Question 6
PCA for Another Image Dataset

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1 Idea

For given question we can say that,
for a given fruit each color of each pixel is a random variable and each instance of those images corresponding to that digit are the values drawn from the random variable in 19200 dimensional space.
That is there are total of 784 random variables for a fruit and total 16 instances of fruit are given to us.

- calculating mean(μ) of images is same as calculating mean of each color of each pixel(i.e. mean of each RV).
- to calculate covariance matrix(C) is same as calculating covariance of any two colors from the vector and inserting it in appropriate matrix place.
- Since covariance matrix is a 19200×19200 real valued symmetric matrix, then according to spectral theorem it has 19200 real eigenvalues and 19200 real valued eigenvectors.
- get the top 4 eigenvalues and corresponding eigenvectors and display the image of mean alongwith these eigenvectors.
- to find closest representation of an image as linear combination of top 4 eigenvectors by minimising Frobenius norm of difference between the representative image and original image(this is taken as measure of closeness).
- To generate new representative images, use random numbers of Gaussian distribution and do linear combination of $\sqrt{\lambda_i} v_i$ multiplied by random numbers.

2 Implementation

First convert every $80 \times 80 \times 3$ integer data to floating-point data
Then reshape each image to a column vector of size 19200.

2.1 To get mean Image of fruits

- Create a mean vector of size 19200
- Get sum of all the image vectors and divide it by 16 to get mean vector.
- Suppose $I_1, I_2, ..., I_{16}$ are the 16 image vector so mean vector of fruits will be

$$\mu = \frac{\sum_{i=1}^{16} I_i}{16} \tag{1}$$

  μ is mean vector of size 19200.

2.2 To get covariance matrix for fruits images

- First step is to store all images corresponding of fruits inside a matrix of size 19200×16. Lets call this matrix D.
- We can calculate covariance of two vector given μ vector by following formula.

$$C = \frac{(X - \mu)(Y - \mu)^T}{N} \tag{2}$$

- By above method we can calculate each entry of covariance matrix of fruits as follows(given below formula calculates entry of covariance matrix for digit 1. Where $X_i and X_j$ are i^{th} and j^{th} column vector of matrix D)

$$C_n(i, j) = \frac{(X_i - \mu'_{ni})(X_j - \mu'_{nj})^T}{N} \tag{3}$$

  where, X_i and X_j are the vectors of size 1×16 and μ'_{ni} and μ'_{nj} are also vectors of same size with each entry of these vectors equal to i^{th} and j^{th} value of μ_n of fruits.

2.3 To get top 4 eigenvectors

- Since covariance matrix is 19200×19200 real valued symmetric matrix, so according to spectral theorem it has 19200 real eigenvalues and 19200 real valued eigenvectors.
- using eigs function in matlab we get diagonal eigenvalue matrix with values arranged in descending order nad corresponding eigenvectors.
- take top 4 eigenvectors from these matrix.

