Dimension_Reduction.R

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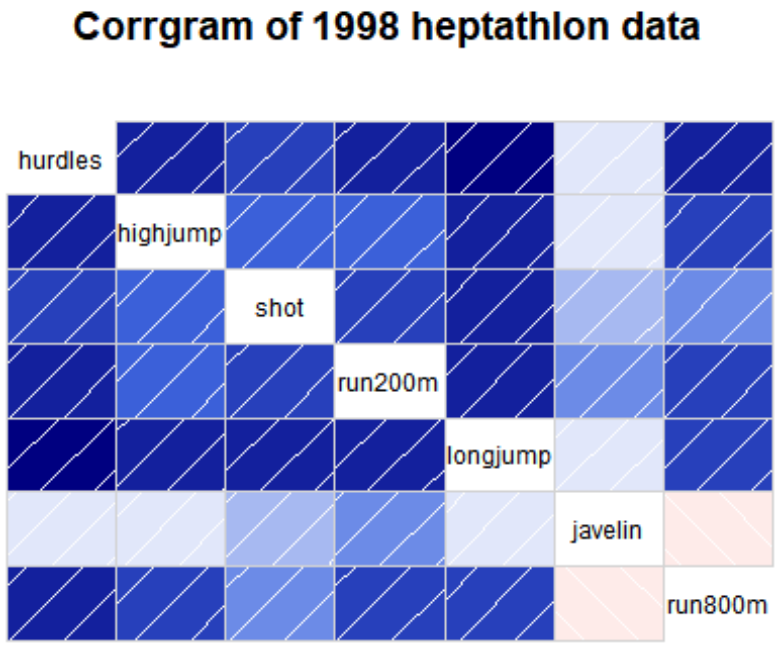
```
hep <- read.csv("C:/Users/Fiona/Desktop/Business Analytics and Decision
Sciences/Forecasting and Business Analytics/FABA-L4-Notes/hep88.csv",
header=TRUE)
hep$hurdles <- max(hep$hurdles) -  hep$hurdles
hep$run200m <- max(hep$run200m) -  hep$run200m
hep$run800m <- max(hep$run800m) - hep$run800m

# cor matrix - rounded to 3DP
round(cor(hep[2:8]), 3)
```

```
##          hurdles highjump  shot run200m longjump javelin run800m
## hurdles    1.000    0.811 0.651   0.774    0.912   0.008   0.779
## highjump   0.811    1.000 0.441   0.488    0.782   0.002   0.591
## shot       0.651    0.441 1.000   0.683    0.743   0.269   0.420
## run200m    0.774    0.488 0.683   1.000    0.817   0.333   0.617
## longjump   0.912    0.782 0.743   0.817    1.000   0.067   0.700
## javelin    0.008    0.002 0.269   0.333    0.067   1.000  -0.020
## run800m    0.779    0.591 0.420   0.617    0.700  -0.020   1.000
```

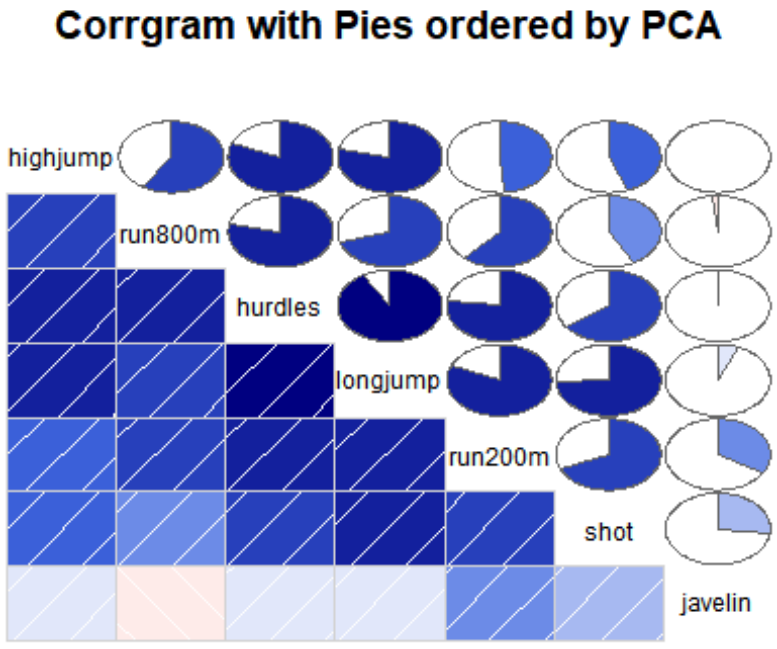
```
# get corrgram plots
#install.packages("corrgram")
library("corrgram")
```

```
## Warning: package 'corrgram' was built under R version 3.3.3
```

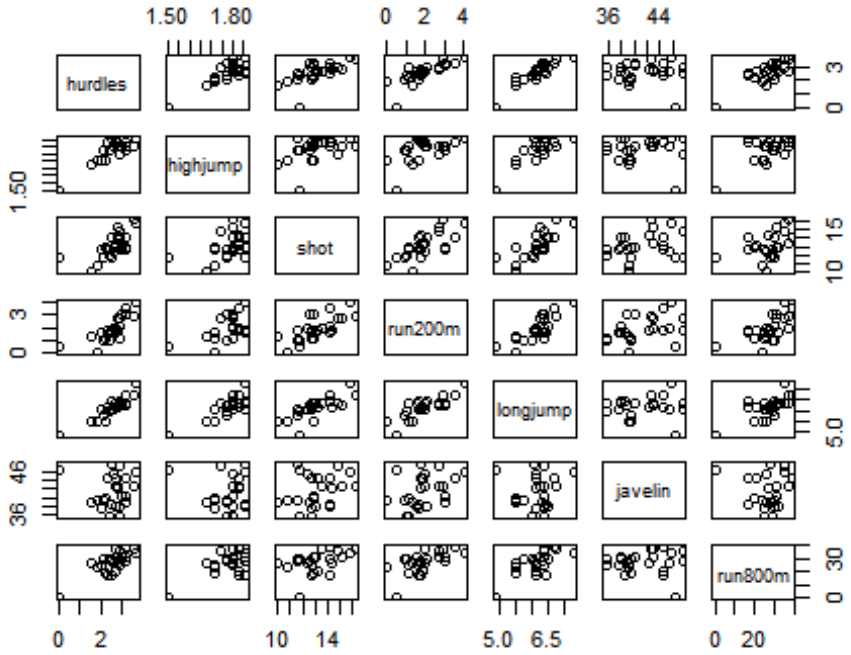
```
corrgram(hep[2:8], main="Corrgram of 1998 heptathlon data")
```




```
corrgram(hep[2:8], order=TRUE, upper.panel=panel.pie, main="Corrgram with
Pies ordered by PCA")
```



```r
# pairs plot
pairs(hep[2:8])
```




```r
# PCA & scale to unit variance
heppca <- prcomp(hep[2:8], scale = TRUE)
summary(heppca)
```

```
## Importance of components:
##                           PC1    PC2     PC3     PC4     PC5     PC6
## Standard deviation     2.1119 1.0928 0.72181 0.67614 0.49524 0.27010
## Proportion of Variance 0.6372 0.1706 0.07443 0.06531 0.03504 0.01042
## Cumulative Proportion  0.6372 0.8078 0.88223 0.94754 0.98258 0.99300
##                           PC7
## Standard deviation     0.2214
## Proportion of Variance 0.0070
## Cumulative Proportion  1.0000
```

```r
heppca
```

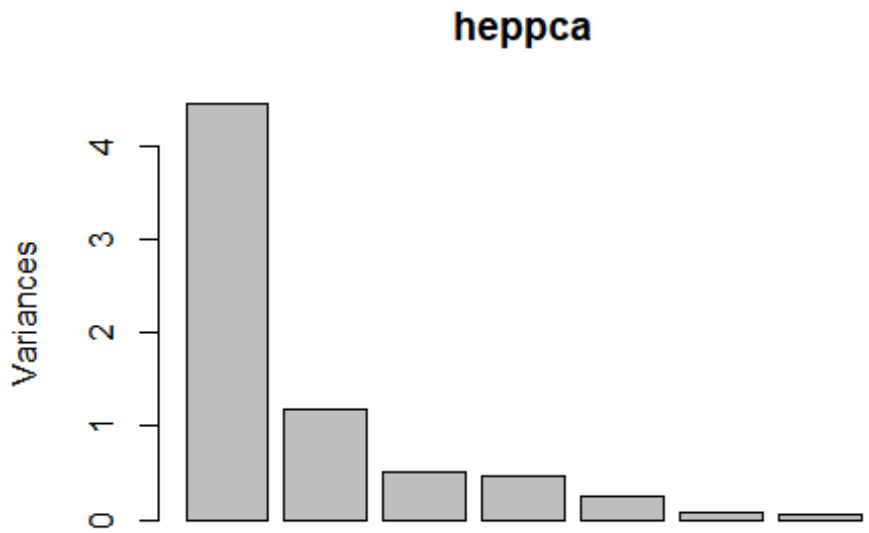
```
## Standard deviations:
## [1] 2.1119364 1.0928497 0.7218131 0.6761411 0.4952441 0.2701029 0.2213617
##
## Rotation:
##                  PC1          PC2          PC3          PC4          PC5
## hurdles  -0.4528710  0.15792058 -0.04514996  0.02653873 -0.09494792
## highjump -0.3771992  0.24807386 -0.36777902  0.67999172  0.01879888
## shot     -0.3630725 -0.28940743  0.67618919  0.12431725  0.51165201
```

```
## run200m  -0.4078950 -0.26038545  0.08359211 -0.36106580 -0.64983404
## longjump -0.4562318  0.05587394  0.13931653  0.11129249 -0.18429810
## javelin  -0.0754090 -0.84169212 -0.47156016  0.12079924  0.13510669
## run800m  -0.3749594  0.22448984 -0.39585671 -0.60341130  0.50432116
##                  PC6         PC7
## hurdles  -0.78334101  0.38024707
## highjump  0.09939981 -0.43393114
## shot     -0.05085983 -0.21762491
## run200m   0.02495639 -0.45338483
## longjump  0.59020972  0.61206388
## javelin  -0.02724076  0.17294667
## run800m   0.15555520 -0.09830963
```

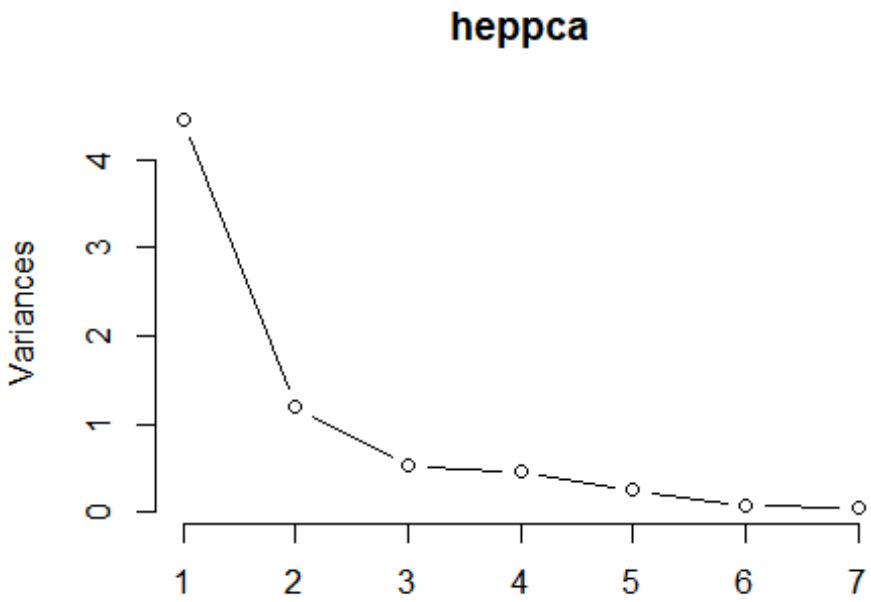
```
# get eiganvalues using SD2
heppca$sdev ^ 2
```

```
## [1] 4.46027516 1.19432056 0.52101413 0.45716683 0.24526674 0.07295558
## [7] 0.04900101
```

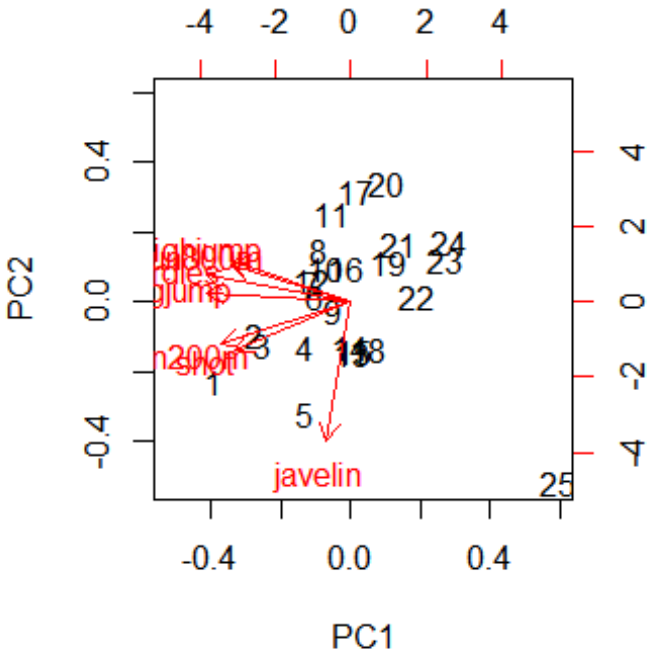
```
# scree plot (barplot)
plot(heppca)
```




```
# normal scree plot (line)
plot(heppca, type="line")
```




```r
# PCA biplot
biplot(heppca)
```

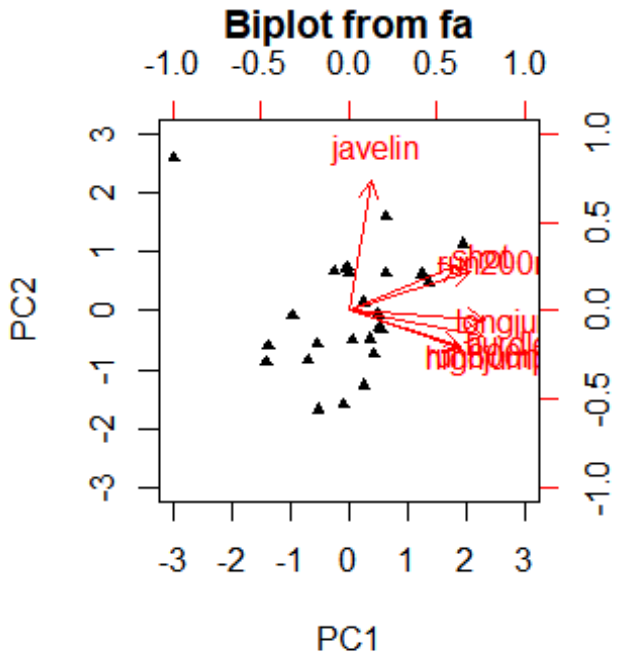


```
# Get scores
heppca$x
```

```
