MATLAB Simulation Exercise

Statistical Signal Processing II

Kalman Filter

Group 13

Athmajan Vivekananthan

University of Oulu
Finland

January 31, 2024

1 Theoretical Background

Kalman filter is generally useful in tracking vector signals in the face of noise which are non stationary. As opposed to Wiener filters which cannot be applied when the signals and noises become non stationary. The recursive algorithm of minimising Mean Square Error of a signal with noise where the system state is characterized by dynamic models gives a far good ability to track the true value of the signal.

This particular task involves a transmission channel characterized as a linear time variant system where the effects of fading are accommodated with delayed taps for the filter. These delayed taps represent the effect of delays which attenuate the signal as system reaches steady state. [1] [2]

2 Results and Discussions

2.1 Task 1 Results

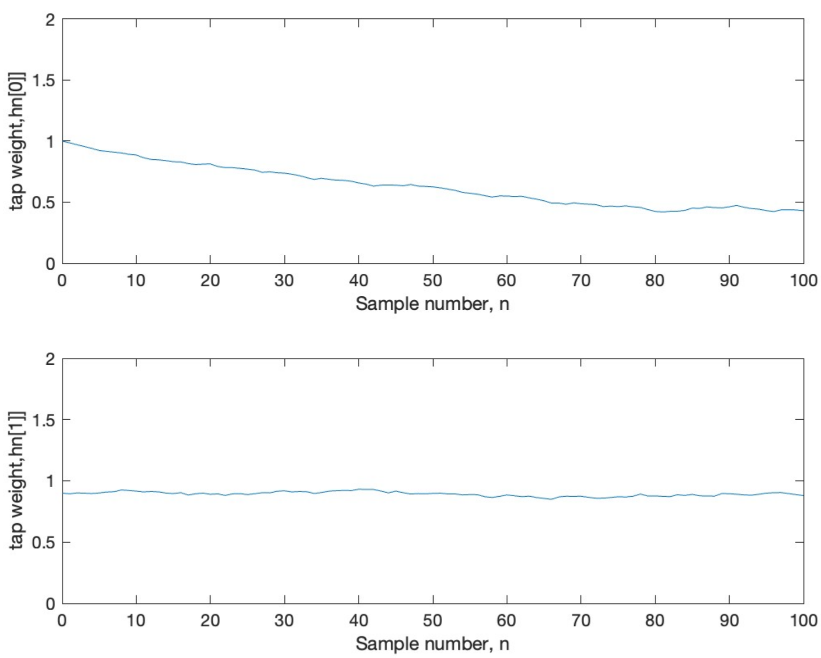

Figure 1: Realization of TDL Coefficients for Task 1

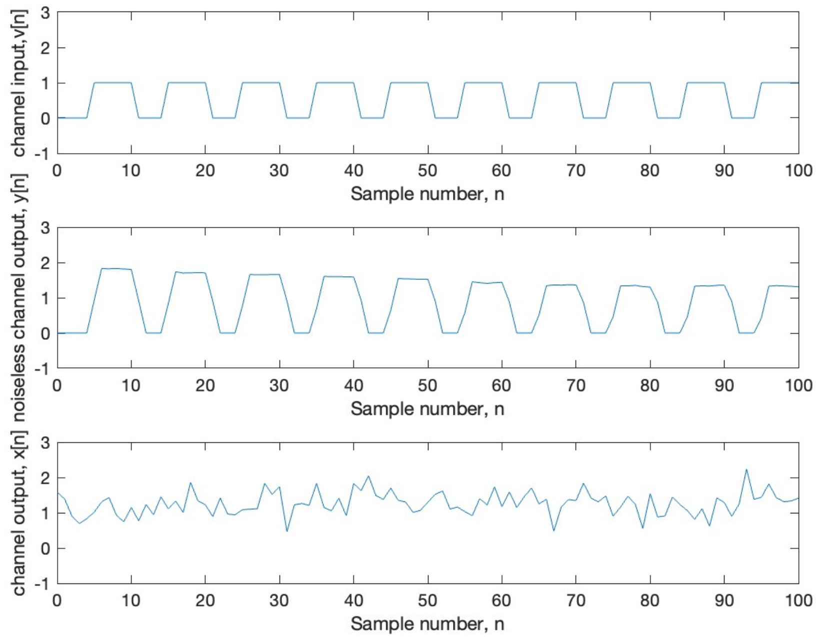

Figure 2: Input Output Waveforms for Task 1

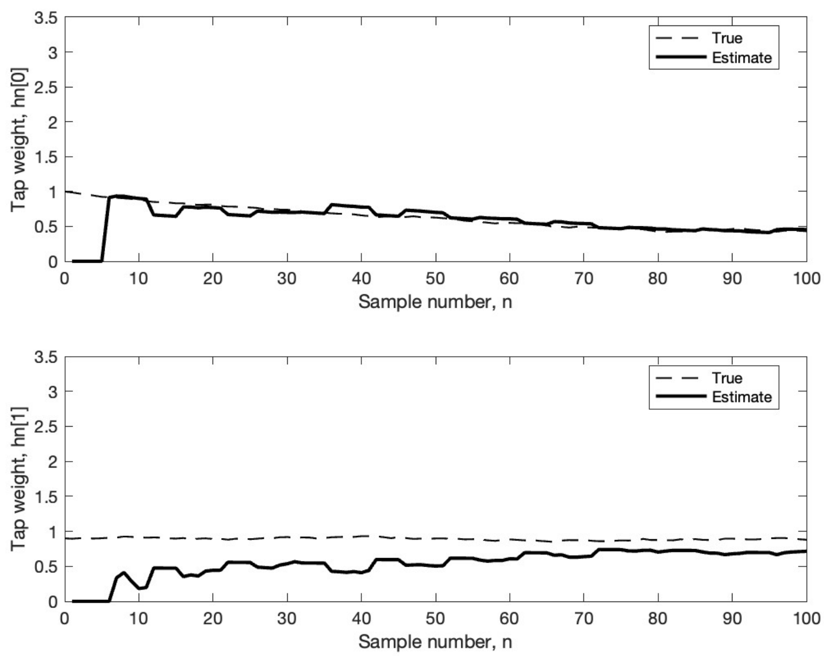

Figure 3: Kalman Filter Estimates for Task 1

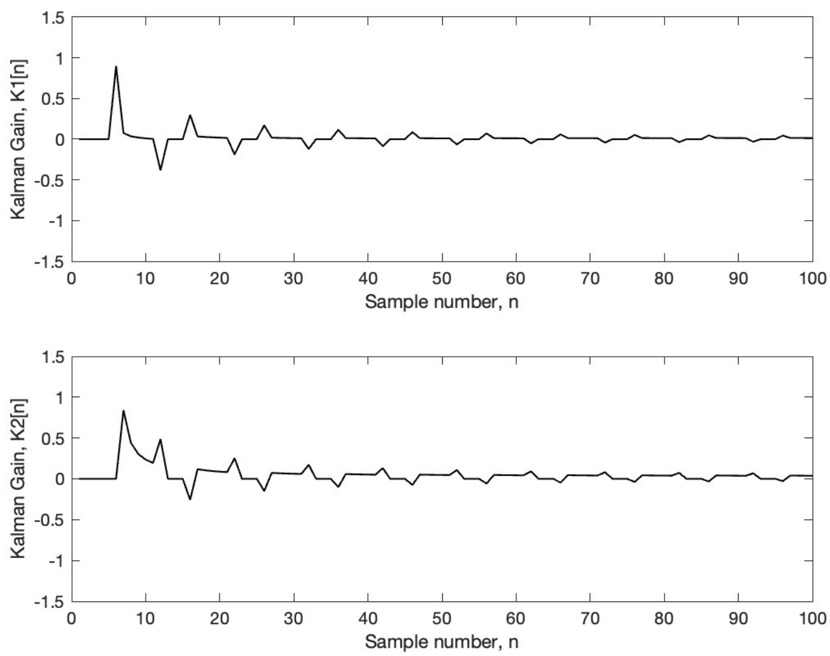

Figure 4: Kalman Filter Gains for Task 1

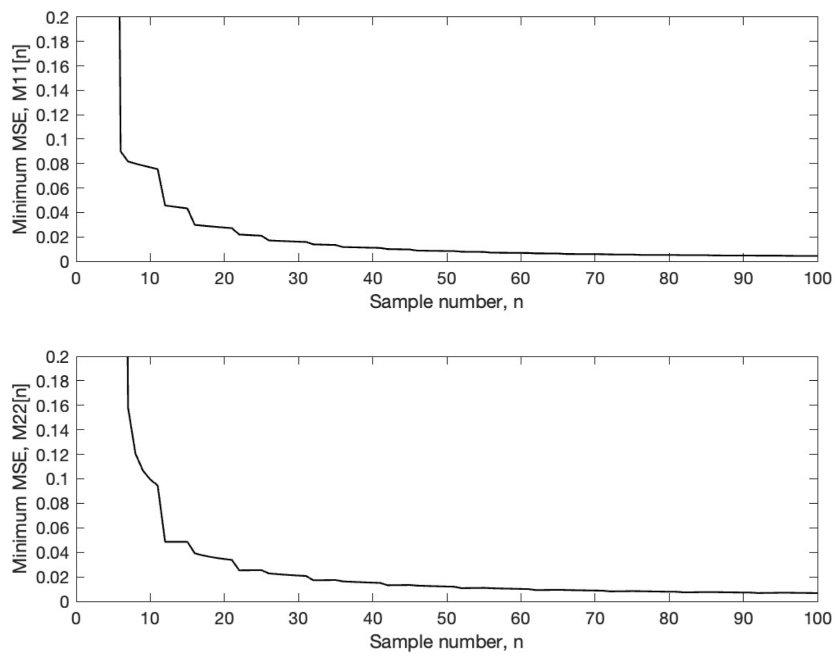

Figure 5: Minimum Mean Square Error for Task 1

2.2 Task 1 Discussion

1. Kalman Filter Estimates
   It can be seen as per Figure 3 $h_n[0]$ is decaying rapidly to zero and $h_n[1]$ becomes fairly constant as the number of samples approaches to larger values. This effect can be explained because the average value of the (p number of weights) weights will be approaching zero when the system is at steady state.

2. Minimum Mean Square Error (MMSE)
   In theory the initial state estimate is actually the mean value of $s[-1]$. But in practical scenarios this is not known. Therefore a random value shall be assigned. However to avoid the bias of the Kalman filter performance, this initial guess is chosen to be some value which is not in the closest vicinity of the true value. This is why the Minimum Mean Square Error is quite high during the initiation of the system. However this converges to zero as the system starts to track the true value rendering the MMSE to approach zero.

3. Kalman Gain
   The pattern of Kalman gain is observed to be a periodic pattern centered around zero. This observation can be explained because when the system has no input, i.e - $v[n] = 0$ the Kalman Filter tends to ignore these inputs by forcing the Kalman Gains to be zero. Therefore this kind of behaviour is observed where the signal only contains noise.

