MATLAB Simulation Exercise

Statistical Signal Processing II

EXTENDED KALMAN FILTER

Group 13

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1 Theoretical Background

Kalman filter is generally useful in tracking vector signals in the face of noise which are non stationary. However when the equation of any state or when any observation equation is non linear the concepts used in Kalman filter cannot be directly applied. An extension to the former shall be needed amid data being non linearly related to the unknown parameters. This extension renders in a new concept called Extended Kalman Filter.

The application of this concept is easily understood with a vehicular tracking where the measurements are done using polar coordinates which introduces the non linearity to the system. [1] [2]

2 Results and Discussions

The results are derived based on the given data in the task description. However changes were required to be made in the parameters. Tables 2 - 10 summarise the parameter changes in their corresponding subsections discussed below. Each of the results are presented first with the original data given in the task sheet and then secondly changing only the true state values without changing the estimates. Thirdly the true state values were kept intact aligned with the original given data but changing only the initial state estimates. Case II has been designed so that the distance between true and estimate values are smaller than case I. Case III has more deviation between true and estimates.

Table 1: Summary of test cases for each task

Case	$\hat{s}$	s	$\|\mathbf{s}-\hat{\mathbf{s}}\|$
Case I	$\begin{bmatrix}5 & 5 & 0 & 0\end{bmatrix}^T$	$\begin{bmatrix}10 & -5 & -0.2 & 0.2\end{bmatrix}^T$	11.1839
Case II	$\begin{bmatrix}5 & 5 & 0 & 0\end{bmatrix}^T$	$\begin{bmatrix}0 & 0 & -0.1 & 0.1\end{bmatrix}^T$	7.0725
Case III	$\begin{bmatrix}10 & -5 & -0.2 & 0.2\end{bmatrix}^T$	$\begin{bmatrix}10 & 10 & 0.1 & 0.1\end{bmatrix}^T$	15.0033

2.1 Task 1 Results

2.1.1 Original Parameters

Table 2: Parameters for section 2.1.1

Parameter	Value
σ_u^2	0.0001
σ_r^2	0.1
σ_b^2	0.01
$\hat{s}[-1\|-1]$	$\begin{bmatrix}5 & 5 & 0 & 0\end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix}10 & -5 & -0.2 & 0.2\end{bmatrix}^T$

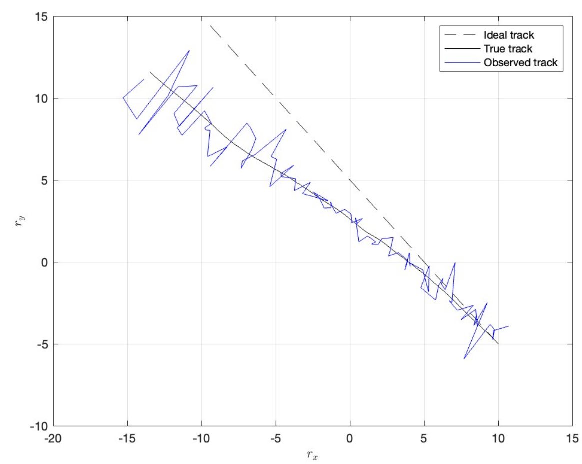

Figure 1: True and observed vehicle tracks

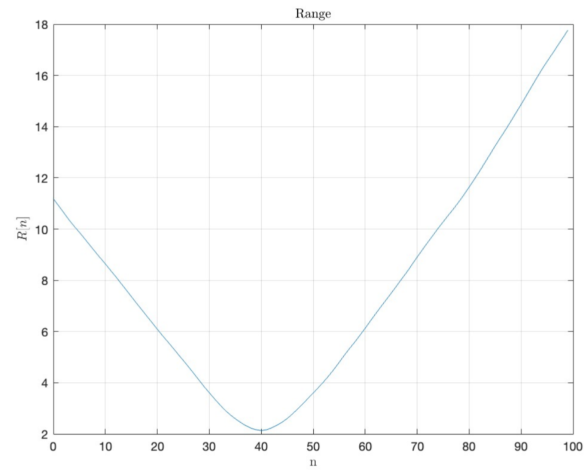

Figure 2: Range of true vehicle track

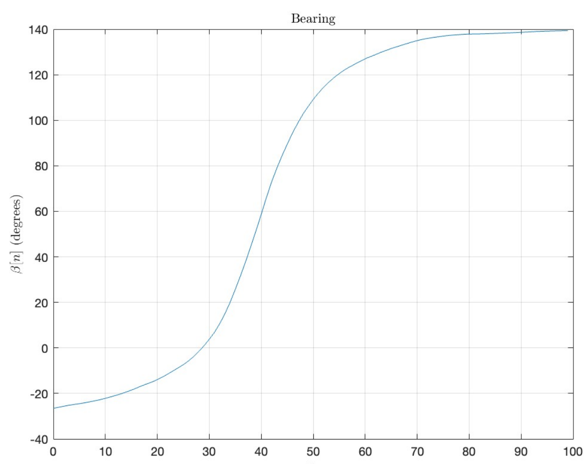

Figure 3: Bearing of true vehicle track

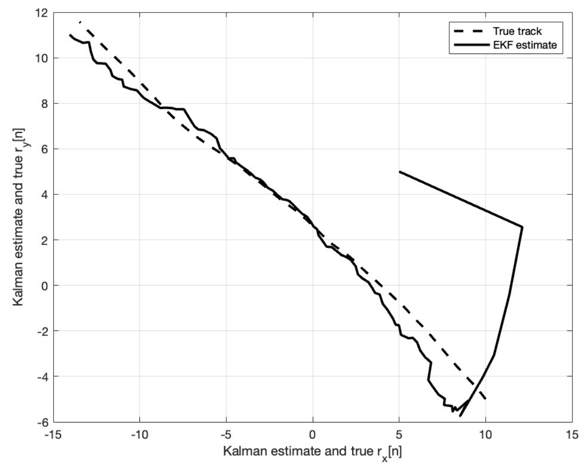

Figure 4: True and Extended Kalman Filter Estimates

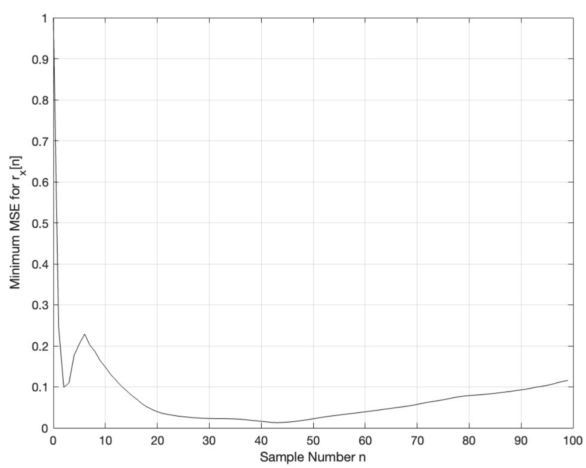

Figure 5: Minimum MSEs for $r_x[n]$

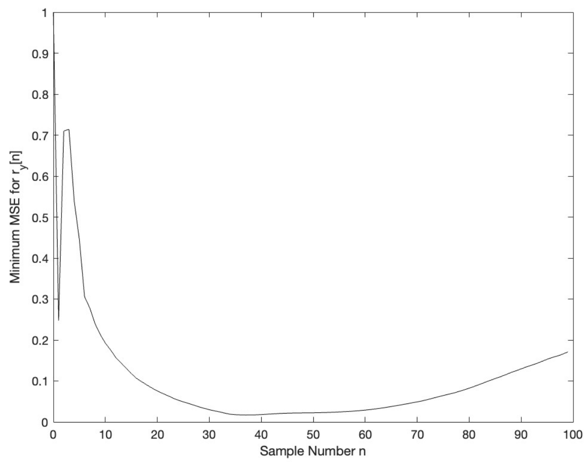

Figure 6: Minimum MSEs for $r_y[n]$

2.1.2 Only changing initial state true values

Table 3: Parameters for section 2.1.2

Parameter	Value
σ_u^2	0.0001
σ_r^2	0.1
σ_b^2	0.01
$\hat{s}[-1\|-1]$	$\begin{bmatrix}5 & 5 & 0 & 0\end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix}0 & 0 & -0.1 & 0.1\end{bmatrix}^T$