##                PC1         PC2         PC3         PC4         PC5
##  [1,] -4.121447626 -1.24240435  0.36991309  0.02300174 -0.42600624
##  [2,] -2.882185935 -0.52372600  0.89741472 -0.47545176  0.70306588
##  [3,] -2.649633766 -0.67876243 -0.45917668 -0.67962860 -0.10552518
##  [4,] -1.343351210 -0.69228324  0.59527044 -0.14067052  0.45392816
##  [5,] -1.359025696 -1.75316563 -0.15070126 -0.83595001  0.68719483
##  [6,] -1.043847471  0.07940725 -0.67453049 -0.20557253  0.73793351
##  [7,] -1.100385639  0.32375304 -0.07343168 -0.48627848 -0.76299122
##  [8,] -0.923173639  0.80681365  0.81241866 -0.03022915  0.09086737
##  [9,] -0.530250689 -0.14632191  0.16122744  0.61590242  0.56851477
## [10,] -0.759819024  0.52601568  0.18316881 -0.66756426 -1.02148109
## [11,] -0.556268302  1.39628179 -0.13619463  0.40503603  0.29221101
## [12,] -1.186453832  0.35376586 -0.08201243 -0.48123479 -0.78103608
## [13,]  0.015461226 -0.80644305 -1.96745373  0.73341733 -0.02177427
## [14,]  0.003774223 -0.71479785 -0.32496780  1.06604134 -0.18389959
## [15,]  0.090747709 -0.76304501 -0.94571404  0.26883477 -0.18416945
## [16,] -0.137225440  0.53724054  1.06529469  1.63144151 -0.21162048
## [17,]  0.171128651  1.74319472  0.58701048  0.47103131 -0.05781435
## [18,]  0.519252646 -0.72696476 -0.31302308  1.28942720 -0.49779301
## [19,]  1.125481833  0.63479040  0.72530080 -0.57961844 -0.15611502
## [20,]  1.085697646  1.84722368  0.01452749 -0.25561691  0.19143514
## [21,]  1.447055499  0.92446876 -0.64596313 -0.21493997  0.49993839
## [22,]  2.014029620  0.09304121  0.64802905  0.02454548  0.24445870
## [23,]  2.880298635  0.66150588 -0.74936718 -1.11903480 -0.47418755
## [24,]  2.970118607  0.95961101 -0.57118753 -0.11547402  0.58055249
## [25,]  6.270021972 -2.83919926  1.03414797 -0.24141489 -0.16568672
##                PC6          PC7
##  [1,]  0.339329222  0.347921325
##  [2,] -0.238087298  0.144015774
##  [3,]  0.239190707 -0.129647756
##  [4,] -0.091805638 -0.486577968
##  [5,] -0.126303968  0.239482044
##  [6,]  0.355789386 -0.103414314
##  [7,] -0.084844490 -0.142871612
##  [8,]  0.151561253  0.034237928
##  [9,] -0.265359696 -0.249591589
## [10,] -0.396397714 -0.020405097
## [11,]  0.344582964 -0.182701990
## [12,] -0.233718538 -0.070605615
## [13,]  0.004249913  0.036155878
## [14,] -0.272903729  0.044351160
## [15,] -0.141403697  0.135136482
## [16,]  0.280043639 -0.171160984
## [17,] -0.147155606  0.520000710
## [18,]  0.071211150 -0.005529394
## [19,]  0.427484048  0.081522940
## [20,]  0.100087033  0.085430091
```

```
## [21,]  0.072673266 -0.125585203
## [22,] -0.640572055 -0.215626046
## [23,]  0.180568513 -0.207364881
## [24,] -0.183940799  0.381783751
## [25,]  0.255722133  0.061044365

# using the principal functiuon in the psych package
#install.packages("psych")
#install.packages("GPArotation")
library(psych)

## Warning: package 'psych' was built under R version 3.3.3

library(GPArotation)
summary(prcomp(hep[2:8], scale = TRUE))

## Importance of components:
##                           PC1    PC2     PC3     PC4     PC5     PC6
## Standard deviation     2.1119 1.0928 0.72181 0.67614 0.49524 0.27010
## Proportion of Variance 0.6372 0.1706 0.07443 0.06531 0.03504 0.01042
## Cumulative Proportion  0.6372 0.8078 0.88223 0.94754 0.98258 0.99300
##                           PC7
## Standard deviation     0.2214
## Proportion of Variance 0.0070
## Cumulative Proportion  1.0000

heppca2 <- principal(hep[2:8], nfactors = 2, rotate="none")
heppca2 # see the output is the same as usuing the prcomp function

