Question 4

CS663 (Digital Image Processing) Assignment 2

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PART

I

Question 4

Problem 1
Prove or disprove: (a) The Laplacian mask with a -4 in the center (see class slides) is a separable filter. (b) The Laplacian mask with a -4 in the center (see class slides) can be implemented entirely using 1D convolutions. [5+5=10 points]

Section 1

part(a)

Subsection 1.1

Separable filter

A separable filter in image processing is a filter that can be written as product of two more simple filters. Typically a 2-dimensional convolution operation is separated into two 1-dimensional filters.

Subsection 1.2

Laplacian as a separable filter

A separable matrix can be expressed as follows:

$$\begin{bmatrix} u_1v_1 & u_1v_2 & u_1v_3 \\ u_2v_1 & u_2v_2 & u_2v_3 \\ u_3v_1 & u_3v_2 & u_3v_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} * \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$

Now, if the 2-D Laplacian mask with a -4 in the center $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ can be expressed as a product of 2 matrices, it should satisfy the following equations:

$$u_1v_1 = 0 \tag{1.1}$$

$$u_1v_2 = 1 \tag{1.2}$$

$$u_1v_3 = 0 \tag{1.3}$$

$$u_2v_1 = 1 \tag{1.4}$$

We can see from equations 1.1, 1.2 and 1.3 that $u_1 \neq 0$. Then v_1 and v_3 should be 0. But then equation 1.4 will not be satisfied. Hence, the 2-D Laplacian mask with a -4 in the center cannot be written as product of two matrices and **is not a separable filter**.

SECTION 2

part(b)

The Laplacian mask with a -4 in the center **cannot be implemented** entirely using 1D convolutions.

As we proved in part(a), The Laplacian filter $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ has non-separable elements, meaning it cannot be expressed as a simple product of two 1D filters (one for the rows and one for the columns). To apply this filter as a convolution operation, we need to consider both horizontal and vertical directions simultaneously.

In contrast, a separable filter can be expressed as a product of two 1D filters, which can be applied separately in the horizontal and vertical directions, making the convolution operation more computationally efficient.