FACULTY OF INFORMATION TECHNOLOGY AND ELECTRICAL ENGINEERING
DEGREE PROGRAMME IN ELECTRONICS AND COMMUNICATIONS ENGINEERING

Statistical Signal Processing I

MATLAB Homework 4

Athmajan Vivekananthan
avivekan23@student.oulu.fi
2305573

Hathurusingha Anjalee Anuththara De Silva
hdesilva23@student.oulu.fi
2303245

1 ESTIMATION OF SINUSOIDAL

The task in hand is to estimate A, f_0, ϕ of the sinusoidal observations done in AWGN channel. In order to estimate the values a complex and long calculation is understood as per explained in the coursebook [1] which is not included here.

However to summarize how the values were obtained,

1. **Estimation of f_0:**
   Given that the range of frequency is between 0 and 0.5 but not close to the extreme values, the value that maximizes the periodogram defined by equation 1 is done. An instant of the behaviour of the said periodogram for the given range is shown in figure 1.1. Therefore arg max was used in the simulation to obtain the frequency values $\hat{f}_0$ that maximizes this curve at each Monte Carlo Loop.

$$I(f) = \frac{1}{N}\left|\sum_{n=1}^{5} x[n]\exp(-j2\pi f n)\right|^2 \tag{1.1}$$

2. **Estimation of A:**
   As per the equation 2 the estimation of the A was done.

$$\hat{A} = \frac{2}{5}\left|\sum_{n=1}^{5} x[n]\exp(-j2\pi \hat{f}_0 n)\right| \tag{1.2}$$

3. **Estimation of ϕ:**
   As per the equation 3 the estimation of the ϕ was done.

$$\phi = \arctan\frac{-\sum_{n=0}^{5} x[n]\sin 2\pi\hat{f}_0 n}{\sum_{n=0}^{5} x[n]\cos 2\pi\hat{f}_0 n} \tag{1.3}$$

Finally the estimated simulation values are gathered in an array and their variance are compared against the CRLB values obtained from the coursebook [1](3.41).