## Principal Components Analysis
## Call: principal(r = hep[2:8], nfactors = 2, rotate = "none")
## Standardized loadings (pattern matrix) based upon correlation matrix
##           PC1   PC2   h2    u2 com
## hurdles  0.96 -0.17 0.94 0.055 1.1
## highjump 0.80 -0.27 0.71 0.292 1.2
## shot     0.77  0.32 0.69 0.312 1.3
## run200m  0.86  0.28 0.82 0.177 1.2
## longjump 0.96 -0.06 0.93 0.068 1.0
## javelin  0.16  0.92 0.87 0.129 1.1
## run800m  0.79 -0.25 0.69 0.313 1.2
##
##                        PC1  PC2
## SS loadings           4.46 1.19
## Proportion Var        0.64 0.17
## Cumulative Var        0.64 0.81
## Proportion Explained  0.79 0.21
## Cumulative Proportion 0.79 1.00
##
## Mean item complexity =  1.2
## Test of the hypothesis that 2 components are sufficient.
##
```

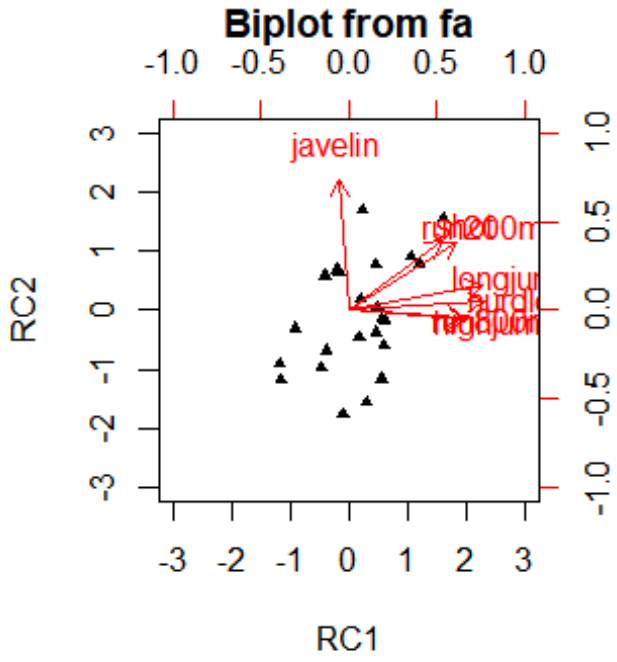
```
## The root mean square of the residuals (RMSR) is  0.07
##  with the empirical chi square  5.31  with prob <  0.72
##
## Fit based upon off diagonal values = 0.99

# h2 = the amount of variance in each variable explained by the components.
# u2 = component uniqueness, i.e. the amount of variance not accounted for by
the components.

# get biplot
biplot(heppca2)
```




```
# Using varimax to get components
principal(hep[2:8], nfactors = 2, rotate="varimax")

## Principal Components Analysis
## Call: principal(r = hep[2:8], nfactors = 2, rotate = "varimax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##             RC1   RC2   h2    u2 com
## hurdles    0.97  0.06 0.94 0.055 1.0
## highjump   0.84 -0.07 0.71 0.292 1.0
## shot       0.67  0.49 0.69 0.312 1.8
## run200m    0.77  0.48 0.82 0.177 1.7
## longjump   0.95  0.17 0.93 0.068 1.1
## javelin   -0.07  0.93 0.87 0.129 1.0
## run800m    0.83 -0.05 0.69 0.313 1.0
##
```

```
##                         RC1  RC2
## SS loadings            4.27 1.39
## Proportion Var         0.61 0.20
## Cumulative Var         0.61 0.81
## Proportion Explained   0.76 0.24
## Cumulative Proportion  0.76 1.00
##
## Mean item complexity =  1.2
## Test of the hypothesis that 2 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.07
##  with the empirical chi square  5.31  with prob <  0.72
##
## Fit based upon off diagonal values = 0.99
```

```
biplot(principal(hep[2:8], nfactors = 2, rotate="varimax"))
```

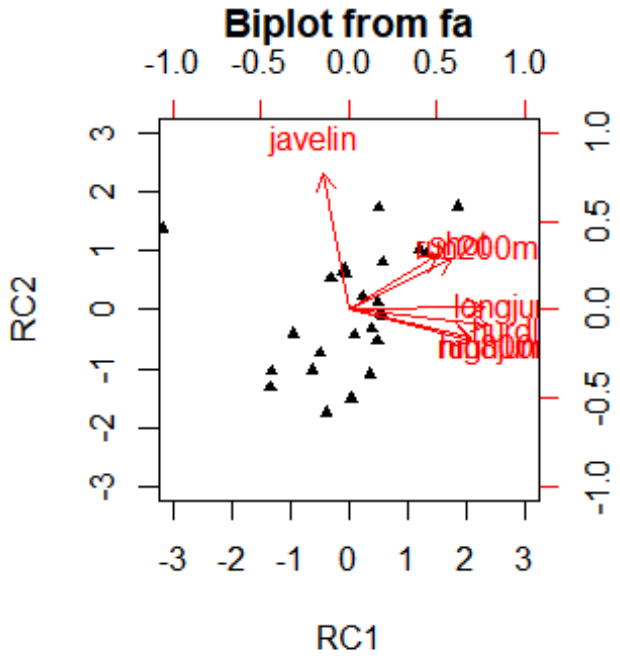



```
# Using promax to get components
principal(hep[2:8], nfactors = 2, rotate="promax")
```

```
## Principal Components Analysis
## Call: principal(r = hep[2:8], nfactors = 2, rotate = "promax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##            RC1   RC2   h2    u2 com
## hurdles   1.00 -0.10 0.94 0.055 1.0
## highjump  0.88 -0.22 0.71 0.292 1.1
## shot      0.63  0.39 0.69 0.312 1.7
```

```
## run200m   0.73  0.37 0.82 0.177 1.5
## longjump  0.96  0.02 0.93 0.068 1.0
## javelin  -0.19  0.97 0.87 0.129 1.1
## run800m   0.86 -0.19 0.69 0.313 1.1
##
##                        RC1  RC2
## SS loadings           4.37 1.29
## Proportion Var        0.62 0.18
## Cumulative Var        0.62 0.81
## Proportion Explained  0.77 0.23
## Cumulative Proportion 0.77 1.00
##
##  With component correlations of
##      RC1  RC2
## RC1 1.00 0.28
## RC2 0.28 1.00
##
## Mean item complexity =  1.2
## Test of the hypothesis that 2 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.07
##  with the empirical chi square  5.31  with prob <  0.72
##
## Fit based upon off diagonal values = 0.99
```

```
biplot(principal(hep[2:8], nfactors = 2, rotate="promax"))
```



```
# Get scores
heppca2$scores
```

```
##                PC1         PC2
##  [1,]  1.951501816  1.13684829
##  [2,]  1.364712498  0.47922965
##  [3,]  1.254599251  0.62109401
##  [4,]  0.636075613  0.63346607
##  [5,]  0.643497468  1.60421471
##  [6,]  0.494260857 -0.07266072
##  [7,]  0.521031629 -0.29624661
##  [8,]  0.437121904 -0.73826586
##  [9,]  0.251073233  0.13389024
## [10,]  0.359773638 -0.48132480
## [11,]  0.263392550 -1.27765212
## [12,]  0.561784843 -0.32370952
## [13,] -0.007320877  0.73792674
## [14,] -0.001787091  0.65406782
## [15,] -0.042968960  0.69821585
## [16,]  0.064976125 -0.49159598
## [17,] -0.081029266 -1.59509094
## [18,] -0.245865670  0.66520101
## [19,] -0.532914655 -0.58085789
## [20,] -0.514076878 -1.69028148
## [21,] -0.685179502 -0.84592486
## [22,] -0.953641249 -0.08513632
## [23,] -1.363818863 -0.60530360
## [24,] -1.406348540 -0.87808138
## [25,] -2.968849873  2.59797770
```

```
# show covariance data
Harman74.cor
```

```
## $cov
##                       VisualPerception Cubes PaperFormBoard Flags
## VisualPerception                 1.000 0.318          0.403 0.468
## Cubes                            0.318 1.000          0.317 0.230
## PaperFormBoard                   0.403 0.317          1.000 0.305
## Flags                            0.468 0.230          0.305 1.000
## GeneralInformation               0.321 0.285          0.247 0.227
## PargraphComprehension            0.335 0.234          0.268 0.327
## SentenceCompletion               0.304 0.157          0.223 0.335
## WordClassification               0.332 0.157          0.382 0.391
## WordMeaning                      0.326 0.195          0.184 0.325
## Addition                         0.116 0.057         -0.075 0.099
## Code                             0.308 0.150          0.091 0.110
## CountingDots                     0.314 0.145          0.140 0.160
## StraightCurvedCapitals           0.489 0.239          0.321 0.327
## WordRecognition                  0.125 0.103          0.177 0.066
## NumberRecognition                0.238 0.131          0.065 0.127
```

```
## FigureRecognition                0.414 0.272          0.263 0.322
## ObjectNumber                     0.176 0.005          0.177 0.187
## NumberFigure                     0.368 0.255          0.211 0.251
## FigureWord                       0.270 0.112          0.312 0.137
## Deduction                        0.365 0.292          0.297 0.339
## NumericalPuzzles                 0.369 0.306          0.165 0.349
## ProblemReasoning                 0.413 0.232          0.250 0.380
## SeriesCompletion                 0.474 0.348          0.383 0.335
## ArithmeticProblems               0.282 0.211          0.203 0.248
##                       GeneralInformation PargraphComprehension
## VisualPerception                   0.321                 0.335
## Cubes                              0.285                 0.234
## PaperFormBoard                     0.247                 0.268
## Flags                              0.227                 0.327
## GeneralInformation                 1.000                 0.622
## PargraphComprehension              0.622                 1.000
## SentenceCompletion                 0.656                 0.722
## WordClassification                 0.578                 0.527
## WordMeaning                        0.723                 0.714
## Addition                           0.311                 0.203
## Code                               0.344                 0.353
## CountingDots                       0.215                 0.095
## StraightCurvedCapitals             0.344                 0.309
## WordRecognition                    0.280                 0.292
## NumberRecognition                  0.229                 0.251
## FigureRecognition                  0.187                 0.291
## ObjectNumber                       0.208                 0.273
## NumberFigure                       0.263                 0.167
## FigureWord                         0.190                 0.251
## Deduction                          0.398                 0.435
## NumericalPuzzles                   0.318                 0.263
## ProblemReasoning                   0.441                 0.386
## SeriesCompletion                   0.435                 0.431
## ArithmeticProblems                 0.420                 0.433
##                       SentenceCompletion WordClassification WordMeaning
## VisualPerception                   0.304              0.332       0.326
## Cubes                              0.157              0.157       0.195
## PaperFormBoard                     0.223              0.382       0.184
## Flags                              0.335              0.391       0.325
## GeneralInformation                 0.656              0.578       0.723
## PargraphComprehension              0.722              0.527       0.714
## SentenceCompletion                 1.000              0.619       0.685
## WordClassification                 0.619              1.000       0.532
## WordMeaning                        0.685              0.532       1.000
## Addition                           0.246              0.285       0.170
## Code                               0.232              0.300       0.280
## CountingDots                       0.181              0.271       0.113
## StraightCurvedCapitals             0.345              0.395       0.280
## WordRecognition                    0.236              0.252       0.260
## NumberRecognition                  0.172              0.175       0.248
```

```
## FigureRecognition                0.180             0.296       0.242
## ObjectNumber                     0.228             0.255       0.274
## NumberFigure                     0.159             0.250       0.208
## FigureWord                       0.226             0.274       0.274
## Deduction                        0.451             0.427       0.446
## NumericalPuzzles                 0.314             0.362       0.266
## ProblemReasoning                 0.396             0.357       0.483
## SeriesCompletion                 0.405             0.501       0.504
## ArithmeticProblems               0.437             0.388       0.424
##                          Addition  Code CountingDots StraightCurvedCapitals
## VisualPerception            0.116 0.308        0.314                  0.489
## Cubes                       0.057 0.150        0.145                  0.239
## PaperFormBoard             -0.075 0.091        0.140                  0.321
## Flags                       0.099 0.110        0.160                  0.327
## GeneralInformation          0.311 0.344        0.215                  0.344
## PargraphComprehension       0.203 0.353        0.095                  0.309
## SentenceCompletion          0.246 0.232        0.181                  0.345
## WordClassification          0.285 0.300        0.271                  0.395
## WordMeaning                 0.170 0.280        0.113                  0.280
## Addition                    1.000 0.484        0.585                  0.408
## Code                        0.484 1.000        0.428                  0.535
## CountingDots                0.585 0.428        1.000                  0.512
## StraightCurvedCapitals      0.408 0.535        0.512                  1.000
## WordRecognition             0.172 0.350        0.131                  0.195
## NumberRecognition           0.154 0.240        0.173                  0.139
## FigureRecognition           0.124 