United International University

School of Science and Engineering

Mid Term Examination Trimester: Fall-2024

Course Title: Advanced Calculus

Course Code: Math 1153 Marks: 30 Time: 1 Hour 30 Mins

Answer all the questions. Answer all parts of a question together.

Q1. (a) Identify the graph of $r(t) = 4\cos t\,\hat{\imath} + 4\sin t\,\hat{\jmath} + 3t\hat{k}$ and find its unit binormal at $t = \frac{\pi}{4}$. **[5]**

(b) Evaluate $\int_C xy\,dx + xy\,dy + z^2\,dz$ along the curve $C: x = \sin t, y = \cos t, z = t^2$ within the interval $0 \le t \le \frac{\pi}{2}$. **[3]**

Q2. (a) Using the implicit partial differentiation, evaluate $\frac{\partial z}{\partial x}$ for $y^2e^{5x} - \cos(\ln z) = 1$. **[2]**

(b) Find the directional derivative of $z = \sqrt{x^2 + y^2}$ at the point $P(-3, -4, 5)$ in the direction of $\vec{a} = -\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$. **[3]**

(c) Consider the surface $x^3 - y^2z - yz^2 = 6$ to find the parametric equations of the line that is normal to the surface at the point $(2,1,1)$. **[2]**

Q3. (a) Describe the set of points z in the complex plane that satisfies $|z - i| = \mathrm{Im}(z) + 1$. **[3]**

(b) Find all the values of $\left(\frac{\sqrt{3}-i}{2i}\right)^{\frac{1}{3}}$. **[3]**

Q4. (a) Evaluate $\lim_{z\to 2} \frac{z-2}{z^3-8}$. **[2]**

(b) Show that $u(x,y) = x^2 - y^2 - 2x$ is harmonic and hence find its harmonic conjugate $v(x,y)$. Also, confirm $w = u(x,y) + iv(x,y) = f(z)$. **[5]**

(c) Solve $e^z + i = 0$ for z. **[2]**

cos

sin −sin

cos