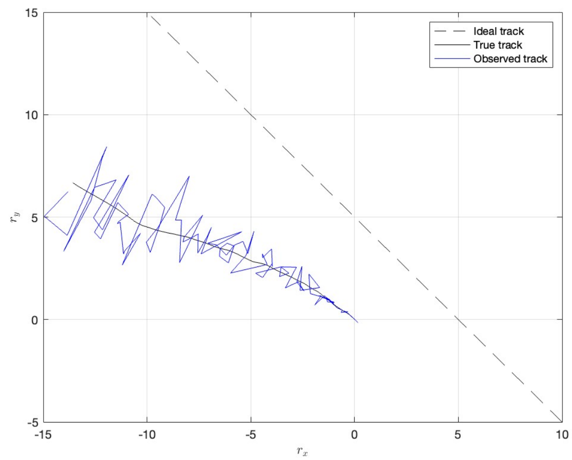

Figure 7: True and observed vehicle tracks

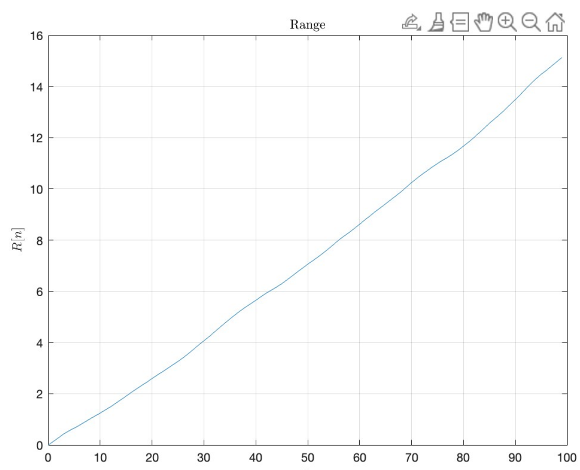

Figure 8: Range of true vehicle track

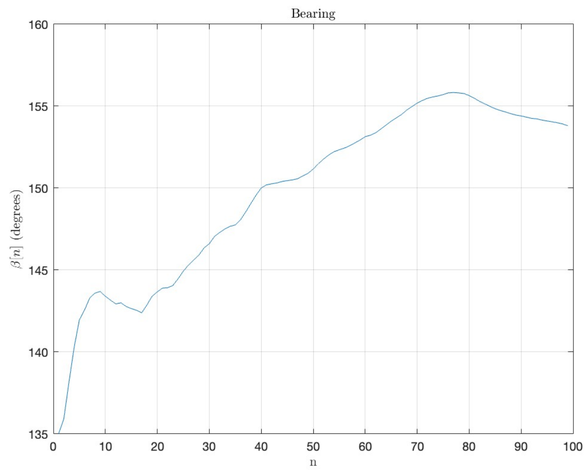

Figure 9: Bearing of true vehicle track

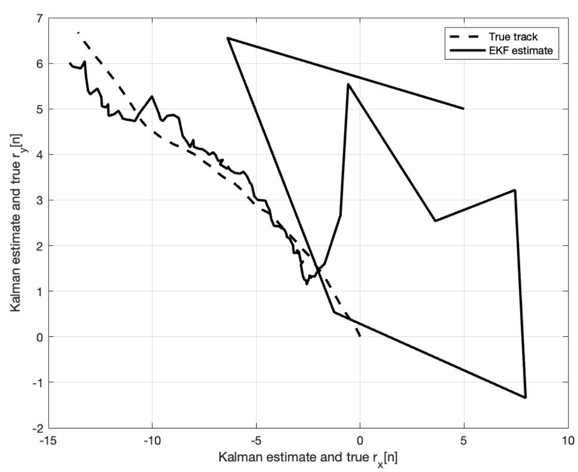

Figure 10: True and Extended Kalman Filter Estimates

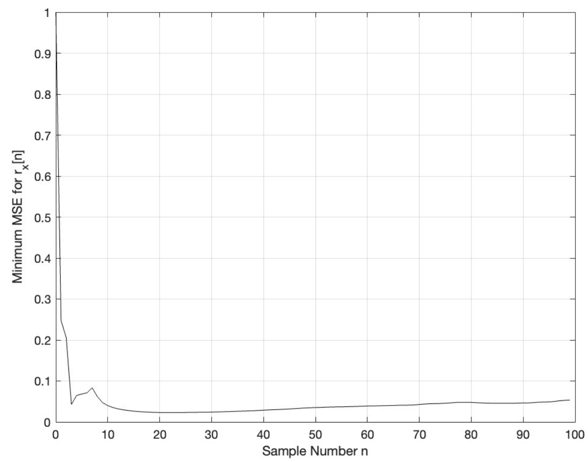

Figure 11: Minimum MSEs for $r_x[n]$

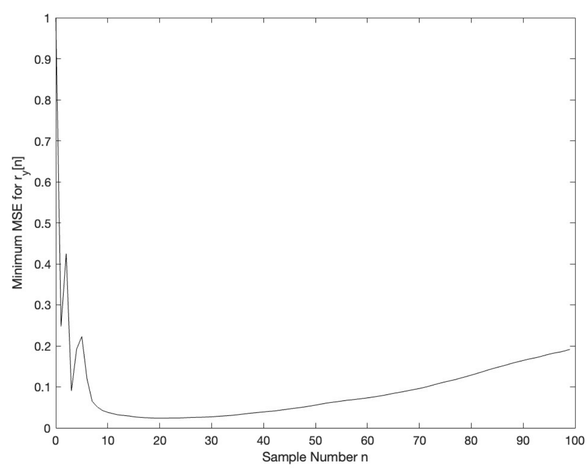

Figure 12: Minimum MSEs for $r_y[n]$

2.1.3 Changing only initial state estimates

Table 4: Parameters for section 2.1.3

Parameter	Value
σ_u^2	0.0001
σ_r^2	0.1
σ_b^2	0.01
$\hat{s}[-1\|-1]$	$\begin{bmatrix}10 & 10 & 0.1 & 0.1\end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix}0 & 0 & -0.1 & 0.1\end{bmatrix}^T$

