Question 6

CS663 (Digital Image Processing) Assignment 3

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PART

I

Problem 1

If $\mathcal{F}$ is the continuous Fourier operator, prove that $\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = f(t)$. Hint: Prove that $\mathcal{F}(\mathcal{F}(f(t))) = f(-t)$ and proceed further from there. [15 points]

SECTION 1

Proof

PROOF Lets start by finding the expression for $\mathcal{F}(\mathcal{F}(f(t)))$

Using definition of Fourier Transform,

$$\mathcal{F}(f(t)) = F(\mu) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}dt$$

Now,

$$\begin{aligned}\mathcal{F}(\mathcal{F}(f(t))) &= \int_{-\infty}^{\infty} F(\mu)e^{-j2\pi\tau\mu}d\mu \\ &= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}dt\right)e^{-j2\pi\tau\mu}d\mu\end{aligned}$$

Using Fibini's theorem, we can exchange the order of integration,

$$\begin{aligned}\mathcal{F}(\mathcal{F}(f(t))) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}e^{-j2\pi\tau\mu}dtd\mu \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu(t+\tau)}dtd\mu \\ &= \int_{-\infty}^{\infty} f(t)\left(\int_{-\infty}^{\infty} e^{-j2\pi\mu(t+\tau)}d\mu\right)dt\end{aligned}$$

Since the inner integral is wrt. μ, the variables t and τ are constants for it. So, on solving the inner integral, we get,

$$\begin{aligned}\mathcal{F}(\mathcal{F}(f(t))) &= \int_{-\infty}^{\infty} f(t)\delta(t+\tau)dt \\ &= f(-\tau) \qquad \textit{(using sifting property of Dirac delta)}\end{aligned}$$

Note that since the t and τ are independent variables and they are just used to represent domains of Fourier transform, the naming of them does not matter, therefore, we can change τ to t,

$$\mathcal{F}(\mathcal{F}(f(t))) = f(-t) \tag{1.1}$$

Now consider $\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t)))))$, using result of above equation, we get,

$$\begin{aligned} \mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) &= \mathcal{F}(\mathcal{F}(f(-t))) \\ &= f(-(-t)) \\ &= f(t) \end{aligned}$$

Hence Proved

$\square$