0.314        0.119                  0.281
## ObjectNumber                0.289 0.362        0.278                  0.194
## NumberFigure                0.317 0.350        0.349                  0.323
## FigureWord                  0.190 0.290        0.110                  0.263
## Deduction                   0.173 0.202        0.246                  0.241
## NumericalPuzzles            0.405 0.399        0.355                  0.425
## ProblemReasoning            0.160 0.304        0.193                  0.279
## SeriesCompletion            0.262 0.251        0.350                  0.382
## ArithmeticProblems          0.531 0.412        0.414                  0.358
##                          WordRecognition NumberRecognition FigureRecognition
## VisualPerception                   0.125             0.238             0.414
## Cubes                              0.103             0.131             0.272
## PaperFormBoard                     0.177             0.065             0.263
## Flags                              0.066             0.127             0.322
## GeneralInformation                 0.280             0.229             0.187
## PargraphComprehension              0.292             0.251             0.291
## SentenceCompletion                 0.236             0.172             0.180
## WordClassification                 0.252             0.175             0.296
## WordMeaning                        0.260             0.248             0.242
## Addition                           0.172             0.154             0.124
## Code                               0.350             0.240             0.314
## CountingDots                       0.131             0.173             0.119
## StraightCurvedCapitals             0.195             0.139             0.281
## WordRecognition                    1.000             0.370             0.412
## NumberRecognition                  0.370             1.000             0.325
```

```
## FigureRecognition                0.412             0.325             1.000
## ObjectNumber                     0.341             0.345             0.324
## NumberFigure                     0.201             0.334             0.344
## FigureWord                       0.206             0.192             0.258
## Deduction                        0.302             0.272             0.388
## NumericalPuzzles                 0.183             0.232             0.348
## ProblemReasoning                 0.243             0.246             0.283
## SeriesCompletion                 0.242             0.256             0.360
## ArithmeticProblems               0.304             0.165             0.262
##                         ObjectNumber NumberFigure FigureWord Deduction
## VisualPerception               0.176        0.368      0.270     0.365
## Cubes                          0.005        0.255      0.112     0.292
## PaperFormBoard                 0.177        0.211      0.312     0.297
## Flags                          0.187        0.251      0.137     0.339
## GeneralInformation             0.208        0.263      0.190     0.398
## PargraphComprehension          0.273        0.167      0.251     0.435
## SentenceCompletion             0.228        0.159      0.226     0.451
## WordClassification             0.255        0.250      0.274     0.427
## WordMeaning                    0.274        0.208      0.274     0.446
## Addition                       0.289        0.317      0.190     0.173
## Code                           0.362        0.350      0.290     0.202
## CountingDots                   0.278        0.349      0.110     0.246
## StraightCurvedCapitals         0.194        0.323      0.263     0.241
## WordRecognition                0.341        0.201      0.206     0.302
## NumberRecognition              0.345        0.334      0.192     0.272
## FigureRecognition              0.324        0.344      0.258     0.388
## ObjectNumber                   1.000        0.448      0.324     0.262
## NumberFigure                   0.448        1.000      0.358     0.301
## FigureWord                     0.324        0.358      1.000     0.167
## Deduction                      0.262        0.301      0.167     1.000
## NumericalPuzzles               0.173        0.357      0.331     0.413
## ProblemReasoning               0.273        0.317      0.342     0.463
## SeriesCompletion               0.287        0.272      0.303     0.509
## ArithmeticProblems             0.326        0.405      0.374     0.366
##                         NumericalPuzzles ProblemReasoning SeriesCompletion
## VisualPerception                   0.369            0.413            0.474
## Cubes                              0.306            0.232            0.348
## PaperFormBoard                     0.165            0.250            0.383
## Flags                              0.349            0.380            0.335
## GeneralInformation                 0.318            0.441            0.435
## PargraphComprehension              0.263            0.386            0.431
## SentenceCompletion                 0.314            0.396            0.405
## WordClassification                 0.362            0.357            0.501
## WordMeaning                        0.266            0.483            0.504
## Addition                           0.405            0.160            0.262
## Code                               0.399            0.304            0.251
## CountingDots                       0.355            0.193            0.350
## StraightCurvedCapitals             0.425            0.279            0.382
## WordRecognition                    0.183            0.243            0.242
## NumberRecognition                  0.232            0.246            0.256
```

```
## FigureRecognition                  0.348            0.283            0.360
## ObjectNumber                       0.173            0.273            0.287
## NumberFigure                       0.357            0.317            0.272
## FigureWord                         0.331            0.342            0.303
## Deduction                          0.413            0.463            0.509
## NumericalPuzzles                   1.000            0.374            0.451
## ProblemReasoning                   0.374            1.000            0.503
## SeriesCompletion                   0.451            0.503            1.000
## ArithmeticProblems                 0.448            0.375            0.434
##                         ArithmeticProblems
## VisualPerception                     0.282
## Cubes                                0.211
## PaperFormBoard                       0.203
## Flags                                0.248
## GeneralInformation                   0.420
## PargraphComprehension                0.433
## SentenceCompletion                   0.437
## WordClassification                   0.388
## WordMeaning                          0.424
## Addition                             0.531
## Code                                 0.412
## CountingDots                         0.414
## StraightCurvedCapitals               0.358
## WordRecognition                      0.304
## NumberRecognition                    0.165
## FigureRecognition                    0.262
## ObjectNumber                         0.326
## NumberFigure                         0.405
## FigureWord                           0.374
## Deduction                            0.366
## NumericalPuzzles                     0.448
## ProblemReasoning                     0.375
## SeriesCompletion                     0.434
## ArithmeticProblems                   1.000
##
## $center
##  [1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
##
## $n.obs
## [1] 145

# convert covariance to correlations
correlations <- cov2cor(Harman74.cor$cov)
correlations

##                         VisualPerception Cubes PaperFormBoard Flags
## VisualPerception                   1.000 0.318          0.403 0.468
## Cubes                              0.318 1.000          0.317 0.230
## PaperFormBoard                     0.403 0.317          1.000 0.305
## Flags                              0.468 0.230          0.305 1.000
```

```
## GeneralInformation                0.321 0.285          0.247 0.227
## PargraphComprehension             0.335 0.234          0.268 0.327
## SentenceCompletion                0.304 0.157          0.223 0.335
## WordClassification                0.332 0.157          0.382 0.391
## WordMeaning                       0.326 0.195          0.184 0.325
## Addition                          0.116 0.057         -0.075 0.099
## Code                              0.308 0.150          0.091 0.110
## CountingDots                      0.314 0.145          0.140 0.160
## StraightCurvedCapitals            0.489 0.239          0.321 0.327
## WordRecognition                   0.125 0.103          0.177 0.066
## NumberRecognition                 0.238 0.131          0.065 0.127
## FigureRecognition                 0.414 0.272          0.263 0.322
## ObjectNumber                      0.176 0.005          0.177 0.187
## NumberFigure                      0.368 0.255          0.211 0.251
## FigureWord                        0.270 0.112          0.312 0.137
## Deduction                         0.365 0.292          0.297 0.339
## NumericalPuzzles                  0.369 0.306          0.165 0.349
## ProblemReasoning                  0.413 0.232          0.250 0.380
## SeriesCompletion                  0.474 0.348          0.383 0.335
## ArithmeticProblems                0.282 0.211          0.203 0.248
##                         GeneralInformation PargraphComprehension
## VisualPerception                     0.321                 0.335
## Cubes                                0.285                 0.234
## PaperFormBoard                       0.247                 0.268
## Flags                                0.227                 0.327
## GeneralInformation                   1.000                 0.622
## PargraphComprehension                0.622                 1.000
## SentenceCompletion                   0.656                 0.722
## WordClassification                   0.578                 0.527
## WordMeaning                          0.723                 0.714
## Addition                             0.311                 0.203
## Code                                 0.344                 0.353
## CountingDots                         0.215                 0.095
## StraightCurvedCapitals               0.344                 0.309
## WordRecognition                      0.280                 0.292
## NumberRecognition                    0.229                 0.251
## FigureRecognition                    0.187                 0.291
## ObjectNumber                         0.208                 0.273
## NumberFigure                         0.263                 0.167
## FigureWord                           0.190                 0.251
## Deduction                            0.398                 0.435
## NumericalPuzzles                     0.318                 0.263
## ProblemReasoning                     0.441                 0.386
## SeriesCompletion                     0.435                 0.431
## ArithmeticProblems                   0.420                 0.433
##                         SentenceCompletion WordClassification WordMeaning
## VisualPerception                     0.304               0.332       0.326
## Cubes                                0.157               0.157       0.195
## PaperFormBoard                       0.223               0.382       0.184
## Flags                                0.335               0.391       0.325
```

