Question 4

CS663 (Digital Image Processing) Assignment 1

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SECTION 1

Part (c) - Plots

The following are plots obtained for NCC, Joint Entropy and QMI. The equations used for them are -

Formulae

$$NCC = \left| \frac{\sum_{(x,y)\in\Omega}(I_1(x,y) - \bar{I}_1)(I_2(x,y) - \bar{I}_2)}{\sqrt{\sum_{(x,y)\in\Omega}(I_1(x,y) - \bar{I}_1)^2 \sum_{(x,y)\in\Omega}(I_2(x,y) - \bar{I}_2)^2}} \right|$$

$$JE = -\sum_{i_1\in\Omega_1}\sum_{i_2\in\Omega_2} p(I_1 = i_1, I_2 = i_2)\log_2 p(I_1 = i_1, I_2 = i_2)$$

$$QMI = \sum_{i_1}\sum_{i_2}(p_{I_1 I_2}(i_1, i_2) - p_{I_1}(i_1)p_{I_2}(i_2))^2$$

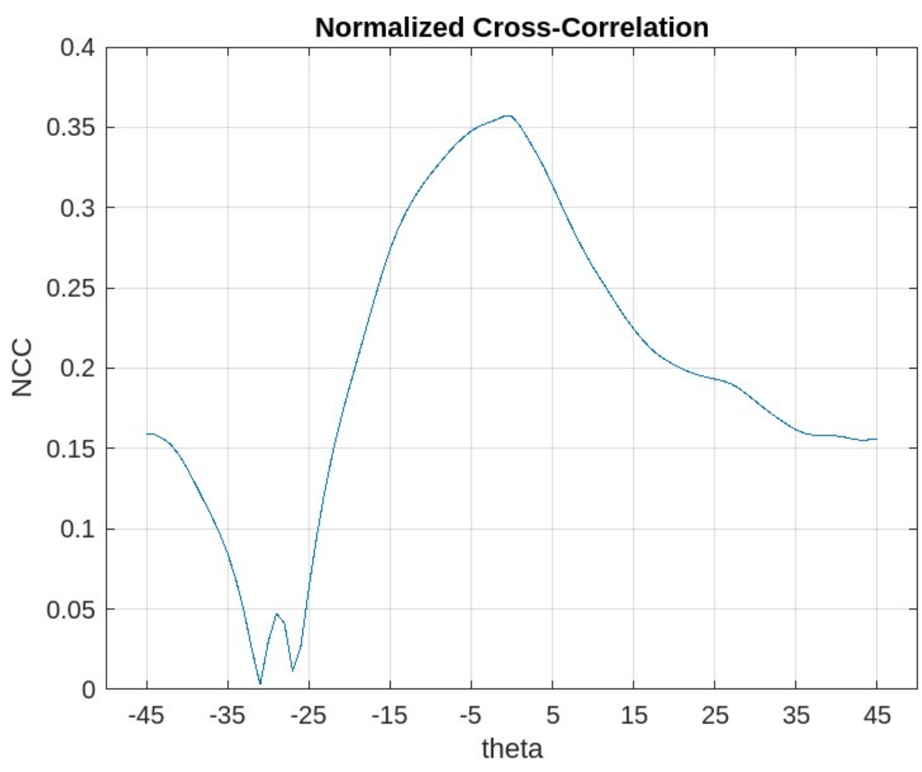

Figure 1. Plot of `NCC` vs θ

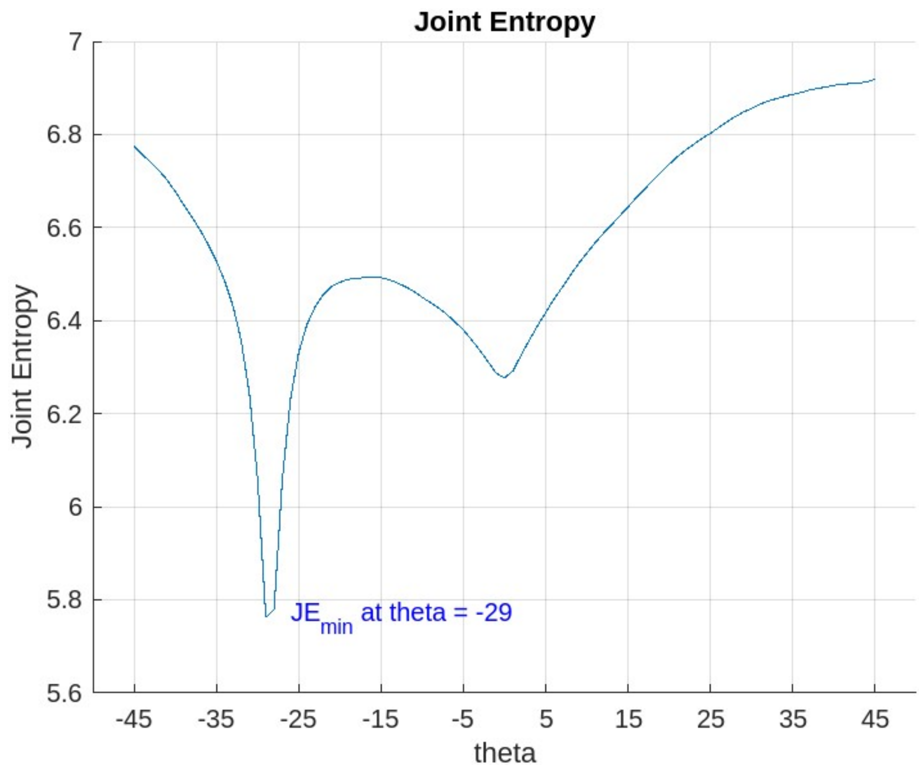

Figure 2. Plot of Joint Entropy vs θ

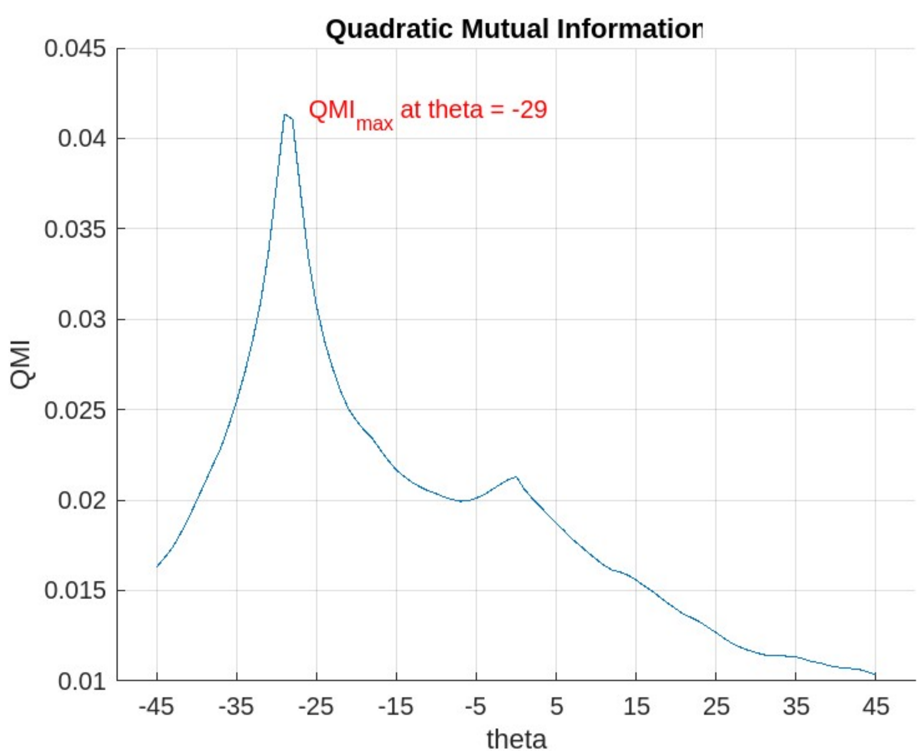

Figure 3. Plot of `QMI` vs θ

Section 2

Part (d) - Optimal Rotation and Observations

The following are the optimal rotation angle and the plot observations that we obtained using each of the 3 measures.

Remark Note that as we rotated the image J2 by $+28.5°$ ($+$ refers to anti-clockwise rotation and $-$ refers to clockwise) to get J3, therefore the correct rotation angle should be -28.5 ideally.

Subsection 2.1

Using NCC

- θ at which NCC maximizes is the optimal rotation angle via NCC.
- From the plot of NCC in figure 1, the maxima is around $\theta = 0°$. But, obviously $0°$ is not the correct rotation as it means that J3 is not rotated at all with respect to J1.
- Even if the global maxima is not correct here, we also observe a local maxima of NCC at $\theta = -29°$, which is close to the correct value.
- But since the global maxima is providing the incorrect answer, we observe that NCC is **not** a good metric for image alignment in this case.