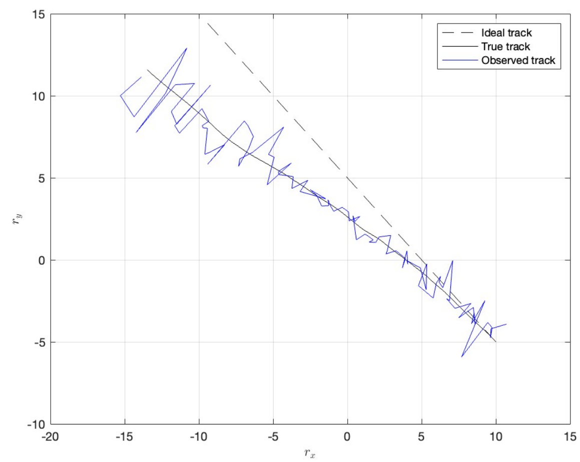

Figure 13: True and observed vehicle tracks

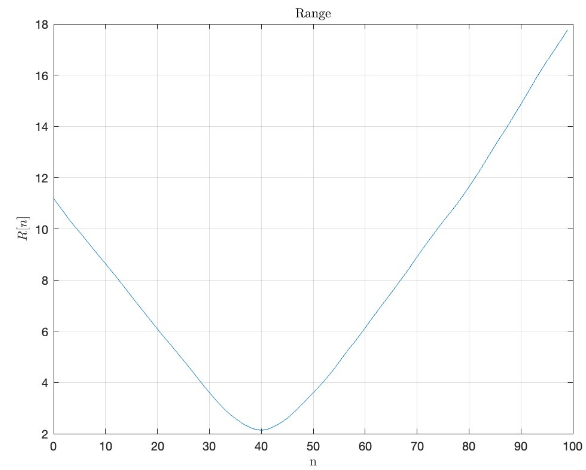

Figure 14: Range of true vehicle track

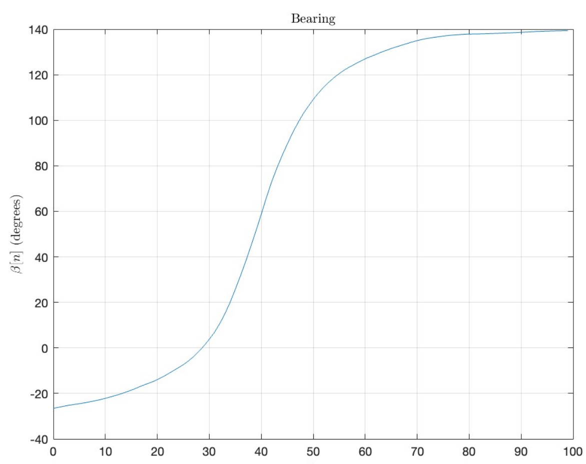

Figure 15: Bearing of true vehicle track

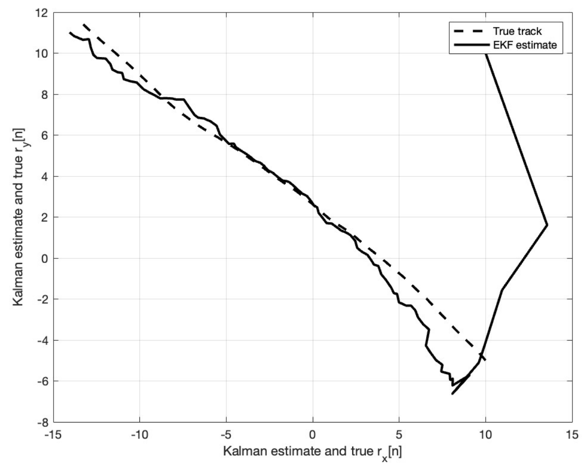

Figure 16: True and Extended Kalman Filter Estimates

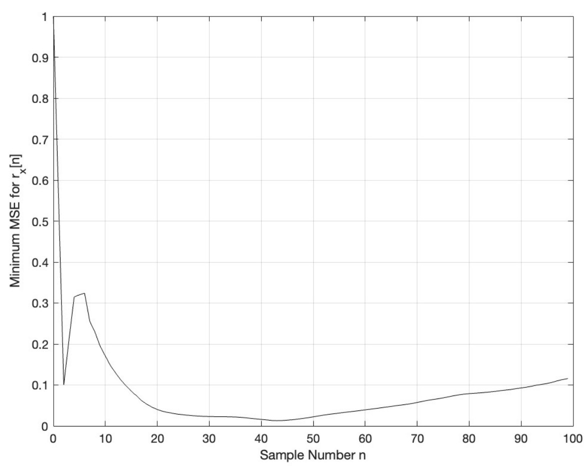

Figure 17: Minimum MSEs for $r_x[n]$

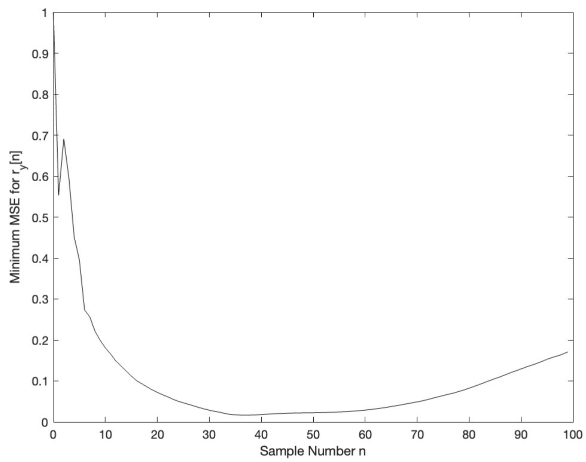

Figure 18: Minimum MSEs for $r_y[n]$

2.1.4 Task 1 Discussion

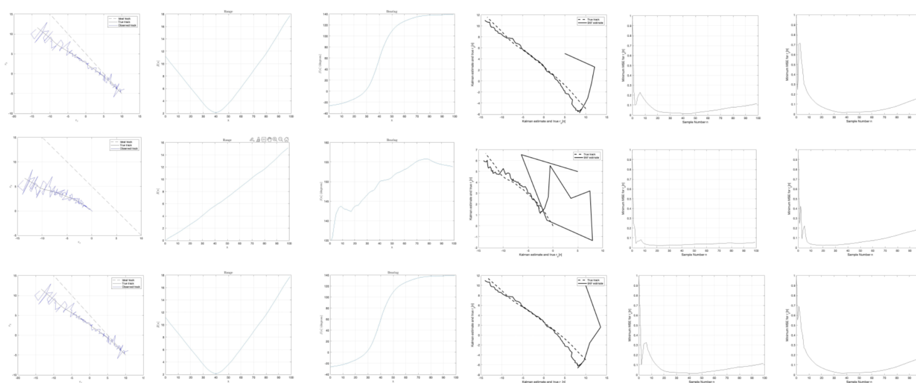

Figure 19: All three cases of Task 1 presented together

As the number of samples increase or in other words, as time progresses the true trajectory deviates from the straight line. The graphs of R_x and R_y show that quickly explode rendering this particular model of Extended Kalman filter to be accurately applicable only to a portion of the trajectory.

Using the different cases analysed in this task, it can be seen the effect of bias between the initial true values and initial estimated values have less of an impact on the convergence of the filter performance.

In all cases it can be seen that initially due to the difference between the true and estimated state values the error is quite large as seen in the MSE curves. In all three cases the sweet spot of attaining tracking seems to be around 40 to 50 samples. But in the second case where the bias is increased between true and estimate initial values the tracking seems to be achieved in earlier stages and does not change much.

2.2 Task 2 Results

2.2.1 Original Parameters

Table 5: Parameters for section 2.2.1

Parameter	Value
σ_u^2	0.01
σ_r^2	0.1
σ_b^2	0.01
$\hat{s}[-1\|-1]$	$\begin{bmatrix} 5 & 5 & 0 & 0 \end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix} 10 & -5 & -0.2 & 0.2 \end{bmatrix}^T$

