

# Sistemas Dinâmicos - Solitões

Laboratório de Física Experimental Avançada

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# Introdução

## Objetivos

1. Verificação experimental da existência de solitões aquando de perturbações numa tina de água para diferentes condições iniciais.
2. Estudo da validade da teoria de Korteweg - de Vries (doravante KdV).
3. Cálculo da velocidade de solitões com vários métodos.
4. Estudo das componentes lineares e não-lineares dos solitões.

## Modelo KdV

- Equação KdV:

$$\frac{\partial \eta}{\partial t} + v_0 \frac{\partial \eta}{\partial x} + \frac{3v_0}{2h} \eta \frac{\partial \eta}{\partial x} + \frac{v_0 h^2}{6} \frac{\partial^3 \eta}{\partial x^3} = 0$$

- Solução para 1 solitão:

$$\eta(t) = \eta_0 \operatorname{sech}^2 \left( \frac{x - vt}{L} \right)$$

onde,

$$v_0 = \sqrt{gh} \quad , \quad L = \sqrt{\frac{4h^3}{3\eta_0}}$$

- Condições de validade do modelo:

$$\epsilon_1 = \frac{\eta}{h} \quad , \quad \epsilon_2 = \left( \frac{h}{l} \right)^2 \quad , \quad U = \frac{\epsilon_1}{\epsilon_2}, \epsilon_1 \ll 1 \quad , \quad \epsilon_2 \ll 1 \quad , \quad U \simeq 1$$

A velocidade teórica dos solitões e o seu erro são calculados através de:

$$v_{\text{teo}} = \sqrt{gh} \left( 1 + \frac{A}{4h} \right)$$

$$\sigma_{v_{\text{teo}}} = \frac{1}{2} \sqrt{\frac{g}{h}} \sqrt{\left( 1 - \frac{A}{4h} \right)^2 \sigma_h^2 + \left( \frac{\sigma_A}{2} \right)^2}$$

$$N = 1 + \text{Int} \left( \frac{S}{\pi} \right) \quad , \quad S = \sqrt{\frac{3A}{h}} \frac{B}{h}$$

## Modelo KdV

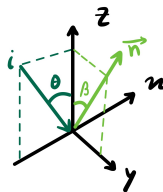
Para 2 solitões obteve-se o seguinte modelo:

$$\eta(t) = \frac{n(v_1 - v_2)(v_1 \cosh^2(\sqrt{c_2}\xi_2/2) + v_2 \sinh^2(\sqrt{c_1}\xi_1/2))}{((\sqrt{c_1} - \sqrt{c_2}) \cosh((\sqrt{c_1}\xi_1 + \sqrt{c_2}\xi_2)/2) + (\sqrt{c_1} + \sqrt{c_2}) \cosh((\sqrt{c_1}\xi_1 - \sqrt{c_2}\xi_2)/2))^2}$$

onde,

$$\begin{cases} v_1 > v_2 > 0, \\ \xi_1 = x_1 - c_1 \times t \\ \xi_2 = x_2 - c_2 \end{cases}$$

## Esquema de montagem e incidência do laser



$$\vec{n}_i = (\sin\theta, 0, -\cos\theta)$$

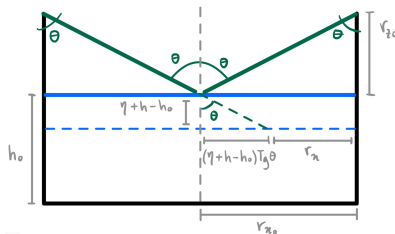
$$\vec{n} = (0, \sin\beta, \cos\beta)$$

$$\vec{n}_r = (\sin(\theta), \cos(\theta)\sin(2\beta), \cos(\theta)\cos(2\beta))$$

$$\vec{n}_r = (\sin(\theta), \cos(\theta)\sin(2\beta), \cos(\theta)\cos(2\beta))$$

$$r_z = \cot(\theta)\cos(2\beta)r_x \quad , \quad r_y = \cot(\theta)\sin(2\beta)r_x$$

$$r_{x0} = r_{z0}\tan(\theta) \quad , \quad r_x = (r_{z0} + \eta + (h - h_0))\tan(\theta)$$