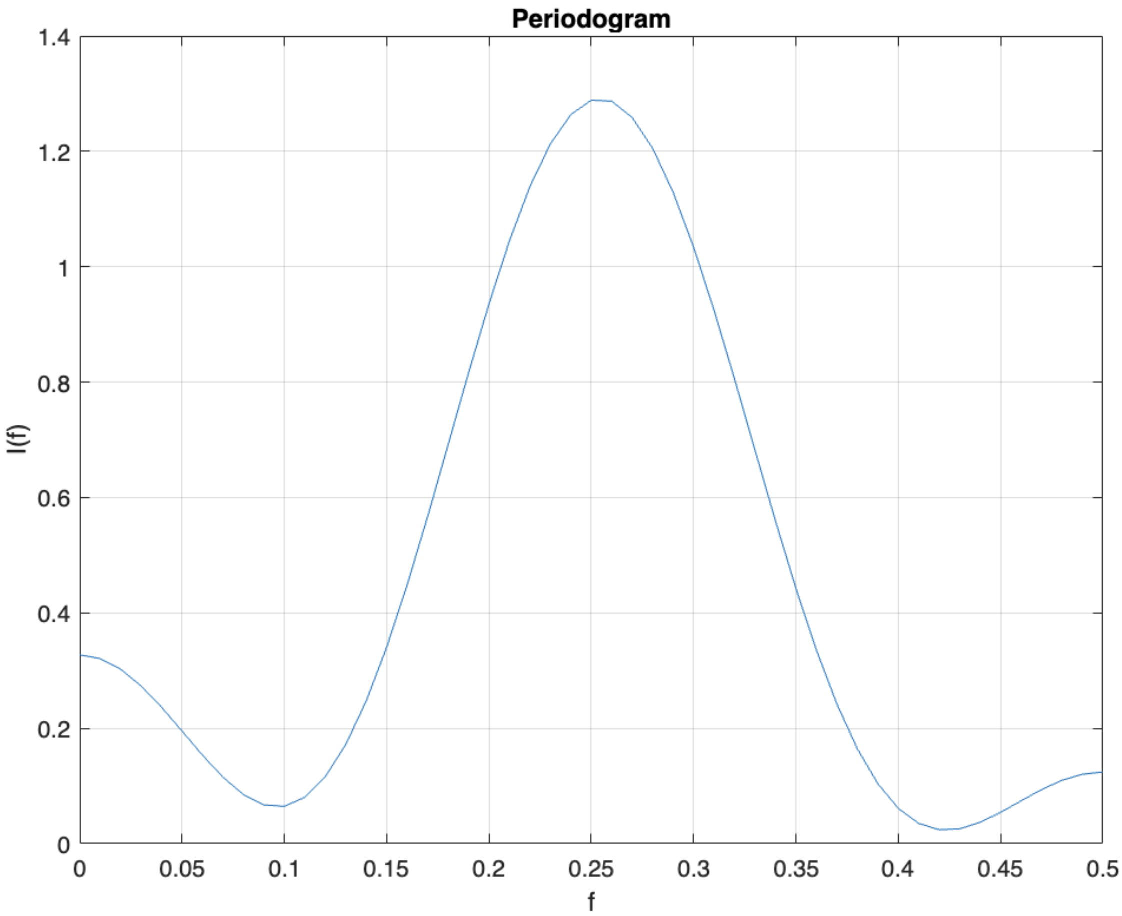

Figure 1.1. Periodogram for the range of frequencies given

```matlab
% Defining parameters

N = 5; % Number of observations
A = 1; % True value of A
sigma2 = 0.001; % Variance of WGN
f0 = 0.25; % True value of fundamental frequency
phi = pi/3; % True value of Phi
numMonteCLoops = 1E5; % Number of monte carlo loops

% Initiate estimates array
estimate_f0 = ones(1,numMonteCLoops)*NaN;
estimate_A = ones(1,numMonteCLoops)*NaN;
estimate_Phi = ones(1,numMonteCLoops)*NaN;
```

```matlab
% Monte Carlo Loops
for MC=1:numMonteCLoops

    w = randn(1,N)*sqrt(sigma2).*(1:N); % the WGN part
    ang1 = 2*pi*f0.*(1:N);
    ang2 = ang1 + phi;
    x = A*cos(ang2) + w;

    % The periogram which is maximized by MLE of f0
    f = 0:0.01:0.5;
    xx = zeros(size(f));
    for n = 1 :5
        xInst = x(n) * exp(-1j * 2 * pi .*f * n);
        xx = xx + xInst;
    end

    If = 0.2* abs(xx).^2;

    if MC == 1

       % Plot squares_of_absolute_values against f
       plot(f, If);
       xlabel('f');
       ylabel('I(f)');
       title('Periodogram');
       grid on;
       disp('Periodogram for one loop to observe the behavior');
    end

    % Find the arg max of If as in (7.66)
    [max_value, index] = max(If);

    % Get the corresponding frequency value
    f0_est = f(index);
```

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Figure 1.2. Task Matlab Code - Page 1

```
    % calculation of A_est as in (7.66)
    A_est = 0;
    for n = 1 :5
        xInst = x(n) * exp(-1j * 2 * pi *f0_est * n);
        A_est = A_est + xInst;
    end
    A_est = 0.4* abs(A_est);

    % calculation of Phi_est (7.66)
    Phi_est_Num = 0;
    Phi_est_Dem = 0;
    for n = 1 :5
        xInst = x(n) *sin(2*pi*f0_est*n);
        Phi_est_Num = Phi_est_Num + xInst;

        xInst1 = x(n) *cos(2*pi*f0_est*n);
        Phi_est_Dem = Phi_est_Dem + xInst1;
    end

    Phi_est = atan(-1*Phi_est_Num/Phi_est_Dem);

    estimate_f0(MC) = f0_est;
    estimate_A(MC) = A_est;
    estimate_Phi(MC) = Phi_est;

end
```

