United International University
School of Science and Engineering
Final Examination Trimester- Spring 2025
Course: Math -2205
Total marks 40 || Duration 2 hours

[Note that the number of marks is given in brackets [ ] at the end of each question or part question. You have to answer all the questions. You are requested to answer in order.]

Q1 Suppose the time (in minutes) it takes to re-heat a cup of coffee is modeled as a continuous random variable X with the following probability density function (PDF):

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2 \\ 0, & otherwise \end{cases}$$

(i) Find the value of k [2]

(ii) Evaluate $P\left(1 \leq x \leq \frac{3}{2}\right)$ [2]

(iii) Sketch the graph of $f(x)$ and state the value of the mode [2]

(iv) Find the mean and median time and the variance. [4]

(v) Find the CDF to the given PDF [2]

Q2 (i) The heights of adult men in a city are approximately normally distributed, with a mean of 175 cm and a standard deviation of 8 cm. what is the probability that a randomly [3] selected man is between 167 cm and 183 cm tall?

(ii) A new email spam filter can correctly detect a spam message 90% of the time. Suppose you receive 20 emails, and each one is checked by the filter independently. [3] What is the probability that the filter correctly identifies at least 18 of the 20 spam emails?

(iii) A small café receives an average of 5 customer arrivals per 10-minute interval. What is the probability that exactly 8 customers arrive in a randomly chosen 10-minute [3] period?

Q3 (a) A company uses an automated system to detect phishing emails (scam emails). According to past data, only 1% of emails are actually phishing. The system correctly flags 99% of phishing emails, but it also wrongly flags 5% of non-phishing emails. Now, if an employee receives an email that has been flagged by the system, the question is: what is the chance that this email is truly a phishing email, given that it was flagged? [3]

(b) A company is planning to launch a new product. The success of the product largely depends on the overall economic condition of the country. Based on past data and expert opinion, the marketing team has identified three possible economic conditions, and their associated probability as follows:

Economics conditions	Boom	Stable	Recession
Profit (in million $)	50	20	-10
Probability	0.3	0.5	0.2

[4]

i. Find the expected profit.
ii. Find the variance of the profit.

(c) In a small company, 40 employees were surveyed. The following data was collected:
25 employees like working remotely (Event A)
20 employees use productivity tracking apps (Event B) [3]
10 employees both like working remotely and use productivity apps (Event A ∩ B)
Are the events "likes working remotely" (A) and "uses productivity apps" (B) independent?

Q4 (a) An IT company is evaluating the average time (in minutes) it takes for its helpdesk team to resolve customer support tickets. A random sample of 10 recently resolved tickets produced the following resolution times (in minutes): 32, 28, 35, 30, 34, 31, 29, 33, 36, 27
Assume that the resolution time is approximately normally distributed. Using a 97% confidence, [4] estimate the mean resolution time for all customer support tickets handled by the team.

(b) A bottled water company claims that its bottles contain, on average, 1 liter of water. However, a consumer rights organization suspects that the actual amount is less than 1 liter. To evaluate this claim, the organization randomly selects a sample of 30 bottles and measures their water content. [5] The sample yields a mean of 0.985 liters, while the population standard deviation is known to be 0.03 liters. Using a 5% level of significance, test whether there is sufficient evidence to conclude that the average amount of water per bottle is less than the claimed 1 liter, assuming the water content is approximately normally distributed. What type of error may occur in this case?

Distribution	**_Probability function_**
Binomial	$f(x) = n_{C_x} p^x (1-p)^{n-x} \; ; \; x = 0, 1, 2, \dots n$
Poisson	$f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!} \; ; \; x = 0, 1, 2, \dots$
Uniform	$f(x) = \dfrac{1}{b-a} \; ; \; a \leq x \leq b$
Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \; ; \; -\infty < x < \infty$