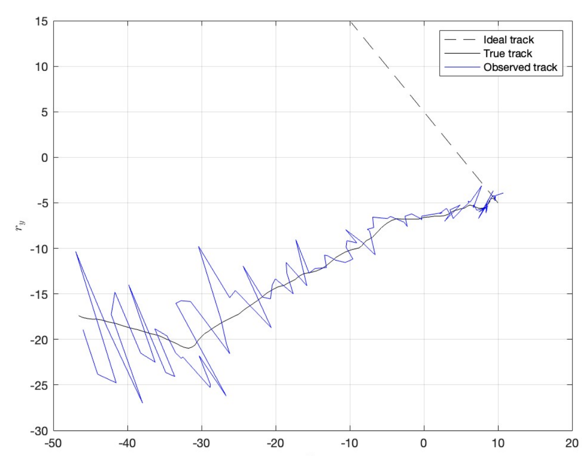

Figure 20: True and observed vehicle tracks

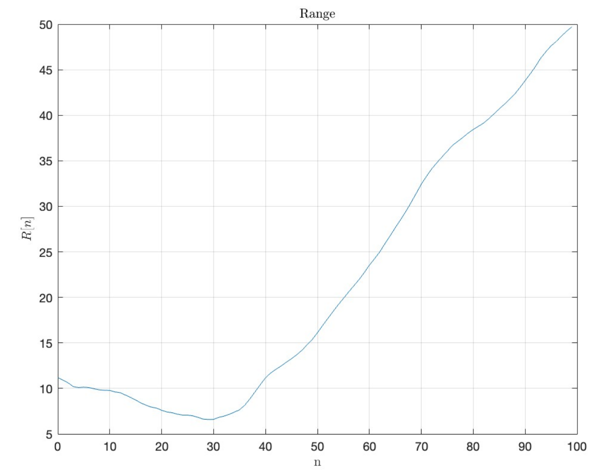

Figure 21: Range of true vehicle track

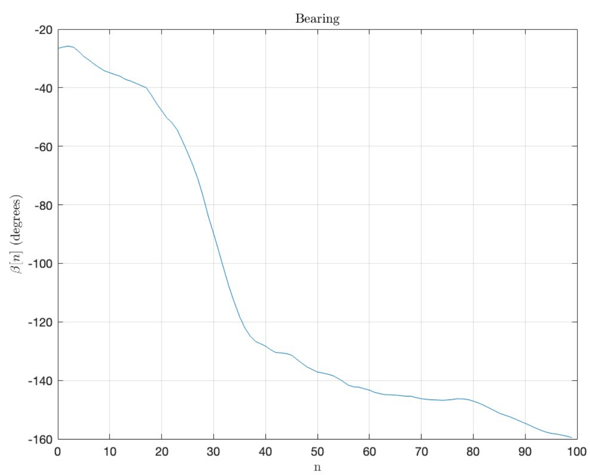

Figure 22: Bearing of true vehicle track

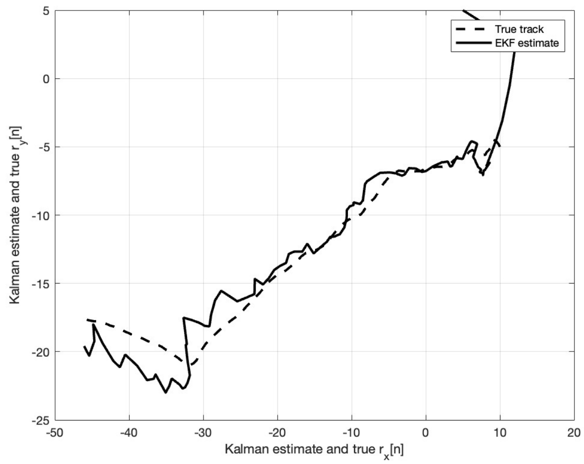

Figure 23: True and Extended Kalman Filter Estimates

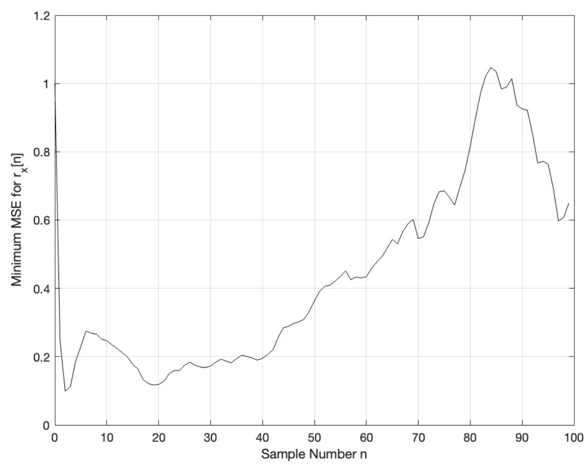

Figure 24: Minimum MSEs for $r_x[n]$

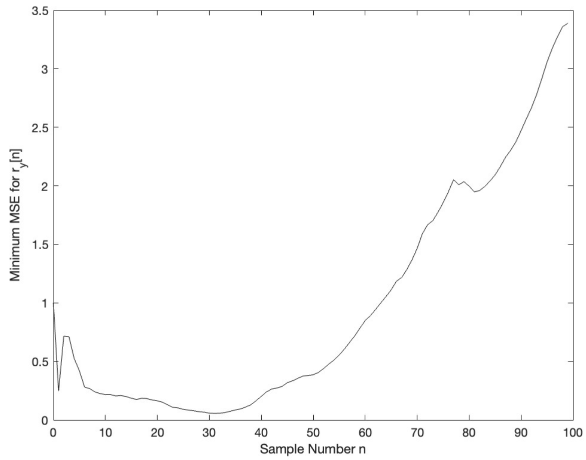

Figure 25: Minimum MSEs for $r_y[n]$

2.2.2 Only changing initial state true values

Table 6: Parameters for section 2.2.2

Parameter	Value
σ_u^2	0.01
σ_r^2	0.1
σ_b^2	0.01
$\hat{s}[-1\|-1]$	$\begin{bmatrix}5 & 5 & 0 & 0\end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix}0 & 0 & -0.1 & 0.1\end{bmatrix}^T$

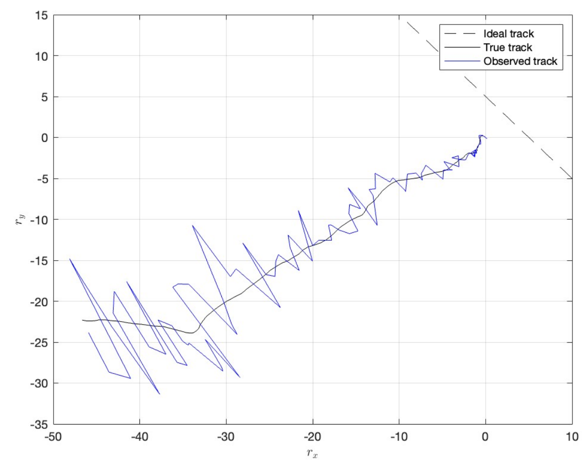

Figure 26: True and observed vehicle tracks

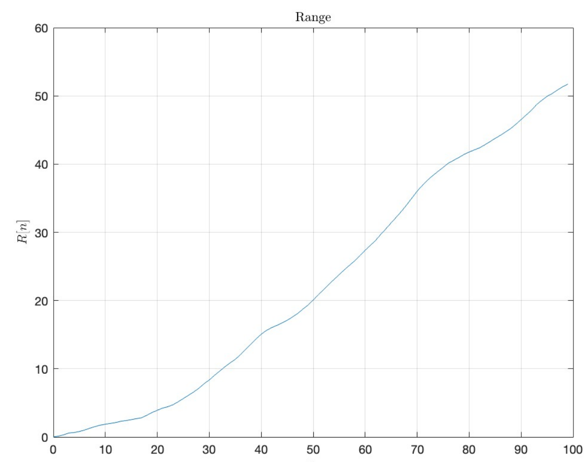

Figure 27: Range of true vehicle track

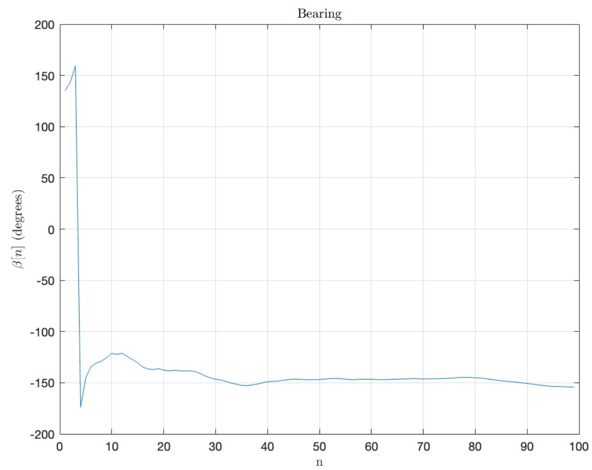

Figure 28: Bearing of true vehicle track

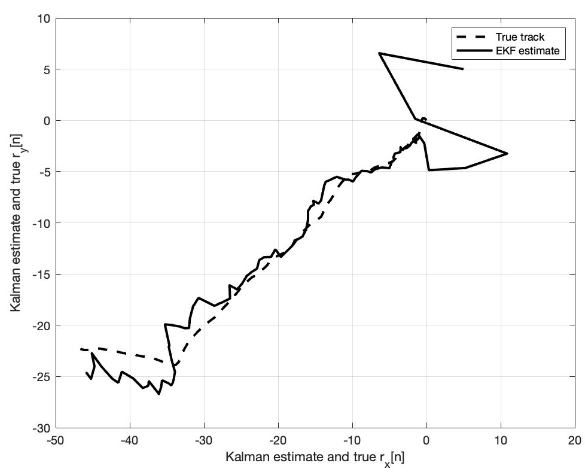

Figure 29: True and Extended Kalman Filter Estimates

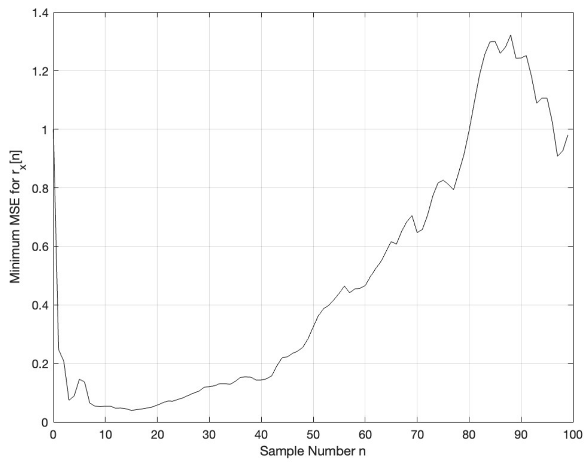

Figure 30: Minimum MSEs for $r_x[n]$

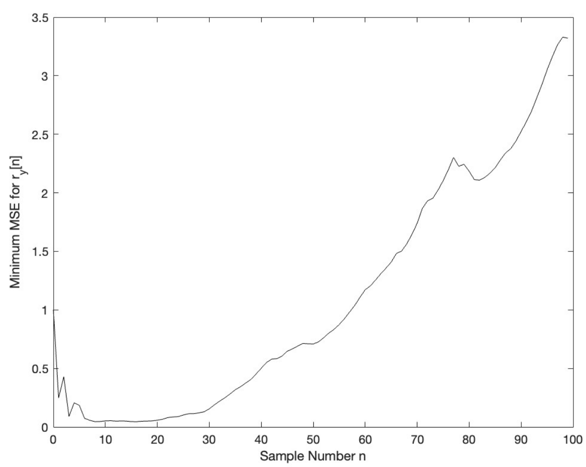

Figure 31: Minimum MSEs for $r_y[n]$

2.2.3 Changing only initial state estimates

Table 7: Parameters for section 2.2.3

Parameter	Value
σ_u^2	0.01
σ_r^2	0.1
σ_b^2	0.01
$\hat{s}[-1\|-1]$	$\begin{bmatrix}10 & 10 & 0.1 & 0.1\end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix}0 & 0 & -0.1 & 0.1\end{bmatrix}^T$