2.3 Task 2

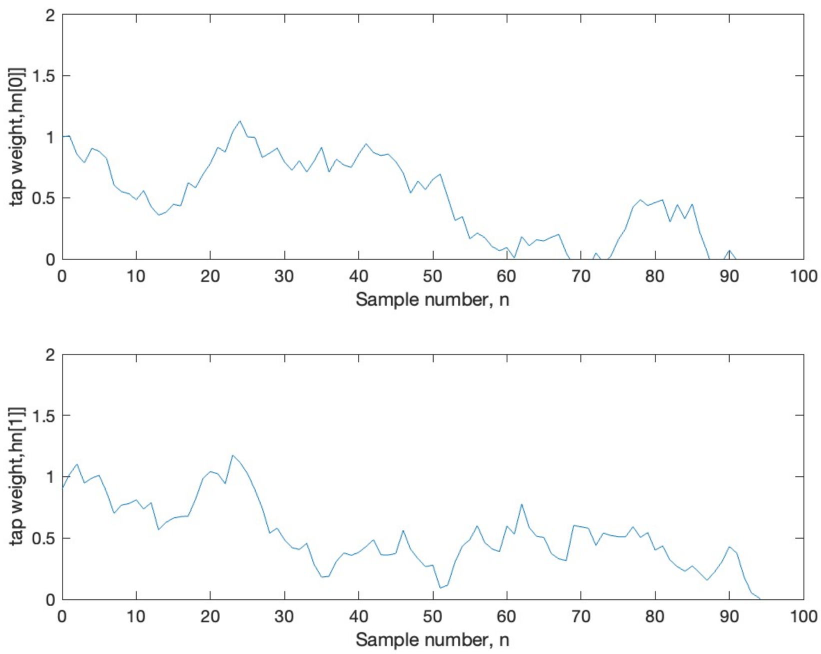

Figure 6: Realization of TDL Coefficients for Task 2

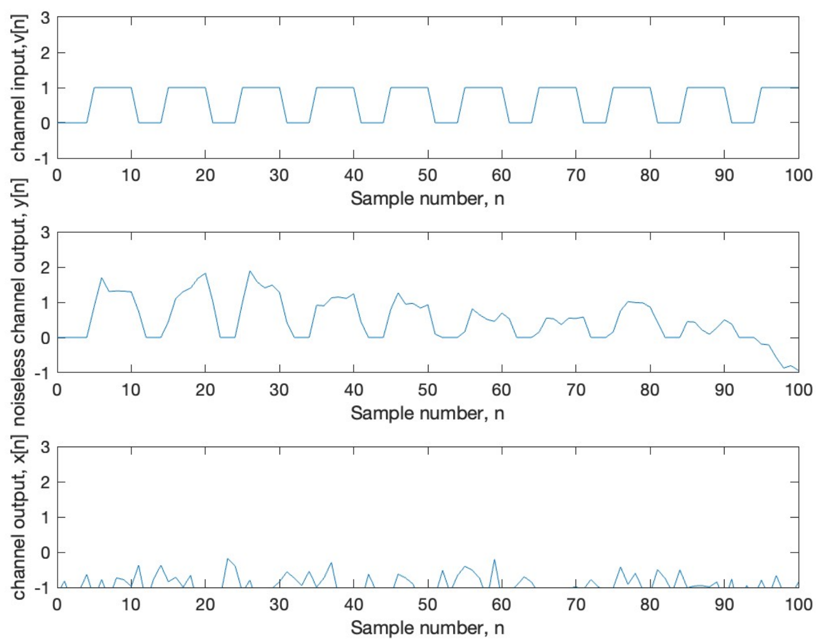

Figure 7: Input Output Waveforms for Task 2

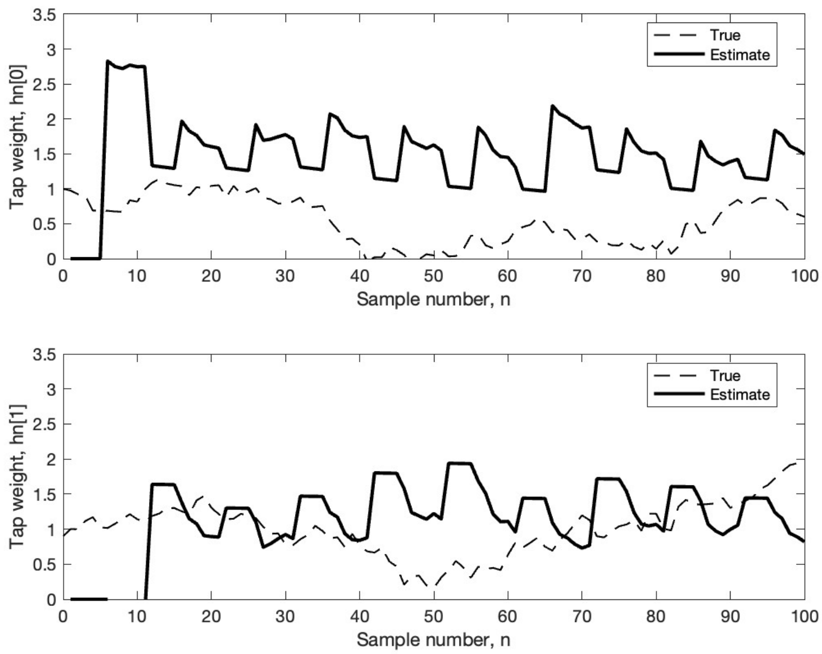

Figure 8: Kalman Filter Estimates for Task 2

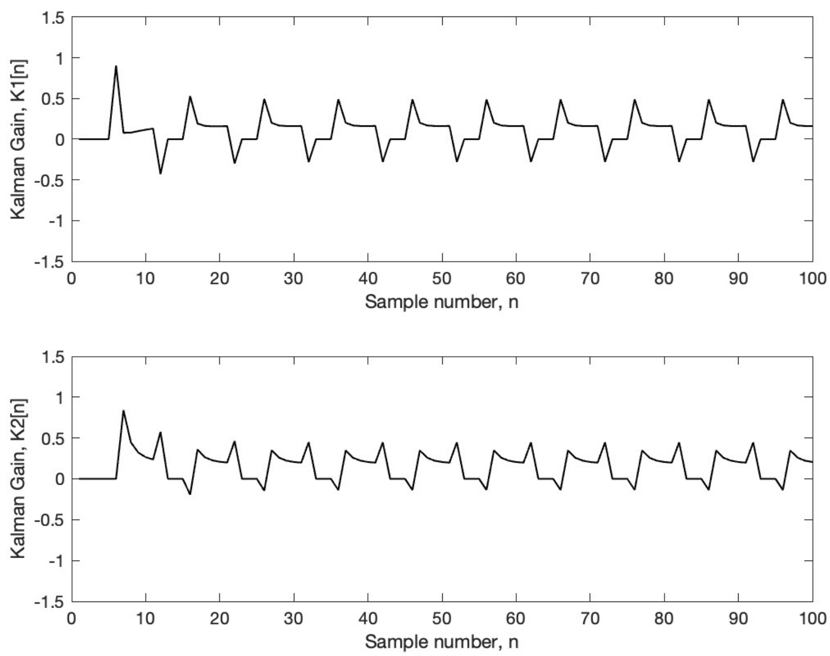