$$\begin{cases} x_L = r_y \\ y_L = \eta + r_z - r_{z0} - (h - h_0) \end{cases}$$



$$\begin{cases} \eta = \frac{x_L}{\sin(2\beta)} - r_{z0} - (h - h_0) \\ \beta = \arctan\left(\frac{x_L}{y_L + 2(r_{z0} + (h - h_0))}\right) \end{cases}$$

$$\mathbf{div\ 0:} \quad \eta(x_L = 0) = \frac{y_L}{2}$$

$$\delta\eta = \left|\frac{\partial\eta}{\partial x_L}\right|\delta x_L + \left|\frac{\partial\eta}{\partial y_L}\right|\delta y_L + \left|\frac{\partial\eta}{\partial h}\right|\delta h + \left|\frac{\partial\eta}{\partial h_0}\right|\delta h_0 + \left|\frac{\partial\eta}{\partial r_{z0}}\right|\delta r_{z0}$$

## **Previsões Teóricas**

## Previsões Teóricas

A ( $\pm 0,2$ cm)	B ( $\pm 0,2$ cm)	h ( $\pm 0,2$ cm)	Nº Solitões	$v_{teo}$ (cm/s)	$\epsilon_1 < < 1$	$\epsilon_2 < < 1$	$U \approx 1$
2.0	10.0	8.0	1	$94.08 \pm 1.18$	0.125	0.16	0.781
2.0	20.0	6.0	2	$83.07 \pm 1.33$	0.167	0.023	7.261
2.0	40.0	6.0	3	$83.07 \pm 1.33$	0.167	0.006	27.833
2.0	40.0	6.0	3	$83.07 \pm 1.33$	0.167	0.006	27.833
2.0	40.0	6.0	3	$83.07 \pm 1.33$	0.167	0.006	27.833
6.0	10.0	6.0	1	$95.85 \pm 1.15$	0.500	0.09	5.556
4.0	10.0	5.0	1	$84.00 \pm 1.32$	0.400	0.063	6.349
2.0	10.0	5.0	1	$77.00 \pm 1.44$	0.200	0.063	3.175
1.0	40.0	4.0	3	$66.52 \pm 1.66$	0.125	0.003	41.667
6.0	40.0	4.0	7	$86.09 \pm 1.25$	0.750	0.003	250
6.0	40.0	4.0	7	$86.09 \pm 1.25$	0.750	0.003	250
6.0	10.0	4.0	2	$86.09 \pm 1.25$	0.750	0.04	18.75
6.0	10.0	4.0	2	$86.09 \pm 1.25$	0.750	0.04	18.75
4.0	10.0	4.0	2	$78.26 \pm 1.41$	0.500	0.04	12.5
4.0	10.0	4.0	2	$78.26 \pm 1.41$	0.500	0.04	12.5
2.0	20.0	4.0	2	$70.44 \pm 1.58$	0.250	0.01	25
6.0	40.0	2.0	20	$77.48 \pm 1.24$	1.500	0.001	1500
6.0	30.0	2.0	15	$77.48 \pm 1.24$	1.500	0.001	1500
6.0	10.0	2.0	5	$77.48 \pm 1.24$	1.500	0.01	150
4.0	40.0	2.0	16	$66.41 \pm 1.57$	1.000	0.001	1000
2.0	40.0	2.0	12	$55.34 \pm 2.00$	0.500	0.001	500

# **Aquisição de Dados**

## Cálculo da altura da água

Fizeram-se 9 medições equidistantes ao longo da tina.

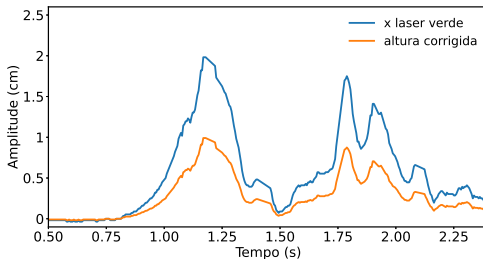
Incerteza é o maior dos desvios à média.

