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AIDS 2 EXP 7

AIM:

To implement and visualize various properties of fuzzy sets, including Union, Intersection, Complement, Algebraic Product, Bounded Sum, and Bounded Difference.

THEORY:

Fuzzy set theory is an extension of classical set theory, where the membership of an element in a set is not restricted to binary values (0 or 1). Instead, it is represented as a value between 0 and 1, indicating the degree of membership.

In classical sets, an element either belongs to a set or does not. Fuzzy sets, on the other hand, allow partial membership, which is useful in modeling situations where things are not clearly defined or have varying degrees of truth.

The following properties of fuzzy sets will be explored in this experiment:

1. **Union of Fuzzy Sets:**
   The union of two fuzzy sets represents the membership of an element in at least one of the sets. The membership value in the union is determined by taking the maximum of the membership values from the two sets.

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

2. **Intersection of Fuzzy Sets:**
   The intersection of two fuzzy sets represents the degree to which an element belongs to both sets. The membership value in the intersection is determined by the minimum of the membership values from the two sets.

$$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

3. **Complement of a Fuzzy Set:**
   The complement of a fuzzy set indicates how much an element does **not** belong to the set. The complement is calculated by subtracting the membership value from 1.

$$\mu_{A'}(x) = 1 - \mu_A(x)$$

4. **Algebraic Sum of Fuzzy Sets:**
   The algebraic sum represents a modified union of two fuzzy sets, taking into account both their membership values and their overlap. It is calculated as:

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

5. **Algebraic Product of Fuzzy Sets:**
   The algebraic product represents the intersection of two fuzzy sets. This is computed by multiplying the membership values of the two sets. It models the situation where the two sets are strongly related.

$$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$$

6. **Bounded Sum of Fuzzy Sets:**
   The bounded sum represents the union of two fuzzy sets, but with the membership value capped at 1. This ensures that the degree of membership in the result does not exceed 1.

$$\mu_{A+B}(x) = \min(1, \mu_A(x) + \mu_B(x))$$

7. **Bounded Difference of Fuzzy Sets:**
   The bounded difference represents how much the membership of an element in set A exceeds its membership in set B. It is computed by subtracting the membership value of B from A, ensuring the result is never less than 0.

