Math-1153 Class Test 05 Section BB

1. Consider the following heat equation. [2]

$$9u_{xx} = u_t \; ; 0 \leq x \leq 5, t > 0$$

$$u(0,t) = 0 \; ; \; u(5,t) = 0$$

$$u(x,0) = 5x$$

If the general solution is $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{9n^2\pi^2 t}{25}} \sin\frac{n\pi x}{5}$, find c_n.

2. A metal rod of length 3 meters whose starting end at the temperature 50^o and the other end is insulated. If initially a heat source is provided satisfying $f(x) = 2x$, formulate a heat conduction problem. Assume the heat diffusivity is provided as 25 units. [2]

3. Give the physical explanation of the following wave equation. [2]

$$4u_{xx} = u_{tt} \; ; 0 \leq x \leq 20, t > 0$$

$$u(0,t) = 0 \; ; \; u(20,t) = 0$$

$$u_t(x,0) = 0 \; ; \; u(x,0) = 5$$

4. An elastic string of length 10 units. The end $x = 0$ is held fixed, while the end $x = 10$ is free. The string is set in motion with no initial velocity from the initial position $f(x) = 10$. Formulate the wave equation according to the given information, where the tension of the string is 4.5 units with a density of 2 units. [2]

5. Write two real-life examples of heat conduction and wave equations for each. [2]