Figure 9: Kalman Filter Gains for Task 2

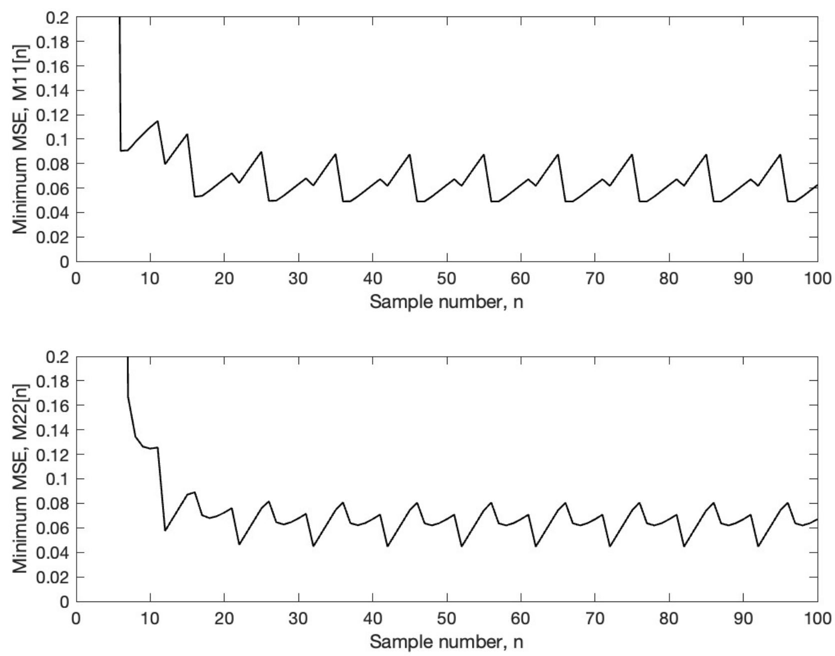

Figure 10: Minimum Mean Square Error for Task 2

2.4 Task 2 Discussion

The only difference for Task 2 is that the degree of corruption introduced by driving noise and the observation noise to the state vector is 100 times higher than Task 1. Everything else is the same.