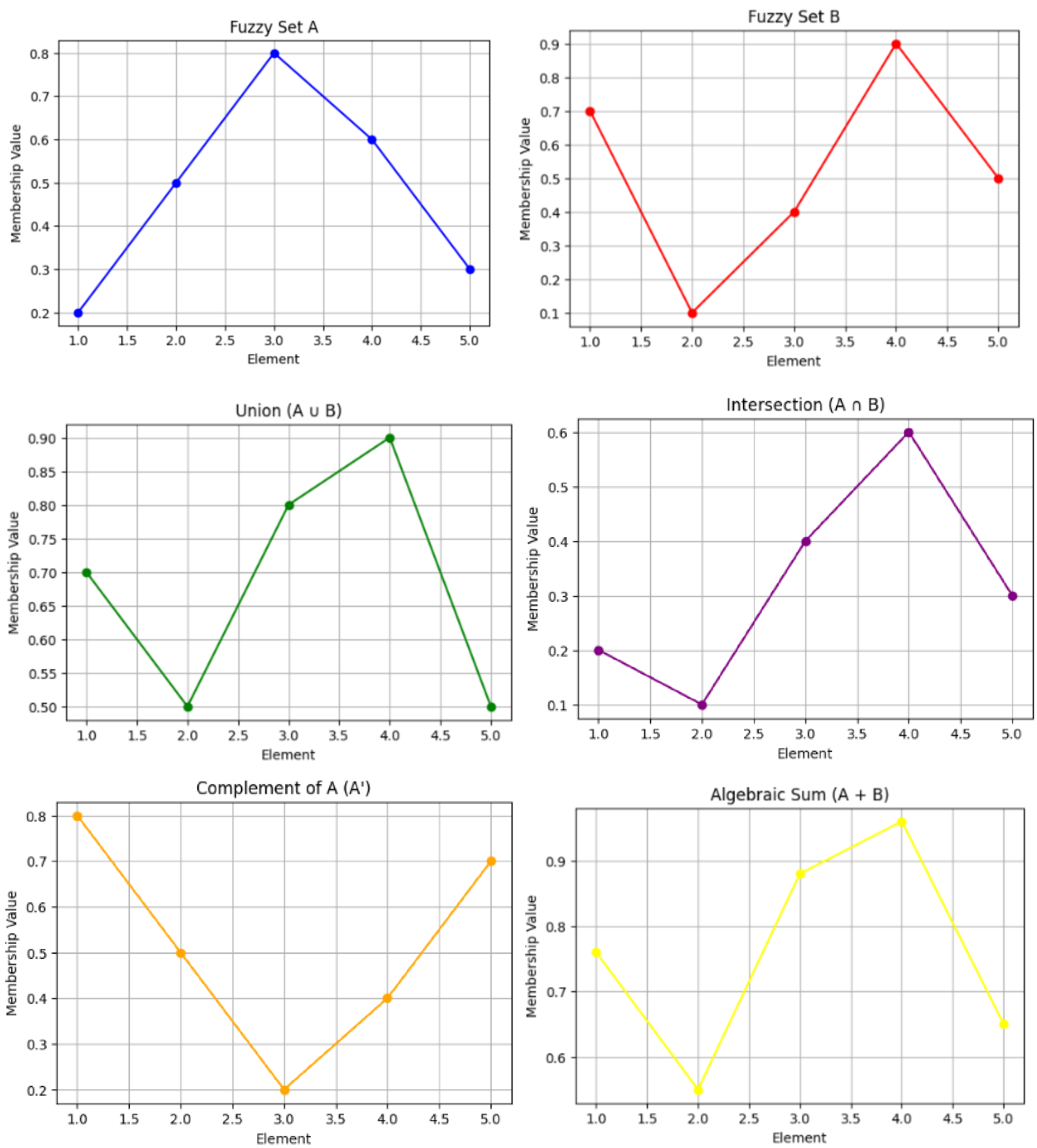
$$\mu_{A-B}(x) = \max(0, \mu_A(x) - \mu_B(x))$$

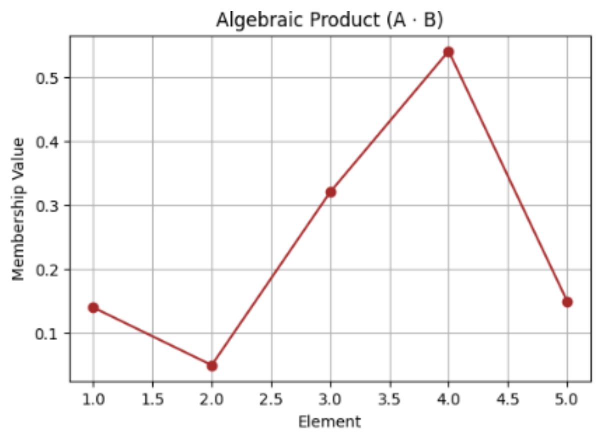
CODE:

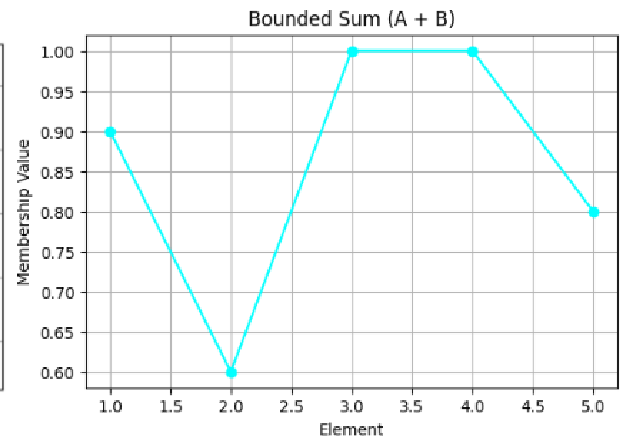
```
import matplotlib.pyplot as plt

# Define new fuzzy sets
x = [1, 2, 3, 4, 5]
A = [0.2, 0.5, 0.8, 0.6, 0.3]
B = [0.7, 0.1, 0.4, 0.9, 0.5]
```

