Exercício 2

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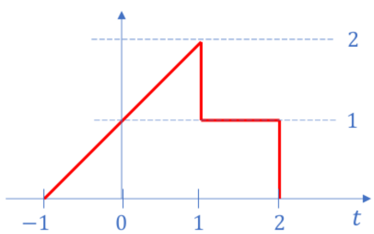
EXERCÍCIO 2

Considere o pulso $f(t)$ ilustrado na figura abaixo:



(a) Determine a transformada de Fourier de $f(t)$ nas formas cartesiana e polar e na forma de diagramas de Bode;

(b) Determine o espectro de energia de $f(t)$;

(c) Calcule a energia de $f(t)$.

Item (a)

A transformada $F(\omega)$ de $f(t)$ na sua forma cartesiana é

$$
\begin{aligned}
F(\omega) &= \int_{-1}^{1} (1+t)e^{-i\omega t} + \int_{1}^{2} 1e^{-i\omega t} = \qquad (1)\\
&= \frac{\sin(\omega)+\sin(2\omega)}{\omega} + i\frac{\cos(\omega)+\cos(2\omega)-\frac{2}{\omega}\sin(\omega)}{\omega} \qquad (2)\\
& \qquad (3)
\end{aligned}
$$

logo,

$$
\begin{aligned}
A(\omega) &= \frac{\sin(\omega)+\sin(2\omega)}{\omega} \qquad (4)\\
B(\omega) &= \frac{\cos(\omega)+\cos(2\omega)-\frac{2}{\omega}\sin(\omega)}{\omega} \qquad (5)\\
& \qquad (6)
\end{aligned}
$$

Assim, já que a forma polar de $F(\omega) = |F(\omega)|e^{i\theta(\omega)}$ com

$$
\begin{aligned}
|F(\omega)| &= \sqrt{A(\omega)^2 + B(\omega)^2} && (7)\\
\theta(\omega) &= \tan^{-1}\left(\frac{B(\omega)}{A(\omega)}\right) && (8)\\
& && (9)
\end{aligned}
$$

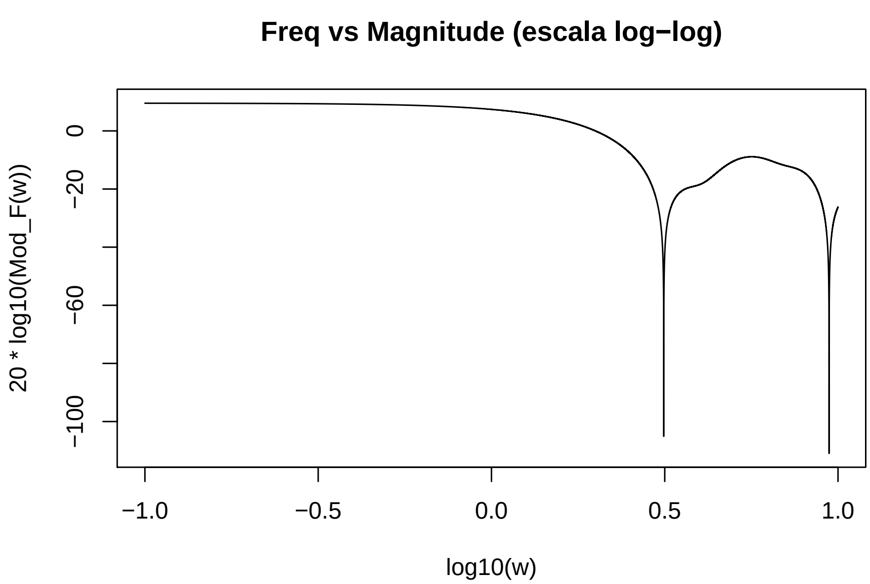
Tem-se,

$$
\begin{aligned}
|F(\omega)| &= \frac{\sqrt{2\big[1-\cos(2\omega)+\omega[\sin(\omega)-\sin(2\omega)-\sin(3\omega)]+\omega^2[1+\cos(\omega)]\big]}}{w^2} && (10)\\
\theta(\omega) &= \tan^{-1}\left(\frac{\cos(\omega)+\cos(2\omega)-\frac{2}{\omega}\sin(\omega)}{\sin(\omega)+\sin(2\omega)}\right) && (11)\\
& && (12)
\end{aligned}
$$