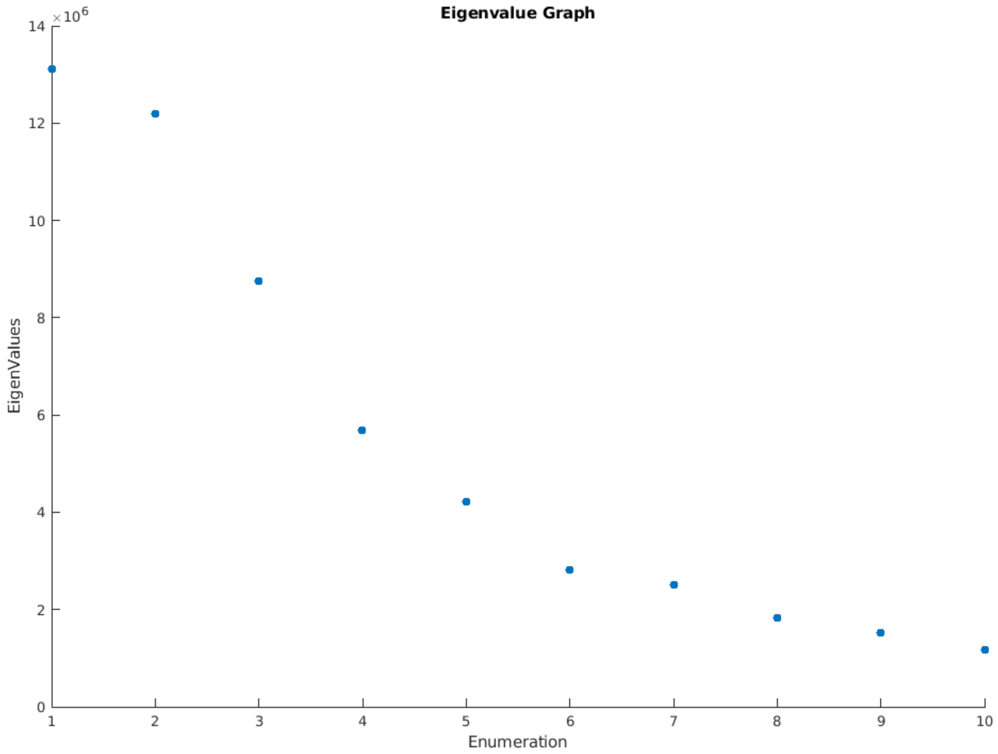

Figure 1: EigenValue graph of top 10 Eigenvalue

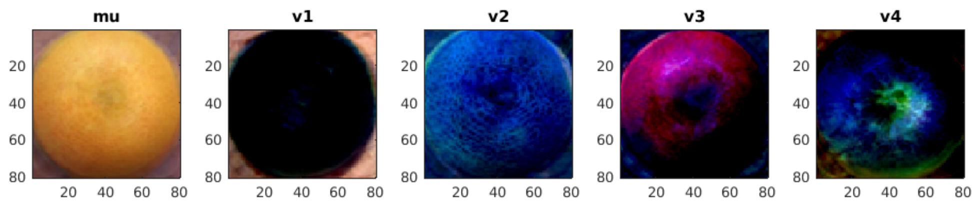

Figure 2: Image of mean and top 4 eigenvectors

2.4 To get closest representation of images

- let $p = \mu + a.v_1 + b.v_2 + c.v_3 + d.v_4$ be the closest representation of image.
- Now by minimising the measure of closeness(here it is Frobenius norm of difference between p and real image) we will get representative image of the given image
- let F be the Frobenius norm of difference between p and X(real image vector) $= ||p - X|| = |p - X|$
- since $v_1, v_2, ..., v_{19200}$ are the orthonormal eigenvectors of covariance matrix, we can write $p - x$ as ,
$$p - x = \sum_{i=1}^{19200} ((p - x).v_1)v_1 \tag{4}$$
So $|p - x| = \sum_{i=1}^{19200} \sqrt{((p - x).v_i)^2}$, since $v_i.v_j = 0$ for $i \neq j$ and 1 for $i = j$.
$(p - x).v_1 = (\mu - x).v_1 + a$
$(p - x).v_2 = (\mu - x).v_2 + b$
$(p - x).v_3 = (\mu - x).v_3 + c$
$(p - x).v_4 = (\mu - x).v_4 + d$
$(p - x).v_5 = (\mu - x).v_5$
.
.
.
$(p - x).v_{19200} = (\mu - x).v_{19200}$
- we can clearly see that the terms after 4^{th} equation in above sequence of equation are constants so to minimize —p - x— we need to equate first four terms to zero.
- thus we get,
$a = (x - \mu).v_1$
$b = (x - \mu).v_2$
$c = (x - \mu).v_3$
$d = (x - \mu).v_4$
thus the closest representation of image is ,
$$p = \mu + ((x - \mu).v_1).v_1 + ((x - \mu).v_2).v_2 + ((x - \mu).v_3).v_3 + ((x - \mu).v_4).v_4 \tag{5}$$

