

**Assignment/Homework 10% of your final grade**  
**Due 18th November 2024 23:59pm**

**Test Instructions:**

**Format** This test includes a mix of question types: Questions in part (a) require manual calculations and handwritten solutions. Questions in part (b) require you to use the R software.

**Submission Requirements:** Handwritten Solutions: For questions requiring calculations, write your solutions clearly on paper. Ensure all steps are shown, as partial credit may be awarded based on your working process. R Script: For questions that require R, create an R script file or R mark-down file that includes all relevant code. Please include comments in your script to explain your steps and reasoning. This file should be well-organized and easy to follow.

**Submission Format:** Scan or take clear photos of your handwritten solutions and save them as a single PDF file. Submit both the PDF file (for handwritten solutions) and the R script (.R file) as part of your final submission.

**Important Notes:**

- **Ensure Accuracy:** Double-check that all calculations, both handwritten and in R, are accurate and complete.
- **File Naming:** Name your files in the following format: '[YourName]\_Handwritten.pdf' and '[YourName]\_RScript.R'.
- **Deadline:** Submit both files via Brightspace by 18th November 2024 at 23:59pm. Late submissions will incur a penalty. Submissions made within 10 minutes of the deadline will not be penalized, allowing for any technical issues. Each day after the deadline will result in a 10% deduction from the total marks. Extensions may be granted under exceptional circumstances, such as documented medical issues. If you believe you have a valid reason for a late submission, please contact michelle.carey@ucd.ie.

**Academic Integrity:** This test is individual. Ensure all work is your own and cite any sources or references as appropriate. Each student must submit an academic integrity declaration with each assignment. This declaration should affirm that they have not committed any acts of plagiarism. For this module, plagiarism includes, but is not limited to, copying from another student, copying from a website, using web-based tools, generative artificial intelligence tools, web-based problem solvers, or web-based code repositories. Please refer to the School's Academic Integrity Protocol for 2024/25 on Brightspace for further details.

**Part (a)**

1. Two machines make identical parts which are combined on a production line. The older machine makes 40% of the units, of which 85% are of satisfactory quality. The newer machine makes 60% of the units, of which 90% are satisfactory. A random check further down the production line shows an unusual fault which suggests the machine which made the unit needs some adjustment. Which machine is more likely to have made the unit? [4]
2. An insurance company classifies its policies as grade  $B_1$ , grade  $B_2$  or grade  $B_3$ . Of all policies issued 30% are grade  $B_1$ , 60% grade  $B_2$  and 10% grade  $B_3$ . One month after the premiums are due 8% of grade  $B_1$  policies remain unpaid, 3% of grade  $B_2$  policies are unpaid and 15% of grade  $B_3$  policies are unpaid. Find the probability that:
  - (a) a randomly selected policy is unpaid; [4]
  - (b) a policy selected at random and found to be unpaid is grade  $B_1$ , grade  $B_2$ , grade  $B_3$ . [6]
  - (c) If unpaid policies are processed at a cost of €0.70 for a grade  $B_1$  policy, €1.00 for a grade  $B_2$  policy and €1.50 for a grade  $B_3$  policy, find the expected cost of processing an unpaid policy [6]
3. According to recent mortality tables, the probability an American citizen will die in his twentieth year is 0.00178. Assume that a life insurance company will sell a €1000 one-year term policy to a 19-year-old for €10 (This means the company will pay the beneficiaries €1000 if the person dies during his twentieth year; if the person does not die, the company retains the €10).
  - (a) What is the expected gain to the insurance company from selling one such policy, ignoring expenses of selling and administration? [5]
  - (b) What is the standard deviation of the gain? [5]
4. Let  $X$  be an exponential random variable with mean  $1/\lambda$  and define  $Y = \lfloor X \rfloor$ , the largest integer in  $X$  (that is the largest integer less than or equal to  $X$ ).
  - (a) Find the probability function for the discrete variable  $Y$ . [6]
  - (b) Show that  $W \sim \text{geo}(p)$  where  $W = Y + 1$ , and evaluate  $p$  in terms of  $\lambda$ . [8]
5. A system with two independent components works correctly if at least one component is functioning properly. Failure rates (failures / hr) of the individual components are  $\lambda_1 = 0.01$ , and  $\lambda_2 = 0.02$ . Assuming exponentially distributed lifetimes:

- (a) determine the pdf of the lifetime  $X$  of the system, and find the expected lifetime  $E(X)$ ; [10]
- (b) determine the probability that the system will work for 250 hours. [10]

**Hint:** Remember  $\int_0^\infty xe^{-\lambda x} dx = 1/\lambda^2$ .

**Part (b)**

- The Irish Aviation Authority (IAA) is a support organization for the principal Irish airlines. Some of the IAA's activities include promoting the air transport industry and conducting industry-wide studies. The IAA also keeps statistics on commercial airline flights, including those involving accidents. From 1979 through 2008, for aircraft with 10 or more seats, there were 76 fatal commercial airplane accidents involving Irish airlines.

Year	1979	1980	1981	1982	1983	1984	1985	1986
Accidents	4	0	1	1	1	3	1	1
Year	1987	1988	1989	1990	1991	1992	1993	1994
Accidents	2	1	6	0	3	2	0	1
Year	1995	1996	1997	1998	1999	2000	2001	2002
Accidents	5	2	1	6	0	2	2	2
Year	2003	2004	2005	2006	2007	2008		
Accidents	3	3	2	3	2	0		

- (a) In 2006, there were about 11 million commercial flights in the Ireland. If one is selected at random, what is the probability that it involved a fatal accident? [4]
- (b) Suppose that the probability of a fatal accident in a given year is 0.00000001. Using R calculate the binomial probability distribution for  $n = 11,000,000$  and  $p = 0.00000001$  with  $x = 0$  to 12. [8]
- (c) Using the information from part (b) answer the following. What is the probability that there will be
- 4 fatal accidents in a year? [2]
  - 10 fatal accidents? [2]
  - between 1 and 5, inclusive? [2]
- (d) Construct a binomial distribution for  $n = 11,000,000$  and  $p = 0.00000001$  with  $x = 0$  to 12. Compare your results with the distribution in part (c). [4]
- (e) Is a binomial distribution a good model for determining the probabilities of various numbers of fatal accidents during a year? Explain your reasoning and include a discussion of the four criteria for a binomial experiment. [8]

- (f) According to analysis by the **Irish Times**, air flight is so safe that a person “would have to fly every day for more than 64,000 years before dying in an accident.” How can such a statement be justified?  
[6]