Question 5

CS663 (Digital Image Processing) Assignment 2

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Question 5

Problem 1

Suppose I convolve an image f with a mean-filter of size $(2a+1)\times(2a+1)$ where $a > 0$ is an integer to produce a result f_1. Suppose I convolve the resultant image f_1 with the same mean filter once again to produce an image f_2, and so on until you get image f_K in the Kth iteration. Can you express f_K as a convolution of f with some kernel. If not, why not? If yes, with what kernel? Justify. [10 points]

Section 1

Expressing f_K as a convolution of f

On applying mean-filter M of size $(2a+1)\times(2a+1)$ to an image f, the resulting image f_1 is (class slides):

$$f_1(x,y) = \frac{1}{(2a+1)^2}\sum_{i=-a}^{a}\sum_{j=-a}^{a} f(x+j, y+i)$$

which can be written in terms of convolution of f with a mask M,

$$f_1 = M * f$$

where

$$M = \frac{1}{(2a+1)^2}\begin{bmatrix} 1 & 1 & \dots & 1 & 1 \\ 1 & 1 & \dots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix}_{(2a+1)\times(2a+1)}$$

Now, when we convolve the image f_1 with the same M, we get

$$f_2 = M * f_1 = M * (M * f)$$

Using the **Associativity** property of Convolution, we can write $f2$ as

$$f_2 = (M * M) * f$$

Let a mask $M_2 = M * M$, so the above expression for f_2 can be seen as taking convolution of f with this mean filter,

$$f_2 = M_2 * f$$

Let $M_1 = M$, we have $f_1 = M_1 * f$ and $f_2 = M_2 * f$. Now, when the same mask M is applied K times, using induction, let $f_{K-1} = M_{K-1} * f$ where $M_{K-1} = (M * M * \ldots * M)|_{K-1}$ times. So, again by using **associative** property for convolution,

$$f_K = M * f_{K-1} = M * (M_{K-1} * f) = (M * M_{K-1}) * f$$

Let $M_K = M * M_{K-1}$, which is essentially $M_K = (M * M * \ldots * M)|_K$ times (using associativity), then we can express the final image f_K as a convolution of f with the kernel M_k -

$$f_K = M_K * f$$

Answer Therefore, yes, we can express f_K as a convolution of f with some kernel. The kernel is $M_K = (M * M * \ldots * M)|_K$ times

Remark We used the **Associative** property of convolution. The proof of which was linked in the class slides and can be found here