521324S SSP II — Matlab Simulation Exercise Task 2

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KALMAN FILTER (7 PTS)

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The goal of the task is to implement the Kalman filter to perform time varying channel estimation *[see Example 13.3, Kay, Vol.1]*. In particular, you are asked to reproduce the results in *[Example 13.3, Kay, Vol.1]*. The channel estimation problem is briefly summarized below.

Transmission channels can often be characterized as time-varying linear discrete-time FIR filters. If the filter sampling is represented at the transmission symbol rate, those are often referred as fading frequency-selective channels. The physical cause of the temporal distortion is often the multipath propagation, which is also a typical reason for the time-variation due to, for example, the movement of the scatterers. The channel is illustrated in Figure 1.

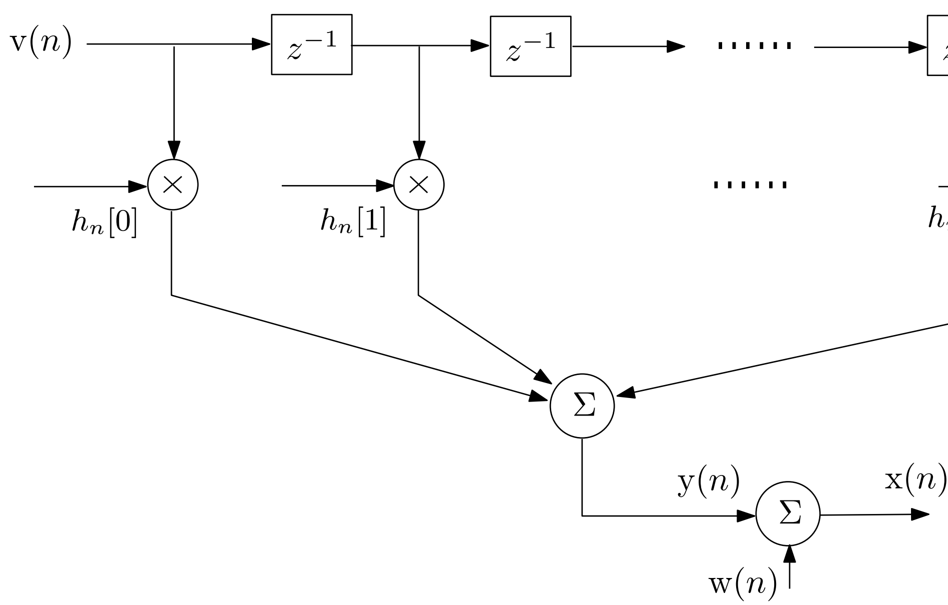

Fig. 1: FIR low-pass filter representation of the channel.

The input-output description can be written as

$$y[n] = \sum_{k=0}^{p-1} h_n[k]v[n-k] \tag{1}$$

where $v[n]$ is the transmit signal at time sample n and $h_n[k]$ is channel impulse response at time sample n of path k. The channel coefficient $h_n[k]$ is time-varying. Thus, (1) is a linear time-varying channel model. The received signal $y[n]$ is corrupted by noise $w[n]$, i.e., at the output of the system, we observe the noise corrupted signal

$$x[n] = \sum_{k=0}^{p-1} h_n[k]v[n-k] + w[n]. \tag{2}$$

If we have knowledge of coefficients $h_n[k]$, we can design a filter/detector/equalizer to detect $v[n]$ from observation $x[n]$, e.g., Wiener filter as done in Task 1. This is the so-called *signal estimation problem*. On the other hand, we can see that it is important to know the channel. As such, channel estimation is also a primary focus in communications.

In contrast to signal estimation problem, the idea of channel estimation is to estimate $h_n[n]$ using the knowledge of $v[n]$ and statistical information of $w[n]$. In particular, given the model in Figure 1, we have the observation equation (2) in the vector form as

$$x[n] = \mathbf{v}^{\mathrm{T}}[n]\mathbf{h}[n] + w[n], \tag{3}$$

where $\mathbf{v}^{\mathrm{T}}[n] = [\ v[n]\ v[n-1]\ v[n-2] \ldots .\ v[n-p+1]\ ]$ is known; $w[n]$ is assumed to be WGN with known variance σ^2, i.e, $w[n] \sim \mathcal{N}(0, \sigma^2)$; and $\mathbf{h}[n] = [\ h_n[0]\ h[1]\ h[2]\ \ldots\ h[p-1]]^{\mathrm{T}}$ is variable vector to be estimated. Assuming a slow-fading channel, i.e., $h_n[k]$ is not changing rapidly from sample to sample. We describe the state vector of $\mathbf{h}[n]$ by Gauss-Markov model as

$$\mathbf{h}[n] = \mathbf{A}\mathbf{h}[n-1] + \mathbf{u}[n] \tag{4}$$

where $\mathbf{A}$ is a known $p \times p$ matrix, and $\mathbf{u}[n]$ is vector WGN with covariance matrix $\mathbf{Q}$, i.e, $\mathbf{u}[n] \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$. It assumes that the tap weights are uncorrelated with each other and hence independent due to the jointly Gaussian assumption. As such we can let $\mathbf{A}$, $\mathbf{Q}$, and $\mathbf{C}_h$–the covariance matrix of $\mathbf{h}[-1]$–be diagonal matrices. At this point, we have (3) and (4) the observation equation and the state equation, respectively. Here Kalman filter can be applied to estimate $\mathbf{h}[n]$.

Task:

1) Given $p = 2$, $\mathbf{A} = \begin{bmatrix} 0.99 & 0 \\ 0 & 0.999 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}$, $\sigma^2 = 0.1$. Reproduce numerical results in Figs. 13.16 – 13.20 in *[Example 13.3, Kay, Vol.1]*.
2) Given $p = 2$, $\mathbf{A} = \begin{bmatrix} 0.99 & 0 \\ 0 & 0.999 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$, $\sigma^2 = 0.1$. Repeat the simulation, and give your comments on the obtained results.
3) Given $p = 2$, $\mathbf{A} = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.999 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}$, $\sigma^2 = 0.1$. Repeat the simulation, and give your comments on the obtained results.