Report - Question 1

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1 Laplace Distribution

1.1 Probability Distribution Function

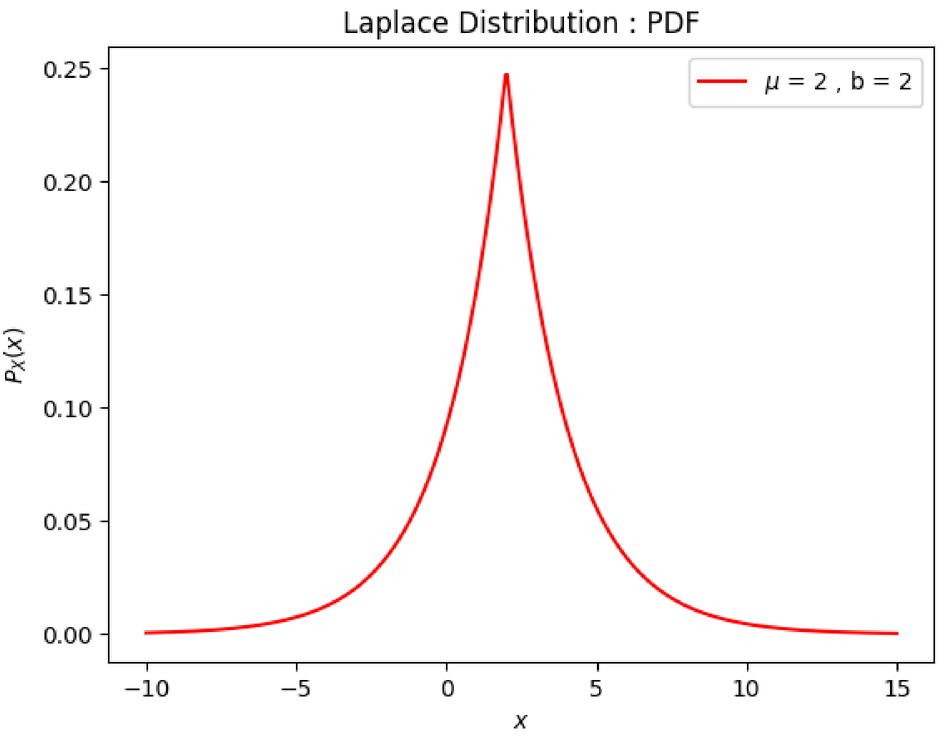


Figure 1: Laplace Distribution - PDF (based on analytical expression)

The PDF for Laplace Distribution is given by -

$$P_X(x) = \frac{e^{-\frac{|x-\mu|}{b}}}{2b} \tag{1}$$

We plotted this PDF using the above equation analytically.

1.2 Cumulative Distribution Function

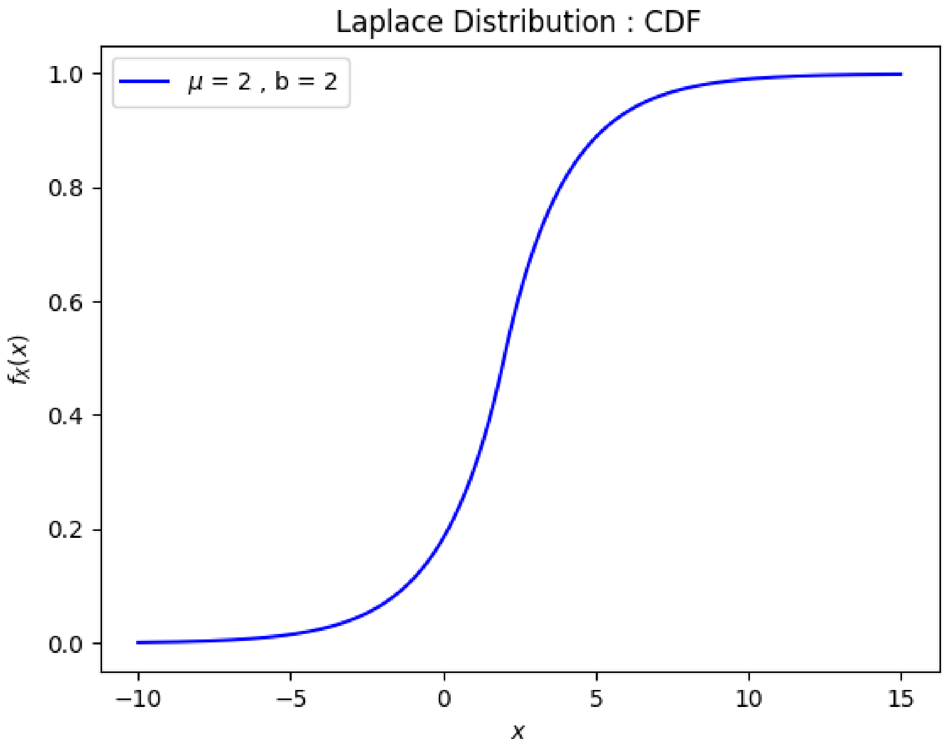


Figure 2: Laplace Distribution - CDF (using Riemann-sum approximation)

