Question 2

CS663 (Digital Image Processing) Assignment 5

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SECTION 1

Part a

SUBSECTION 1.1

Procedure

The paper describes a method for image registration that uses the Fourier domain approach to match images that are translated, rotated, and scaled with respect to one another. The technique is an extension of the phase correlation technique, which is particularly insensitive to translation, rotation, scaling, and noise, and also characterized by low computational cost.

The method relies on the translation property of the Fourier transform, known as the **Fourier shift theorem**. This theorem facilitates the determination of displacement between two images that differ only by a translation. We assume the images are already in alignment with respect to rotation and scaling

Firstly, after calculating the **2D Fourier transforms** for both pictures, two complex-valued frequency domain representations, F1 and F2, are produced. By taking the element-wise product of F1 and the complex conjugate of F2, one can find the **cross-power spectrum**. The cross-power spectrum's **inverse Fourier transform** is used to compute the **phase correlation**. As a result, the spatial domain displays an impulse-like function that is almost zero everywhere but at the displacement (translation) required to correctly pair the two images. The translation vector (X, Y) that aligns the two images correlates with the location of the peak in the phase correlation function.

SUBSECTION 1.2

Time Complexity

- **Fourier Method**
  2D Fourier Transform of an NxN image is of order $O(N^2 logN)$. The cross-power spectrum calculation is of time complexity $O(N^2)$. The Inverse Fourier Transform step also takes time of order $O(N^2 logN)$. Thus, the overall time complexity of this procedure is $O(N^2 logN)$

- **Pixel-wise image comparison**
  Each pixel of Image1 is compared with each pixel of Image2 in the spatial domain. So the net time complexity is $O(N^2)$ x $O(N^2)$, which is $O(N^4)$ which is significantly larger than the Fourier method.

Section 2

Part b

If $f_2(x, y)$ is a translated and rotated replica of $f_1(x, y)$, with translation (x_0, y_0) and rotation θ_0, then the relationship between the two images can be described as:

$$f_2(x, y) = f_1(x\cos\theta_0 + y\sin\theta_0 - s_0, -x\sin\theta_0 + y\cos\theta_0 - y_0)$$

According to the Fourier translation and rotation properties, the Fourier transforms of f_1 and f_2 are related by:

$$F_2(u, v) = e^{-j2\pi(ux_0+vy_0)}F_1(u\cos\theta_0 + v\sin\theta_0, -u\sin\theta_0 + v\cos\theta_0)$$

Let M_1 and M_2 be the magnitudes of F_1 and F_2. From the above equation, we can deduce that:

$$M_2(u, v) = M_1(u\cos\theta_0 + v\sin\theta_0, -u\sin\theta_0 + v\cos\theta_0)$$

This implies that the magnitudes of both the Fourier spectra are the same but one is a rotated replica of the other. Rotational movement without translation can be deduced using the phase correlation method by representing the rotation as a translational displacement in polar coordinates:

$$M_1(p, \beta) = M_2(p, \beta - \theta_0)$$

Using phase correlation, the angle θ_0 can be easily determined.