Question 4

CS663 (Digital Image Processing) Assignment 3

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Question 4

Problem 1
Consider a 201×201 image whose pixels are all black except for the central row (i.e. row index 101 beginning from 1 to 201) in which all pixels have the value 255. Derive the Fourier transform of this image analytically, and also plot the logarithm of its Fourier magnitude using `fft2` and `fftshift` in `MATLAB`. Use appropriate colorbars. [8+2=10 points]

Section 1

Fourier transform of the image

By definition, the 2D DFT of a image f(x,y) of size W_1 x W_2 is given by:

Formulae

$$F(u,v) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y) e^{-j2\pi(\frac{ux}{W_1} + \frac{vy}{W_2})}$$

In this case, $W_1 = W_2 = 201$. Substituting the given image values into the formula, we get:

$$F(u,v) = \frac{1}{\sqrt{201 * 201}} \sum_{x=0}^{200} \sum_{y=0}^{200} f(x,y) e^{-j2\pi(\frac{ux}{201} + \frac{vy}{201})}$$

$$F(u,v) = \frac{1}{201} \sum_{x=0}^{200} \sum_{y=0}^{200} f(x,y) e^{-j2\pi(\frac{ux}{201} + \frac{vy}{201})}$$

In an image, at black pixels, f(x, y) = 0.
Now since the image has all black pixels except for the central row (row index 101) where all pixels have the value 255, we can write

$$f(x,y) = 0 \qquad \forall y, \forall x \in \{1,2,...,201\} - \{101\}$$

$$f(101, y) = 255 \qquad \forall y \in \{1,2,...,201\}$$

Therefore, the DFT of this image can be represented as:

$$F(u,v) = \frac{1}{201}\sum_{y=0}^{200} f(101,y)e^{-j2\pi(\frac{u101}{201}+\frac{vy}{201})}$$

$$F(u,v) = \frac{1}{201}\sum_{y=0}^{200} 255e^{-j2\pi(\frac{u101}{201}+\frac{vy}{201})}$$

$$F(u,v) = \frac{255}{201}e^{-j2\pi(\frac{u101}{201})}\sum_{y=0}^{200} e^{-j2\pi(\frac{vy}{201})}$$

Now we can see that $\sum_{y=0}^{200} e^{-j2\pi(\frac{vy}{201})}$ is a geometric series with 201 terms, first term as 1 and a common ratio as $e^{-j2\pi(\frac{v}{201})}$.
The sum of a geometric series with 'n' terms, first term as 'a' and a common ratio 'r' is given by:

Formulae Sum of geometric series:

$$sum = \frac{a(1-r^n)}{1-r}$$

Applying this formula to our sum, we get:

$$\sum_{y=0}^{200} e^{-j2\pi(\frac{vy}{201})} = \frac{1-e^{-j2\pi(\frac{v201}{201})}}{1-e^{-j2\pi(\frac{v}{201})}} = \frac{1-e^{-j2\pi v}}{1-e^{-j2\pi(\frac{v}{201})}}$$

Therefore we can write our DFT as

$$F(u,v) = \frac{255}{201}e^{-j2\pi(\frac{u101}{201})}\frac{1-e^{-j2\pi v}}{1-e^{-j2\pi(\frac{v}{201})}}$$

Section 2

plot of the logarithm of Fourier magnitude

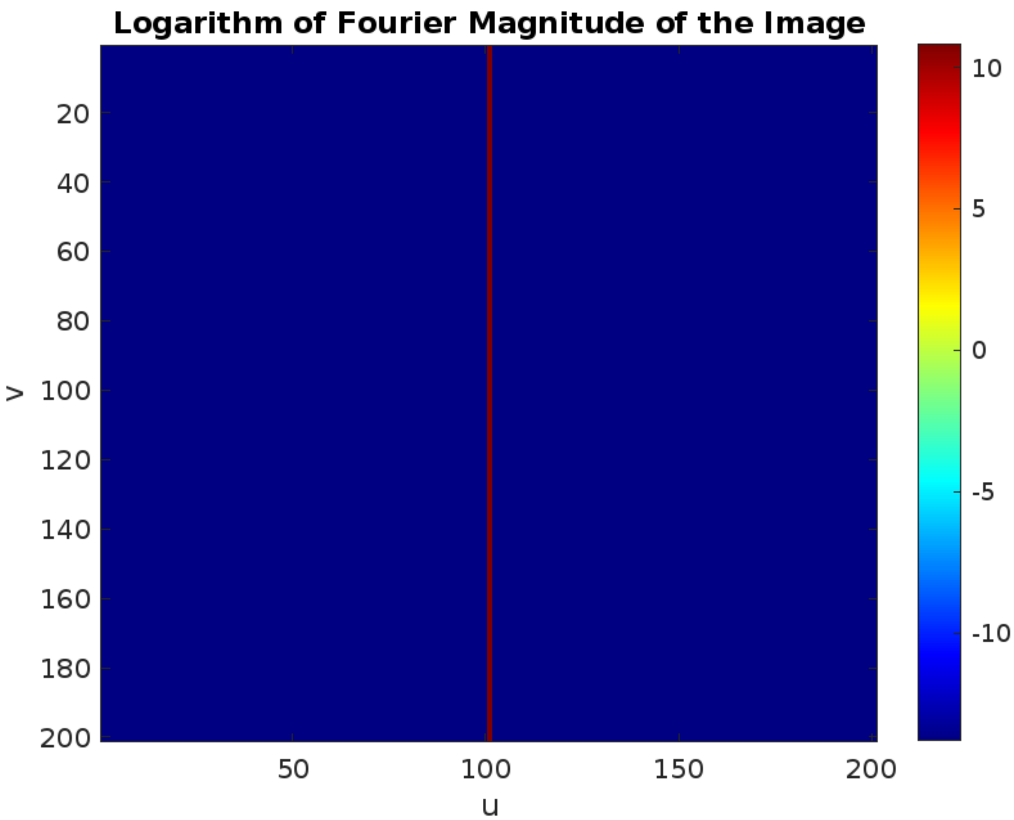

Figure 1. plot of the logarithm of the Fourier magnitude of the given image