```
## GeneralInformation                0.656             0.578       0.723
## PargraphComprehension             0.722             0.527       0.714
## SentenceCompletion                1.000             0.619       0.685
## WordClassification                0.619             1.000       0.532
## WordMeaning                       0.685             0.532       1.000
## Addition                          0.246             0.285       0.170
## Code                              0.232             0.300       0.280
## CountingDots                      0.181             0.271       0.113
## StraightCurvedCapitals            0.345             0.395       0.280
## WordRecognition                   0.236             0.252       0.260
## NumberRecognition                 0.172             0.175       0.248
## FigureRecognition                 0.180             0.296       0.242
## ObjectNumber                      0.228             0.255       0.274
## NumberFigure                      0.159             0.250       0.208
## FigureWord                        0.226             0.274       0.274
## Deduction                         0.451             0.427       0.446
## NumericalPuzzles                  0.314             0.362       0.266
## ProblemReasoning                  0.396             0.357       0.483
## SeriesCompletion                  0.405             0.501       0.504
## ArithmeticProblems                0.437             0.388       0.424
##                          Addition  Code CountingDots StraightCurvedCapitals
## VisualPerception            0.116 0.308       0.314                   0.489
## Cubes                       0.057 0.150       0.145                   0.239
## PaperFormBoard             -0.075 0.091       0.140                   0.321
## Flags                       0.099 0.110       0.160                   0.327
## GeneralInformation          0.311 0.344       0.215                   0.344
## PargraphComprehension       0.203 0.353       0.095                   0.309
## SentenceCompletion          0.246 0.232       0.181                   0.345
## WordClassification          0.285 0.300       0.271                   0.395
## WordMeaning                 0.170 0.280       0.113                   0.280
## Addition                    1.000 0.484       0.585                   0.408
## Code                        0.484 1.000       0.428                   0.535
## CountingDots                0.585 0.428       1.000                   0.512
## StraightCurvedCapitals      0.408 0.535       0.512                   1.000
## WordRecognition             0.172 0.350       0.131                   0.195
## NumberRecognition           0.154 0.240       0.173                   0.139
## FigureRecognition           0.124 0.314       0.119                   0.281
## ObjectNumber                0.289 0.362       0.278                   0.194
## NumberFigure                0.317 0.350       0.349                   0.323
## FigureWord                  0.190 0.290       0.110                   0.263
## Deduction                   0.173 0.202       0.246                   0.241
## NumericalPuzzles            0.405 0.399       0.355                   0.425
## ProblemReasoning            0.160 0.304       0.193                   0.279
## SeriesCompletion            0.262 0.251       0.350                   0.382
## ArithmeticProblems          0.531 0.412       0.414                   0.358
##                          WordRecognition NumberRecognition FigureRecognition
## VisualPerception                   0.125             0.238             0.414
## Cubes                              0.103             0.131             0.272
## PaperFormBoard                     0.177             0.065             0.263
## Flags                              0.066             0.127             0.322
```

```
## GeneralInformation               0.280             0.229             0.187
## PargraphComprehension            0.292             0.251             0.291
## SentenceCompletion               0.236             0.172             0.180
## WordClassification               0.252             0.175             0.296
## WordMeaning                      0.260             0.248             0.242
## Addition                         0.172             0.154             0.124
## Code                             0.350             0.240             0.314
## CountingDots                     0.131             0.173             0.119
## StraightCurvedCapitals           0.195             0.139             0.281
## WordRecognition                  1.000             0.370             0.412
## NumberRecognition                0.370             1.000             0.325
## FigureRecognition                0.412             0.325             1.000
## ObjectNumber                     0.341             0.345             0.324
## NumberFigure                     0.201             0.334             0.344
## FigureWord                       0.206             0.192             0.258
## Deduction                        0.302             0.272             0.388
## NumericalPuzzles                 0.183             0.232             0.348
## ProblemReasoning                 0.243             0.246             0.283
## SeriesCompletion                 0.242             0.256             0.360
## ArithmeticProblems               0.304             0.165             0.262
##                          ObjectNumber NumberFigure FigureWord Deduction
## VisualPerception                0.176        0.368      0.270     0.365
## Cubes                           0.005        0.255      0.112     0.292
## PaperFormBoard                  0.177        0.211      0.312     0.297
## Flags                           0.187        0.251      0.137     0.339
## GeneralInformation              0.208        0.263      0.190     0.398
## PargraphComprehension           0.273        0.167      0.251     0.435
## SentenceCompletion              0.228        0.159      0.226     0.451
## WordClassification              0.255        0.250      0.274     0.427
## WordMeaning                     0.274        0.208      0.274     0.446
## Addition                        0.289        0.317      0.190     0.173
## Code                            0.362        0.350      0.290     0.202
## CountingDots                    0.278        0.349      0.110     0.246
## StraightCurvedCapitals          0.194        0.323      0.263     0.241
## WordRecognition                 0.341        0.201      0.206     0.302
## NumberRecognition               0.345        0.334      0.192     0.272
## FigureRecognition               0.324        0.344      0.258     0.388
## ObjectNumber                    1.000        0.448      0.324     0.262
## NumberFigure                    0.448        1.000      0.358     0.301
## FigureWord                      0.324        0.358      1.000     0.167
## Deduction                       0.262        0.301      0.167     1.000
## NumericalPuzzles                0.173        0.357      0.331     0.413
## ProblemReasoning                0.273        0.317      0.342     0.463
## SeriesCompletion                0.287        0.272      0.303     0.509
## ArithmeticProblems              0.326        0.405      0.374     0.366
##                          NumericalPuzzles ProblemReasoning SeriesCompletion
## VisualPerception                    0.369             0.413            0.474
## Cubes                               0.306             0.232            0.348
## PaperFormBoard                      0.165             0.250            0.383
## Flags                               0.349             0.380            0.335
```

```
## GeneralInformation                0.318             0.441             0.435
## PargraphComprehension             0.263             0.386             0.431
## SentenceCompletion                0.314             0.396             0.405
## WordClassification                0.362             0.357             0.501
## WordMeaning                       0.266             0.483             0.504
## Addition                          0.405             0.160             0.262
## Code                              0.399             0.304             0.251
## CountingDots                      0.355             0.193             0.350
## StraightCurvedCapitals            0.425             0.279             0.382
## WordRecognition                   0.183             0.243             0.242
## NumberRecognition                 0.232             0.246             0.256
## FigureRecognition                 0.348             0.283             0.360
## ObjectNumber                      0.173             0.273             0.287
## NumberFigure                      0.357             0.317             0.272
## FigureWord                        0.331             0.342             0.303
## Deduction                         0.413             0.463             0.509
## NumericalPuzzles                  1.000             0.374             0.451
## ProblemReasoning                  0.374             1.000             0.503
## SeriesCompletion                  0.451             0.503             1.000
## ArithmeticProblems                0.448             0.375             0.434
##                         ArithmeticProblems
## VisualPerception                     0.282
## Cubes                                0.211
## PaperFormBoard                       0.203
## Flags                                0.248
## GeneralInformation                   0.420
## PargraphComprehension                0.433
## SentenceCompletion                   0.437
## WordClassification                   0.388
## WordMeaning                          0.424
## Addition                             0.531
## Code                                 0.412
## CountingDots                         0.414
## StraightCurvedCapitals               0.358
## WordRecognition                      0.304
## NumberRecognition                    0.165
## FigureRecognition                    0.262
## ObjectNumber                         0.326
## NumberFigure                         0.405
## FigureWord                           0.374
## Deduction                            0.366
## NumericalPuzzles                     0.448
## ProblemReasoning                     0.375
## SeriesCompletion                     0.434
## ArithmeticProblems                   1.000