Medições de A e B feitas por zoom, pelo que se considerou incerteza maior.

h ( $\pm 0.2$ cm)	média h (cm)
3.5	3.7 $\pm$ 0.2
3.6	
3.8	
4.0	
3.9	
3.8	
3.8	
3.7	
3.5	

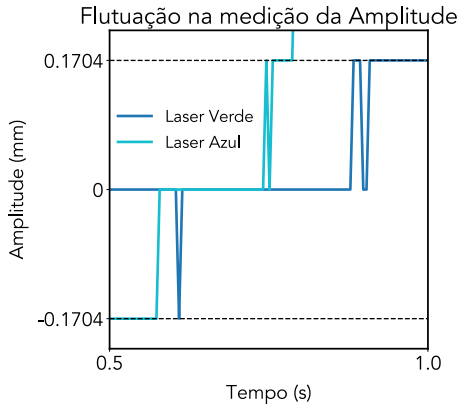
## Correção da altura do laser

Parâmetro	Valor
$h_0$	$6.0 \pm 0.1 \text{ cm}$
$r_{z0}$	$2.1 \pm 0.1 \text{ cm}$
$r_{x0}$	$2.8 \pm 0.2 \text{ cm}$
$\theta$	$0.927 \pm 0.057 \text{ rad}$

Exemplo para o ensaio **h5p5a4b10**

## Tracker

Os valores para o perfil do solitão foram obtidos recorrendo a um tracker. Assumiu-se como erro metade do intervalo mínimo de espaço percorrido detetado pelo software (*Cinéris*).



# **Análise de Dados**



## Cálculo da velocidade dos solitões

1. Utilizando o intervalo de tempo entre detecção nos lasers

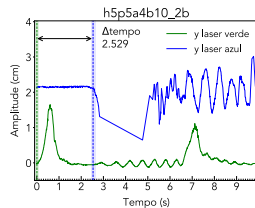
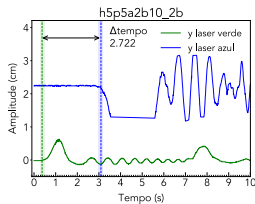
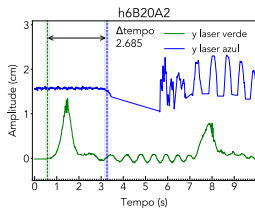
$$v_{laser} = \frac{d}{\Delta t} = \frac{d}{t_{azul} - t_{verde}}$$

$$\sigma_{v_{laser}} = \frac{\sqrt{\sigma_d^2 + v_{laser}^2 \left( \sigma_{t_{azul}}^2 + \sigma_{t_{verde}}^2 \right)}}{\Delta t}$$

$$d = 213.7 \pm 0.2 mm$$

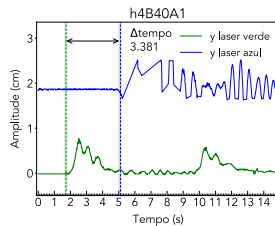
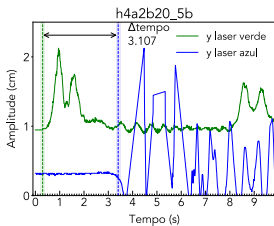
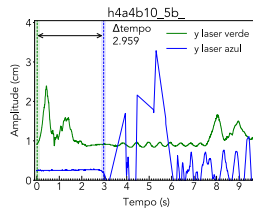
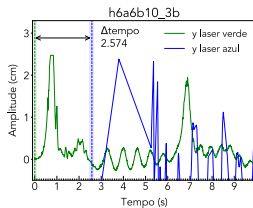
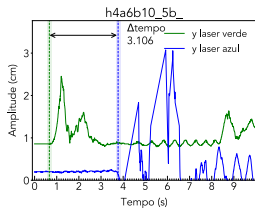
2. Extraíndo a velocidade como uma parâmetro do ajuste