Below are the images of closest representation of the images for all Fruit images -

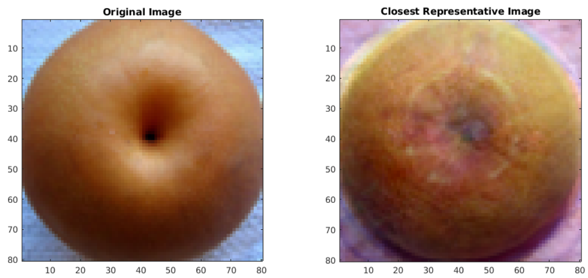

Figure 3: Image of fruit 1

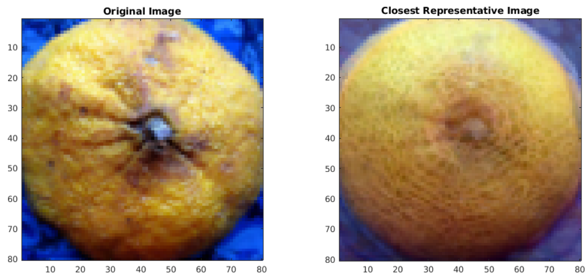

Figure 4: Image of fruit 2

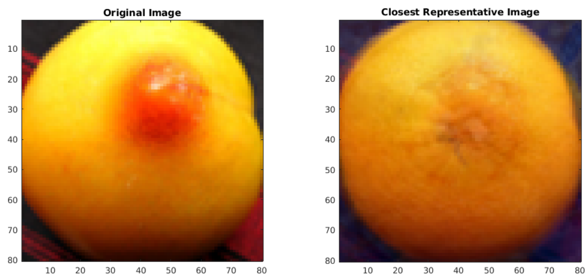

Figure 5: Image of fruit 3

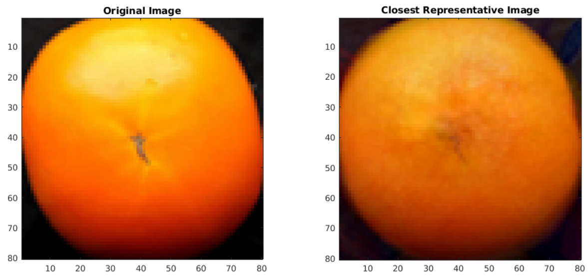

Figure 6: Image of fruit 4

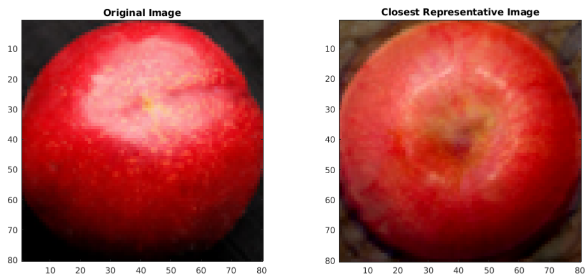

Figure 7: Image of fruit 5

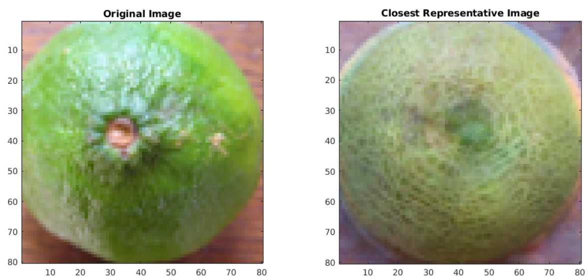

Figure 8: Image of fruit 6

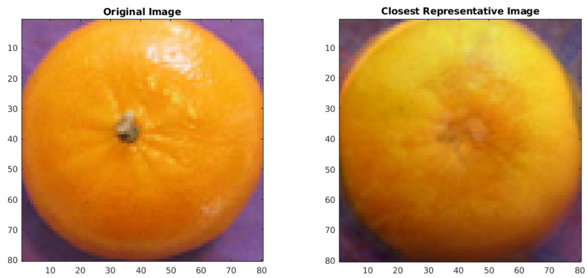

Figure 9: Image of fruit 7

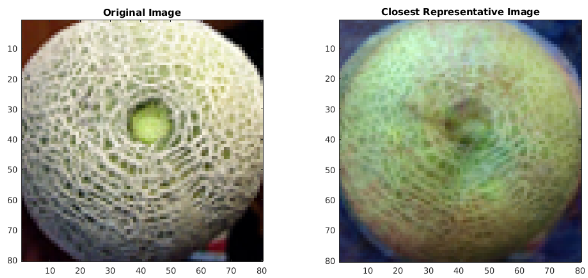

Figure 10: Image of fruit 8

