Page No.
Date

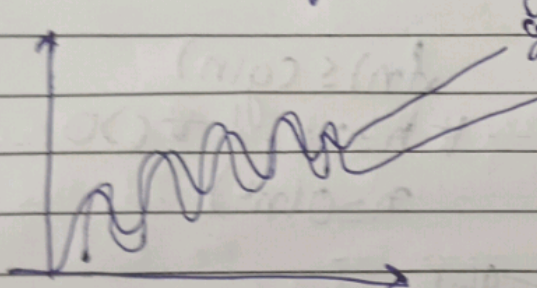
ASSIGNMENT NO:- 01

Q1.) Asymptotic Notation

→ These notations are used to tell the complexity of an algorithm when the input is very large.

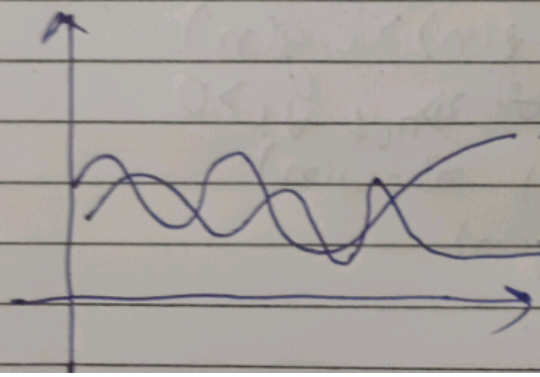
→ It describes the algorithm efficiency & performance in a meaningful way. It describes the behaviour of time or space complexity for large instance characteristics. (5 types)

(i) Big Oh Notation (O):- The function $f(n) = O(g(n))$, iff & only if there exists a +ve constant C & K such that $f(n) \leq C^* g(n)$ for all n, $n \geq K$.



$f(n) = O(g(n))$
iff
$f(n) \leq c \cdot g(n)$
$\forall n \geq n_0$,
for every constant $c > 0$

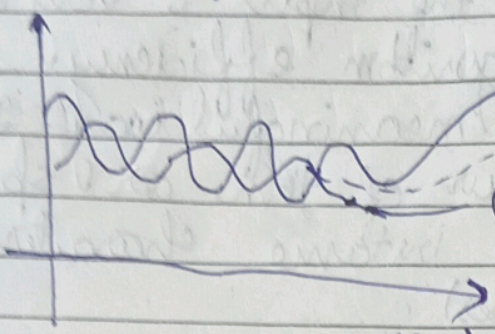
(ii) Big Omega notation (Ω):- The function $f(n) = \Omega(g(n))$ iff there exists a +ve constant C & K such that $f(n) \geq C^* g(n)$ for all n, $n \geq K$



$f(n) = \Omega g(n)$ iff
$f(n) \geq \Omega g(n)$
$\forall n \geq n_0$ & some const. $c > 0$

(iii) Big theta notation (Θ): The function $f(n) = \Theta(g(n))$, iff there exists a +ve constant c_1, c_2 & k such that

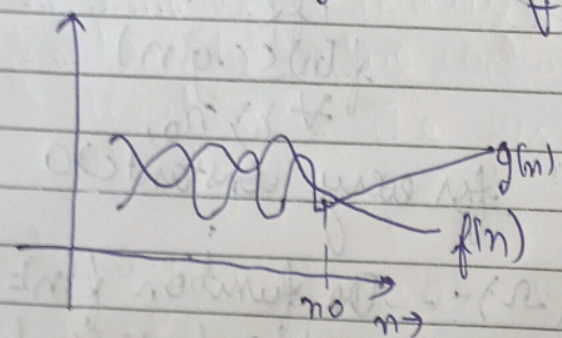
$$c_1 g(n) \le f(n) \le c_2 g(n) \text{ for all } n,$$



$f(n) = \Theta(g(n))$ iff
$c_1 g(n) \le f(n) \le c_2 \cdot g(n)$
$\forall n \ge \max(n_1, n_2)$

