldsymbol{w}$ to a new Gaussian random vector $\boldsymbol{x}$ such that $\boldsymbol{x} = \boldsymbol{L}\boldsymbol{w}$.

5. Compute the sample covariance matrix of $\boldsymbol{x}$ and verify that the sample covariance matrix is approximately equal to $\boldsymbol{\Sigma}$.