Question 3

CS663 (Digital Image Processing) Assignment 2

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Contents

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PART

I

Problem 1
Consider a clean image $I(x, y)$ which gets corrupted by additive noise randomly and independently from a zero mean Gaussian distribution with standard deviation σ. Derive an expression for the PDF of the resulting noisy image. Assume continuous-valued intensities. [10 points]

Section 1

PDF Derivation of Noisy Image

Given a clean Image $I(x, y)$, after it gets corrupted by a Gaussian noise $\eta(x, y)$ randomly and independently, the noisy image we have is

$$I_{noisy}(x, y) = I(x, y) + \eta(x, y), \quad \eta \sim N(0, \sigma)$$

Formulae

Gaussian distribution with mean μ, standard deviation σ :

$$G(x; \mu, \sigma) = \frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

Putting $\mu = 0$, we have

$$N(0, \sigma) = \frac{e^{\frac{-x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

Let $p_I(i)$ be the PDF for the clean image, and $p_{noisy}(j)$ be the PDF of noisy image, we have

$$p_{noisy}(j) = P(I_{noisy} = j) = P(I + \eta = j) = P(I = j - k, \eta = k)$$

Since, it is given that noise is added independently,

$$
\begin{aligned}
p_{noisy}(j) &= \int_{-\infty}^{\infty} P(I = j-k, \eta = k)\, dk \\
&= \int_{-\infty}^{\infty} P(I = j-k) P(\eta = k)\, dk \\
&= \int_{-\infty}^{\infty} p_I(j-k) p_\eta(k)\, dk \\
\therefore p_{noisy}(j) &= \int_{-\infty}^{\infty} p_I(j-k) \frac{e^{\frac{-k^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}\, dk
\end{aligned}
$$

Thus, $p_{noisy}(j)$ is the convolution of the PDF of the clean image and PDF of the Gaussian Noise