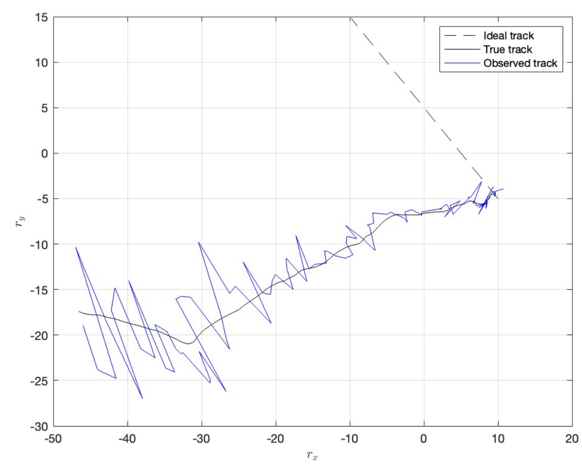

Figure 32: True and observed vehicle tracks

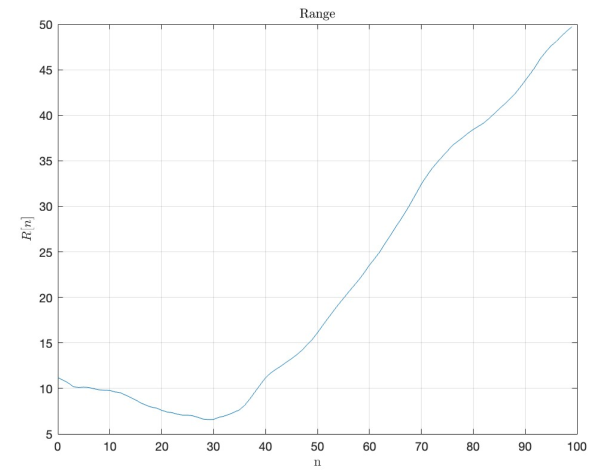

Figure 33: Range of true vehicle track

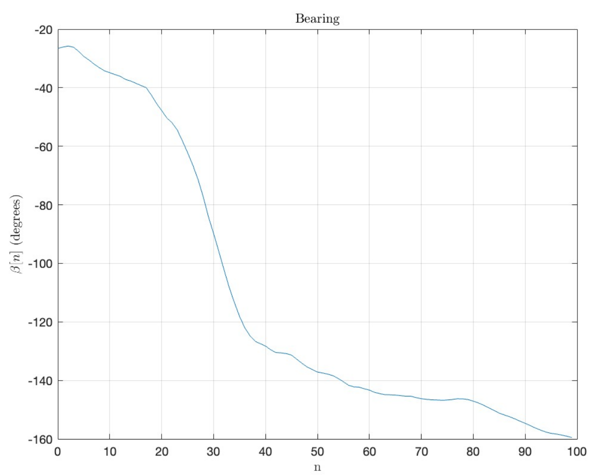

Figure 34: Bearing of true vehicle track

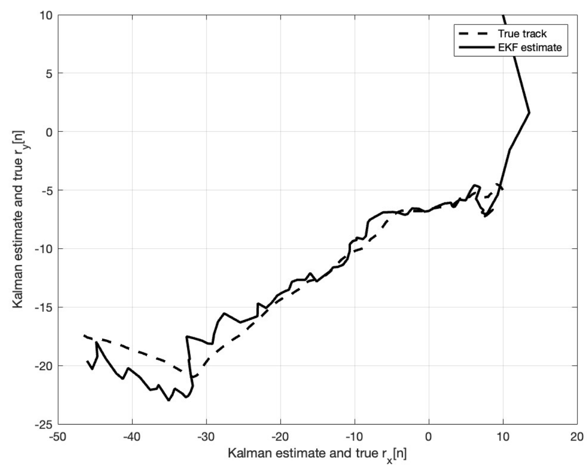

Figure 35: True and Extended Kalman Filter Estimates

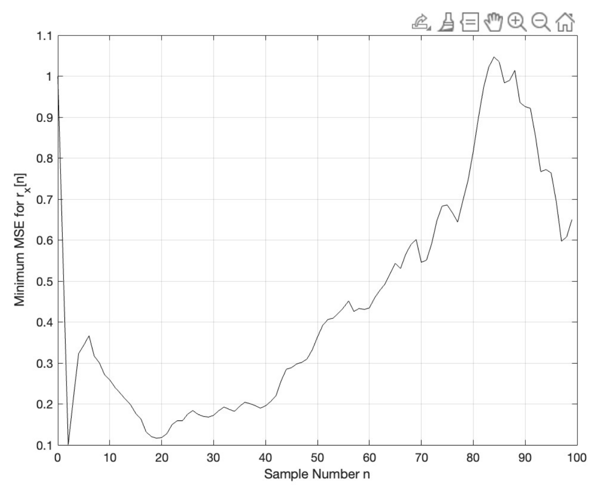

Figure 36: Minimum MSEs for $r_x[n]$

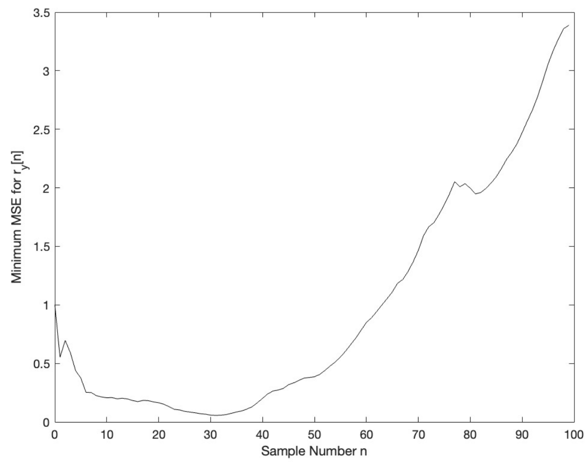

Figure 37: Minimum MSEs for $r_y[n]$

2.2.4 Task 2 Discussion

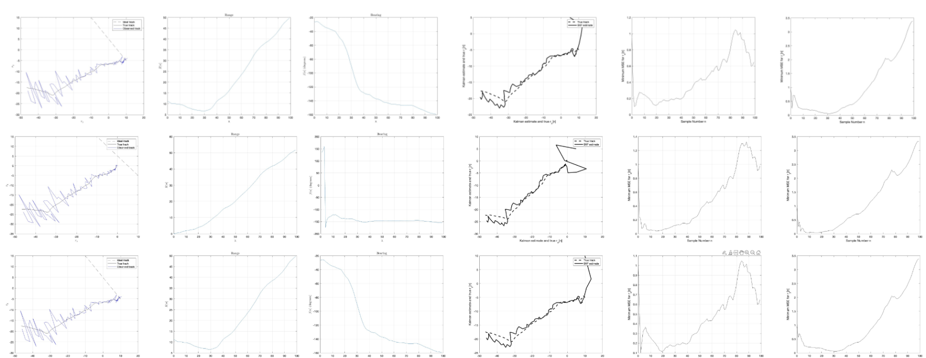

Figure 38: All three cases of Task 2 presented together

Compared to task 1 the impact of driving noise has been increased without changing the measurement noise variances in task 2. This effect can be studies using the resultant curves. The impact of the driving noise or the plant noise can be clearly seen in the difference between the true track and the observed track.

As the number of samples increase or in other words, as time progresses the true trajectory deviates from the straight line. The graphs of R_x and R_y show that quickly explode compared to task 1. But however the tracking has been attained without losing the value of the model. But still the tracking is valid only to a portion of the track. This effect can be validated using the observed MSE curved too.