Subsection 2.2

Using Joint Entropy

- θ at which Joint Entropy is minimized is the optimal rotation angle.
- From the plot of Joint Entropy in figure 2, we can see that JE is minimized at $\theta = -29°$, which is very close to the correct value of $-28.5°$.
- Note that we brute-forced for θ with a step-size of $1°$, so anyway getting $\theta = -28.5°$ exactly was not possible.
- Hence, this solution is **correct** (up to precision of $1°$, which is caused due to the step-size taken) and thus JE is a good metric here.

Subsection 2.3

Using QMI

- θ at which QMI is maximized is the optimal rotation angle.
- From the plot of QMI in figure 3, we can see that it is maximized at $\theta = -29°$, which is very close to the correct value of $-28.5°$.
- Similar to the JE case, here also the solution is **correct** (upto precision of $1°$, which is caused due to the step-size taken) and thus QMI is also a good metric in this case.

Section 3

Part (e) - Joint Histogram

The optimal rotation angle which we got from JE is $\theta = -29°$. Corresponding Joint histogram using this angle is -

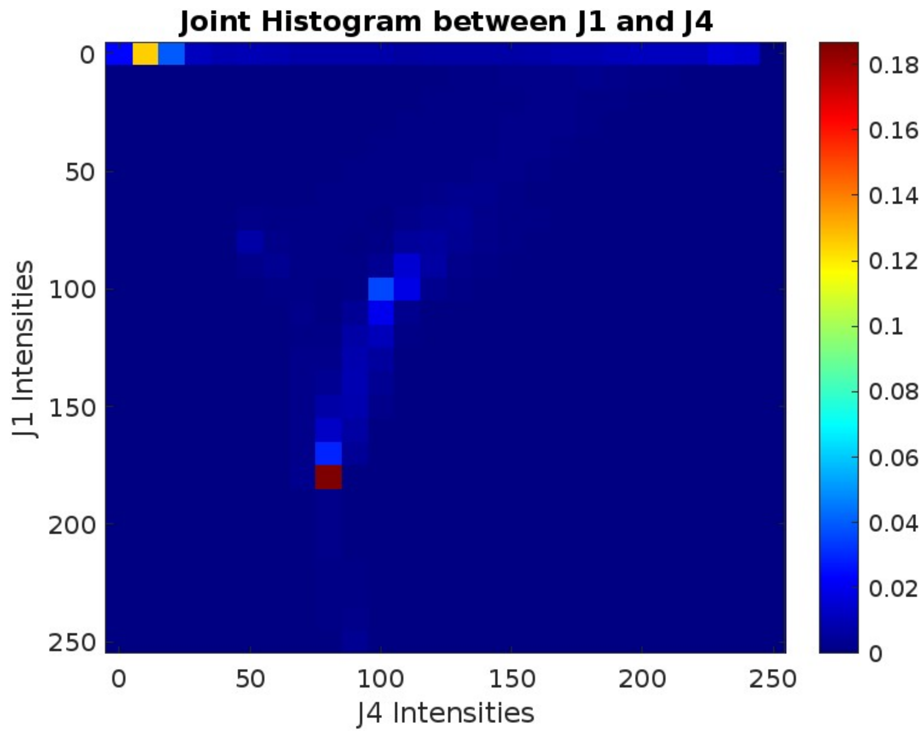

Figure 4. Joint Histogram of $J1$ and $J4$ for optimal rotation using `JE`

Section 4

Part (f) - Intuition Regarding QMI

The Quadratic Mutual Information (QMI) between two images I_1 and I_2 is defined as

Formulae

$$QMI = \sum_{i_1} \sum_{i_2} (p_{I_1 I_2}(i_1, i_2) - p_{I_1}(i_1) p_{I_2}(i_2))^2$$

- It is a measure of dependence between 2 images. This is because when two images I_1 and I_2 are statistically independent, then their joint histogram $p_{I_1 I_2}(i_1, i_2)$ is equal to the product of their marginal histograms $(p_{I_1}(i_1) p_{I_2}(i_2))$.
- Now, as QMI is a measure of difference between these two quantities, more the magnitude of difference between these terms, the images would be more dependent or correlated to each other, and QMI would be higher.

- In other words, if the joint histogram significantly deviates from the product of the marginals, it indicates a dependence between the images' pixel intensities. This could mean that certain patterns, structures, or relationships exist between the images. In this case, the QMI will be higher, indicating a stronger dependence.
- On the other hand, if QMI is low, this indicates that joint histogram and product of marginals are getting close to each other which means the images are statistically independent and are not aligned with each other.

Thus, on θ which maximizes the QMI, we are getting the image which is more related/similar/depended on the other image, which means they are more **aligned** with respect to each other as compared to other rotated images.