## Ensaio dentro do regime de validade da teoria KdV



Ensaio	$\Delta t (\pm 0.05 \text{ s})$	$v_{laser} (\text{cm/s})$	$\delta v_{laser} (\%)$	$\delta v_{laser} (\sigma)$	$\epsilon_1$	$\epsilon_2$	U
h6B20A2	2.685	$79.590 \pm 4.193$	4.189	0.830	0.167	0.023	7.261
h5p5a2b10_2b	2.722	$78.508 \pm 4.080$	2.075	0.392	0.200	0.063	3.175
h5p5a4b10_2b	2.529	$84.500 \pm 4.726$	7.973	1.320	0.400	0.063	6.349

## Ensaio fora do regime de validade da teoria KdV



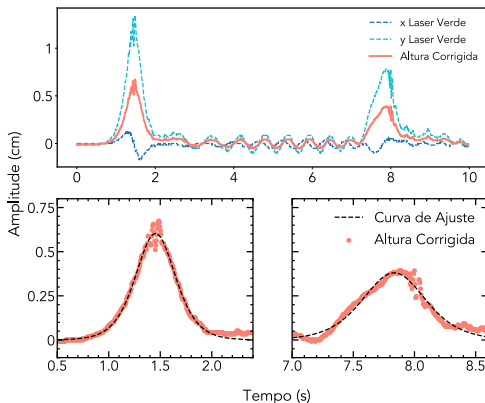
## Ensaio fora do regime de validade da teoria KdV

Ensaio	$\Delta t (\pm 0.05 \text{ s})$	$v_{laser} (cm/s)$	$\delta_{v_{laser}} (\%)$	$\delta_{v_{laser}} (\sigma)$	$\epsilon_1$	$\epsilon_2$	U
h4a6b10_5b	3.106	$68.802 \pm 3.133$	20.088	5.518	0.75	0.04	18.75
h6a6b10_3b	2.574	$83.023 \pm 4.562$	13.382	2.812	0.5	0.09	5.556
h4a4b10_5b	2.959	$72.220 \pm 3.452$	7.718	1.75	0.5	0.04	12.5
h4a2b20_5b	3.107	$68.780 \pm 3.131$	2.357	0.53	0.25	0.01	25
h4B40A1	3.381	$63.206 \pm 2.644$	4.982	1.253	0.125	0.003	41.667

## Influência de $\epsilon_1$ , $\epsilon_2$ e $U$ nas grandezas experimentais

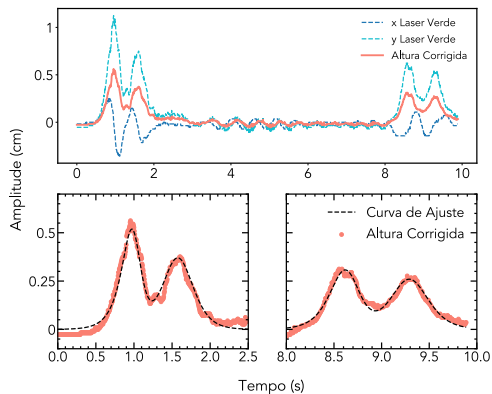
- ▶ menor  $\epsilon_1 \Rightarrow$  menor desvio da velocidade ao teórico
- ▶  $\epsilon_2$  é quase sempre uma ordem de grandeza abaixo de  $\epsilon_1$  e é quase constante: logo não é possível avaliar a sua influência
- ▶ hipótese de  $U$  menor conduzir a uma melhor estimativa do número de solitões: não foi possível corroborar

## Ajustes à solução para 1 solitão

Ensaio **h6B20A2**

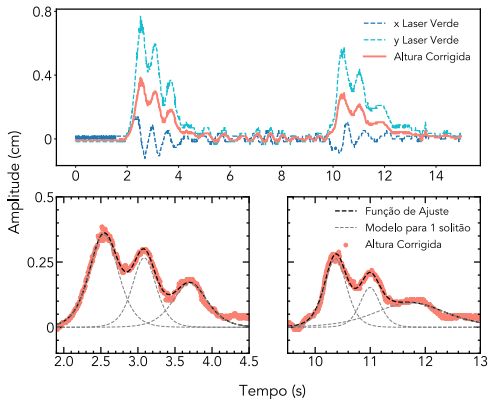
Solitão	$\eta_0$ (cm)	$x$ (m)	$v$ ( $cm \cdot s^{-1}$ )	$\chi^2/g.l$
1º	$0.6 \pm 2e-05$	$1.223 \pm 0.0049$	$84.13 \pm 0.33$	3.186
2º	$0.4 \pm 3e-05$	$6.229 \pm 0.0576$	$79.52 \pm 0.73$	9.170