The impact of the bias between the initial state values and estimate values can be still explained using the reasoning in task 1. When the bias is increased (as per in the Case II) the tracking has been achieved much quicker then the other cases.

2.3 Task 3 Results

2.3.1 Original Parameters

Table 8: Parameters for section 2.3.1

Parameter	Value
σ_u^2	0.0001
σ_r^2	1
σ_b^2	0.5
$\hat{s}[-1\|-1]$	$\begin{bmatrix}5 & 5 & 0 & 0\end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix}10 & -5 & -0.2 & 0.2\end{bmatrix}^T$

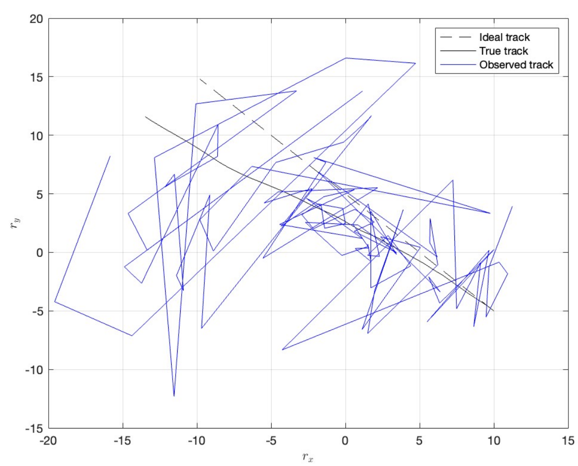

Figure 39: True and observed vehicle tracks

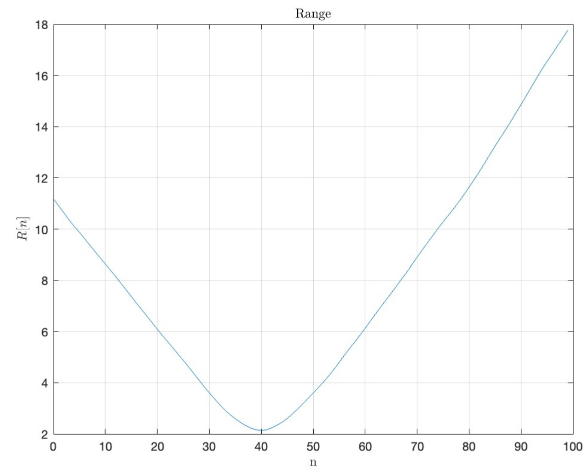

Figure 40: Range of true vehicle track

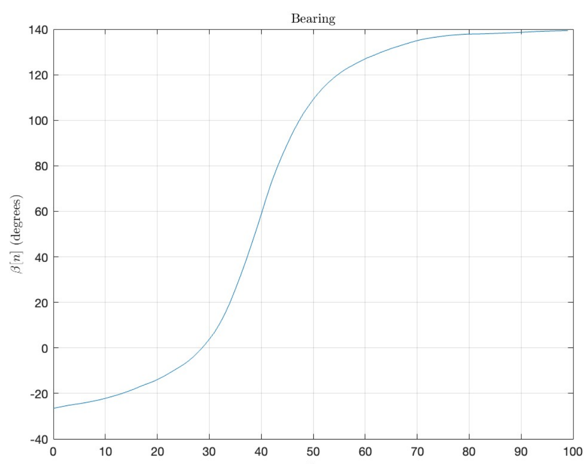

Figure 41: Bearing of true vehicle track

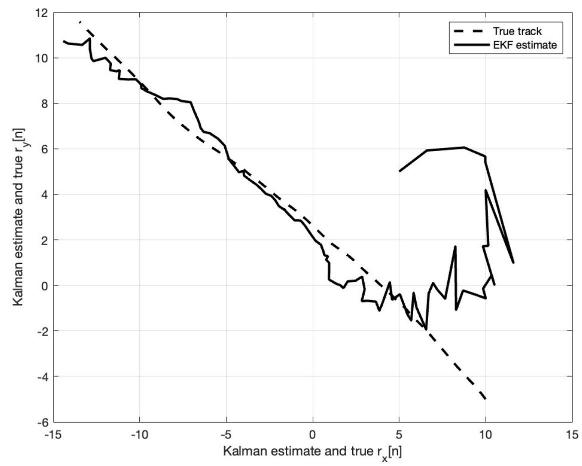

Figure 42: True and Extended Kalman Filter Estimates

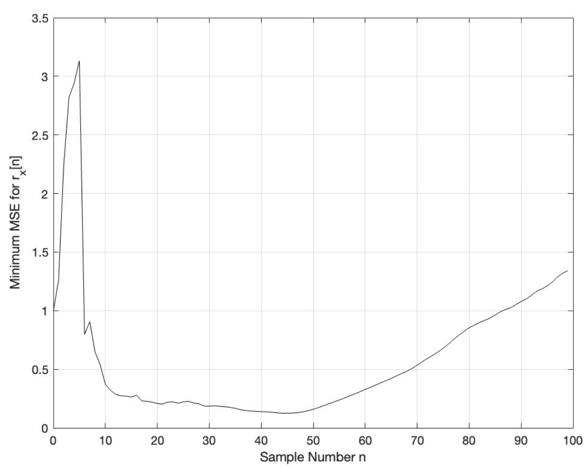

Figure 43: Minimum MSEs for $r_x[n]$

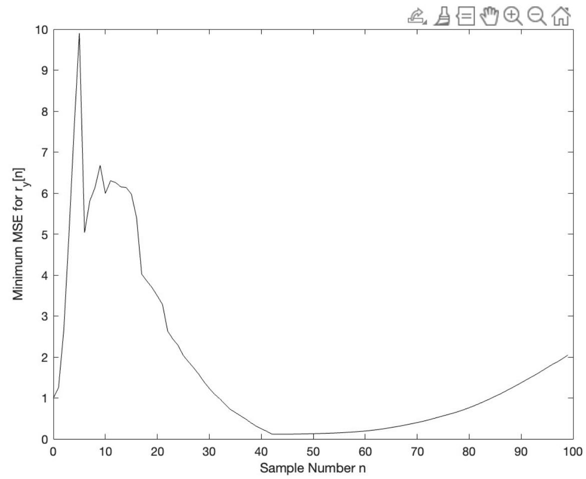

Figure 44: Minimum MSEs for $r_y[n]$

2.3.2 Only changing initial state true values

Table 9: Parameters for section 2.3.2

Parameter	Value
σ_u^2	0.0001
σ_r^2	1
σ_b^2	0.5
$\hat{s}[-1\|-1]$	$\begin{bmatrix}5 & 5 & 0 & 0\end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix}0 & 0 & -0.1 & 0.1\end{bmatrix}^T$