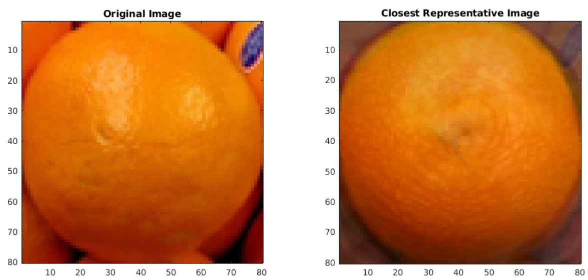

Figure 11: Image of fruit 9

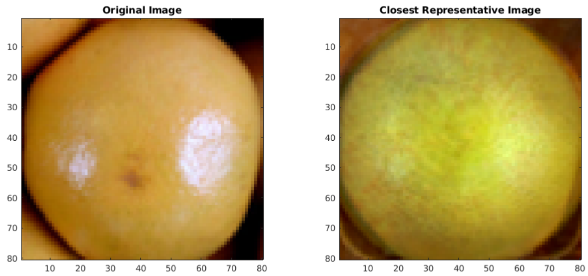

Figure 12: Image of fruit 10

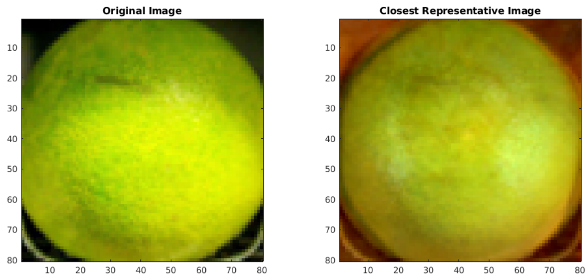

Figure 13: Image of fruit 11

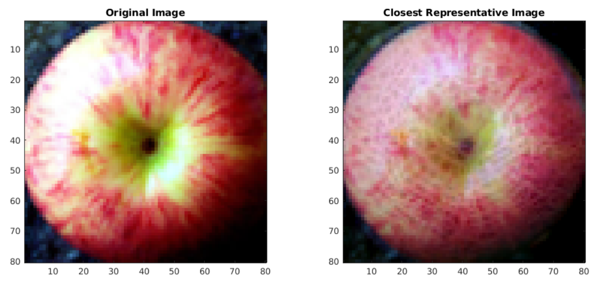

Figure 14: Image of fruit 12

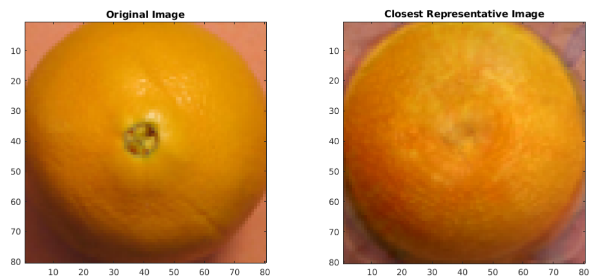

Figure 15: Image of fruit 13

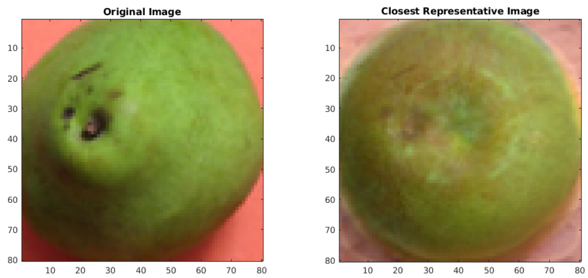

Figure 16: Image of fruit 14

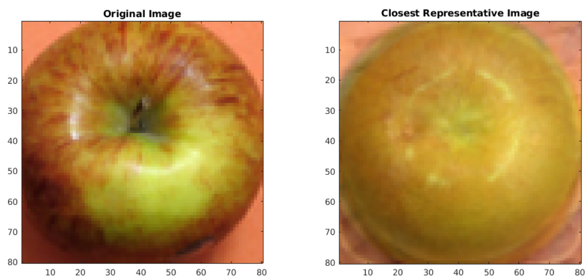

Figure 17: Image of fruit 15

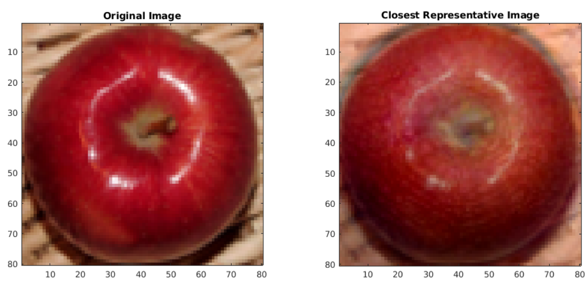

Figure 18: Image of fruit 16

3 Generating new images of fruit