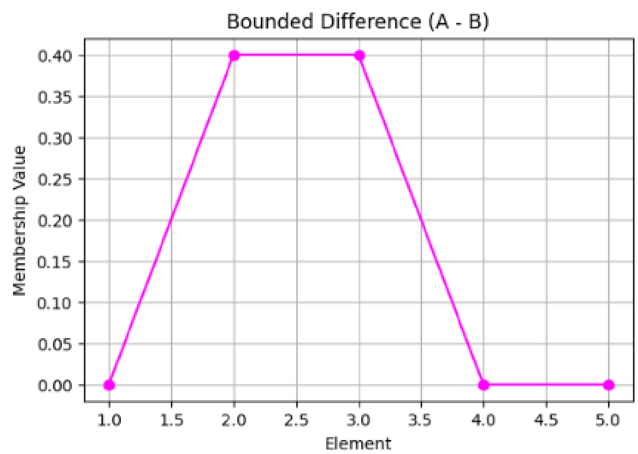
```python
# Calculate Union, Intersection, Complement, Algebraic Product, Bounded
Sum, Bounded Difference
A_union_B = [max(a, b) for a, b in zip(A, B)]
A_intersection_B = [min(a, b) for a, b in zip(A, B)]
A_complement = [1 - a for a in A]

# Algebraic Sum: algebraic sum of membership values
A_algebraic_sum_B = [a + b - a * b for a, b in zip(A, B)]

# Algebraic Product: product of membership values
A_product_B = [a * b for a, b in zip(A, B)]

# Bounded Sum: ensures membership does not exceed 1
A_plus_B = [min(1, a + b) for a, b in zip(A, B)]

# Bounded Difference: ensures result is never negative
A_minus_B = [max(0, a - b) for a, b in zip(A, B)]

# List of plots to generate (Title, Data, Color)
plots = [
    ("Fuzzy Set A", A, 'blue'),
    ("Fuzzy Set B", B, 'red'),
    ("Union (A ∪ B)", A_union_B, 'green'),
    ("Intersection (A ∩ B)", A_intersection_B, 'purple'),
    ("Complement of A (A')", A_complement, 'orange'),
    ("Algebraic Sum (A + B)", A_algebraic_sum_B, 'yellow'),
    ("Algebraic Product (A · B)", A_product_B, 'brown'),
    ("Bounded Sum (A + B)", A_plus_B, 'cyan'),
    ("Bounded Difference (A - B)", A_minus_B, 'magenta')
]

# Generate each plot
for title, y_values, color in plots:
    plt.figure(figsize=(6, 4))
    plt.plot(x, y_values, 'o-', color=color)
    plt.title(title)
    plt.xlabel('Element')
    plt.ylabel('Membership Value')
    plt.grid(True)
    plt.show()
```

OUTPUT:


Fuzzy Set A
Membership Value
Element
Fuzzy Set B
Membership Value
Element
Union (A ∪ B)
Membership Value
Element
Intersection (A ∩ B)
Membership Value
Element
Complement of A (A')
Membership Value
Element
Algebraic Sum (A + B)
Membership Value
Element







CONCLUSION:

The inclusion of the **Algebraic Sum** operation improves our understanding of how fuzzy sets combine, allowing us to better visualize their individual contributions and overlaps. Fuzzy set theory, unlike classical set theory, allows for partial membership, where elements can belong to multiple sets to varying degrees. The Algebraic Sum enhances this by aggregating memberships from different sets in a way that reflects both their individual and shared characteristics.

This experiment highlights how fuzzy set theory is particularly useful for modeling **uncertainty** and **imprecision**, offering a flexible approach to situations where categories are not strictly defined. By applying operations like the Algebraic Sum, we can more accurately represent complex relationships, such as in **decision-making** and **control systems**, where partial membership is key. Ultimately, this demonstrates the power of fuzzy set theory in capturing the subtleties of real-world data, where uncertainty is often inherent.