The CDF can be written as the integral of PDF. We used Riemann Sum approximation for calculating the integral for CDF. The CDF is defined as -

$$f_X(x) = \int_{-\infty}^{x} P_X(t)dt \tag{2}$$

Using Riemann Sum approximation for evaluating the above integral,

$$f_X(x) = \sum_{i=1}^{n} P_X(c_i)\Delta x_i \tag{3}$$

We plotted the CDF as the above summation for very small intervals of Δx_i's and then plotted the graph of CDF.

1.3 Variance

We computed the expectation E(X) as,

$$E(X) = \int xP(X)dx \tag{4}$$

The Variance can be calculated by using any of these formulas for Variance -

$$Var(X) = E[X - (E[X])^2] \tag{5}$$

$$Var(X) = E[X^2] - (E[X])^2 \tag{6}$$

$$Var(X) = \int (X - E(X))^2 P(X)dx \tag{7}$$

We used the expectation calculated by a function and the eq(7) for finding the Variance (both using Riemann Sum approximations) to calculate the Variance. The code for Computing Variance using Riemann Sum is given in the code folder. Here is the result that we got for Variance of Laplace Distribution-

```
Variance (using Reimann Sum) :  8.0000000000037
Variance (theoretically)     :  8
Tolerance                    :  3.700151296470722e-12
```

Figure 3: Variance for Laplace Distribution and its comparison with theoretical value

The theoretical expression for Variance of Laplace Distribution is -

$$Var(X) = 2b^2 \tag{8}$$

2 Gumbel Distribution

2.1 Probability Distribution Function

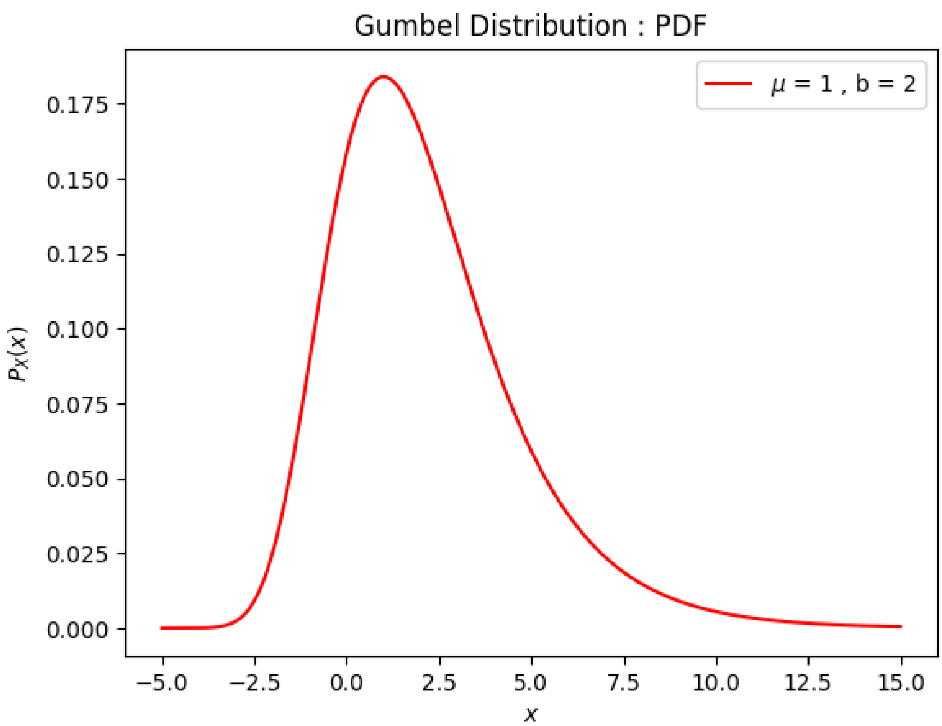


Figure 4: Gumbel Distribution - PDF (based on analytical expression)

The PDF for Gumbel Distribution is given by -

$$P_X(x) = \frac{e^{-(z+e^{-z})}}{\beta} \tag{9}$$

We plotted this PDF using the above equation analytically.