- Generating new images around mean using linear combination of eigenvectors is used to create images that represent the whole set of images but is not equal to any of the image. This can be said as perturbation of image around mean using 4 eigenvectors.
- Suppose $I = \mu + a.\sqrt{\lambda_1}.v_1 + b.\sqrt{\lambda_2}.v_2 + c.\sqrt{\lambda_3}.v_3 + d.\sqrt{\lambda_4}.v_4$ is our new image.
- the values of a, b, c and d are obtained from standard normal distribution.

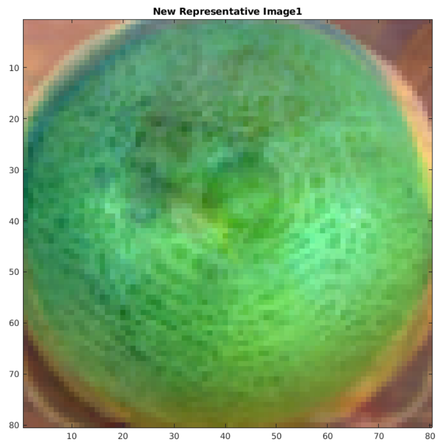

Figure 19: New Representative image 1

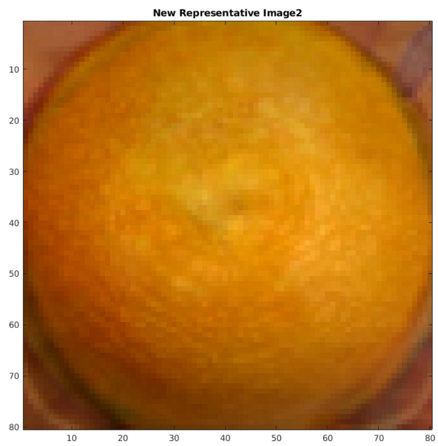

Figure 20: New Representative image 2

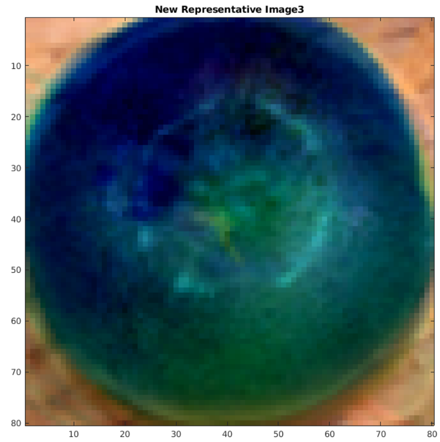

Figure 21: New Representative image 3