Statistical Signal Processing I
MATLAB Homework

Instructions (Please read carefully)

Return your solution to Moodle before 27.11.2023 16:15. Late returning will lead to deduction of points.

Upload your solutions to Moodle as ZIPPED file containing all files needed to run your codes. Your codes must be ready to run without any additional user configuration.

Use filename `MATLAB_Group.zip`. In addition to the well-commented program, make sure the report includes the following:

- The full name and the student number for each group member.
- Clear answers to the problems posted (including for example figures obtained by running the simulation program).
- Plagiarism is strictly forbidden and will lead to immediate rejection!
- You will get separate grade for each MATLAB exercise and you must need passing grade in order to pass the whole course.

Tasks

1. **Estimation using linear model**

   We observe two samples of a DC level A in correlated zero-meanGaussian noise

   $$x[0] = A + \ w[0]$$

   $$x[1] = A + \ w[1]$$

   where $\mathbf{w} = [w[0] \ w[1]]^{\mathrm{T}}$ is zero-mean Gaussian random vector with covariance matrix.

   $$\mathbf{C} = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

   The parameter $-1 \leq \rho \leq 1$ is the correlation coefficient between $w[0]$ and $w[1]$.

   - derive the expression of the linear model estimator (simplify as much as you can and include all the details in the report)

- Create a Monte-Carlo simulation program for the problem using different values of ρ as follow $\rho = \{-1, 0, 0.5, 1\}$. Include in your report Table showing simulated variance and theoretical variance for each these values of ρ. Include also Table showing simulated mean and theoretical mean for each value of ρ.
  Hint: Before you implement the estimator, for each value of ρ, check if the linear model is applicable. If it is not, you can implement any other efficient estimator.
- What is special about plot with $\rho = -1$ why is it like this? Handle this value also properly in simulations.

Use these parameter values:

$A = 2$;
$\sigma^2 = 0.5$;
$MC = 100000$; % number of Monte Carlo loops

2. **Maximum likelihood estimator** For N IID observations of $U[0, \theta]$ PDF (uniform distribution) **Create a Monte-Carlo simulation program for the problem.**

   - Find the maximum likelihood estimator of θ.
   - Implement the maximum likelihood estimator for θ in MATLAB. Perform MC Monte Carlo loops. In each loop, generate N random variables following the given uniform distribution.
   - Implement also the following the estimator $A\_est\_mean = samplemean * 2$ in MATLAB. Are both estimators unbiased?
   - Find out the fully theoretical PDF for both estimators and plot against simulated PDFs/histograms (all in the same figure). For estimator using A_est_mean you may use Gaussian approximation for theoretical PDF. For ML estimator plot the exact fully theoretical PDF.

     Hint: For theoretical PDF for ML estimator: 1) check section "Order statistics sampled from an uniform distribution" in the below link. 2) you can, for example, use MATLAB command *pdf('beta',...)* to generate the theoretical PDF for ML estimator.

     `https://en.wikipedia.org/wiki/Order_statistic#Probability_distributions_of_order_statistics`

   Use the following parameters for the Monte Carlo simulation

   $MC = 100000$;
   $N = 100$;
   $\theta = 1$;