United International University

School of Science and Engineering

Final Examination Trimester: Fall-2024

Course Title: Advanced Calculus

Course Code: Math 1153 Marks: 40 Time: 2 Hours

Answer all the questions. Answer all parts of a question together.

Q1. (a) Write 4 differential equations involving the derivatives about time that have significance in the practical applications. [2]

(b) Show that $u(x,t) = e^{-t}\sin 2x$ is a solution of the equation $u_{xx} = 4u_t$. [2]

(c) If $(axy - \sin x)dx + (x^2 - \cos y)dy = 0$ is an exact differential equation find the value of a and hence solve it. [3]

Q2. (a) Solve $2xy\frac{dy}{dx} + y^2 = x^2$. [5]

(b) Use the method of reduction of order to find a second solution of $x^2y'' - xy' + y = 0$ with the stated first solution $y_1 = x$. [5]

Q3. (a) If a fossilized bone is found to disintegrate **a quarter** of the original amount of **C-14**. If the half-life time of **C-14** is **5730 years**, determine the age of the fossil. [5]

(b) A mass weighing **16 lb** stretches a spring **6 in**. Suppose that the mass is displaced an additional **4 in**. in the positive direction and then **released**. The mass is in a medium that exerts a viscous resistance of **10 lb** when the mass has a velocity of **5 ft/s**. Formulate the initial value problem that governs the motion of the mass and determines the position of the mass at any later time. [5]

Q4. (a) Solve the following Heat equation. [6]

$$9u_{xx} = u_t\,;\, 0 < x < 10, t > 0$$
$$u(0,t) = 0 = u_x(10,t)\,;\, t > 0$$
$$u(x,0) = x\,;\, 0 < x < 10$$

(b) Solve the following Wave equation. [7]

$$16u_{xx} = u_{tt}\,;\, 0 < x < 5, t > 0$$
$$u(0,t) = 0 = u(5,t)\,;\, t > 0$$
$$u(x,0) = 10 - x, u_t(x,0) = 0\,;\, 0 < x < 5$$