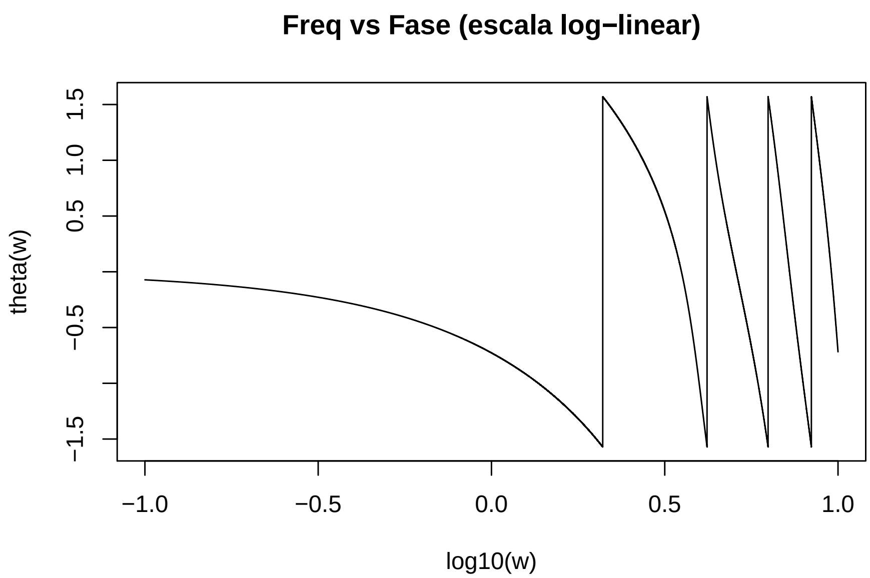
diagramas de Bode

```
A <- function(w) (sin(w) + sin(2*w))/w
B <- function(w) (cos(w) + cos(2*w) - (2/w)*sin(w))/w
Mod_F <- function(w) sqrt(A(w)^2 + B(w)^2)
theta <- function(w) atan(B(w)/A(w))
```

```
w <- seq(0.1, 10, l = 100000)
plot(log10(w), 20*log10(Mod_F(w)), type = "l",
     main = "Freq vs Magnitude (escala log-log)")
```



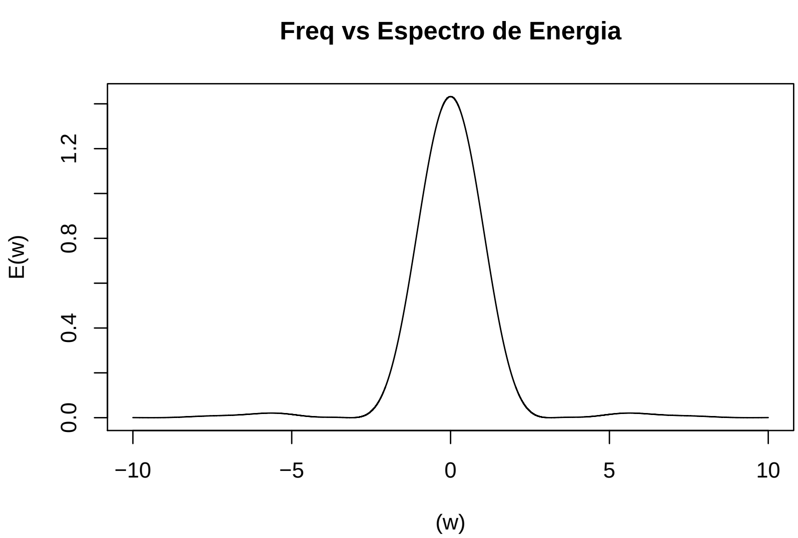
```r
plot(log10(w), theta(w), type = "l",
     main = "Freq vs Fase (escala log-linear)")
```



Item (b)

O espectro de energia é definido como $E(\omega) = \frac{1}{2\pi}|F(\omega)|^2$.

```r
w <- seq(-10, 10, l = 100000)
E <- function(w) (1/(2*pi)) * Mod_F(w)^2
plot(
  (w), E(w), type = "l",
    main = "Freq vs Espectro de Energia"
)
```



Item (c)

A energia de $f(t)$ é $E = \int_{\omega=-\infty}^{\infty} E(\omega) d\omega \approx 3.665$

```r
safe_E <- function(w) {
  r <- E(w)
  ifelse(is.finite(r), r, 0)
}
integrate(safe_E, -500, 500)
```

```
## 3.665392 with absolute error < 0.00043
```