# Round to 3DP - still too difficult to read!
round(correlations, 3)
```

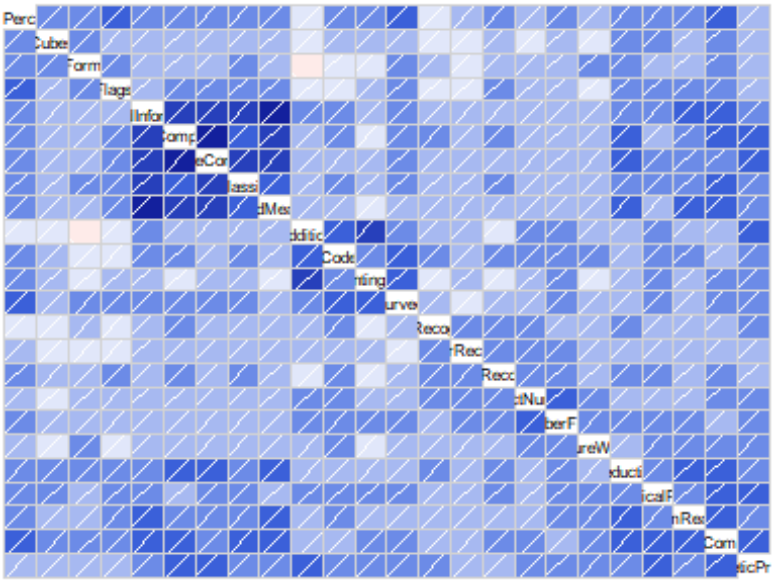
```
##                         VisualPerception Cubes PaperFormBoard Flags
## VisualPerception                   1.000 0.318          0.403 0.468
## Cubes                              0.318 1.000          0.317 0.230
## PaperFormBoard                     0.403 0.317          1.000 0.305
## Flags                              0.468 0.230          0.305 1.000
## GeneralInformation                 0.321 0.285          0.247 0.227
## PargraphComprehension              0.335 0.234          0.268 0.327
## SentenceCompletion                 0.304 0.157          0.223 0.335
## WordClassification                 0.332 0.157          0.382 0.391
## WordMeaning                        0.326 0.195          0.184 0.325
## Addition                           0.116 0.057         -0.075 0.099
## Code                               0.308 0.150          0.091 0.110
## CountingDots                       0.314 0.145          0.140 0.160
## StraightCurvedCapitals             0.489 0.239          0.321 0.327
## WordRecognition                    0.125 0.103          0.177 0.066
## NumberRecognition                  0.238 0.131          0.065 0.127
## FigureRecognition                  0.414 0.272          0.263 0.322
## ObjectNumber                       0.176 0.005          0.177 0.187
## NumberFigure                       0.368 0.255          0.211 0.251
## FigureWord                         0.270 0.112          0.312 0.137
## Deduction                          0.365 0.292          0.297 0.339
## NumericalPuzzles                   0.369 0.306          0.165 0.349
## ProblemReasoning                   0.413 0.232          0.250 0.380
## SeriesCompletion                   0.474 0.348          0.383 0.335
## ArithmeticProblems                 0.282 0.211          0.203 0.248
##                         GeneralInformation PargraphComprehension
## VisualPerception                     0.321                 0.335
## Cubes                                0.285                 0.234
## PaperFormBoard                       0.247                 0.268
## Flags                                0.227                 0.327
## GeneralInformation                   1.000                 0.622
## PargraphComprehension                0.622                 1.000
## SentenceCompletion                   0.656                 0.722
## WordClassification                   0.578                 0.527
## WordMeaning                          0.723                 0.714
## Addition                             0.311                 0.203
## Code                                 0.344                 0.353
## CountingDots                         0.215                 0.095
## StraightCurvedCapitals               0.344                 0.309
## WordRecognition                      0.280                 0.292
## NumberRecognition                    0.229                 0.251
## FigureRecognition                    0.187                 0.291
## ObjectNumber                         0.208                 0.273
## NumberFigure                         0.263                 0.167
## FigureWord                           0.190                 0.251
## Deduction                            0.398                 0.435
## NumericalPuzzles                     0.318                 0.263
## ProblemReasoning                     0.441                 0.386
## SeriesCompletion                     0.435                 0.431
## ArithmeticProblems                   0.420                 0.433
```

```
##                         SentenceCompletion WordClassification WordMeaning
## VisualPerception                    0.304              0.332       0.326
## Cubes                               0.157              0.157       0.195
## PaperFormBoard                      0.223              0.382       0.184
## Flags                               0.335              0.391       0.325
## GeneralInformation                  0.656              0.578       0.723
## PargraphComprehension               0.722              0.527       0.714
## SentenceCompletion                  1.000              0.619       0.685
## WordClassification                  0.619              1.000       0.532
## WordMeaning                         0.685              0.532       1.000
## Addition                            0.246              0.285       0.170
## Code                                0.232              0.300       0.280
## CountingDots                        0.181              0.271       0.113
## StraightCurvedCapitals              0.345              0.395       0.280
## WordRecognition                     0.236              0.252       0.260
## NumberRecognition                   0.172              0.175       0.248
## FigureRecognition                   0.180              0.296       0.242
## ObjectNumber                        0.228              0.255       0.274
## NumberFigure                        0.159              0.250       0.208
## FigureWord                          0.226              0.274       0.274
## Deduction                           0.451              0.427       0.446
## NumericalPuzzles                    0.314              0.362       0.266
## ProblemReasoning                    0.396              0.357       0.483
## SeriesCompletion                    0.405              0.501       0.504
## ArithmeticProblems                  0.437              0.388       0.424
##                         Addition  Code CountingDots StraightCurvedCapitals
## VisualPerception           0.116 0.308        0.314                  0.489
## Cubes                      0.057 0.150        0.145                  0.239
## PaperFormBoard            -0.075 0.091        0.140                  0.321
## Flags                      0.099 0.110        0.160                  0.327
## GeneralInformation         0.311 0.344        0.215                  0.344
## PargraphComprehension      0.203 0.353        0.095                  0.309
## SentenceCompletion         0.246 0.232        0.181                  0.345
## WordClassification         0.285 0.300        0.271                  0.395
## WordMeaning                0.170 0.280        0.113                  0.280
## Addition                   1.000 0.484        0.585                  0.408
## Code                       0.484 1.000        0.428                  0.535
## CountingDots               0.585 0.428        1.000                  0.512
## StraightCurvedCapitals     0.408 0.535        0.512                  1.000
## WordRecognition            0.172 0.350        0.131                  0.195
## NumberRecognition          0.154 0.240        0.173                  0.139
## FigureRecognition          0.124 0.314        0.119                  0.281
## ObjectNumber               0.289 0.362        0.278                  0.194
## NumberFigure               0.317 0.350        0.349                  0.323
## FigureWord                 0.190 0.290        0.110                  0.263
## Deduction                  0.173 0.202        0.246                  0.241
## NumericalPuzzles           0.405 0.399        0.355                  0.425
## ProblemReasoning           0.160 0.304        0.193                  0.279
## SeriesCompletion           0.262 0.251        0.350                  0.382
## ArithmeticProblems         0.531 0.412        0.414                  0.358
```