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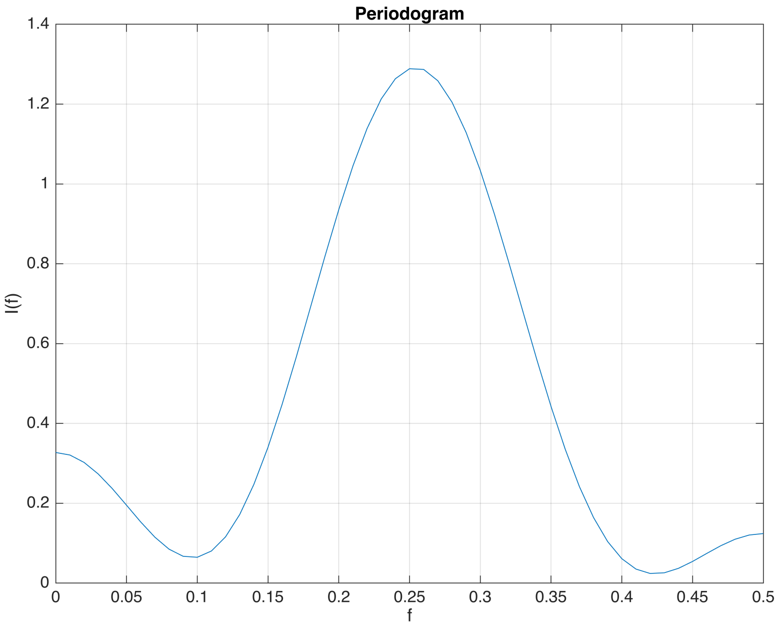
Figure 1.3. Task Matlab Code - Page 2




```
Periodogram for one loop to observe the behavior
```

```
% Variance of MLE of f0
std(estimate_f0)^2
```

```
ans = 2.3713e-05
```

```
% Variance of MLE of A
std(estimate_A)^2
```

```
ans = 0.0053
```

```
% Variance of MLE of Phi
std(estimate_Phi)^2
```

```
ans = 0.0062
```

```
%% Defining CRLB Values as per (3.41)
eta = A^2/(2*sigma2);
CRLB_A = 2*sigma2/N;
CRLB_f0 = 12 / (((2*pi)^2)*eta*N*((N^2)-1));
CRLB_phi = (2*(2*N -1)) / (eta*N*(N+1));



if std(estimate_f0)^2 >= CRLB_f0
    disp('The CRLB is satisfied for MLE of f0');
```

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Figure 1.4. Task Matlab Code - Page 3

```
end
```

```
The CRLB is satisfied for MLE of f0
```

```
if std(estimate_A)^2 >= CRLB_A
    disp('The CRLB is satisfied for MLE of A');
end
```

```
The CRLB is satisfied for MLE of A
```

```
if std(estimate_Phi)^2 >= CRLB_phi
    disp('The CRLB is satisfied for MLE of Phi');
end
```

```
The CRLB is satisfied for MLE of Phi
```

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Figure 1.5. Task Matlab Code - Page 4

2 Least Sqaures Estimation

2. The same data

$$x[n] = A\cos(2\pi f o n + \varphi) + w[n]$$

Implement LSE estimator for frequency *fo* and phase φ as a phase locked loop with 10 times iteration.
with $n = -M,..,0,..,M$.
Parameter values are used as follows:
$M = 2;$
$A = 1;$
$\sigma^2 = 0.001;$
$fo = 1/4;$
$\varphi = \pi/3;$ *Iteration = 10;*

w[n] is the AWGN with zero mean and σ^2 variance.

LSE estimator for frequency and phase is calculated in a phase locked loop. A new estimate for frequency and phase is being calculated in each iteration. The equations for frequency and phase are obtained as follows.

The LSE is used. H(θ) is determined where $\theta = [f_o\ \varphi]^T$. Then $H^T(\theta)*H(\theta)$ is obtained.

$$x[n] = A\cos(2\pi f o n + \varphi) + w[n]$$

Let's derivate it with respect to f_o and φ.

1. f_o

$$\frac{\partial w(n)}{\partial fo} = 0, \text{ Because w(n) is a constant with respect to } f_o$$

$$\frac{\partial x[n]}{\partial fo} = \text{-A}2\pi\text{n sin }(2\pi f_o + \varphi)$$