Therefore this Task enables us to study the impact of the WGN noise in the performance of the Kalman Filter purely.

1. Kalman Filter Estimates
   The filter estimates $h_n[0]$ and $h_n[1]$ do not show the behaviour shown in Task 1. A pattern of fluctuation can be seen in $h_n[0]$ as number of samples approaches larger values but it never converges to the true value.

   Similarly $h_n[1]$ does not become constant for larger values since the signal is hugely corrupted with noise but the expected value of the corrupted signal is still zero rendering these estimates in general to fluctuate within a specific range than blowing up.

2. Minimum Mean Square Error (MMSE)
   As states in Task 1 discussions the initial guess is quite huge, hence the initial MMSE values are quite high. But as number of samples grow, even though the estimates do not exactly converge to the true value the corruption range contributes for the MMSE to also fluctuate within a certain range.

3. Kalman Gain
   Similar to Task 1 the Kalman gain shows a periodic pattern around zero. The same explanation can be used to reason this behaviour. But the instances where Kalman Gains become zero are quite seldom here because the inputs are not always zero due to the impact of high noise.

2.5 Task 3

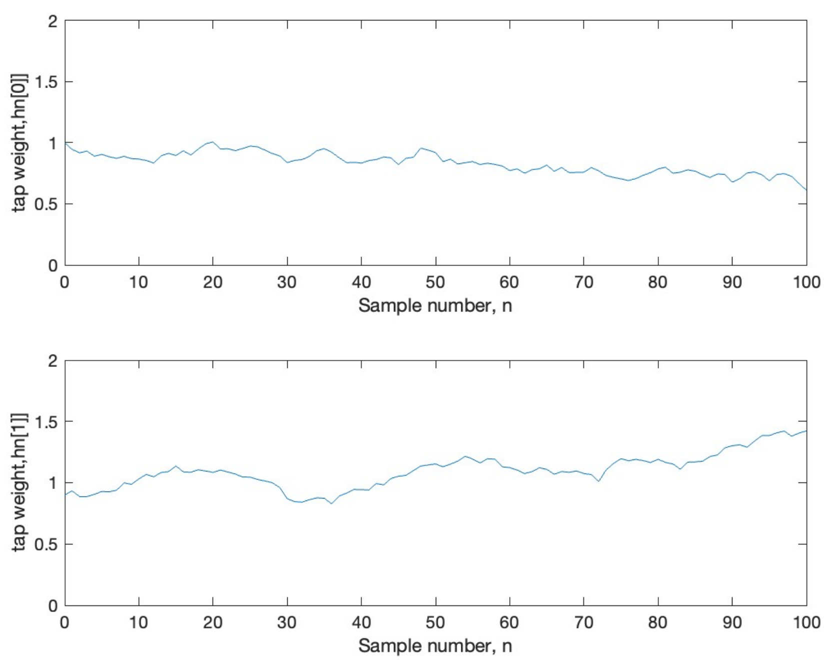

Figure 11: Realization of TDL Coefficients for Task 3

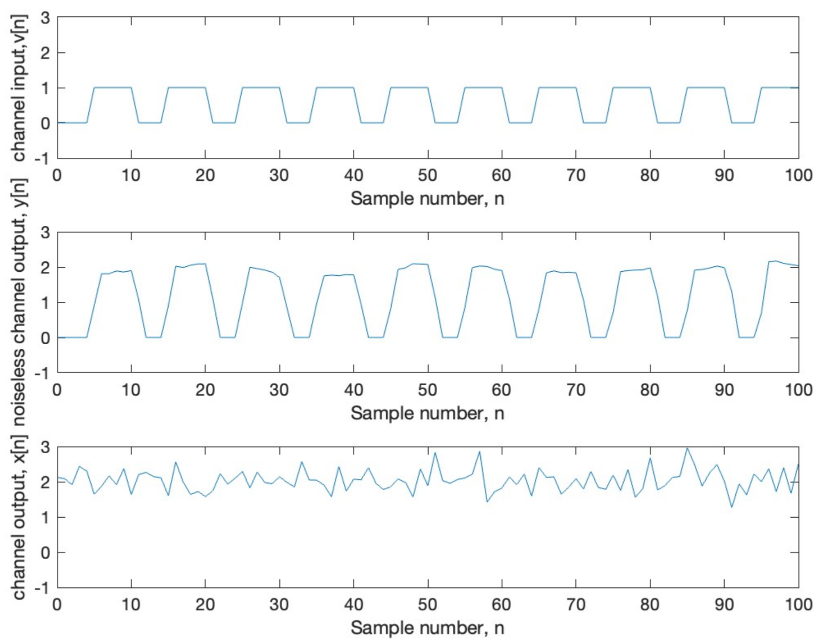

Figure 12: Input Output Waveforms for Task 3

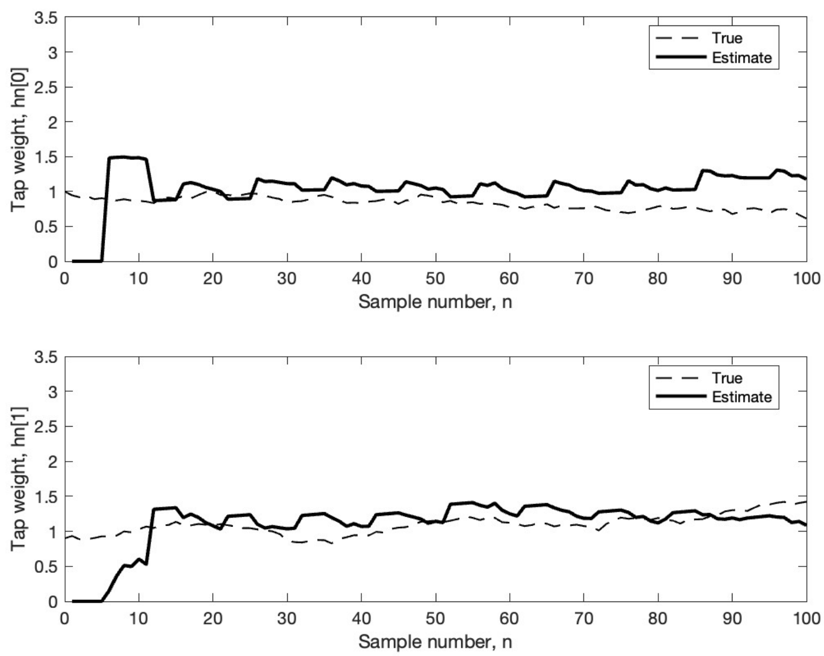

Figure 13: Kalman Filter Estimates for Task 3

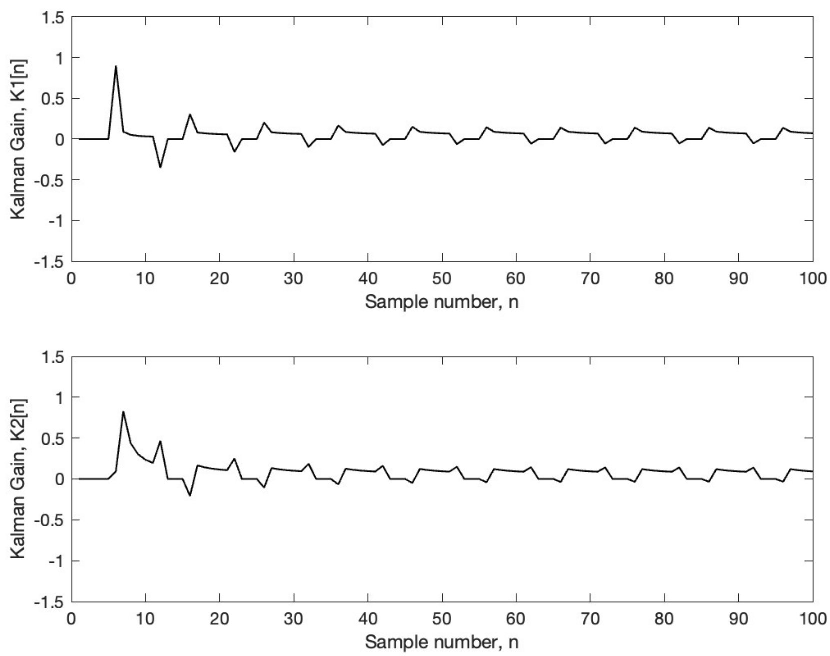

Figure 14: Kalman Filter Gains for Task 3

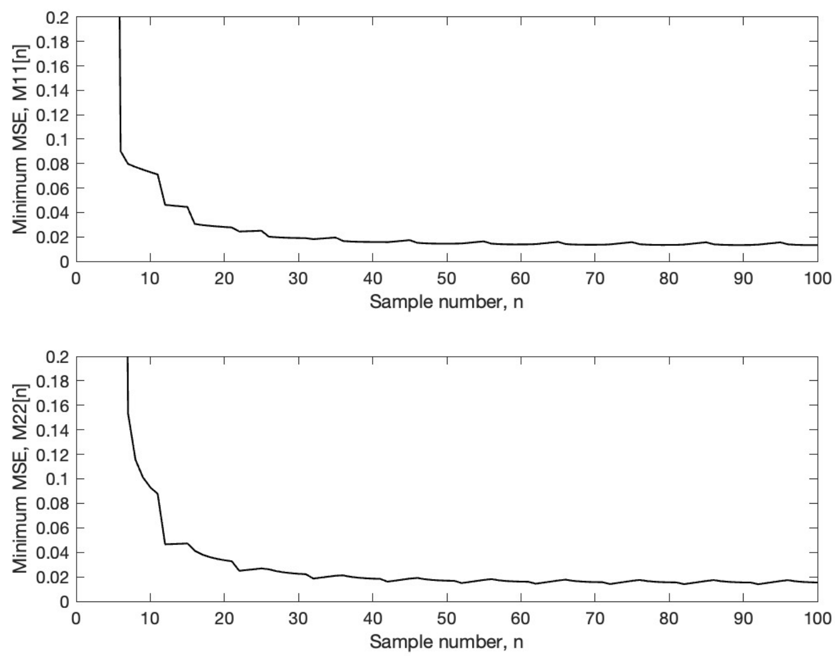

Figure 15: Minimum Mean Square Error for Task 3

2.6 Task 3 Discussion

The only difference for Task 3 compared to Task 1 and 2 is that the impact of driving and observation noise to the state vector is 10 times larger than it was in Task 1 but 10 times lower than task 2. Therefore in the context of noise, this task can be considered to be in the middle of Task 1 and Task2.

However the correlation between the successive values of of a given tap weight is not diagonal anymore. That means that the assumption of uncorrelated scattering is not valid anymore in this realization. The tap weights are going to be correlated with each other.

The results of such correlations can be studies using the observations made in this Task.

1. Kalman Filter Estimates
   The filter estimates $h_n[0]$ is approaching zero as number of samples becomes larger. But $h_n[1]$ does not converge to a constant but explodes as number of samples grow. This behaviour is due to the correlation between the tap weights.

However the estimates generated by the Kalman filter seem to manage this correlation fairly.

2. Minimum Mean Square Error (MMSE)
   Since the tracking is fairly correct as seen in the graph the MMSE coverges to a value very closer to zero but not exactly zero as number of samples grow.

3. Kalman Gain
   Similar to all behaviours observed in other tasks here also Kalman Gain shows a periodic pattern which is quite closer to zero and fluctuates around zero.

References

[1] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall International, 1993.

[2] S. Haykin, *Adaptive Filter Theory*. Pearson, 2014. [Online]. Available: https://books.google.fi/books?id=J4GRKQEACAAJ