United International University
Department of Computer Science and Engineering
CSE 2213/CSI 219: Discrete Mathematics (BSCSE and BSDS)
Final Examination Fall 2024
Total Marks: 40 Time: 2 hours

Any examinee found adopting unfair means will be expelled from the university / banned as per UIU disciplinary rule.

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1. Using mathematical induction, prove the following formula: [5]

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}, \quad \text{for all natural numbers, } n \geq 2.$$

2. (a) Place the following 7 words into the nodes of the tree so that it forms a valid binary search tree (BST): `Grape`, `Orange`, `Strawberry`, `Banana`, `Cherry`, `Mango`, `Apple`. Provide the post-order traversal of this BST you prepared. [1.5 + 1.5 = 3]

   (b) Look at the following prefix mathematical notation: `- * + 3 1 - 9 + 3 4 8`

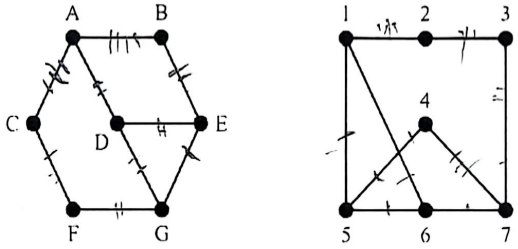
   i. Draw the ordered rooted tree corresponding to the arithmetic expression written above. [2]

   ii. Write its infix notation and find the value. [2]

   (c) A full m-ary tree with 7 internal nodes has a total of 43 vertices. How many leaves does a full $(m+1)$-ary tree with 9 internal nodes have? [2]

3. (a) In a network of 15 computers, each computer is connected to an odd number of other computers. Is it possible to represent this situation as a graph? If so, how would you structure the network? If not, explain why. [2]

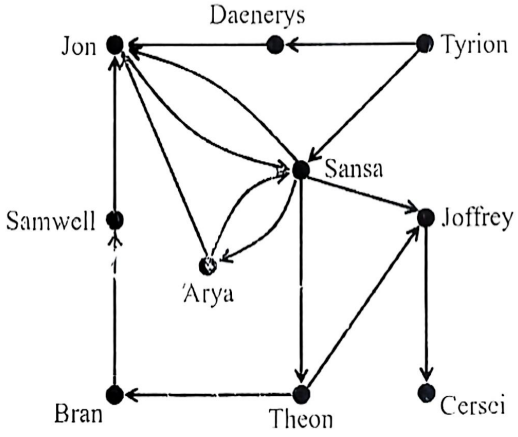
   (b) Determine whether the following two graphs are isomorphic or not. [3]



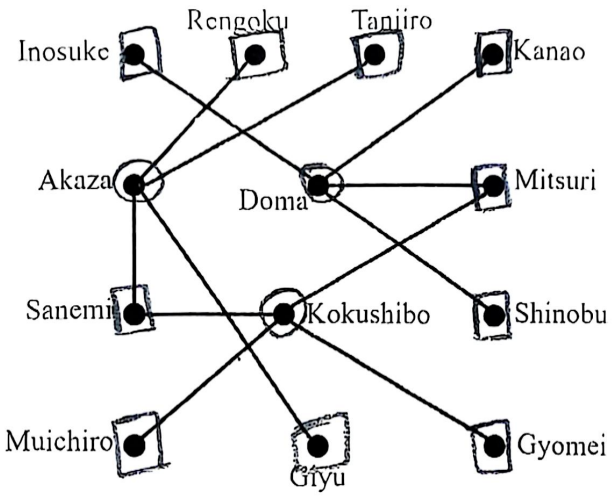
   (c) Consider the following adjacency matrix for a directed graph with 5 vertices. Draw the graph from the adjacency matrix. Find the in-degree and out-degree of each vertex. [3]

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

4. (a) Find out the strongly connected components of the following graph. [3]



(b) Find out if the graph in the following figure is bipartite or not, using two coloring algorithm. If the graph is bipartite, show its bipartite form. [4]



(c) A graph has four vertices (namely A, B, C and D), and the directed edges of this graph are $A \to B, B \to C$ and $C \to D$. Represent the graph using an adjacency matrix. [2]

5. (a) A Discrete Mathematics section of UIU consists of students of 231, 233, 241, 242 and 243 batches. What is the minimum number of students in the class if there are at least 9 students from some batch? [2]

(b) How many strings of uppercase English letters of length 5 can you generate if letters cannot be repeated, and there can neither be two consecutive vowels nor two consecutive consonants? [4]

(c) On the occasion of the grand finale of the Quiddich Cup, 15 students from Hogwarts School of Witchcraft and Wizardry are going to perform a choir performance. The performers must stand in a single line so that a female performer must always be followed by a male performer. If there are 9 male performers and 6 female performers, how many ways can the choir be arranged? [3]