2. φ

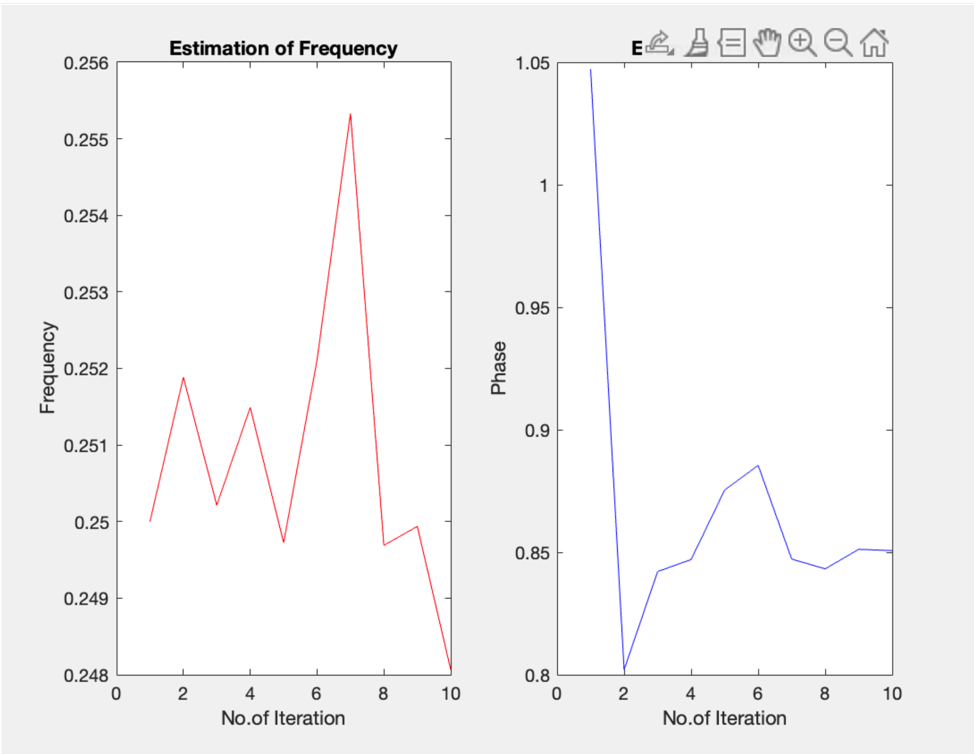
$$\frac{\partial x(n)}{\partial \varphi} = \text{-Asin }(2\pi\text{fon} + \varphi)$$

$$f_{0_{k+1}} = f_{0_k} - \frac{3}{4\pi M^3}\sum_{n=-M}^{M} \text{nx[n]sin }(2\pi f_{0_k} n + \varphi_k)$$

$$\varphi_{k+1} = \varphi_k - \sum_{n=-M}^{M} \text{x[n]sin }(2\pi f_{0_k} n + \varphi_k)$$

Figure 2.1. Task 2 Page 1

Phase locked loop is operated for 10 iterations. After the completion of 10 iterations frequency and phase converge to a value as shown in the graph below.




```
%Task 2

clear all;
close all;

M = 2;
A = 1;
variance = 0.001;
fo = 1/4;
phase = pi/3;
Iteration = 10;
n = -M:1:M;

freq_estimation = zeros(1,Iteration);
phase_estimation = zeros(1,Iteration);

for i=1:Iteration
    wn = sqrt(variance)*randn(1,2*M+1); % Zero Mean Additive White Gaussian noice N(0, var)
    xn = A*cos(2*pi*fo*n + phase) + wn; % Signal

    if(i == 1)
        freq_estimation(i) = fo; % Initial Frequency Estimate
        phase_estimation(i) = phase; % Initial Phase Estimate
    else
        for y=-M:1:M
            freq_estimation(i) = freq_estimation(i) + y*xn(y+M+1)*sin(2*pi*freq_estimation(i-1)*y + phase_estimation(i-1)); %Fr
            phase_estimation(i) = phase_estimation(i) + xn(y+M+1)*sin(2*pi*freq_estimation(i-1)*y + phase_estimation(i-1)); %Ph
        end

        freq_estimation(i) = freq_estimation(i-1) - freq_estimation(i)*3/(4*pi*(M^3)); % substract from estimate of Previous It
        phase_estimation(i) = phase_estimation(i-1) - phase_estimation(i)/M; % substract from estimate of Previous Iteration
```

Figure 2.2. Task 2 Page 2

```
        else
            for y=-M:1:M
                freq_estimation(i) = freq_estimation(i) + y*xn(y+M+1)*sin(2*pi*freq_estimation(i-1)*y + phase_estimation(i-1)); %Fr
                phase_estimation(i) = phase_estimation(i) + xn(y+M+1)*sin(2*pi*freq_estimation(i-1)*y + phase_estimation(i-1)); %Ph
            end

            freq_estimation(i) = freq_estimation(i-1) - freq_estimation(i)*3/(4*pi*(M^3)); % substract from estimate of Previous It
            phase_estimation(i) = phase_estimation(i-1) - phase_estimation(i)/M; % substract from estimate of Previous Iteration
        end
    end
    subplot(1,2,1);
    plot([1:Iteration],freq_estimation,'color',[1,0,0]);
    xlabel('No.of Iteration');
    ylabel('Frequency');
    title('Estimation of Frequency');

    subplot(1,2,2);
    plot([1:Iteration],phase_estimation,'b');
    xlabel('No.of Iteration');
    ylabel('Phase');
    title('Estimation of Phase');
```

Figure 2.3. Task 2 Page 3

3 REFERENCES

[1] Kay S.M. (1993) Fundamentals of Statistical Signal Processing, Volume 1: Estimation Theory. Pearson Education.