Oscillons

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1 Introduction

We start by examining the models described by Amin et al[1] where they discuss flat top oscillon solutions which are spatially localised and are long lived in time. With the intuitive reasoning from Sfakianakis[2]. We begin by understanding oscillons in 1+1D, i.e one spatial and one time dimension.

1.1 Oscillon model in 1+1D

We define the lagrangian density for a scalar field, in analogous to ref[2]

$$\mathcal{L} = \frac{1}{2}((\partial_t \phi)^2 - (\partial_x \phi)^2) - V(\phi) \tag{1}$$

with

$$V = \frac{1}{2}\phi^2 - \frac{1}{4}\phi^4 + \frac{\Lambda}{6\epsilon^2}\phi^6$$

the ϵ parameter is a small (constant) number which we will encounter later. Here it just means that the strength of the ϕ^6 term is somehow dependant on the ratio between the Λ parameter and the ϵ parameter. The Λ parameter is proportional to the sixth order coupling strength, which is usually denoted by g. The equation of motion for the field becomes:

$$\frac{d^2\phi}{dt^2} - \frac{d^2\phi}{dx^2} + \phi - \phi^3 + \frac{\Lambda}{\epsilon}\phi^5 = 0 \tag{2}$$

with the change of variables from t to τ given by $t = \epsilon^2\tau$ and from x to ρ given by $x = \epsilon\rho$ and also noting that the oscillons are oscillating only in time and are localized in space, we can write eqn(2) as:

$$\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial t}\right) = \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial \tau}\frac{\partial \tau}{\partial t}\right)\left(\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial \tau}\epsilon^2\right) = \frac{\partial^2\phi}{\partial t^2} + 2\epsilon^2\frac{\partial^2\phi}{\partial\tau\partial t} + \frac{\partial^2\phi}{\partial\tau^2}\epsilon^4 \tag{3}$$

with these the equation of motion becomes:

$$\frac{\partial^2\phi}{\partial t^2} + 2\epsilon^2\frac{\partial^2\phi}{\partial\tau\partial t} + \epsilon^4\frac{\partial^2\phi}{\partial\tau^2} - \epsilon^2\frac{\partial^2}{\partial\rho^2} + \phi - \epsilon^2\phi^3 + \Lambda\epsilon^2\phi^5 \tag{4}$$

to this equation, we add the most general solution in orders of ϵ of the form:

$$\phi = \phi_0 + \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \dots \tag{5}$$

we get the corresponding equations of motion as:

$$\frac{\partial^2\phi_0}{\partial t^2} + \epsilon\frac{\partial^2\phi_1}{\partial t^2} + \epsilon^2\frac{\partial^2\phi_2}{\partial t^2} + 2\epsilon^2\frac{\partial^2\phi_0}{\partial\tau\partial t} - \epsilon^2\frac{\partial^2\phi_0}{\partial\rho^2} + \phi_0 + \epsilon\phi_1 + \epsilon^2\phi_2 + \Lambda\epsilon^2\phi_0^5 + \mathcal{O}(\epsilon^3) = 0 \tag{6}$$

counting in powers of ϵ we have for $\mathcal{O}(1)$:

$$\frac{\partial^2\phi_0}{\partial t^2} + \phi_0 = 0 \tag{7}$$

at $\mathcal{O}(\epsilon)$, we have:

$$\frac{\partial^2\phi_1}{\partial t^2} + \phi_1 = 0 \tag{8}$$

for $\mathcal{O}(\epsilon)$, we then get:

$$\frac{\partial^2\phi_2}{\partial t^2} + 2\frac{\partial^2\phi_0}{\partial t\partial\tau} - \frac{\partial^2\phi_0}{\partial\rho^2} + \phi_2 - \phi_0^3 + \Lambda\phi_0^5 = 0 \tag{9}$$

The general solution for ϕ_0 from eqn(7) is of the form:

$$\phi_0 = \frac{1}{2}(Ae^{-it} + A^*e^{it}) \tag{10}$$

If we substitute this into eqn(9) and taking only the decaying mode solutions, we get:

$$\frac{A_{\rho\rho}}{2} + iA_\tau + \frac{3}{8}|A|^2A = 0 \tag{11}$$

and with the ansatz $A = a(\rho)e^{i\tau/2}$ we get:

$$a_{\rho\rho} - a + \frac{3}{4}a^3 = 0 \tag{12}$$

References

[1] Mustafa A. Amin and David Shirokoff *Flat-top oscillons in an expanding universe.* 2010.

[2] Evangelos I. Sfakianakis *Analysis of Oscillons in the SU (2) Gauged Higgs Model* 2012