## Ajustes à solução para 2 solitões



## Ensaio h4a2b20\_5b

Solitão	$x_1$ (m)	$x_2$ (m)	$c_1$ (m/s)	$c_2$ (m/s)	$n$	$\chi^2/\text{g.l.}$
1 <sup>o</sup>	$7.28 \pm 0.04$	$4.97 \pm 0.05$	$6.52 \pm 0.06$	$3.81 \pm 0.03$	$0.0039 \pm 2.41\text{e-}05$	2.85
2 <sup>o</sup>	$49.78 \pm 0.33$	$34.42 \pm 0.28$	$5.62 \pm 0.04$	$3.83 \pm 0.03$	$0.0027 \pm 1.62\text{e-}05$	1.26



## Ensaio h6B20A2

Solitão	$\eta_0$ (cm)	$x$ (cm)	$v$ (cm $\cdot$ s $^{-1}$ )	$\chi^2$
1º Incidente	$0.2 \pm 2e-03$	$-492 \pm 10.0$	$132.8 \pm 2.6$	0.706
2º Incidente	$0.3 \pm 3e-03$	$-437.3 \pm 10.5$	$153.6 \pm 3.4$	
3º Incidente	$0.4 \pm 3e-03$	$-274.6 \pm 2.8$	$108.1 \pm 1.2$	
1º Refletido	$0.1 \pm 1e-03$	$-755.2 \pm 26.9$	$64.4 \pm 2.2$	0.624
2º Refletido	$0.2 \pm 4e-03$	$-1955.4 \pm 71.4$	$177.8 \pm 6.5$	
3º Refletido	$0.3 \pm 2e-03$	$-1288.5 \pm 15.5$	$124.2 \pm 1.5$	

Tabela: Parâmetros do ajuste ao ensaio h6B20A2



## Comparação das velocidades - solitões dentro da teoria

Ensaio	$v_{teo}$ (cm/s)	$v_{fit}$ (cm/s)	$\delta_{v_{fit}}$ (%)	$\delta_{v_{fit}}$ ( $\sigma$ )	$\eta_0$ (cm)	$x$ (cm)
h6B20A2	$79.590 \pm 0.41$	$83.56 \pm 0.41$	5.0	9.68	$0.6 \pm 0.003$	$-121.5 \pm 0.61$
h5p5a2b10.2b	$78.508 \pm 4.080$	$106.80 \pm 1.32$	36.0	21.43	$0.3 \pm 0.004$	$-119.1 \pm 1.50$
h5p5a4b10.2b	$84.500 \pm 4.726$	$85.74 \pm 0.43$	1.5	2.88	$0.8 \pm 0.004$	$-51.2 \pm 0.27$

## Conclusões

- ▶ Número de solitões observados e velocidade (e consequentemente amplitude) tendem a aumentar com o aumento de A e B, de acordo com a teoria.
- ▶ Medimos experimentalmente a velocidade dos solitões através de 2 métodos distintos. As incertezas experimentais tendem a não cobrir o valor teórico. Maior influência de  $\epsilon_1$  e  $\epsilon_2$ . Os dois métodos usados revelaram igual sucesso.
- ▶ Sugere-se manter a tina perfeitamente na horizontal, usar uma comporta que isole melhor os 2 compartimentos e calcular a velocidade do solitão por outros métodos (cronómetro).

