Statistical Signal Processing I

MATLAB Homework

Instructions (Please read carefully)

Return your solution to Moodle before 31.12.2023 16:15. Late returning will lead to deduction of points.

Upload your solutions to Moodle as ZIPPED file containing all files needed to run your codes. Your codes must be ready to run without any additional user configuration.

Use filename `MATLAB_Group.zip`. In addition to the well-commented program, make sure the report includes the following:

- The full name and the student number for each group member.
- Clear answers to the problems posted (including for example figures obtained by running the simulation program).
- Plagiarism is strictly forbidden and will lead to immediate rejection!
- You will get separate grade for each MATLAB exercise and you must need passing grade in order to pass the whole course.

Tasks

1. **Task 1**

   The data is given as

   $$X = H\theta + W$$

   where $\theta \in \{-1, 1\}$ and $H \sim \mathcal{CN}(0, \sigma^2 I)$ with $\sigma^2 = 1$. Find LS and LMMSE estimator for θ.

   Simulate average square error (ASE) for both LS and LMMSE. Compare the theoretical MSE of LMMSE.

   Plot the ASE SE for LS, LMMSE and theoretical MSE and **discuss the results**.

   Hint: See Theorem 12.1, equation (15.65), (15.67) for LMMSE estimator and theoretical MSE formula.

   Note: You don't need to write the whole code. For this task, you can find the code on **moodle** in **exercises** section with the name **Task1Code**. You just have to fill the blank parts to plot.

2. **Task 2**

Signal detection is very critical task in several signal processing applications. The goal of this assignment is apply detection theory to a signal $\mathbf{x}$ in a noisy environment. Let assume that the receiver observes the following signal

$$x[n] = As[n] + w[n], \quad n = 0, \ldots, N-1,$$

where both A and $\mathbf{S} = \left[s[0], s[1], \ldots, s[N-1]\right]^{\mathrm{T}}$ are deterministic variables and **known to the Receiver**. let us consider that $\mathbf{S}$ is a unit-norm Dc level and $\mathbf{w}$ is a white Gaussian noise such that $w[n] \sim \mathcal{N}(0, \sigma^2), n = 0, \ldots, N-1$.

The receiver aims to detect whether the signal $A\mathbf{S}$ is present. This can be formulated using the following two hypotheses

$$H_0 : \mathbf{x} = \mathbf{w}$$

$$H_1 : \mathbf{x} = A\mathbf{s} + \mathbf{w},$$

(a) Derive a detector of your choice and explain clearly when you chose that detector. In the derivation, you need to show: the complete PDF of test statistics under both H_0 and H_1, probability of false alarm P_{FA} and probability of detection P_{D}.

(b) Implement Monte Carlo simulation for your detector of choice in MATLAB, plot and compare the theoretical and the simulated PDFs. Using $A = \sqrt{20}$, $\sigma^2 = 2$, $N = 100$.

(c) Plot the ROC curve of your estimator. To get better looking curves, use the command *loglog* in MATLAB instead of plot.
Hint: generate 1000 equispaced values of P_{FA} between 0 and 1, and for each value, calculate both P_{FA} and P_{D}.

(d) Change the values of A to plot the P_{D} versus P_{FA} for the following signal-to-noise ratio (SNR) values $\{-5, 0, 5, 10, 15, 20\}$dB .