521324S SSP II — Matlab Simulation Exercise Task 3

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EXTENDED KALMAN FILTER (6 PTS)

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The goal of the task is to implement the Extended Kalman filter to perform vehicle tracking *[see Example 13.4, Kay, Vol.1]*. In particular, you are asked to reproduce the results in *[Example 13.4, Kay, Vol.1]*.

In this task we use an extended Kalman filter to track the position and velocity of a vehicle moving in a nominal given direction and at a nominal speed. We assume a constant velocity, perturbed only by wind gusts, slight speed corrections, etc., as might occur in an aircraft. We model these perturbations as noise inputs, so that the velocity components in the x and y directions at time n are

$$\begin{aligned} v_x[n] &= v_x[n-1] + u_x[n] \\ v_y[n] &= v_y[n-1] + u_y[n]. \end{aligned} \tag{1}$$

From the equations of motion, the position at time n is

$$\begin{aligned} r_x[n] &= r_x[n-1] + v_x[n-1]\Delta \\ r_y[n] &= r_y[n-1] + v_y[n-1]\Delta, \end{aligned} \tag{2}$$

where Δ is the time interval between samples. Therefore, we can write the signal vector as consisting of the position and velocity components as

$$\mathbf{s}[n] = \begin{bmatrix} r_x[n] \\ r_y[n] \\ v_x[n] \\ v_y[n] \end{bmatrix} \tag{3}$$

The measurements are noisy versions of the range and bearing. Thus, the observation equation is written as

$$\mathbf{x}[n] = \underbrace{\begin{bmatrix} \sqrt{r_x^2[n] + r_y^2[n]} \\ \arctan \frac{r_y[n]}{r_x[n]} \end{bmatrix}}_{\mathbf{h}(\mathbf{s}[n])} + \underbrace{\begin{bmatrix} w_r[n] \\ w_b[n] \end{bmatrix}}_{\mathbf{w}}, \tag{4}$$

with $w_r[n] \sim \mathcal{N}(0, \sigma_r^2)$ and $w_b[n] \sim \mathcal{N}(0, \sigma_b^2)$. Assume that the wind gusts, speed corrections, etc., are independent and just as likely to occur in any direction and with the same magnitude, then it seems reasonable to assign the same variances to $u_x[n]$ and $u_y[n]$. Call the common variance σ_u^2.

Task:

1) Let $\sigma_u^2 = 0.0001$, $\sigma_r^2 = 0.1$, $\sigma_b^2 = 0.01$. Implement the extended Kalman filter to estimate the position of the vehicle with initial state $\hat{s}[-1|-1] = [5\ 5\ 0\ 0]^{\mathrm{T}}$. Plot the figures for the estimated position vs true one and the minimum MSE versus samples. Repeat the simulation with a different initial state. Give your comments on the performance of the extended Kalman filter.
2) Let $\sigma_u^2 = 0.01$, $\sigma_r^2 = 0.1$, $\sigma_b^2 = 0.01$. Repeat task 1) and give your comments.
3) Let $\sigma_u^2 = 0.0001$, $\sigma_r^2 = 1$, $\sigma_b^2 = 0.5$. Repeat task 1) and give your comments.