## **Bibliografia**



### **Material disponibilizado pelos docentes**

alguém que enumere se acharem que faz sentido



### **Github**

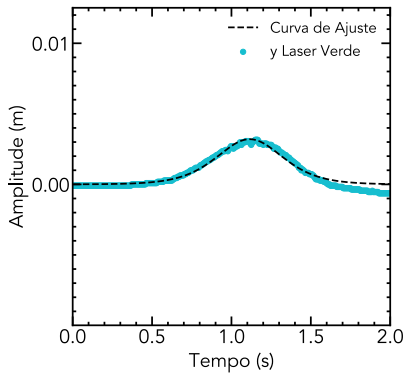
<https://github.com/AthomsG/LFEA>



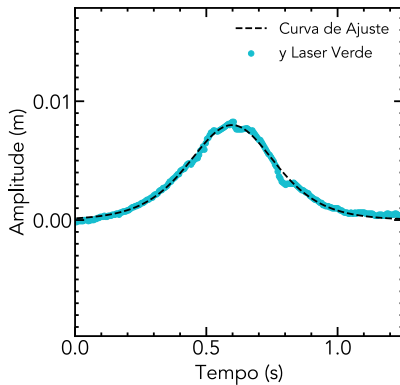
### **KdV 2-Soliton Solution**

[http://lie.math.brocku.ca/~sancho/solitons/kdv\\_solitons.php](http://lie.math.brocku.ca/~sancho/solitons/kdv_solitons.php)

# Anexos

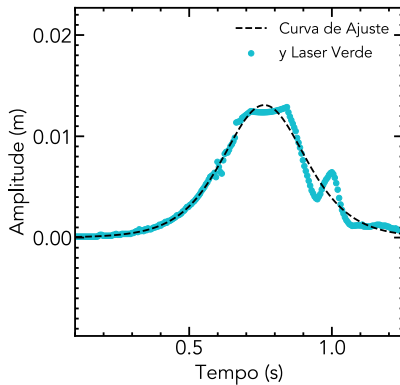
**Ensaio h5p5a2b10**

$\eta_0$ (cm)	$x$ (m)	$v$ ( $cm \cdot s^{-1}$ )	$\chi^2/g.l$
$0.6 \pm 2e-05$	$1.223 \pm 0.0049$	$84.13 \pm 0.33$	17.14

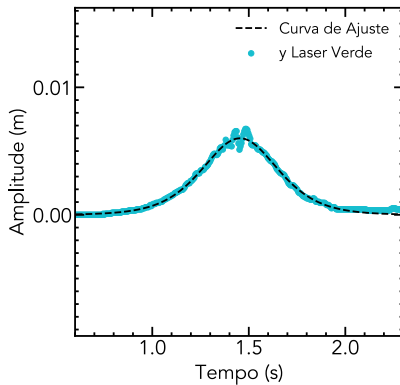
**Ensaio h5p5a4b1**

$\eta_0$ (cm)	$x$ (m)	$v$ ( $cm \cdot s^{-1}$ )	$\chi^2/g.l$
$0.6 \pm 3e-05$	$-1.22 \pm 0.00609$	$-0.84 \pm 0.00414$	163.54

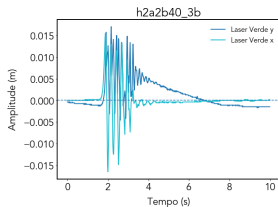
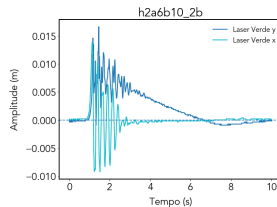
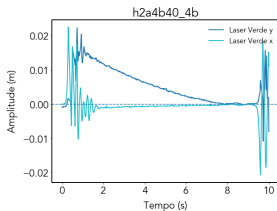
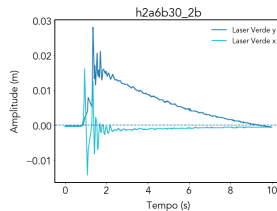


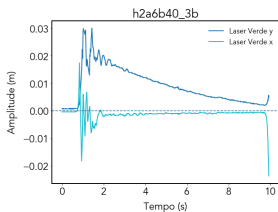
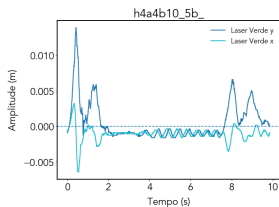
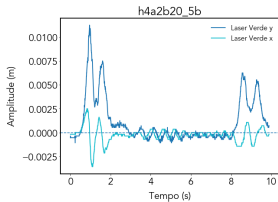
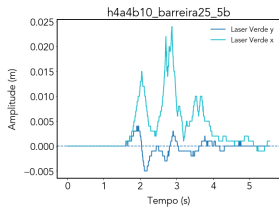
Ensaio **h6a6b10**