2.2 Cumulative Distribution Function

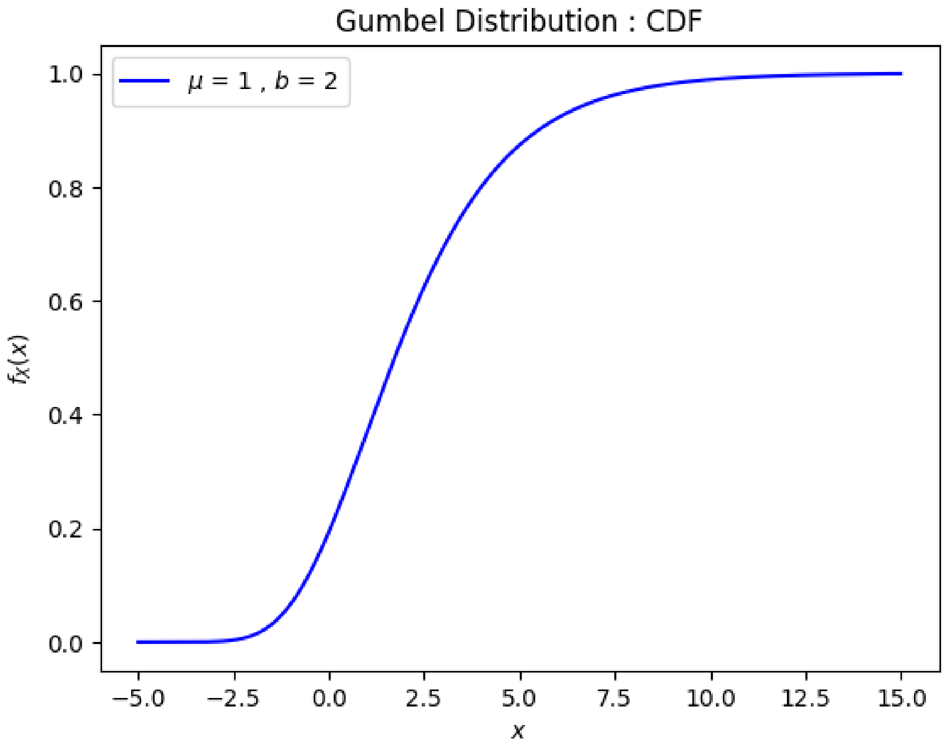
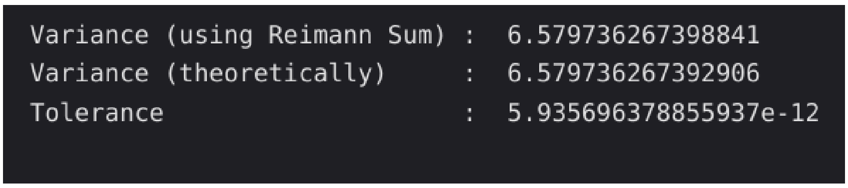


Figure 5: Gumbel Distribution - CDF (using Riemann-sum approximation)

Here also, we computed the CDF as integral of PDF using Riemann Sum approximation.

2.3 Variance

Variance is also calculated here similar to previous case of Laplace Distribution using Riemann Sum. The value of Variance we got is -



```
Variance (using Reimann Sum) :  6.579736267398841
Variance (theoretically)     :  6.579736267392906
Tolerance                    :  5.935696378855937e-12
```

Figure 6: Variance for Gumbel Distribution and its comparison with theoretical value

