Question 6

CS663 (Digital Image Processing) Assignment 2

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Contents

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PART

I

Problem 1

Consider a 1D ramp image of the form $I(x) = cx + d$ where c, d are scalar coefficients. Derive an expression for the image J which results when I is filtered by a zero-mean Gaussian with standard deviation σ. Derive an expression for the image that results when I is treated with a bilateral filter of parameters σ_s, σ_r. (Hint: in both cases, you get back the same image.) Ignore any border issues, i.e. assume the image had infinite extent. [10 points]

Section 1

part(a) Gaussian filter

We have to derive the expression for the image J when the 1D ramp image I(x) is filtered by a zero-mean Gaussian with standard deviation σ.

The 1D Gaussian filter in continuous space can be represented as:

Formulae

$$G(x, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{(-\frac{x^2}{2\sigma^2})}$$

Now, we want to filter the image I(x) = cx + d with this Gaussian filter. The final image J(x) with the Gaussian filter can be represented and calculated as follows:

$$J(x) = \frac{1}{w_g(x)} \sum (cx' + d) G(x - x', \sigma)$$

$$J(x) = \frac{1}{w_g(x)} \sum (cx' + d) (\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x')^2}{2\sigma^2}})$$

$$J(x) = \frac{1}{\sigma\sqrt{2\pi} w_g(x)} \sum (cx' + d) e^{-\frac{(x-x')^2}{2\sigma^2}}$$

Here $w_g(x)$ is written as:

$$w_g(x) = \sum \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x')^2}{2\sigma^2}}$$

$$w_g(x) = \frac{1}{\sigma\sqrt{2\pi}} \sum e^{-\frac{(x-x')^2}{2\sigma^2}}$$

Therefore, the final expression for the image J(x) when I(x) is filtered by a zero-mean Gaussian with standard deviation σ is:

Answer

$$J(x) = \frac{1}{\sum e^{-\frac{(x-x')^2}{2\sigma^2}}} \sum (cx' + d) e^{-\frac{(x-x')^2}{2\sigma^2}}$$

Section 2

part(b) Bilateral filter

Now, let's derive the expression for the image that results when I is treated with a bilateral filter of parameters σ_s and σ_r.
The final image after applying bilateral filter can be represented as:

Formulae

$$I_B(x) = \frac{1}{w_B(x)} \sum I(x') G_s(x - x', \sigma_s) G_r(I(x) - I(x'), \sigma_r)$$

$$I_B(x) = \frac{1}{w_B(x)} \sum (cx' + d)\left(\frac{1}{\sigma_s\sqrt{2\pi}} e^{-\frac{(x-x')^2}{2\sigma_s^2}}\right)\left(\frac{1}{\sigma_r\sqrt{2\pi}} e^{-\frac{(I(x)-I(x'))^2}{2\sigma_r^2}}\right)$$

$$I_B(x) = \frac{1}{w_B(x)} \sum (cx' + d)\left(\frac{1}{\sigma_s\sqrt{2\pi}} e^{-\frac{(x-x')^2}{2\sigma_s^2}}\right)\left(\frac{1}{\sigma_r\sqrt{2\pi}} e^{-\frac{(cx+d-cx'-d)^2}{2\sigma_r^2}}\right)$$

$$I_B(x) = \frac{1}{2\pi\sigma_s\sigma_r w_B(x)} \sum (cx' + d) e^{-(\frac{1}{2\sigma_s^2} + \frac{c^2}{2\sigma_r^2})(x-x')^2}$$

Here $w_B(x)$ is written as:

$$w_B(x) = \sum\left(\frac{1}{\sigma_s\sqrt{2\pi}} e^{-\frac{(x-x')^2}{2\sigma_s^2}}\right)\left(\frac{1}{\sigma_r\sqrt{2\pi}} e^{-\frac{(I(x)-I(x'))^2}{2\sigma_r^2}}\right)$$

$$w_B(x) = \frac{1}{2\pi\sigma_s\sigma_r} \sum e^{-\frac{(x-x')^2}{2\sigma_s^2}} e^{-\frac{(cx+d-cx'-d)^2}{2\sigma_r^2}}$$

$$w_B(x) = \frac{1}{2\pi\sigma_s\sigma_r} \sum e^{-(\frac{1}{2\sigma_s^2} + \frac{c^2}{2\sigma_r^2})(x-x')^2}$$

Therefore, the final image is:

$$I_B(x) = \frac{1}{e^{-(\frac{1}{2\sigma_s^2} + \frac{c^2}{2\sigma_r^2})(x-x')^2}} \sum (cx' + d) e^{-(\frac{1}{2\sigma_s^2} + \frac{c^2}{2\sigma_r^2})(x-x')^2}$$

Answer

$$I_B(x) = \frac{1}{e^{-\frac{(x-x')^2}{2\sigma_s^2}}} \sum (cx' + d) e^{-\frac{(x-x')^2}{2\sigma_s^2}}$$

where:

$$\frac{1}{\sigma'^2} = \frac{1}{\sigma_s^2} + \frac{c^2}{\sigma_r^2}$$

Both part(a) and part(b) will give same results if $\sigma' = \sigma$