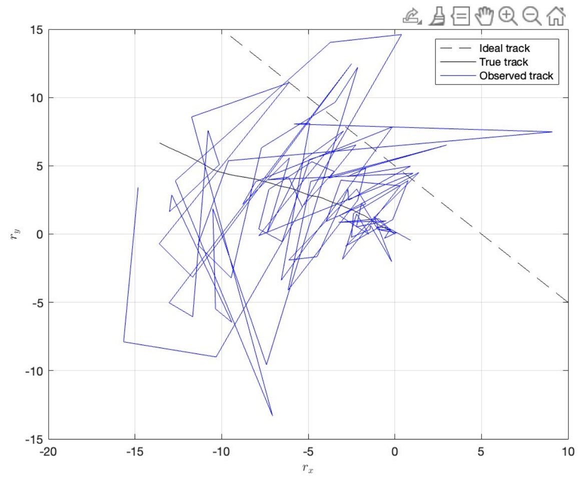

Figure 45: True and observed vehicle tracks

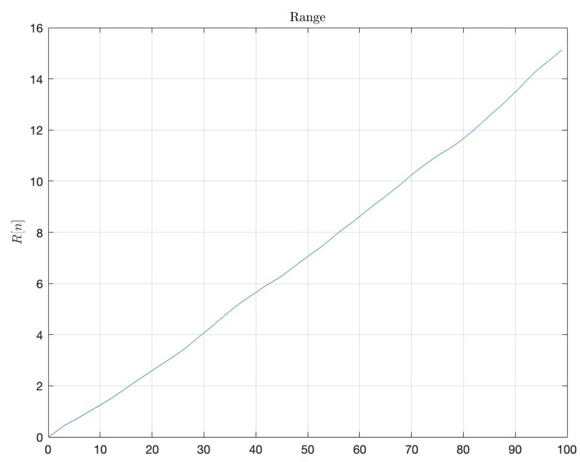

Figure 46: Range of true vehicle track

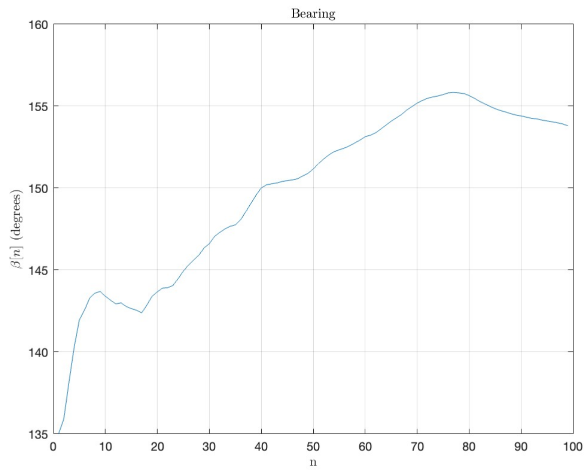

Figure 47: Bearing of true vehicle track

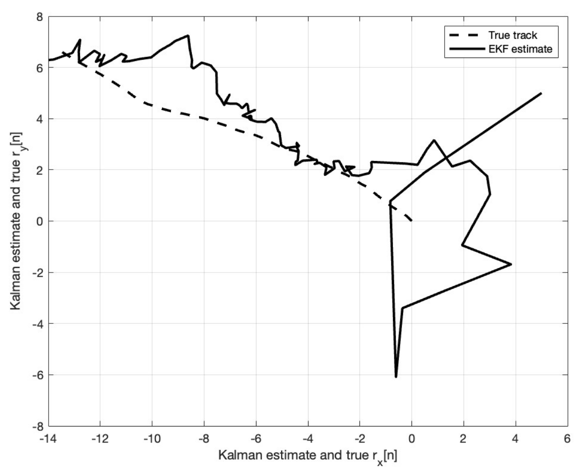

Figure 48: True and Extended Kalman Filter Estimates

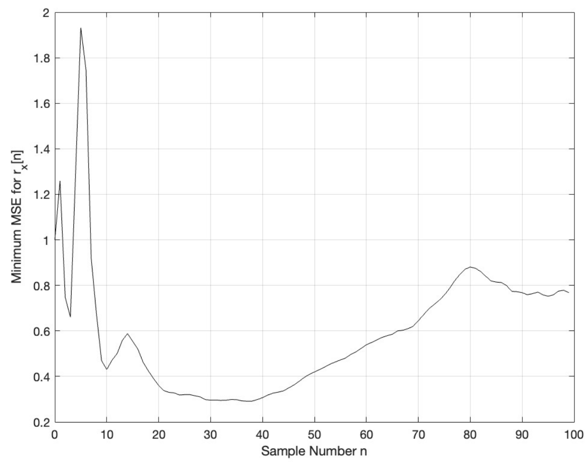

Figure 49: Minimum MSEs for $r_x[n]$

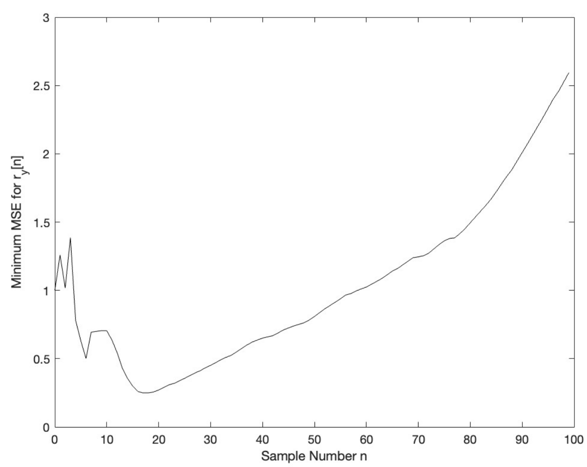

Figure 50: Minimum MSEs for $r_y[n]$

2.3.3 Changing only initial state estimates

Table 10: Parameters for section 2.3.3

Parameter	Value
σ_u^2	0.0001
σ_r^2	1
σ_b^2	0.5
$\hat{s}[-1\|-1]$	$\begin{bmatrix}10 & 10 & 0.1 & 0.1\end{bmatrix}^T$
$s[-1\|-1]$	$\begin{bmatrix}0 & 0 & -0.1 & 0.1\end{bmatrix}^T$

