Question 7

CS663 (Digital Image Processing) Assignment 3

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PART

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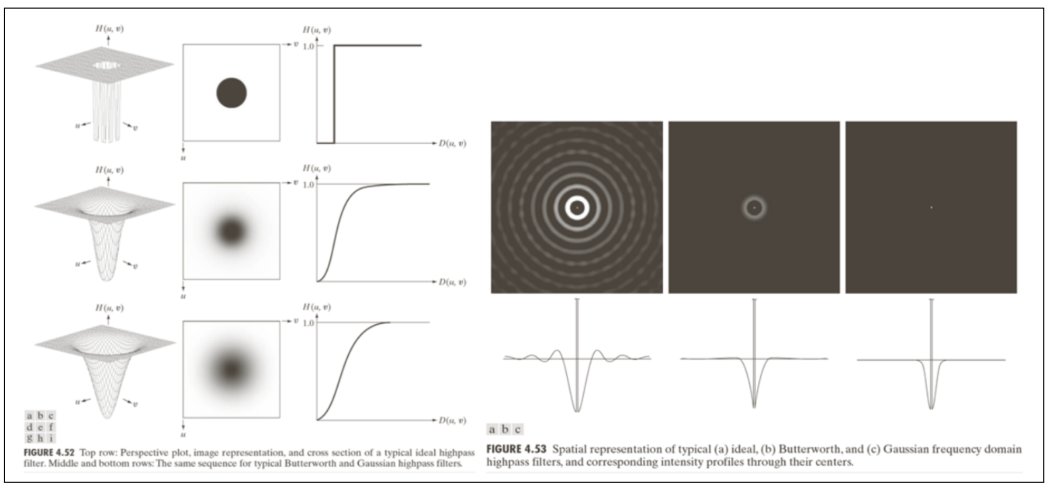
Problem 1

Provide an explanation for the presence of strong spikes in the center of the filters in the second sub-figure Of Fig. ??. Note that the fourier transform magnitudes of these filters are plotted in the first figure. [10 points]

SECTION 1

Explanation



(a) Fourier domains and Spatial domains of various filters

According to definition of high pass filters,

$$\mathcal{H}_{HP}(u,v) = 1 - \mathcal{H}_{LP}(u,v)$$

$$\therefore f(x,y) = \mathcal{F}^{-1}(\mathcal{H}_{HP}(u,v)) = \mathcal{F}^{-1}(1 - \mathcal{H}_{LP}(u,v))$$

With the help of linearity of Fourier inverse Function,

$$
\begin{aligned}
\therefore f(x,y) &= \mathcal{F}^{-1}(1) - \mathcal{F}^{-1}(\mathcal{H}_{LP}(u,v)) \\
&= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} e^{j2\pi(ux+vy)}du\right)dv - \mathcal{F}^{-1}(\mathcal{H}_{LP}(u,v)) \\
&= \left(\int_{-\infty}^{\infty} e^{j2\pi ux}du\right)\left(\int_{-\infty}^{\infty} e^{j2\pi vy}dv\right) - \mathcal{F}^{-1}(\mathcal{H}_{LP}(u,v)) \\
&= \delta(x).\delta(y) - \mathcal{F}^{-1}(\mathcal{H}_{LP}(u,v))
\end{aligned}
$$

Here, $\delta(x)$ is Dirac-delta function, which gives very large values at x=0 Thus $\delta(x).\delta(y)$ produces an unbounded value at the origin, which leads to a strong spike here. Also, $\mathcal{F}^{-1}(\mathcal{H}_{LP}(u,v))$ gives a finite value at the center.

Secondly, we can write f(x,y) in the following way -

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_{HP}(u,v)e^{j2\pi ux}e^{j2\pi vy}dudv$$

At x=0 and y=0, the above equation yields -

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_{HP}(u,v)dudv$$

Now, as $\mathcal{H}_{HP}(u,v)$ is a high-pass filter, it yields finite values for infinitely many frequencies. As f(x,y) contains an integral from $-\infty$ to ∞, f(x,y) gets a very large value at the origin (because of the Dirac-delta function), leading to spiking at the center of the filter.