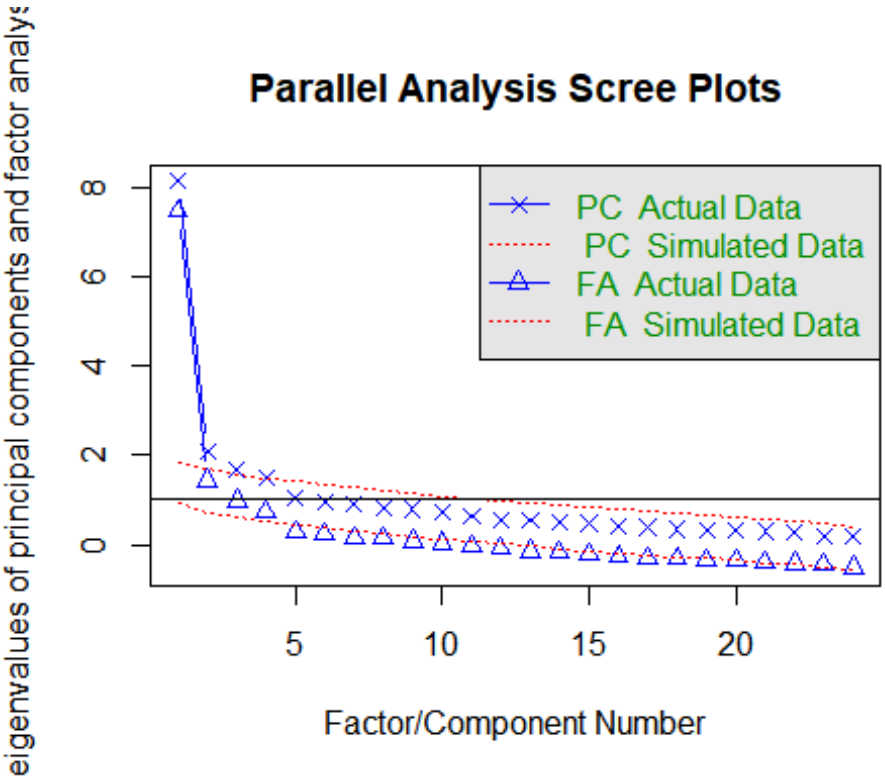
```
##                         WordRecognition NumberRecognition FigureRecognition
## VisualPerception                  0.125             0.238             0.414
## Cubes                             0.103             0.131             0.272
## PaperFormBoard                    0.177             0.065             0.263
## Flags                             0.066             0.127             0.322
## GeneralInformation                0.280             0.229             0.187
## PargraphComprehension             0.292             0.251             0.291
## SentenceCompletion                0.236             0.172             0.180
## WordClassification                0.252             0.175             0.296
## WordMeaning                       0.260             0.248             0.242
## Addition                          0.172             0.154             0.124
## Code                              0.350             0.240             0.314
## CountingDots                      0.131             0.173             0.119
## StraightCurvedCapitals            0.195             0.139             0.281
## WordRecognition                   1.000             0.370             0.412
## NumberRecognition                 0.370             1.000             0.325
## FigureRecognition                 0.412             0.325             1.000
## ObjectNumber                      0.341             0.345             0.324
## NumberFigure                      0.201             0.334             0.344
## FigureWord                        0.206             0.192             0.258
## Deduction                         0.302             0.272             0.388
## NumericalPuzzles                  0.183             0.232             0.348
## ProblemReasoning                  0.243             0.246             0.283
## SeriesCompletion                  0.242             0.256             0.360
## ArithmeticProblems                0.304             0.165             0.262
##                         ObjectNumber NumberFigure FigureWord Deduction
## VisualPerception               0.176        0.368      0.270     0.365
## Cubes                          0.005        0.255      0.112     0.292
## PaperFormBoard                 0.177        0.211      0.312     0.297
## Flags                          0.187        0.251      0.137     0.339
## GeneralInformation             0.208        0.263      0.190     0.398
## PargraphComprehension          0.273        0.167      0.251     0.435
## SentenceCompletion             0.228        0.159      0.226     0.451
## WordClassification             0.255        0.250      0.274     0.427
## WordMeaning                    0.274        0.208      0.274     0.446
## Addition                       0.289        0.317      0.190     0.173
## Code                           0.362        0.350      0.290     0.202
## CountingDots                   0.278        0.349      0.110     0.246
## StraightCurvedCapitals         0.194        0.323      0.263     0.241
## WordRecognition                0.341        0.201      0.206     0.302
## NumberRecognition              0.345        0.334      0.192     0.272
## FigureRecognition              0.324        0.344      0.258     0.388
## ObjectNumber                   1.000        0.448      0.324     0.262
## NumberFigure                   0.448        1.000      0.358     0.301
## FigureWord                     0.324        0.358      1.000     0.167
## Deduction                      0.262        0.301      0.167     1.000
## NumericalPuzzles               0.173        0.357      0.331     0.413
## ProblemReasoning               0.273        0.317      0.342     0.463
## SeriesCompletion               0.287        0.272      0.303     0.509
## ArithmeticProblems             0.326        0.405      0.374     0.366
```

```
##                         NumericalPuzzles ProblemReasoning SeriesCompletion
## VisualPerception                   0.369            0.413            0.474
## Cubes                              0.306            0.232            0.348
## PaperFormBoard                     0.165            0.250            0.383
## Flags                              0.349            0.380            0.335
## GeneralInformation                 0.318            0.441            0.435
## PargraphComprehension              0.263            0.386            0.431
## SentenceCompletion                 0.314            0.396            0.405
## WordClassification                 0.362            0.357            0.501
## WordMeaning                        0.266            0.483            0.504
## Addition                           0.405            0.160            0.262
## Code                               0.399            0.304            0.251
## CountingDots                       0.355            0.193            0.350
## StraightCurvedCapitals             0.425            0.279            0.382
## WordRecognition                    0.183            0.243            0.242
## NumberRecognition                  0.232            0.246            0.256
## FigureRecognition                  0.348            0.283            0.360
## ObjectNumber                       0.173            0.273            0.287
## NumberFigure                       0.357            0.317            0.272
## FigureWord                         0.331            0.342            0.303
## Deduction                          0.413            0.463            0.509
## NumericalPuzzles                   1.000            0.374            0.451
## ProblemReasoning                   0.374            1.000            0.503
## SeriesCompletion                   0.451            0.503            1.000
## ArithmeticProblems                 0.448            0.375            0.434
##                         ArithmeticProblems
## VisualPerception                     0.282
## Cubes                                0.211
## PaperFormBoard                       0.203
## Flags                                0.248
## GeneralInformation                   0.420
## PargraphComprehension                0.433
## SentenceCompletion                   0.437
## WordClassification                   0.388
## WordMeaning                          0.424
## Addition                             0.531
## Code                                 0.412
## CountingDots                         0.414
## StraightCurvedCapitals               0.358
## WordRecognition                      0.304
## NumberRecognition                    0.165
## FigureRecognition                    0.262
## ObjectNumber                         0.326
## NumberFigure                         0.405
## FigureWord                           0.374
## Deduction                            0.366
## NumericalPuzzles                     0.448
## ProblemReasoning                     0.375
## SeriesCompletion                     0.434
## ArithmeticProblems                   1.000
```

```
library(corrgram) # Could also try plotcorr() in ellipse package?
corrgram(correlations)
```



```
fa.parallel(correlations, n.obs=145, fa="both", n.iter=100)

## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate =
## rotate, : A loading greater than abs(1) was detected. Examine the loadings
## carefully.

## The estimated weights for the factor scores are probably incorrect.  Try a
different factor extraction method.

## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate =
## rotate, : An ultra-Heywood case was detected. Examine the results
carefully
```




```
## Parallel analysis suggests that the number of factors =  4  and the number
of components =  4

# FA with 4 factors using maximum likelihood
fa(correlations, nfactors=4, rotate="none", fm="ml")

## Factor Analysis using method =  ml
## Call: fa(r = correlations, nfactors = 4, rotate = "none", fm = "ml")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                          ML1   ML2   ML3   ML4   h2   u2 com
## VisualPerception        0.55  0.04  0.45 -0.22 0.56 0.44 2.3
## Cubes                   0.34 -0.01  0.29 -0.13 0.22 0.78 2.3
## PaperFormBoard          0.38 -0.11  0.42 -0.16 0.36 0.64 2.4
## Flags                   0.46 -0.07  0.30 -0.20 0.35 0.65 2.2
## GeneralInformation      0.74 -0.23 -0.22 -0.04 0.65 0.35 1.4
## PargraphComprehension   0.74 -0.35 -0.15  0.06 0.69 0.31 1.5
## SentenceCompletion      0.74 -0.32 -0.24 -0.10 0.72 0.28 1.7
## WordClassification      0.70 -0.12 -0.03 -0.12 0.51 0.49 1.1
## WordMeaning             0.75 -0.39 -0.16  0.06 0.74 0.26 1.6
## Addition                0.49  0.62 -0.38 -0.01 0.76 0.24 2.6
## Code                    0.54  0.37 -0.04  0.14 0.45 0.55 1.9
## CountingDots            0.45  0.57 -0.04 -0.19 0.56 0.44 2.2
## StraightCurvedCapitals 0.58  0.31  0.12 -0.26 0.51 0.49 2.1
## WordRecognition         0.40  0.04  0.08  0.43 0.35 0.65 2.1
## NumberRecognition       0.36  0.07  0.16  0.37 0.30 0.70 2.4
## FigureRecognition       0.45  0.07  0.42  0.26 0.45 0.55 2.6
## ObjectNumber            0.44  0.19  0.08  0.41 0.40 0.60 2.4
```

```
## NumberFigure           0.46  0.31  0.24  0.18 0.41 0.59 2.7
## FigureWord             0.42  0.09  0.17  0.16 0.24 0.76 1.8
## Deduction              0.60 -0.09  0.19  0.04 0.41 0.59 1.3
## NumericalPuzzles       0.56  0.27  0.15 -0.09 0.42 0.58 1.7
## ProblemReasoning       0.59 -0.08  0.19  0.04 0.40 0.60 1.3
## SeriesCompletion       0.67  0.00  0.22 -0.09 0.50 0.50 1.2
## ArithmeticProblems     0.65  0.24 -0.11  0.06 0.50 0.50 1.3
##
##                        ML1  ML2  ML3  ML4
## SS loadings           7.52 1.70 1.33 0.92
## Proportion Var        0.31 0.07 0.06 0.04
## Cumulative Var        0.31 0.38 0.44 0.48
## Proportion Explained  0.66 0.15 0.12 0.08
## Cumulative Proportion 0.66 0.80 0.92 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective
function was  11.44
## The degrees of freedom for the model are 186  and the objective function
was  1.71
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                 ML1  ML2  ML3  ML4
## Correlation of scores with factors             0.97 0.91 0.86 0.78
## Multiple R square of scores with factors       0.95 0.83 0.75 0.61
## Minimum correlation of possible factor scores  0.89 0.66 0.49 0.23
```

```
# with varimax rotation
fa(correlations, nfactors=4, rotate="varimax", fm="ml")
```

```
## Factor Analysis using method =  ml
## Call: fa(r = correlations, nfactors = 4, rotate = "varimax", fm = "ml")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                         ML1   ML3   ML2  ML4   h2   u2 com
## VisualPerception       0.16  0.69  0.19 0.16 0.56 0.44 1.4
## Cubes                  0.12  0.44  0.08 0.10 0.22 0.78 1.3
## PaperFormBoard         0.14  0.57 -0.02 0.11 0.36 0.64 1.2
## Flags                  0.23  0.53  0.10 0.08 0.35 0.65 1.5
## GeneralInformation     0.74  0.19  0.21 0.15 0.65 0.35 1.4
## PargraphComprehension  0.77  0.20  0.07 0.23 0.69 0.31 1.4
## SentenceCompletion     0.81  0.20  0.15 0.07 0.72 0.28 1.2
## WordClassification     0.57  0.34  0.24 0.13 0.51 0.49 2.2
## WordMeaning            0.81  0.20  0.04 0.23 0.74 0.26 1.3
## Addition               0.17 -0.12  0.83 0.17 0.76 0.24 1.2
```

```
## Code                   0.18  0.12  0.51 0.37 0.45 0.55 2.2
## CountingDots           0.02  0.21  0.72 0.09 0.56 0.44 1.2
## StraightCurvedCapitals 0.19  0.44  0.53 0.08 0.51 0.49 2.3
## WordRecognition        0.20  0.05  0.08 0.55 0.35 0.65 1.3
## NumberRecognition      0.12  0.12  0.07 0.52 0.30 0.70 1.3
## FigureRecognition      0.07  0.41  0.06 0.53 0.45 0.55 2.0
## ObjectNumber           0.14  0.06  0.22 0.57 0.40 0.60 1.4
## NumberFigure           0.03  0.29  0.34 0.46 0.41 0.59 2.6
## FigureWord             0.15  0.24  0.16 0.37 0.24 0.76 2.6
## Deduction              0.38  0.40  0.12 0.30 0.41 0.59 3.0
## NumericalPuzzles       0.17  0.38  0.44 0.22 0.42 0.58 2.8
## ProblemReasoning       0.37  0.40  0.12 0.30 0.40 0.60 3.1
## SeriesCompletion       0.37  0.50  0.24 0.24 0.50 0.50 2.9
## ArithmeticProblems     0.37  0.16  0.50 0.30 0.50 0.50 2.8
##
##                         ML1  ML3  ML2  ML4
## SS loadings            3.65 2.87 2.66 2.29
## Proportion Var         0.15 0.12 0.11 0.10
## Cumulative Var         0.15 0.27 0.38 0.48
## Proportion Explained   0.32 0.25 0.23 0.20
## Cumulative Proportion  0.32 0.57 0.80 1.00
##
## Mean item complexity =  1.9
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective
function was  11.44
## The degrees of freedom for the model are 186  and the objective function
was  1.71
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                 ML1  ML3  ML2  ML4
## Correlation of scores with factors             0.93 0.87 0.91 0.82
## Multiple R square of scores with factors       0.87 0.76 0.83 0.68
## Minimum correlation of possible factor scores  0.73 0.52 0.66 0.36

# with promax roation
fa(correlations, nfactors=4, rotate="promax", fm="ml")

## Factor Analysis using method =  ml
## Call: fa(r = correlations, nfactors = 4, rotate = "promax", fm = "ml")
## Standardized loadings (pattern matrix) based upon correlation matrix
##                         ML1   ML3   ML2   ML4   h2   u2 com
## VisualPerception      -0.07  0.78  0.05 -0.04 0.56 0.44 1.0
## Cubes                 -0.02  0.50 -0.01 -0.03 0.22 0.78 1.0
## PaperFormBoard        -0.02  0.69 -0.17 -0.02 0.36 0.64 1.1
```

```
## Flags                    0.10  0.59 -0.02 -0.10 0.35 0.65 1.1
## GeneralInformation       0.80 -0.03  0.09 -0.05 0.65 0.35 1.0
## PargraphComprehension    0.84 -0.01 -0.12  0.09 0.69 0.31 1.1
## SentenceCompletion       0.91 -0.01  0.03 -0.14 0.72 0.28 1.1
## WordClassification       0.54  0.21  0.13 -0.08 0.51 0.49 1.5
## WordMeaning              0.89 -0.02 -0.15  0.08 0.74 0.26 1.1
## Addition                 0.08 -0.41  0.99  0.00 0.76 0.24 1.3
## Code                     0.03 -0.09  0.50  0.29 0.45 0.55 1.7
## CountingDots            -0.17  0.10  0.83 -0.13 0.56 0.44 1.2
## StraightCurvedCapitals   0.01  0.40  0.53 -0.17 0.51 0.49 2.1
## WordRecognition          0.10 -0.15 -0.07  0.65 0.35 0.65 1.2
## NumberRecognition        0.00 -0.03 -0.07  0.61 0.30 0.70 1.0
## FigureRecognition       -0.16  0.37 -0.13  0.55 0.45 0.55 2.1
## ObjectNumber             0.00 -0.15  0.10  0.65 0.40 0.60 1.2
## NumberFigure            -0.20  0.19  0.25  0.43 0.41 0.59 2.6
## FigureWord               0.01  0.15  0.05  0.35 0.24 0.76 1.4
## Deduction                0.27  0.33 -0.05  0.19 0.41 0.59 2.6
## NumericalPuzzles        -0.01  0.31  0.40  0.05 0.42 0.58 1.9
## ProblemReasoning         0.26  0.32 -0.04  0.19 0.40 0.60 2.6
## SeriesCompletion         0.22  0.45  0.10  0.05 0.50 0.50 1.6
## ArithmeticProblems       0.27 -0.07  0.46  0.16 0.50 0.50 1.9
##
##                         ML1  ML3  ML2  ML4
## SS loadings            3.72 2.88 2.72 2.15
## Proportion Var         0.16 0.12 0.11 0.09
## Cumulative Var         0.16 0.27 0.39 0.48
## Proportion Explained   0.32 0.25 0.24 0.19
## Cumulative Proportion 0.32 0.58 0.81 1.00
##
##  With factor correlations of
##      ML1  ML3  ML2  ML4
## ML1 1.00 0.59 0.48 0.54
## ML3 0.59 1.00 0.53 0.59
## ML2 0.48 0.53 1.00 0.56
## ML4 0.54 0.59 0.56 1.00
##
## Mean item complexity =  1.5
## Test of the hypothesis that 4 factors are sufficient.
##
## The degrees of freedom for the null model are  276  and the objective
function was  11.44
## The degrees of freedom for the model are 186  and the objective function
was  1.71
##
## The root mean square of the residuals (RMSR) is  0.04
## The df corrected root mean square of the residuals is  0.05
##
## Fit based upon off diagonal values = 0.98
## Measures of factor score adequacy
##                                                 ML1  ML3  ML2  ML4
```

```
## Correlation of scores with factors             0.96 0.93 0.94 0.90
## Multiple R square of scores with factors       0.92 0.86 0.89 0.81
## Minimum correlation of possible factor scores  0.85 0.72 0.78 0.62
```