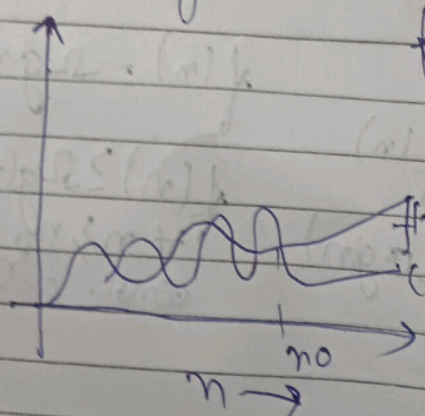
(iv) Small-Oh (o):-

$f(n) = o(g(n))$ $\quad f(n) \le c\,g(n)$
$\forall n \ge n_0$ & $\forall c > 0$
$n = o(n^2)$



(v) Small-omega

lower bound
$f(n) - \omega(g(n))$
$f(n) \not> c \cdot g(n)$
$\forall n > n_0$ & $\forall c > 0$
$n^2 = \omega(n)$



Page No.
Date

2. Time complexity of a loop means no. of times it has run.

i	1	2	4	8	16	32	...	2^k
value	2	2^2	2^3	2^4	2^5	2^6		n

$i = 1, 2, 4, 8, 16, 32 \ldots 2^k$ this means k times

i.e $2^k = n$

$\log_2 2^k = \log_2 n$ [$\log_2 2 = 1$]

$T.C = O(\log n)$

3. $T(n) = \{3T(n-1), n > 0\}$

By forward substitution

$T(n) = 3T(n-1)$

$T(0) = 3T(-1) = 0$

$T(1) = 3T(1-1) = 3T(0) = 3$

$T(2) = 3T(2-1) = 3T(1) = 3*3 = 3^2$

$T(3) = 3T(3-1) = 3T(2) = 3*3^2 = 3^3$

$T(n) = 3^n$

$\therefore T.C = O(3^n)$

4. $T(n) = \begin{cases} 2T(n-1)-1, & n>0 \end{cases}$

By forward substitution,

$T(0) = 1$

$T(1) = 2T(1-1)-1 = 2^1-1 = 1$

$T(2) = 2T(2-1)-1 = 2^2-2^1-1$

$T(3) = 2T(3-1)-1 = 2^3-2^2-2^1-1$

$2^n-(2^{n-1}-2^{n-2}-2^{n-3}\ldots 2^3-2^2-2^1-2^0)$

$= 2^n-(2^n-1)$

$= 1$

$T.C = O(1)$

5. The value of 'i' increases by one for each iteration. The value contained in 's' at the ith iteration is the sum of the first 'i' +ve integers. If k is the total no. of iterations taken by an program then while loop terminate if:

$1+2+3+\ldots+k \Rightarrow [k(k+1)/2] > n$

So, $k = O(\sqrt{n})$

$T.C = O(\sqrt{n})$

6. $O(n) = T.(c)$

Page No.
Date

7. void function(int n)
{
int i, j, k, count=0;
for(i=n/2; i<=n; i++) O(n)
for(j=1; j<=n; j=j*2) O(log n)
for(k=1; k<=n; k=k*2) O(log n)
count++;
}

T.C = O(n log² n)

8. function(int n)
{
if (n==1)
return;
for(i=1 to n) O(n)
{
for(j=1 to n) O(n)
{
printf("*");
}
}
function(n-3);
}

T.C = O(n²)

9. void function(int n)
{
for(i=1 to n) O(n)
{
for(i=1; j<=n; j=j+1) O(n)
printf("*");
}

T.C = O(n²)

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Page No.
Date

10. For the function n^k & c^n what is the asymptotic relationship b/w these functions:-

Assume that $k >= 1$ & $c > 1$ are constants.
Find out the value of c & no. for which relation holds.

n^k is $O(c^n)$

As c^n is the upper bound