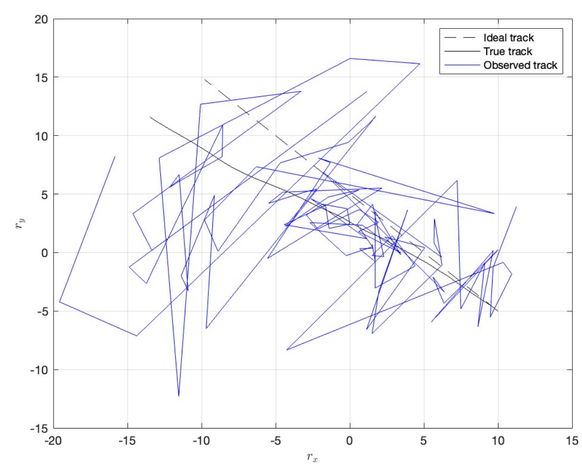

Figure 51: True and observed vehicle tracks

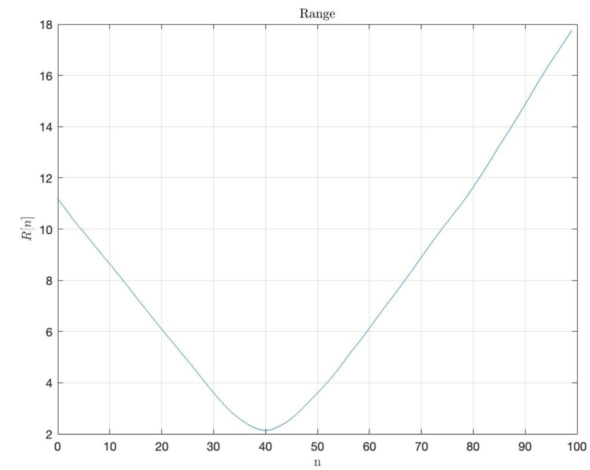

Figure 52: Range of true vehicle track

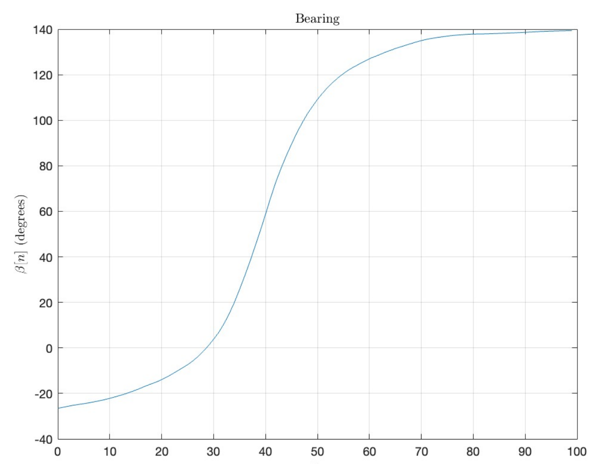

Figure 53: Bearing of true vehicle track

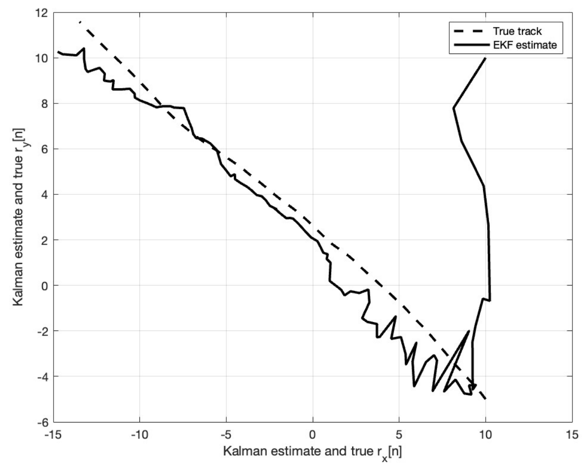

Figure 54: True and Extended Kalman Filter Estimates

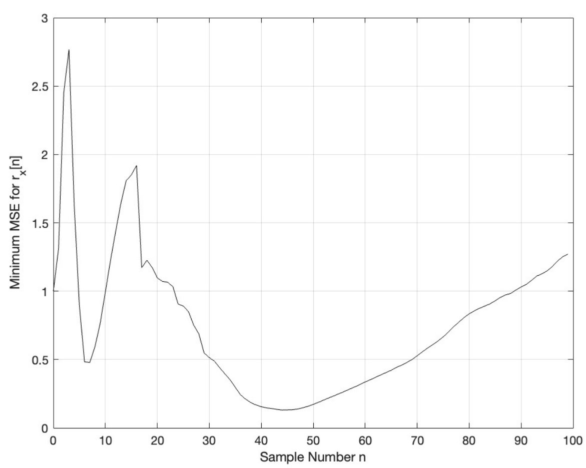

Figure 55: Minimum MSEs for $r_x[n]$

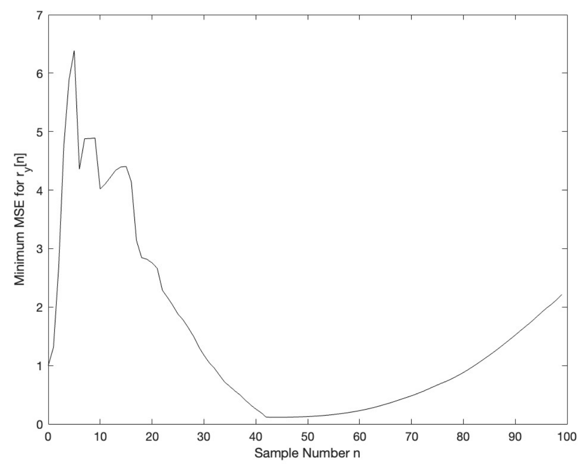

Figure 56: Minimum MSEs for $r_y[n]$

2.3.4 Task 3 Discussion

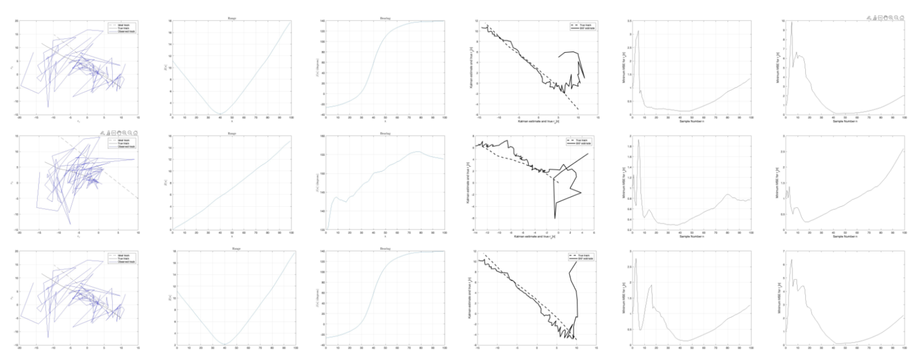

Figure 57: All three cases of Task 3 presented together

Compared to task 1 and task 2, the impact of driving noise has been brought back to the original value but the observation noise on both bearing and range has been increased in task 3.

This effect can be studies using the resultant curves. This can be clearly seen in the huge deviation between the true track and the observation track.

As opposed to task 2 the impact of the increment of observation noise seem to impact the tracking ability largely based on the observed results.

However the filter is able to track accurately for the most parts of the trajectory.

The impact of the bias between the initial state values and estimate values can be still explained using the reasoning in task 1 and 2. However in the case of more bias the MSE is not being reduced closer to zero at the most tracking points.

3 Appendix - Matlab Code

```
%% Lab 3 - Extended Kalman Filter
close all;
clear;
clc;

rng(1);
N = 100;
delta = 1;
%%-----------------------------------
% % Case I
% sig_u_sqr = 0.0001;
% sig_r_sqr = 0.1;
% sig_b_sqr = 0.01;
% %%-----------------------------------
% % % Case II
% sig_u_sqr = 0.01;
% sig_r_sqr = 0.1;
% sig_b_sqr = 0.01;
%
% %%-----------------------------------
% % %Case III
sig_u_sqr = 0.0001;
sig_r_sqr = 1;
sig_b_sqr = 0.5;

A = eye(4) + diag([delta delta],2);
auxvar = [0 0 sig_u_sqr sig_u_sqr];
Q = diag(auxvar);
C = diag([sig_r_sqr sig_b_sqr]);
```

```
Un = sqrt(sig_u_sqr)*randn(2,N);
U = [zeros(2,N); Un];
R_ideal = zeros(2,N);

vn =[-0.2; 0.2]; % vn[-1]
rn = [10; -5];  % rn[-1]

rn_ideal = rn;
R_ideal(:,1) = rn;

% % Change Initial State
%%----------------------
S(:,1)= [rn; vn]; % Given in the task
%S(:,1)= [0;0;-0.1;0.1]; % My change to init state

h_sn(:,1) = [ sqrt(S(1,1)^2 + S(2,1)^2); atan( S(2,1)/S(1,1) ) ];
for n=2:N
    rn_ideal = rn_ideal + [-0.2; 0.2]*delta;
    R_ideal(:,n) = rn_ideal;
    S(:,n) = A*S(:,n-1) + U(:,n);
    h_sn(:,n) = [ sqrt(S(1,n)^2 + S(2,n)^2); atan2( S(2,n),S(1,n) ) ];
end

w = [sqrt(sig_r_sqr)*randn(1,N); sqrt(sig_b_sqr)*randn(1,N)];
xn = h_sn + w;

R_obs = [xn(1,:).*cos(xn(2,:)); xn(1,:).*sin(xn(2,:))];

figure
plot(R_ideal(1,:),R_ideal(2,:),'--k',S(1,:),S(2,:),'-k',R_obs(1,:),R_obs(2,:),'-b');
xlabel('$r_x$','interpreter','latex')
ylabel('$r_y$','interpreter','latex')
legend('Ideal track', 'True track', 'Observed track')
grid on;

figure
plot(0:N-1,h_sn(1,:));
xlabel('n','interpreter','latex')
ylabel('$R[n]$','interpreter','latex')
title('Range','interpreter','latex')
```

```
grid on;

figure
plot(0:N-1,h_sn(2,:)*180/pi);
xlabel('n','interpreter','latex')
ylabel('$\beta [n]$ (degrees)','interpreter','latex')
title('Bearing','interpreter','latex')
grid on;

%% Extended Kalman filter

S_hat = zeros(4,N);

% Initial Estimates to change
%S_hat(:,1) = [5 5 0 0]';
S_hat(:,1) = [10 10 0.1 0.1]';
M = zeros(4,4,N);
M(:,:,1) = eye(4);

for n=2:N
    S_hat(:,n) = A*S_hat(:,n-1);
    M(:,:,n) = A*M(:,:,n-1)*A' + Q;
    rxn = S_hat(1,n);
    ryn = S_hat(2,n);
    lenRxy = sqrt(rxn^2 + ryn^2);
    H = [rxn/lenRxy ryn/lenRxy 0 0;
         -1*ryn/lenRxy^2 rxn/lenRxy^2 0 0];
    K_num = M(:,:,n)*H.';
    K_den = C + H*M(:,:,n)*H.';
    K = K_num/K_den;
    hsn = [lenRxy; atan2(ryn,rxn)];
    S_hat(:,n) = S_hat(:,n) + K*( xn(:,n) - hsn );
    M(:,:,n) = (eye(4)-K*H)*M(:,:,n);
end

figure
plot(S(1,:),S(2,:),'k--',S_hat(1,:),S_hat(2,:),'k','LineWidth',2);
xlabel(['Kalman estimate and true ' 'r_{x}[n]'])
ylabel(['Kalman estimate and true ' 'r_{y}[n]'])
legend('True track','EKF estimate')
```

```
grid on
figure
plot(0:N-1,squeeze(M(1,1,:)),'k');
xlabel('Sample Number n')
ylabel('Minimum MSE for r_{x}[n]')

grid on
figure
plot(0:N-1,squeeze(M(2,2,:)),'k');
xlabel('Sample Number n')
ylabel('Minimum MSE for r_{y}[n]')
```

References

[1] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall International, 1993.

[2] S. Haykin, *Adaptive Filter Theory*. Pearson, 2014. [Online]. Available: https://books.google.fi/books?id=J4GRKQEACAAJ