The Theoretical formula for Variance of Gumbel is given by -

$$Var(X) = \frac{\pi^2 \beta^2}{6} \tag{10}$$

3 Cauchy Distribution

3.1 Probability Distribution Function

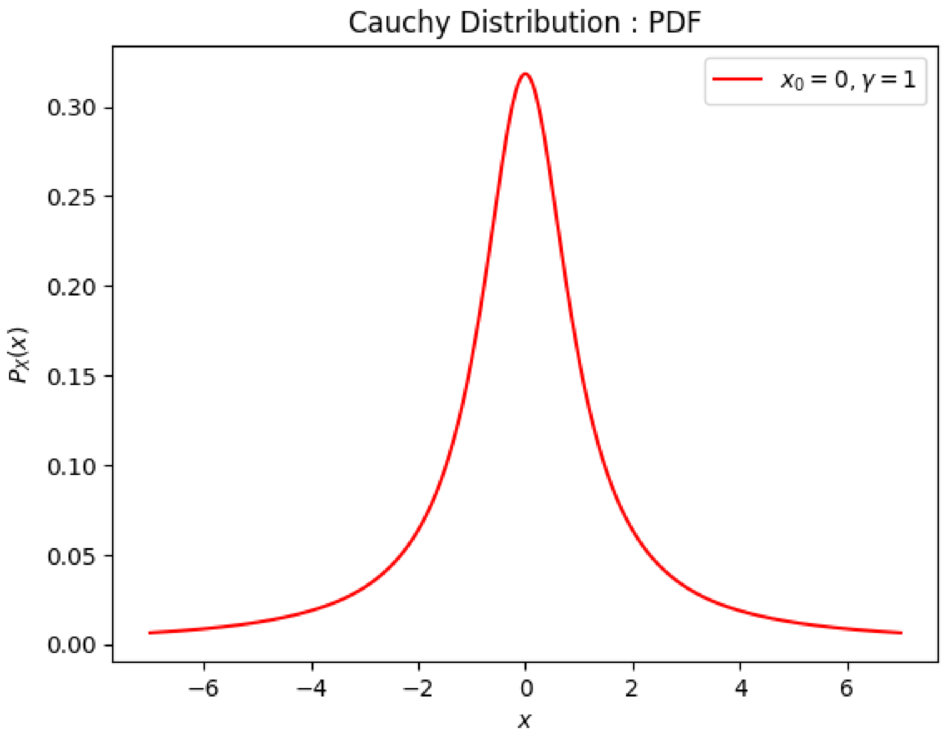


Figure 7: Cauchy Distribution - PDF (based on analytical expression)

The PDF for Cauchy Distribution is given by -

$$P_X(x) = \frac{1}{\pi\gamma[1 + (\frac{x - x_0}{\gamma})^2]} \tag{11}$$

We plotted this PDF using the above equation analytically.

3.2 Cumulative Distribution Function

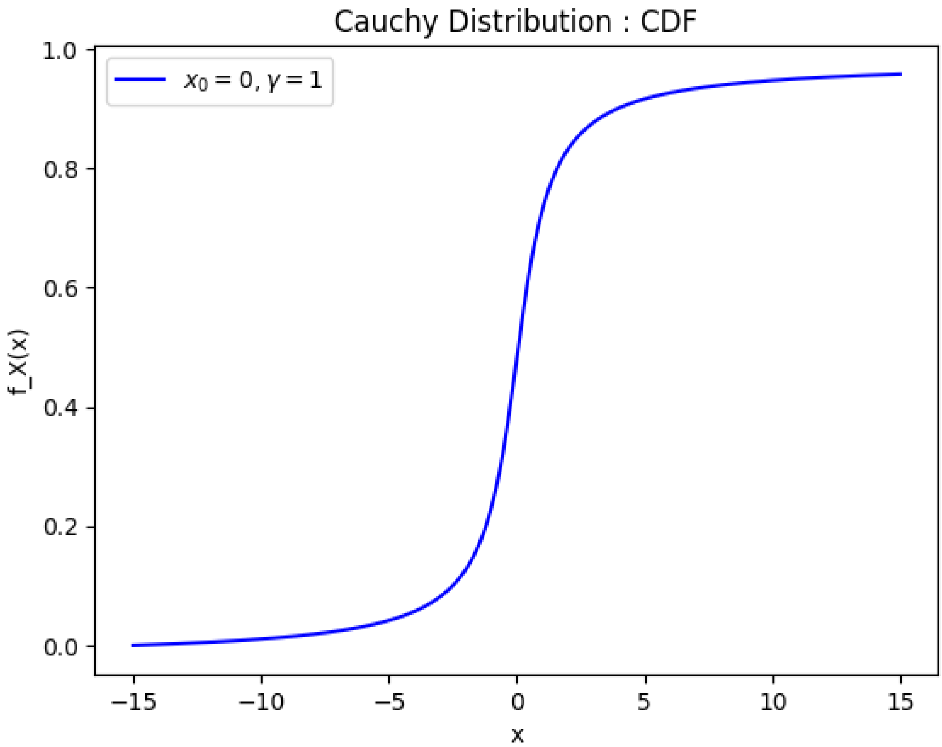


Figure 8: Cauchy Distribution - CDF (using Riemann-sum approximation)

The CDF is calculated as integral of PDF using Riemann Sum approximation.

3.3 Variance

The Variance for Cauchy Distribution is undefined as its Mean is undefined. That is because it diverges.