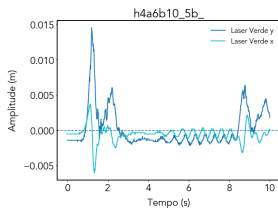
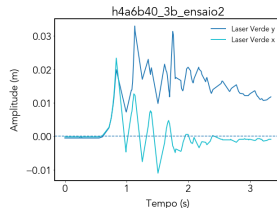
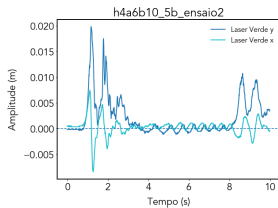
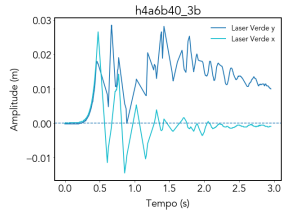
$\eta_0$ (cm)	$x$ (m)	$v$ ( $cm \cdot s^{-1}$ )	$\chi^2/g.l$
$0.8 \pm 4e-05$	$-0.51 \pm 0.00271$	$-0.86 \pm 0.00431$	19.30

**Ensaio h6B20A2**

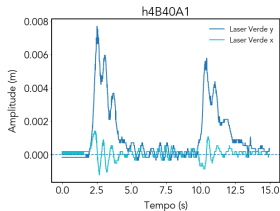
$\eta_0$ (cm)	$x$ (m)	$v$ ( $cm \cdot s^{-1}$ )	$\chi^2/g.l$
$0.3 \pm 4e-05$	$-1.19 \pm 0.01504$	$-1.07 \pm 0.01318$	38.04

**Ensaio h2a2b40\_3b****Ensaio h2a6b10\_2b****Ensaio h2a4b40\_4b****Ensaio h2a6b30\_2b**

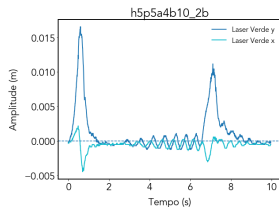
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**Ensaio h4a6b10\_5b\_****Ensaio h4a6b40\_3b\_ensaio2****Ensaio h4a6b10\_5b\_ensaio2****Ensaio h4a6b40\_3b**

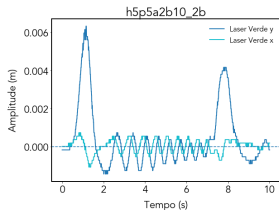
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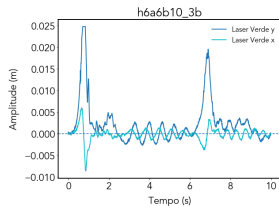
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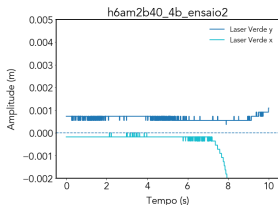
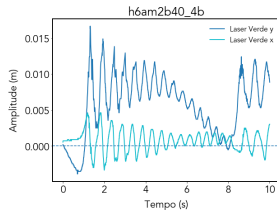
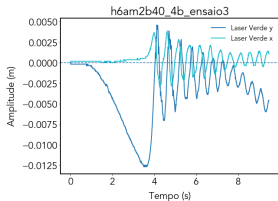
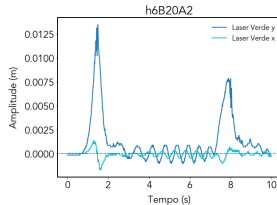


## Ensaio h5p5a2b10\_2b



## Ensaio h6a6b10\_3b



**Ensaio h6am2b40\_4b\_ensaio2****Ensaio h6am2b40\_4b****Ensaio h6am2b40\_4b\_ensaio3****Ensaio h6B20A2**

## Ensaio h6B10A2

