Question 2

CS663 (Digital Image Processing) Assignment 2

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Problem 1

In bicubic interpolation, the image intensity value is expressed in the form $v(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}x^i y^j$ where a_{ij} are the coefficients of interpolation and (x,y) are spatial coordinates. This uses sixteen nearest neighbors of a point (x,y). Given the intensity values of these 16 neighbors, explain with the help of matrix-based equations, how one can determine the coefficients a_{ij} that determine the function $v(x,y)$? Why do you require 16 neighbors for determining the coefficients? [10 points]

Section 1

Determining the coefficients a_{ij}

Formulae

Image intensity value in Bicubic Interpolation is expressed as -

$$\begin{aligned} v(x,y) &= \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}x^i y^j \\ &= a_{00} + a_{10}x + a_{20}x^2 + a_{30}x^3 \\ &\quad + a_{01}y + a_{11}xy + a_{21}x^2y + a_{31}x^3y \\ &\quad + a_{02}y^2 + a_{12}xy^2 + a_{22}x^2y^2 + a_{32}x^3y^2 \\ &\quad + a_{03}y^3 + a_{13}xy^3 + a_{23}x^2y^3 + a_{33}x^3y^3 \end{aligned}$$

We have to find the values of the coefficients a_{ij}.

The above equation contains 16 unknown coefficients and hence, we need at least 16 linear equations to solve for finding them.

We have given the intensity values of 16 nearest neighbor points of (x,y), which we can use for getting 16 linear equations to find these 16 variables.

Say, the 16 intensity values given are -
$v(x_0,y_0)$, $v(x_1,y_0)$, $v(x_2,y_0)$, $v(x_3,y_0)$, $v(x_0,y_1)$, $v(x_1,y_1)$, $v(x_2,y_1)$, $v(x_3,y_1)$, $v(x_0,y_2)$, $v(x_1,y_2)$, $v(x_2,y_2)$, $v(x_3,y_2)$, $v(x_0,y_3)$, $v(x_1,y_3)$, $v(x_2,y_3)$, $v(x_3,y_3)$.

Putting them in the Bicubic interpolation equation will provide these 16 linear equations -

$$
\begin{aligned}
v(x_0, y_0) &= \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x_0^i y_0^j \\
v(x_1, y_0) &= \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x_1^i y_0^j \\
&\vdots \qquad \vdots \\
&\vdots \qquad \vdots \\
v(x_2, y_3) &= \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x_2^i y_3^j \\
v(x_3, y_3) &= \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x_3^i y_3^j
\end{aligned}
$$

Remark Note that the variables to find in the above equations are a_{ij} and the rest are just coefficients

We can represent the intensity value terms of LHS of these equations and the unknown variables a_{ij} as 16×1 dimension **column** vectors as -

$$
v = \begin{bmatrix} v(x_0, y_0) \\ v(x_1, y_0) \\ \vdots \\ \vdots \\ v(x_2, y_3) \\ v(x_3, y_3) \end{bmatrix}_{16\times 1} , \qquad a = \begin{bmatrix} a_{00} \\ a_{10} \\ \vdots \\ \vdots \\ a_{23} \\ a_{33} \end{bmatrix}_{16\times 1}
$$

We would like to represent all the 16 linear equations in the matrix-equation form of type $Aa = v$, where A is 16×16 coefficient matrix, v is the column matrix as shown above, and a is the variable column matrix which contains the variables to be found. Consider the matrix-based equation :

$$
\begin{bmatrix}
1 & x_0 & x_0^2 & x_0^3 & y_0 & x_0y_0 & x_0^2y_0 & \dots & x_0^2y_0^3 & x_0^3y_0^3 \\
1 & x_1 & x_1^2 & x_1^3 & y_0 & x_1y_0 & x_1^2y_0 & \dots & x_1^2y_0^3 & x_1^3y_0^3 \\
1 & x_2 & x_2^2 & x_2^3 & y_0 & x_0y_0 & x_2^2y_0 & \dots & x_2^2y_0^3 & x_2^3y_0^3 \\
1 & x_3 & x_3^2 & x_3^3 & y_0 & x_3y_0 & x_3^2y_0 & \dots & x_3^2y_0^3 & x_3^3y_0^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & x_2 & x_2^2 & x_2^3 & y_3 & x_2y_3 & x_2^2y_3 & \dots & x_2^2y_3^3 & x_2^3y_3^3 \\
1 & x_3 & x_3^2 & x_3^3 & y_3 & x_3y_3 & x_3^2y_3 & \dots & x_3^2y_3^3 & x_3^3y_3^3
\end{bmatrix}_{16\times 16}
\begin{bmatrix} a_{00} \\ a_{10} \\ a_{20} \\ \vdots \\ \vdots \\ a_{23} \\ a_{33} \end{bmatrix}_{16\times 1}
=
\begin{bmatrix} v(x_0, y_0) \\ v(x_1, y_0) \\ v(x_2, y_0) \\ \vdots \\ \vdots \\ v(x_2, y_3) \\ v(x_3, y_3) \end{bmatrix}_{16\times 1}
$$

$$
Aa = v \implies a = A^{-1}v
$$

So, we determined the values of column matrix a by the method shown above using the construction of appropriate matrix A, and these elements of column matrix a represent each of the original coefficients a_{ij} of Bicubic Interpolation.

Section 2

Why require 16 neighbors?

Answer The equation of $v(x, y)$ contains 16 unknown coefficients a_{ij}. If we have intensity value at a point (x, y), then putting it in the formula of Bicubic interpolation gives a linear equation with $a_{i,j}$ as variables. As we need at least 16 linear equations to solve a system with 16 unknown variables, we require at least 16 neighbors for determining the coefficients a_{ij}.